Passive reduced order multiport modelling: the Padé-Laguerre, Krylov-Arnoldi-SVD connection (and its application to FDTD)

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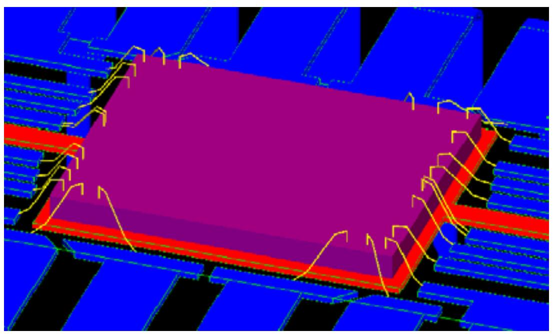
Outline

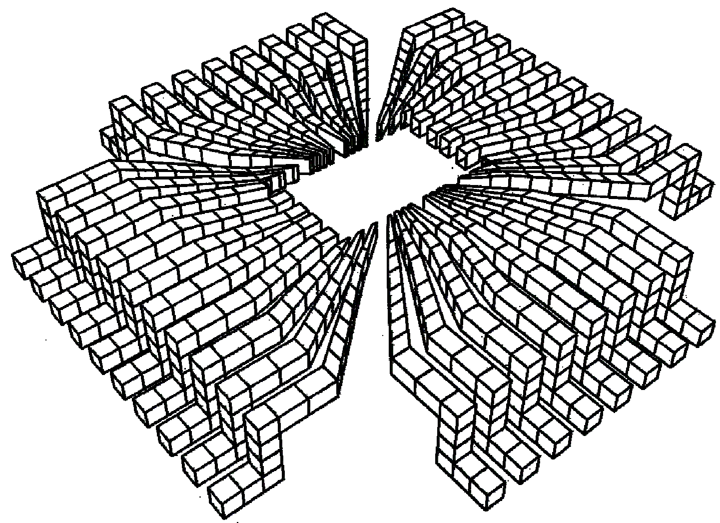
- Introduction
- Laguerre-SVD Reduced Order Modelling (ROM)
- Example: PEEC-circuit
- ROM and explicit reciprocity
- Example: two coupled lossy transmission lines
- Application to FDTD: automatic subcell generation
- Example: a grid of dielectric wires
- Conclusion

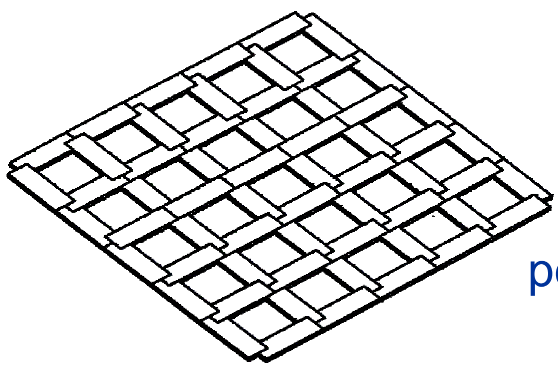
Introduction

- The time-domain behaviour of a circuit or a physical device (e.g. a package) can be described by a state space description
- in electromagnetics this description typically emerges from numerical techniques such as FDTD, TLM or PEEC



package





perforated ground plane

State-space description

- Typical representation as a set of first-order differential equations

$$\mathbf{C}\,\dot{\mathbf{x}} = -\mathbf{G}\,\mathbf{x} + \mathbf{B}\,\mathbf{u} \qquad (\mathbf{C}, \mathbf{G}: \mathrm{N \times N})$$

$$\mathbf{y} = \mathbf{L}^{T}\mathbf{x} \qquad (\mathbf{B}, \mathbf{L}: \mathrm{N \times P})$$

x: a vector of N internal variables (e.g. currents and voltages*)

u: a vector of P input (source) variables (e.g. port voltages)

y: a vector of P output variables (e.g port currents)

***Note*: the variables could also be fields → FDTD!

Reduced Order Modeling

- The purpose of ROM techniques is to reformulate the previous state-space description as

$$\hat{\mathbf{C}}\,\dot{\mathbf{w}} = -\hat{\mathbf{G}}\,\mathbf{w} + \hat{\mathbf{B}}\,\mathbf{u} \qquad (\hat{\mathbf{C}}, \hat{\mathbf{G}}: Q \times Q)$$

$$\mathbf{y} = \hat{\mathbf{L}}^{T}\,\mathbf{w} \qquad (\hat{\mathbf{B}}, \hat{\mathbf{L}}: Q \times P)$$

w: a **new** vector of Q (Q < N) internal variables

u: the **same** vector of P input (source) variables (i.e. port voltages)

y: the **same** vector of P output variables (i.e. port currents)

ROM-techniques: state-of-the-art

- 4 major techniques are found in literature
  - Asymptotic Waveform Evaluation (AWE)
  - Matrix Padé via Lanczos
  - Arnoldi-PRIMA
  - Congruence transformation
- these techniques mostly work for early time responses but are less performant for low frequences!
- a new solution (based on an expansion in Laguerre polynomials) was developed by L. Knockaert, circumventing this low frequency problem.

Expansion in scaled Laguerre functions (1)

- In most techniques described in literature the transfer matrix $\mathbf{H}(s)$ with

$$\mathbf{H}(s) = \mathbf{L}^T(\mathbf{G} + s\mathbf{C})^{-1}\mathbf{B}$$

is used as the starting point to obtain Padé approximations by means of moment matching.

- Here we start from $\mathbf{h}(t) = \mathrm{Lapl}^{-1}(\mathbf{H}(s))$ and expand $\mathbf{h}(t)$ in a set of scaled Laguerre functions La_n:

$$\mathbf{h}(t) = \sum_{n=0}^{\infty} \mathbf{F}_n \, \phi_n^{\alpha}(t)$$

with

$$\phi_n^{\alpha}(t) = (2\alpha)^{1/2} e^{-\alpha t} \mathrm{La}_n(2\alpha t) \qquad \text{with } \mathrm{La}_n(t) = \frac{e^t}{n!}\frac{d^n}{dt^n}(e^{-t} t^{n})$$

Expansion in scaled Laguerre functions (2)

- The Laplace transform of the scaled Laguerre functions is

$$\Phi_n^{\alpha}(s) = (2\alpha)^{1/2}\frac{1}{s+\alpha}\left(\frac{s-\alpha}{s+\alpha}\right)^n$$

- Consequently:

$$\mathbf{H}(s) = \mathbf{L}^T(\mathbf{G} + s\mathbf{C})^{-1}\mathbf{B}$$

$$= \sum_{n=0}^{\infty} \mathbf{F}_n \Phi_n^{\alpha}(s) = (2\alpha)^{1/2}\frac{1}{s+\alpha}\sum_{n=0}^{\infty}\mathbf{F}_n\left(\frac{s-\alpha}{s+\alpha}\right)^n$$

This implies that **H**(s) is written as the **product** of a simple **low-pass filter** and a weighted sum of **all-pass filters**!

The u-domain

- We now map the s-domain onto the u-domain with the bilinear transformation (right half-plane $\rightarrow$ unit circle)

$$u = \frac{s - \alpha}{s + \alpha}$$

leading to

$$\mathbf{H}(u) = \mathbf{L}^T[(\alpha\mathbf{C} + \mathbf{G}) + u\,(\alpha\mathbf{C} - \mathbf{G})]^{-1}\mathbf{B}$$

Consequently:

an m-th order Padé approximation in the u-domain of **H**(u)

is equivalent to

an m-th order Laguerre approximation of **H**(s) in the s-domain!

Convergence issues

- The previous statement implies that **H**(s) can be optimally approximated in the Hardy space by the truncated Laguerre-expansion, i.e.

$$\mathbf{H}(s) \approx \sum_{n=0}^{m} \mathbf{F}_n \, \Phi_n^{\alpha}(s) = \mathbf{H}_m(s)$$

- The point-wise convergence $\mathbf{H}_m(i\omega) \rightarrow \mathbf{H}(i\omega)$ can be proven

(see L. Knockaert and D. De Zutter, “Passive Reduced Multiport Modelling…”, Int. J. Electr. Commun. (AEÜ), pp. 254-260, no. 5, **53**, 1999. [1])

- Mathematical arguments show that a good choice for α is

$$\alpha \approx 2\pi \, f_{max}$$

with f_{max} the bandwidth of the considered system.

The Krylov-Arnoldi-SVD connection (1)

- Reduced order modelling via Krylov-Arnoldi

Starting point: original system (N x N)

$$\mathbf{C}\dot{\mathbf{x}} = -\mathbf{G}\,\mathbf{x} + \mathbf{B}\,\mathbf{u}$$
$$\mathbf{y} = \mathbf{L}^{T}\mathbf{x}$$

- via the u-domain a modified **A** and **R** matrix can be defined:

$$\mathbf{A} = -(\alpha\mathbf{C} + \mathbf{G})^{-1}(\alpha\mathbf{C} - \mathbf{G}) \qquad \text{(classical approach: } \mathbf{A} = -\mathbf{G}^{-1}\mathbf{C}\text{)}$$
$$\mathbf{R} = (\alpha\mathbf{C} + \mathbf{G})^{-1}\,\mathbf{B} \qquad \text{(classical approach: } \mathbf{R} = \mathbf{G}^{-1}\mathbf{B}\text{)}$$

- **End point: reduced order system (Q x Q)**

$$\hat{\mathbf{C}}\dot{\mathbf{w}} = -\hat{\mathbf{G}}\,\mathbf{w} + \hat{\mathbf{B}}\,\mathbf{u}$$
$$\mathbf{y} = \hat{\mathbf{L}}^{T}\mathbf{w}$$

with

- $\hat{\mathbf{C}} = \mathbf{X}^{T}\mathbf{C}\mathbf{X}$
- $\hat{\mathbf{G}} = \mathbf{X}^{T}\mathbf{G}\mathbf{X}$
- $\hat{\mathbf{B}} = \mathbf{X}^{T}\mathbf{B}$
- $\hat{\mathbf{L}} = \mathbf{X}^{T}\mathbf{L}$

The Krylov-Arnoldi-SVD connection (2)

- **X**: a N x Q matrix that orthogonalizes the columns of the N x Q Krylov matrix

  $\mathbf{K}_M = [\mathbf{R}, \mathbf{AR}, \mathbf{A}^2\mathbf{R}, \ldots., \mathbf{A}^{M-1}\mathbf{R}, (\boldsymbol{A}^M\boldsymbol{R})]$ with $Q = P \times M + r$

- **X** can be generated using the block Arnoldi algorithm which is equivalent to so-called 'thin' QR factorization based on modified Gram-Schmidt orthogonalization.

- to avoid precision problems we use an SVD approach i.e. write $\mathbf{K}_M$ as

  $\mathbf{K}_M = \mathbf{U}\, \Sigma\, \mathbf{V}^T$ implying that $\mathbf{X}^T \mathbf{X} = \mathbf{U}^T \mathbf{U}$

  and further implying that the role of **X can be replaced by U**.

  (see [1])

The complete algorithm

- select the values for α ($\approx 2\pi f_{max}$) and Q (reduction factor N/Q)
- solve $(\mathbf{G} + \alpha\mathbf{C})\mathbf{R}_0 = \mathbf{B}$
- for k = 1, M - 1 solve (with M = Q/P)

  $(\mathbf{G} + \alpha\mathbf{C})\mathbf{R}_k = (\mathbf{G} - \alpha\mathbf{C})\mathbf{R}_{k-1}$
- $\mathbf{U}\,\Sigma\,\mathbf{V}^T = \mathrm{SVD}[\mathbf{R}_0, \mathbf{R}_1, \ldots\ldots, \mathbf{R}_{M-1}]$
- finally,

$$\hat{\mathbf{C}} = \mathbf{U}^T\mathbf{C}\mathbf{U}$$
$$\hat{\mathbf{G}} = \mathbf{U}^T\mathbf{G}\mathbf{U}$$
$$\hat{\mathbf{B}} = \mathbf{U}^T\mathbf{B}$$
$$\hat{\mathbf{L}} = \mathbf{U}^T\mathbf{L}$$

Example: a PEEC-circuit

- Typical example: a PEEC circuit

  A.E. Ruehli, “Equivalent circuit models for three-dimensional multiconductor systems”, IEEE Trans. MTT, March 1974

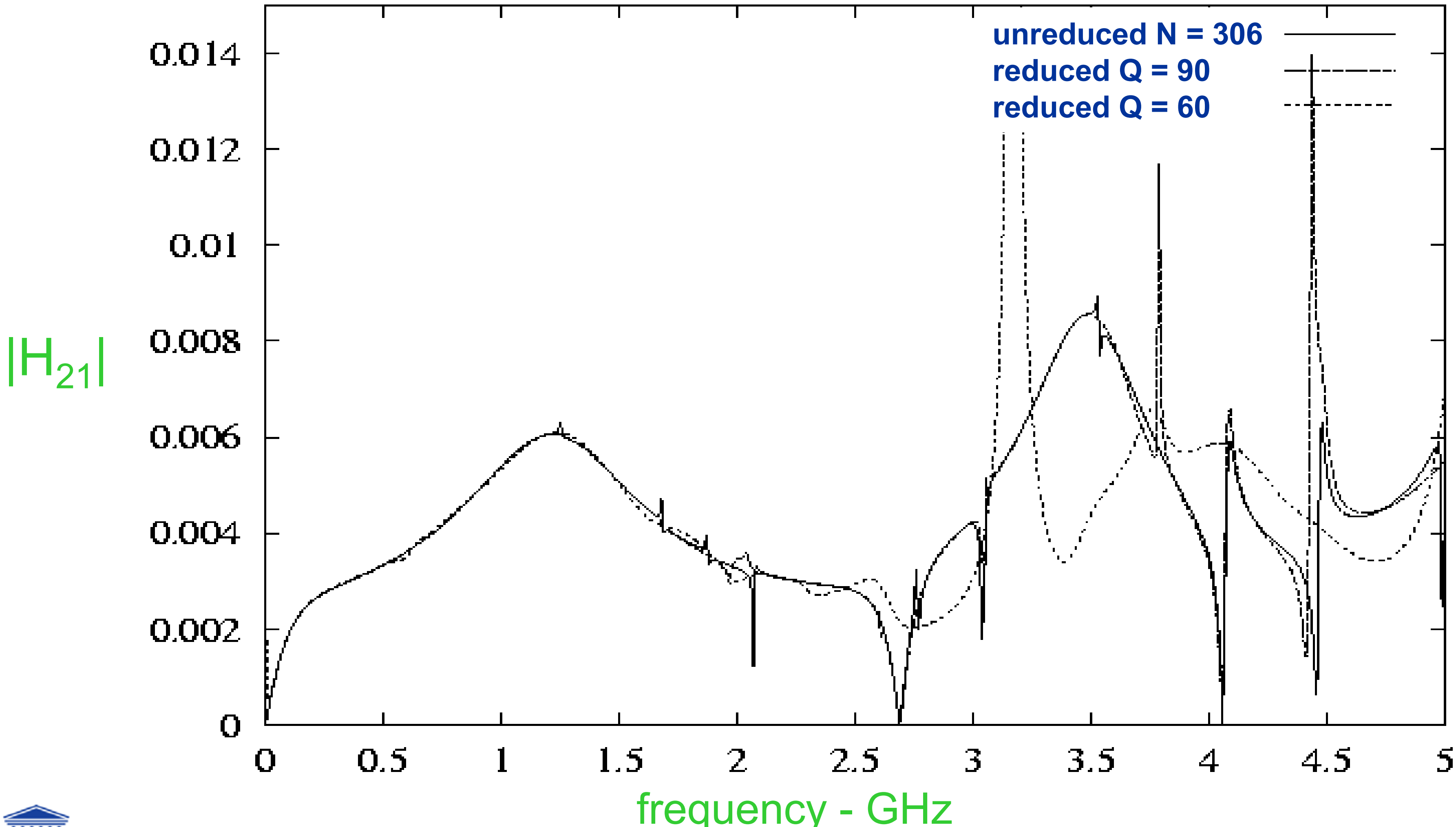
and

  P. Feldmann and R.W. Freund, “Efficient linear circuit analysis by Padé approximation via the Lanczos process”, IEEE Trans. Computer-Aided Design, May 1995)

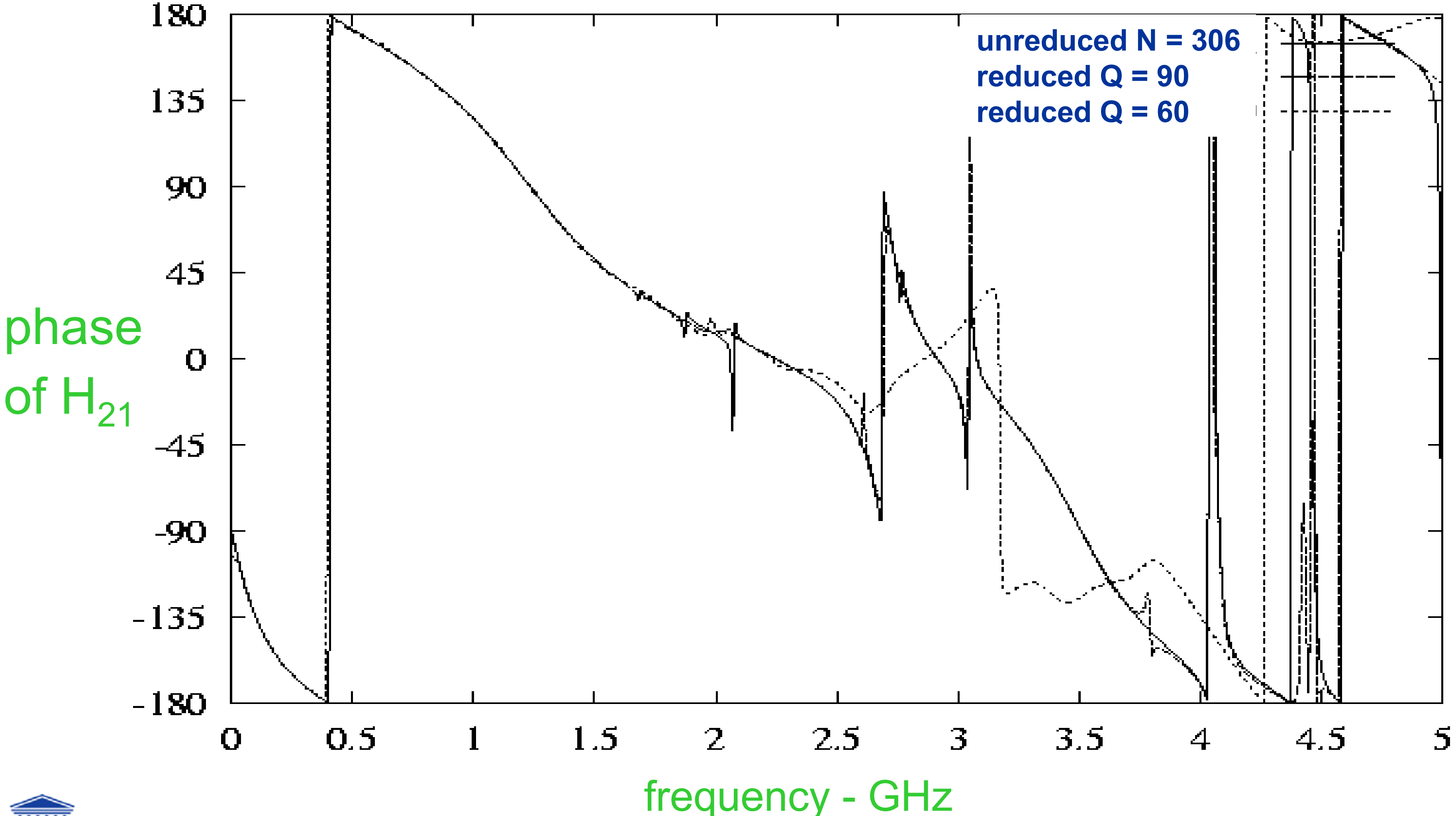
  - 2100 capacitors
  - 127 inductors
  - 6990 mutual inductive couplings
  - **N = 306**

- reduced to **Q < N = 60, 90**

PEEC-circuit: numerical results (1)



PEEC-circuit: numerical results (2)



State-space description and reciprocity

- Suppose now that we start from the following state-space description

$$\begin{cases} \mathbf{C}\dot{\mathbf{x}} = -\mathbf{G}\mathbf{x} + \mathbf{L}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^{T}\mathbf{x} \end{cases}$$

with

$$\mathbf{C} = \mathbf{C}^{T}; \ \mathbf{G} = \mathbf{G}^{T}$$

Consequently, the transfer matrix $\mathbf{H}(s)$ (Laplace domain) is reciprocal, where

$$\mathbf{H}(s) = \mathbf{L}^{T} (\mathbf{G} + s\mathbf{C})^{-1} \mathbf{L}$$

and

$$\mathbf{H}^{T}(s) = \mathbf{H}(s)$$

Explicit reciprocity

Reversely:

- start from standard minimal state-space description: $\mathbf{H}(s) = \mathbf{L}^T (s\mathbf{I} - \mathbf{A})^{-1} \mathbf{R}$
- suppose that $\mathbf{H}^T(s) = \mathbf{H}(s)$ (reciprocity)
- then Frobenius: $\mathbf{A} = -\mathbf{C}^{-1} \mathbf{G}$ with $\mathbf{C}^T = \mathbf{C}$ and $\mathbf{G}^T = \mathbf{G}$
- consequence: $\mathbf{H}(s) = \mathbf{L}^T (s\mathbf{C} + \mathbf{G})^{-1} \mathbf{C}\mathbf{R}$
- since $\mathbf{H}(s) = \mathbf{H}^T(s)$ we have $\mathbf{C}\mathbf{R} = \mathbf{L}$
- or $\mathbf{H}(s) = \mathbf{L}^T (s\mathbf{C} + \mathbf{G})^{-1} \mathbf{L}$

Reciprocity $\Rightarrow$ explicitly reciprocal state space description

Congruent ROM and explicit reciprocity

Using congruent reduced order modeling:

$$\hat{\mathbf{G}} = \mathbf{U}^T\mathbf{G}\mathbf{U} \qquad \hat{\mathbf{C}} = \mathbf{U}^T\mathbf{C}\mathbf{U} \qquad \hat{\mathbf{L}} = \mathbf{U}^T\mathbf{L}$$

e.g. **U** left factor of the SVD of the Krylov matrix (as in our method)

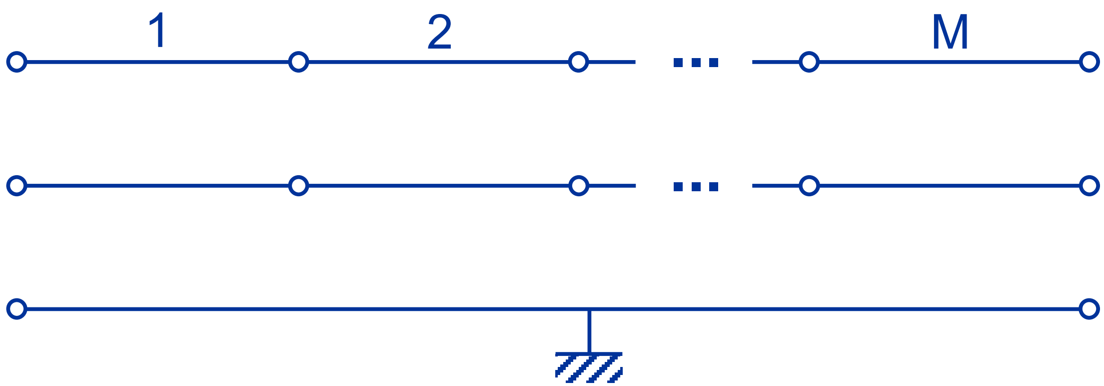
Then reduced state space description is still **explicitly** reciprocal:

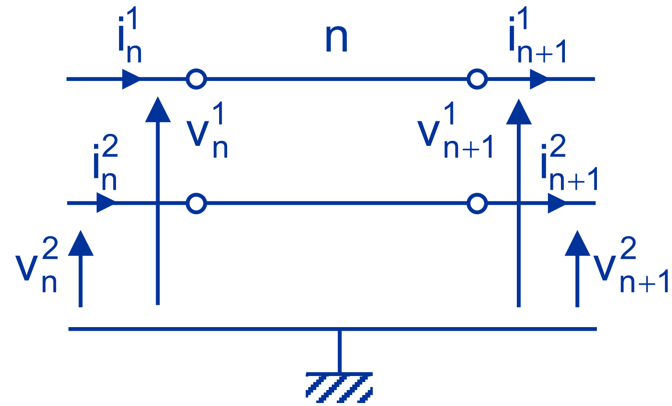
$$\hat{\mathbf{H}}(s) = \hat{\mathbf{L}}^T(s\hat{\mathbf{C}} + \hat{\mathbf{G}})^{-1}\hat{\mathbf{L}} = \hat{\mathbf{H}}^T(s)$$

For the non-reciprocal form this is not the case!

$$\hat{\mathbf{H}}'(s) = \hat{\mathbf{L}}^T\mathbf{U}(s\mathbf{I} - \mathbf{U}^T\mathbf{A}\mathbf{U})^{-1}\mathbf{U}^T\mathbf{R} \neq \hat{\mathbf{H}}'^T(s)$$

Example: 2 coupled lines





2 coupled transmission lines represented by M discrete sections

total length: 5 cm

$$\mathbf{L} = \begin{bmatrix} 494.6 & 63.3 \\ 63.3 & 494.6 \end{bmatrix} \mathrm{nH/m} \qquad \mathbf{R} = \begin{bmatrix} 0.1 & 0.02 \\ 0.02 & 0.1 \end{bmatrix} \Omega/\mathrm{m}$$

$$\mathbf{C} = \begin{bmatrix} 62.8 & -4.9 \\ -4.9 & 62.8 \end{bmatrix} \mathrm{pF/m} \qquad \mathbf{G} = \begin{bmatrix} 0.1 & -0.01 \\ -0.01 & 0.1 \end{bmatrix} \mathrm{S/m}$$

Normal format

Internal state space variables: $\{\mathbf{v}_n, \mathbf{i}_n\}$

$$\mathbf{C}_n \frac{d\mathbf{v}_n}{dt} = -\mathbf{G}_n \mathbf{v}_n + \mathbf{i}_n - \mathbf{i}_{n+1}$$

$$\mathbf{L}_n \frac{d\mathbf{i}_n}{dt} = -\mathbf{R}_n \mathbf{i}_n + \mathbf{v}_{n-1} - \mathbf{v}_n$$

$$\underbrace{\begin{pmatrix} \mathbf{C}_1 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{C}_2 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{L}_1 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{L}_2 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{L}_3 \end{pmatrix}}_{\mathbf{C}}, \underbrace{\begin{pmatrix} \mathbf{G}_1 & 0 & -\mathbf{I} & \mathbf{I} & 0 \\ 0 & \mathbf{G}_2 & 0 & -\mathbf{I} & \mathbf{I} \\ \mathbf{I} & 0 & \mathbf{R}_1 & 0 & 0 \\ -\mathbf{I} & \mathbf{I} & 0 & \mathbf{R}_2 & 0 \\ 0 & -\mathbf{I} & 0 & 0 & \mathbf{R}_3 \end{pmatrix}}_{\mathbf{G}}, \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \mathbf{I} & 0 \\ 0 & 0 \\ 0 & -\mathbf{I} \end{pmatrix}}_{\mathbf{B}}$$

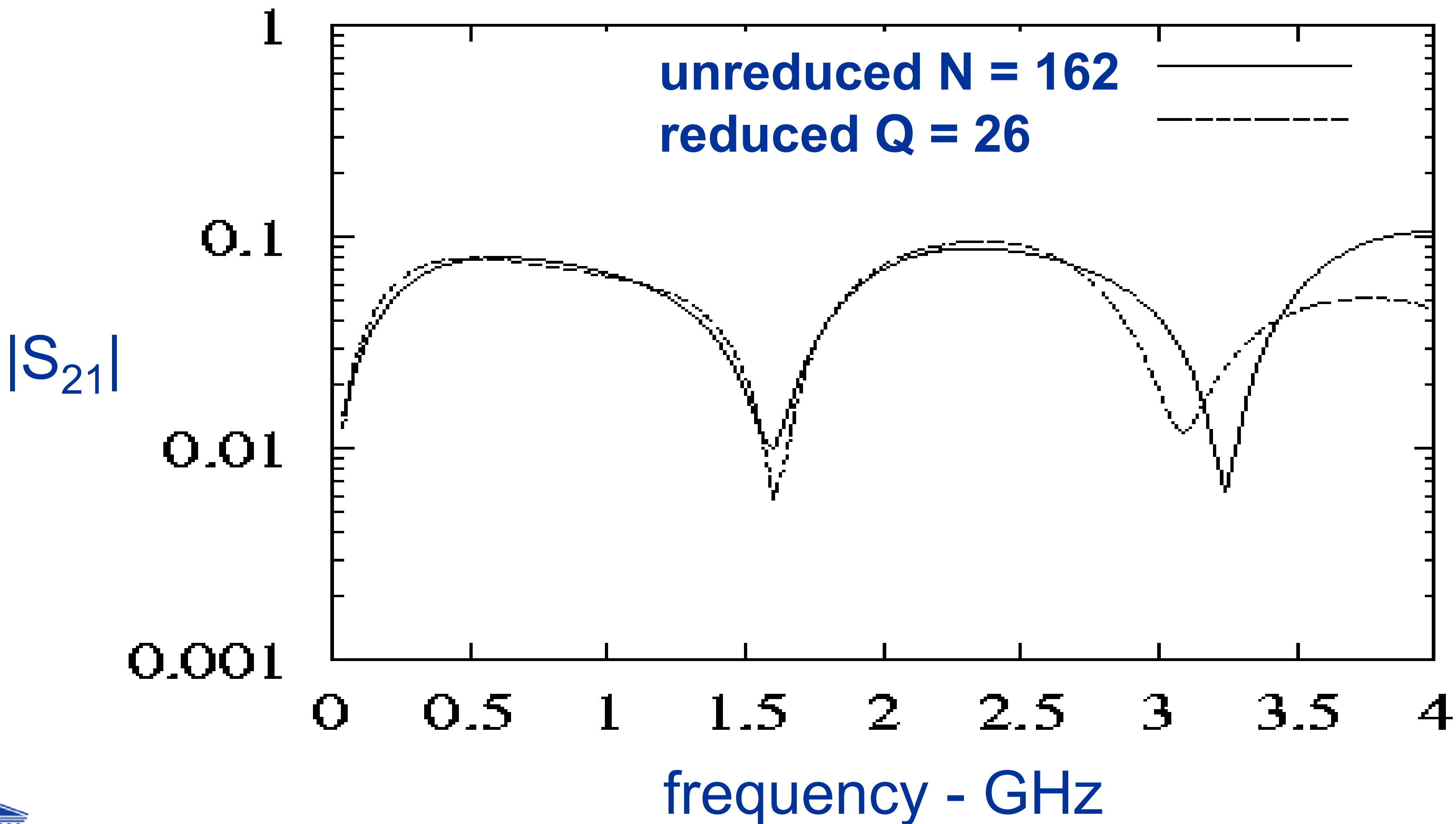
(for M = 2)

Explicitly reciprocal format

Internal state space variables: $\{-\mathbf{v}_n, \mathbf{i}_n\}$

$$\underbrace{\begin{pmatrix} -\mathbf{C}_1 & 0 & 0 & 0 & 0 \\ 0 & -\mathbf{C}_2 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{L}_1 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{L}_2 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{L}_3 \end{pmatrix}}_{\mathbf{C}}, \underbrace{\begin{pmatrix} -\mathbf{G}_1 & 0 & -\mathbf{I} & \mathbf{I} & 0 \\ 0 & -\mathbf{G}_2 & 0 & -\mathbf{I} & \mathbf{I} \\ -\mathbf{I} & 0 & \mathbf{R}_1 & 0 & 0 \\ \mathbf{I} & -\mathbf{I} & 0 & \mathbf{R}_2 & 0 \\ 0 & \mathbf{I} & 0 & 0 & \mathbf{R}_3 \end{pmatrix}}_{\mathbf{G}}, \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \mathbf{I} & 0 \\ 0 & 0 \\ 0 & -\mathbf{I} \end{pmatrix}}_{\mathbf{B}}$$

Coupled lossy lines: numerical result



ROM - Conclusion

- we have discussed a reduced order modeling technique which remains accurate for low frequencies i.e. late time responses
- we have drawn attention to the fact that reciprocity can be explicitly represented in the state-space representation
- we have shown how this explicit reciprocity can be maintained when applying a ROM technique

2-D FDTD Automatic subcell generation

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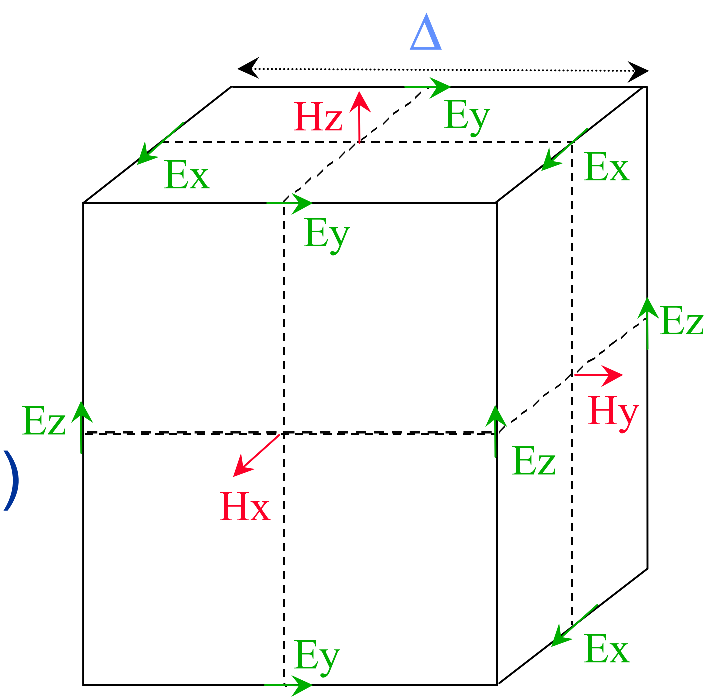
Introduction

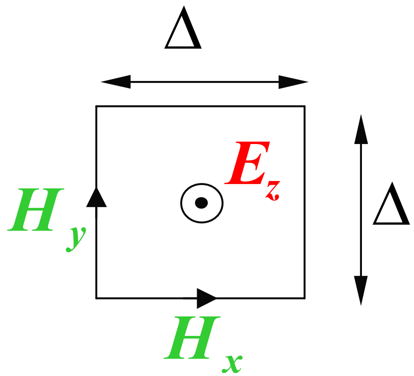
- In normal FDTD: cells of constant size
- Space step Δ (size of cell) is dependent on
  - wavelength (unavoidable)
  - smallest geometrical feature
- Techniques to choose Δ independent of small geometrical features
  - subcell models (thin wire, thin slot,..)
  - non-uniform orthogonal grid
  - subgridding
  - …



Yee cell

2D FDTD - the TM-case

- 2-D FDTD, TM-case, no variation in z-direction
- field variables E_z, H_x, H_y
- Typical cell:

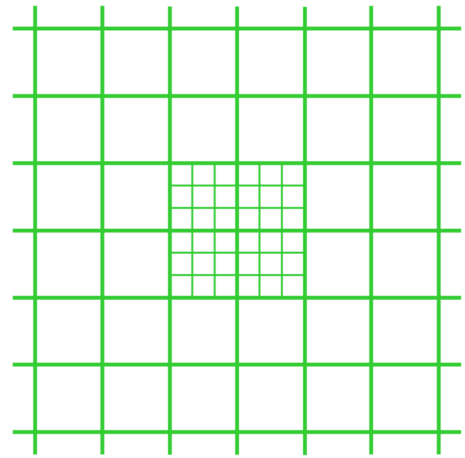


- equations:

$$\partial E_z/\partial x = \mu \, \partial H_y/\partial t$$

$$\partial E_z/\partial y = -\mu \, \partial H_x/\partial t$$

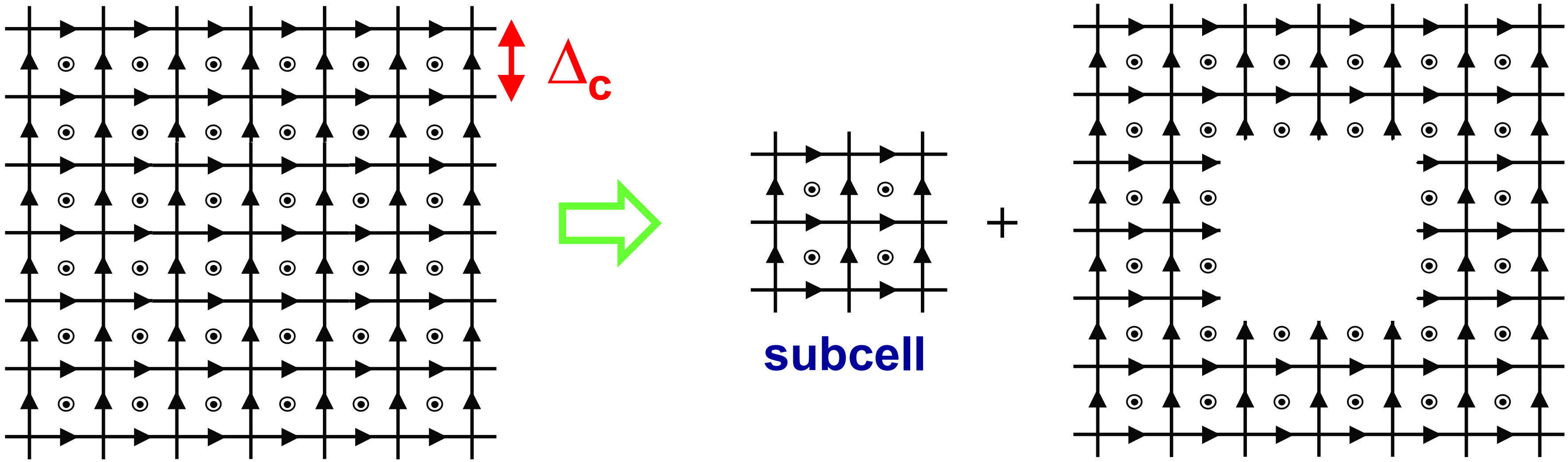
$$\partial H_y/\partial x - \partial H_x/\partial y = -\sigma E_z + \varepsilon \partial E_z/\partial t$$



- subindex c refers to coarse grid

  subindex f refers to fine grid

The subcell idea (1)

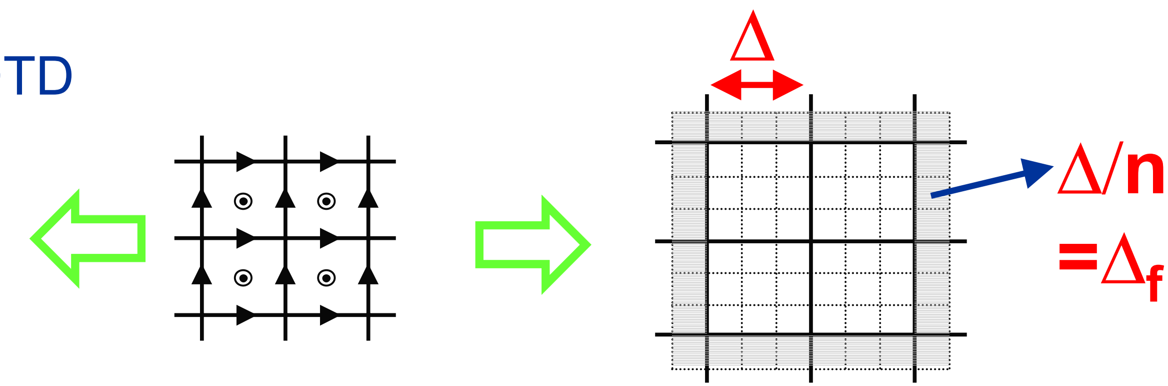


subcell

remaining grid

original FDTD grid

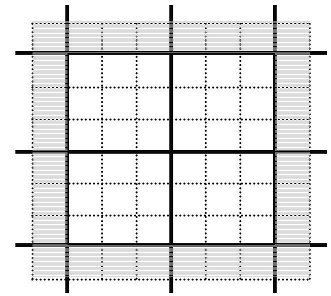
- state-space model through FDTD on fine grid
- only the space derivatives are discretised in the FDTD-way



The subcell idea (2)

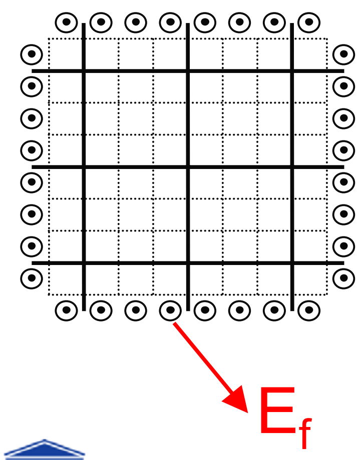
- State-space description of subcell

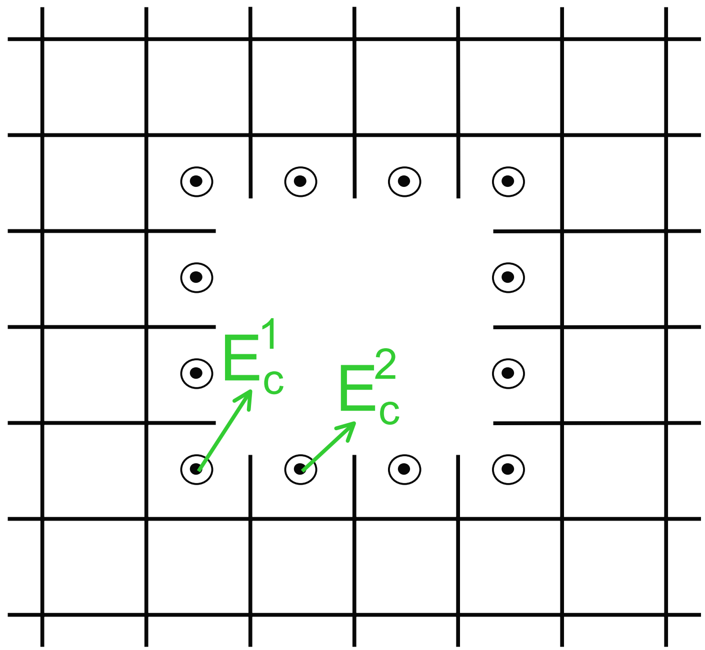
$$\begin{cases} \mathbf{C}\,\dot{\mathbf{x}} = -\mathbf{G}\,\mathbf{x} + \mathbf{B}\,\mathbf{u} \\ \mathbf{y} = \mathbf{L}^T\mathbf{x} \end{cases}$$



x: internal E_z, H_x, H_y fields

u: inputs → fine grid E_z-values derived from E_z fields of coarse grid through a suitable interpolation





$\mathbf{r = \Delta_c/\Delta_f = 3}$

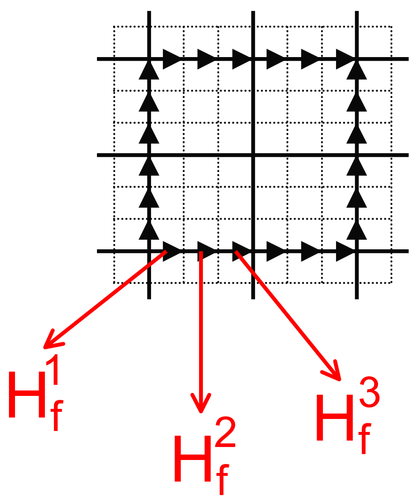
$E_f = \frac{2}{3}E_c^{\,1} + \frac{1}{3}E_c^{\,2}$

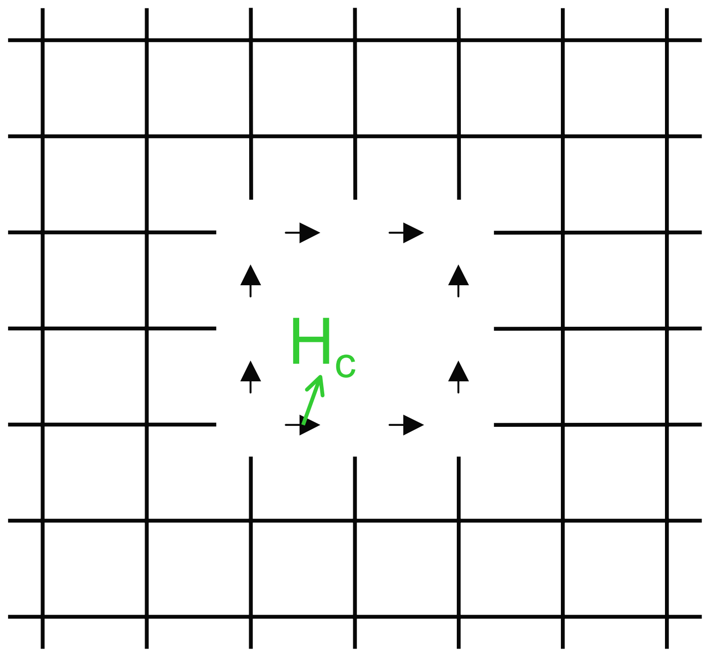
The subcell idea (3)

- State-space description of subcell

$$\begin{cases} \mathbf{C}\,\dot{\mathbf{x}} = -\mathbf{G}\,\mathbf{x} + \mathbf{B}\,\mathbf{u} \\ \mathbf{y} = \mathbf{L}^T\mathbf{x} \end{cases}$$

y: outputs → fine grid H-values leading to
H-fields of coarse grid through suitable averaging



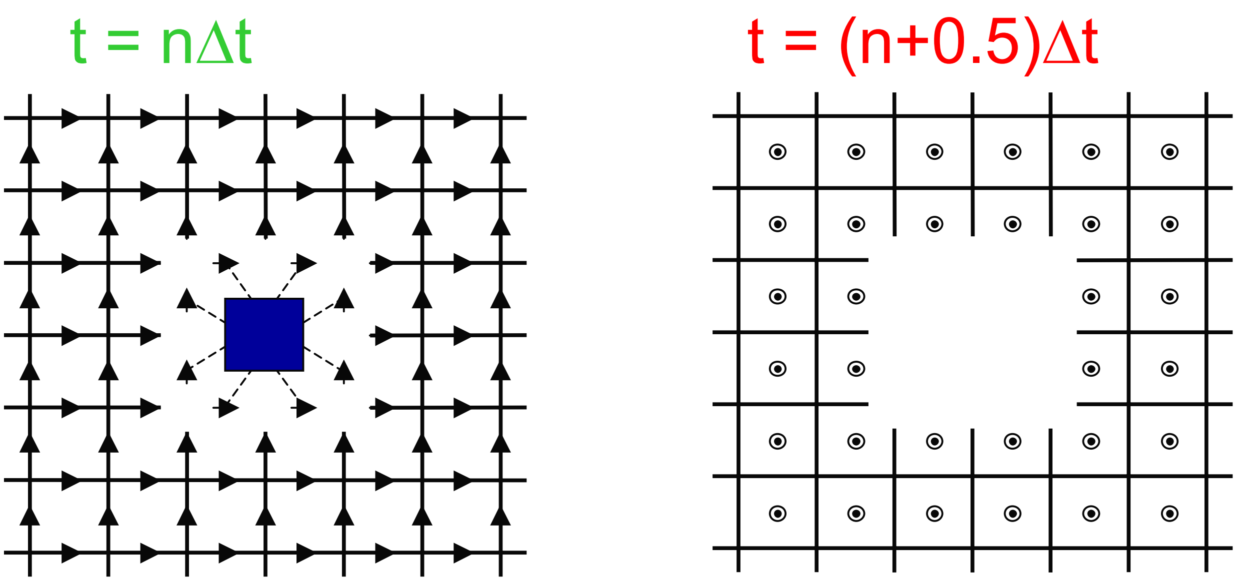


$r = \Delta_c/\Delta_f = 3$

$H_c = \frac{1}{3}(H_f^1 + H_f^2 + H_f^3)$

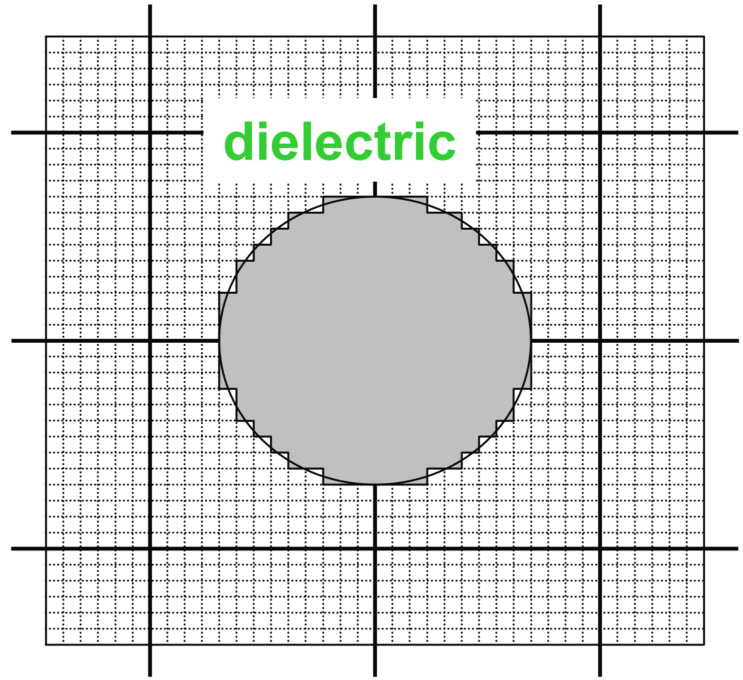
The subcell idea (4) - algorithm

- generate MIMO state-space description of subcell $\begin{cases} \mathbf{C}\,\dot{\mathbf{x}} = -\mathbf{G}\,\mathbf{x} + \mathbf{B}\,\mathbf{u} \\ \mathbf{y} = \mathbf{L}^T\,\mathbf{x} \end{cases}$
- apply ROM-algorithm $\begin{cases} \hat{\mathbf{C}}\,\dot{\mathbf{w}} = -\hat{\mathbf{G}}\,\mathbf{w} + \hat{\mathbf{B}}\,\mathbf{u} \\ \mathbf{y} = \hat{\mathbf{L}}^T\,\mathbf{w} \end{cases}$
- discretise time in the reduced state-space description
- solve the overall problem using FDTD with updating as below



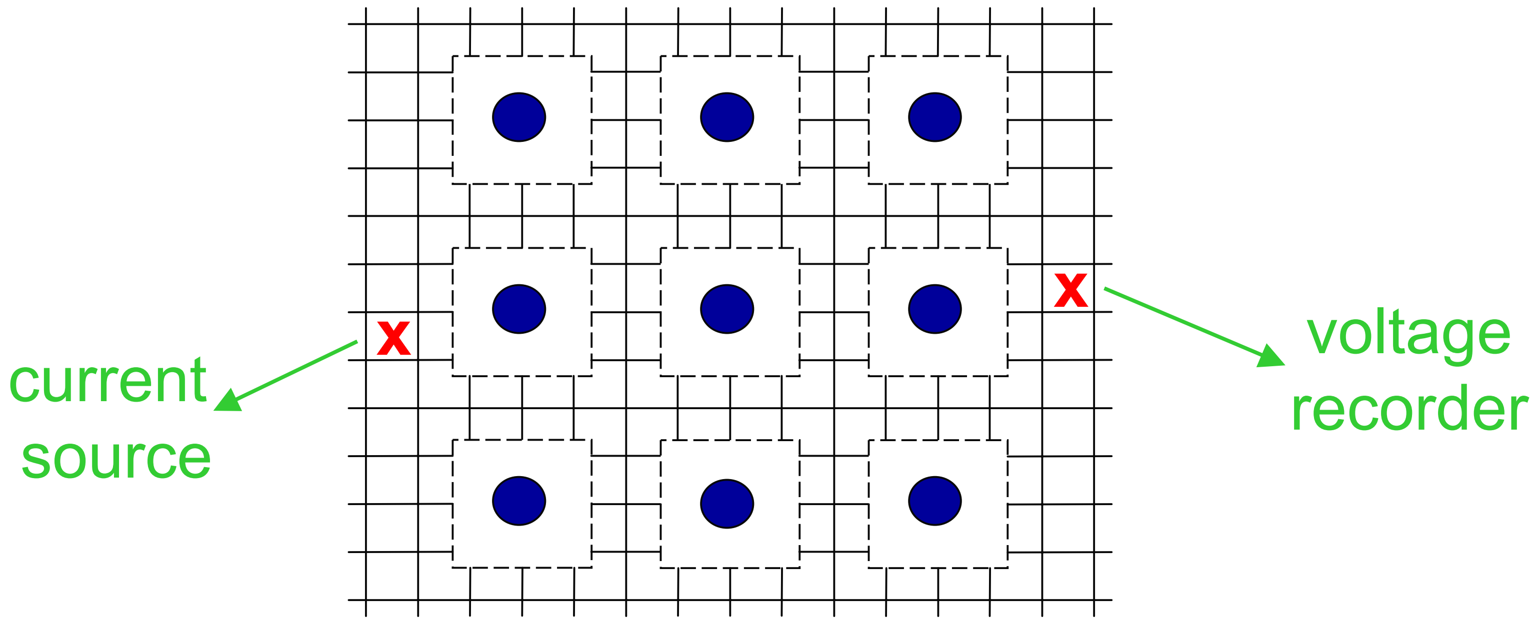
Example: subcell model for a dielectric wire (1)

- dielectric wire of radius = 0.9mm and ε_r = 10
- Δ_c = 1.3mm Δ_f = 0.1mm r = 13
- original number of state-space variables, N = 4408
- number of variables after ROM: either 12, 24 or 48
- subcell consists of 2 x 2 coarse cells

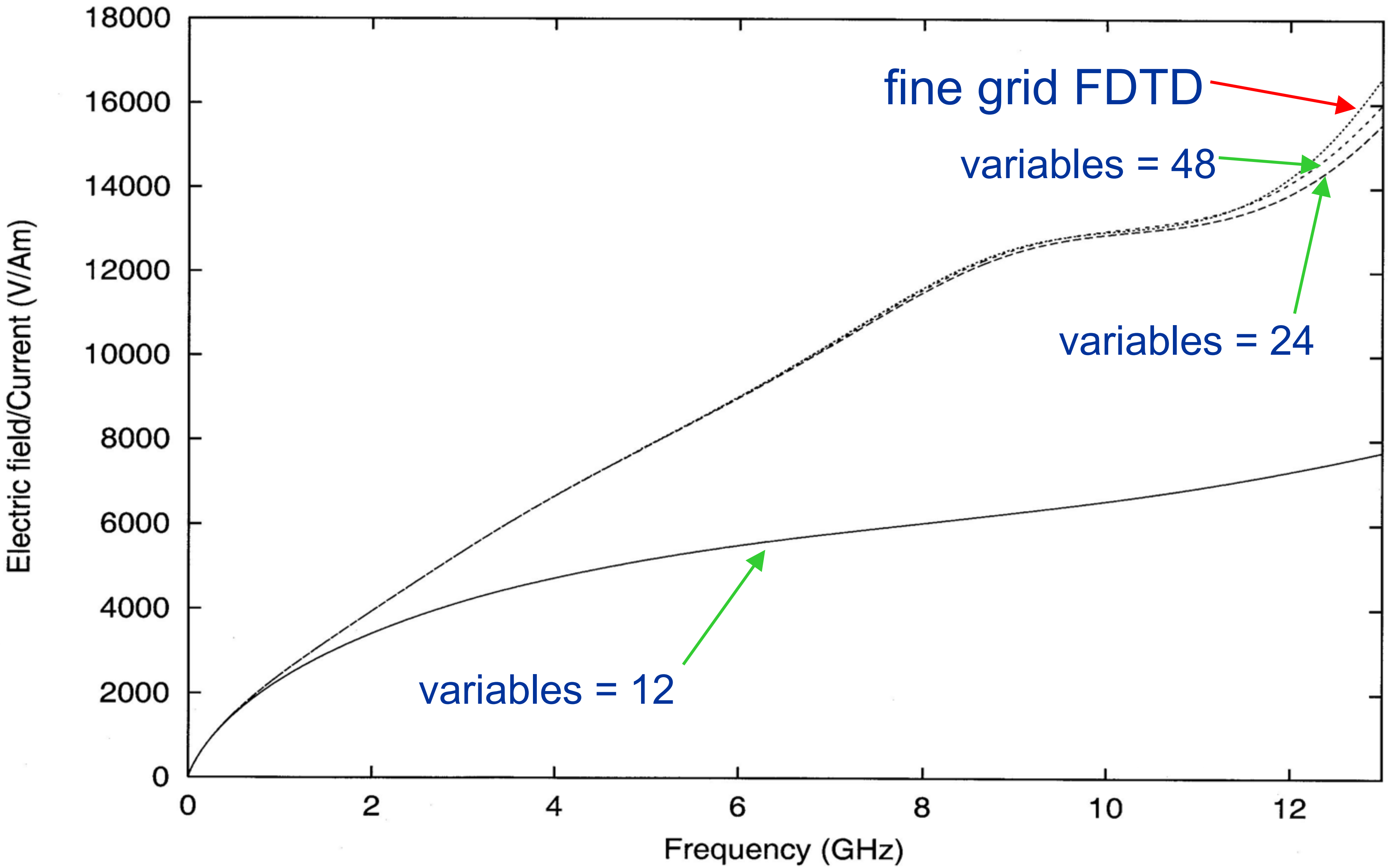


Example: dielectric wire (2)

- reuse the subcell model e.g. in a periodic arrangement
- line source as excitation
- electric field recorded at other side of periodic structure
- comparison: normal FDTD with fine grid

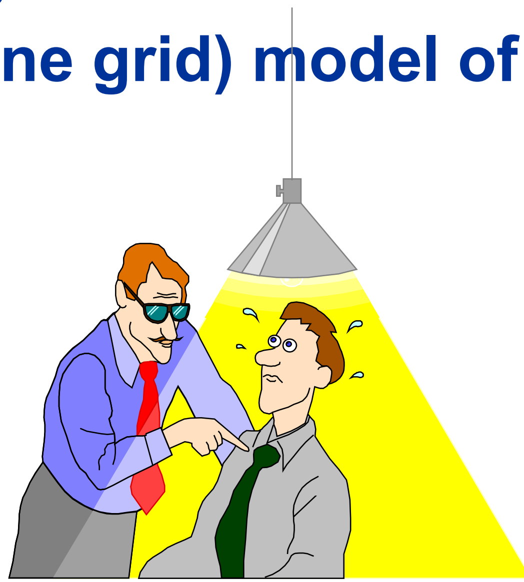


Example: dielectric wire (3)



Conclusions

- A new reduced order modelling (ROM) technique based on representing time-domain signals in terms of a sum of Laguerre functions was proposed.
- It was shown how ROM can be used in EM field simulations using (2D) FDTD to combine an overall coarse grid with a subcell (fine grid) model of arbitrary objects.



Questions?