# Lab 5 – Image Restoration

*Goal: Introduction to image restoration.*

**Introduction**

In practical imaging systems the acquired image often suffers from effects of blurring and noise. Image restoration algorithms are aimed to restore the original undistorted image from its blurry and noisy version. The lab experiment demonstrates the evolution of restoration algorithms from the simple Inverse Filter,

$$H^{I}(u,v) = \frac{1}{H(u,v)},$$

through its improved variant, the Pseudo Inverse Filter

$$H^{PI}(u,v) = \begin{cases} \dfrac{1}{H(u,v)}, & H(u,v) > \varepsilon \\ 0, & H(u,v) \le \varepsilon \end{cases}$$

to the Wiener Filter

$$H^{WI}(u,v) = \frac{H^{*}(u,v)}{\left|H(u,v)\right|^{2} + \dfrac{S_{nn}(u,v)}{S_{ff}(u,v)}},$$

where $S_{ff}(u,v)$ is the spectral density of the original signal and $S_{nn}(u,v)$ is the spectral density of the noise.

Now let's assume that the noise is white, that is, it has a constant spectral density for all spatial frequencies:

$$S_{nn}(u,v) = S_{n}^{2},$$

where $S_{n}^{2}$ is a constant.

Furthermore, let's assume that the spectral density of the original signal is in inverse proportion to the square of the spatial frequency (the squared distance from an origin in the frequency space):

$$S_{ff}(u,v) \propto \frac{1}{u^{2}+v^{2}},$$

that is,

$$S_{ff}(u,v) = \frac{1}{k} \cdot \frac{1}{u^{2}+v^{2}},$$

where $k$ is a constant.

Substituting these two assumptions into the general formula of the Wiener Filter, we obtain:

$$H^{WI}(u,v) = \frac{H^*(u,v)}{\left|H(u,v)\right|^2 + \dfrac{S_{nn}(u,v)}{S_{ff}(u,v)}} = \frac{H^*(u,v)}{\left|H(u,v)\right|^2 + kS_n^2(u^2+v^2)}.$$

In the Preliminary report you will be asked to prove one more formula regarding Wiener Filter.

## Preliminary report

Prepare the following tasks and python files and bring them to the lab:

1. Derive the following formula for Wiener filter:

   $$H^{WI}(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + kS_n^2(u^2+v^2)} = \frac{H^*(u,v)}{|H(u,v)|^2 + \alpha\sigma_n^2(u^2+v^2)},$$

   where $\sigma_n^2$ is the variance of the noise and $\alpha$ is a constant. Derive a relation between the constants $k$ and $\alpha$.

2. Implement in python the image acquisition and degradation process:
   a. Read image from file.
   b. Blur the image using an averaged filter 5X5 (use *cv2.filter2D*).
   c. Add Gaussian noise to the blurred image (use *skimage.util.random_noise*).

   Write the resulting images to image files.

3. Implement in python the following restoration filters:
   a. Inverse Filter.
   b. Pseudo Inverse Filter.
   c. Wiener Filter.

   Pay attention to numerical accuracy issues.

4. Read about Colab from [1] and try to create new notebook and run few commands (no need to flow all the tutorial or submit this section)

## Description of the experiment

Open the Jupyter notebook in colab: click here and follow the README.md

Download the pythons files that you prepared at home.

Follow the instructions in the 4 sections of the lab:

Part 1 - Inverse Filter

Part 2 – Pseudo Inverse Filter

Part 3 – Wiener Filter

Part 4 – Deep learning (DnCNN)

**Final report**

Submit the results of testing and demonstrations from the 4 parts of the experiment, with the image of your choice. Explanation your results in each step.

**Answer all the questions appeared in the final report in your Jupyter notebook - in the end of each section, save the files as PDF or HTML format.**
**Use: File -> Download as -> PDF/HTML. Make sure to submit all your plots and results.**

**References**

1. https://medium.com/deep-learning-turkey/google-colab-free-gpu-tutorial-e113627b9f5d
2. R. C. Gonzalez and R. E. Woods, Digital Image Processing. Prentice-Hall, Inc., 2002, Second Edition (Library Dewey number 621.368 GON).
3. R. C. Gonzalez and R. E. Woods, Digital Image Processing. Addison-Wesley Publishing Company, Inc., 1992 (Library Dewey number 621.368 GON).