Σ-PAS Convergence: Revised Mathematical Specification v2.0

Objective: Establish almost sure convergence of the Phase Alignment Score (S_t) to an ethical equilibrium under bounded noise and hybrid DMAIC control.

1. State Space & Objectives

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space filtered by $\mathcal{F}_t$. Let $S_t \in [0, 1]$ be the scalar Phase Alignment Score at step t.
Lyapunov Candidate: We define the "Distance to Perfection" as:
Objective Function (Harmony): We define the Harmony function H(S) such that its maximum is at $S^* = 1$.
Where $\kappa > 0$ is the "restoring force" coefficient (representing the *Improve* phase gain).

2. The Corrected Update Dynamics

The update rule is a **Projected Stochastic Approximation** process:
Where:

- $\Pi_{[0,1]}(x) = \min(1, \max(0, x))$ is the projection operator.
- λ_t: The step size sequence.
- ξ_t: The aggregate noise/penalty term, defined as:
    - We assume $\mathbb{E}[\xi_t | \mathcal{F}_t] = 0$ (centered noise) or absorbs bias into the drift.
    - Boundedness: $|\xi_t| \le C_\xi$ almost surely.

3. Hybrid DMAIC Dynamics (The "Contraction Boost")

To formalize the "Control Phase" (rollback), we model the system as a **Hybrid Stochastic Process**. Let $\tau = 5$ be the DMAIC cycle period.
If $t \mod \tau \neq 0$: Apply Standard Update (above). If $t \mod \tau = 0$: Apply **Control Operator** Φ:
Where:

- $S_{anchor} \approx 1$ is the cryptographically verified checkpoint.
- $\alpha \in (0, 1)$ is the mixing rate.
- This acts as a **periodic contraction mapping** on the variance.

4. Assumptions for Convergence

A1. Robbins-Monro Step Sizes:
A2. Bounded Noise: There exists a constant $K < \infty$ such that $\mathbb{E}[\xi_t^2 | \mathcal{F}_t] \le K(1 + V(S_t))$ a.s.
A3. Mean Reversion (Drift): For any $\epsilon > 0$, there exists $\delta > 0$ such that:

(This is satisfied by our linear definition $\kappa(1-S)$).

5. Main Theorem (Revised)

Theorem: Under Assumptions A1-A3, and utilizing the Projected Update Rule with DMAIC Hybrid Control:

1. $S_t \to 1$ almost surely as $t \to \infty$ (assuming centered noise).
2. If noise has a bias bound $|\mathbb{E}[\xi_t]| \le B$, then:

Proof Strategy: We utilize the **Robbins-Siegmund Supermartingale Lemma** on the Lyapunov function V_t, showing that the negative drift term $-\lambda_t \kappa (1-S_t)^2$ dominates the noise variance term $\lambda_t^2 K$ for large t.