Model Reduction within Control Systems Design

Pepijn Wortelboer
PHILIPS OPTICAL STORAGE

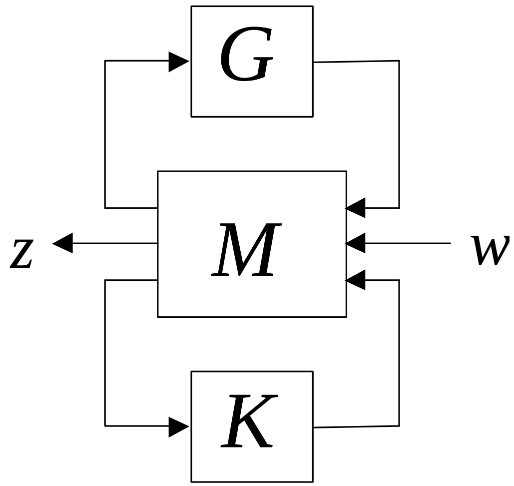
January 2001

Introduction

- **Control problem formulation**
- **Need for order reduction algorithms**
- **Introduction of CD-player mechanism**
- **Standard reduction techniques: modal and balanced**
- **Closed-loop reduction**
- **Iterative low-order control design for CD-player**
- **Concluding remarks**

Control problem formulation

- **Linear time-invariant**
- **model based**
- **standard plant formulation**
- **tuning by weighting function design**



Goal:

simplest K achieving “small” *z*

$$z = \mathrm{l}\,(G, M, K)\, w$$

Need for order reduction algorithms

- **High-speed high-accuracy servo-mechanisms imply high-order models**
- **Highest frequency modes in model are not reliable**
- **High-order model is not suited for optimal control**
- **Controller order exceeds model order**

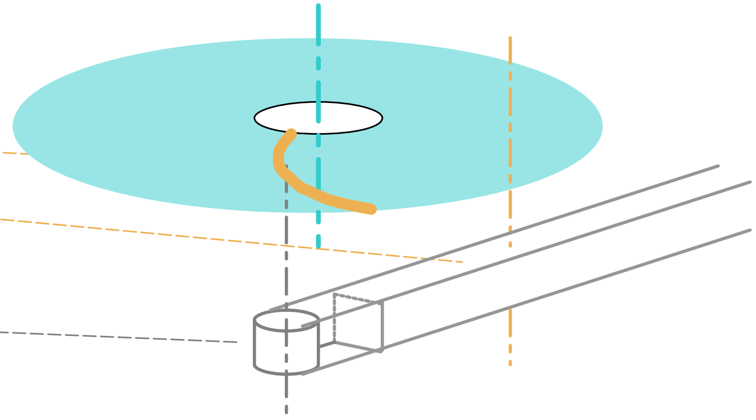
But

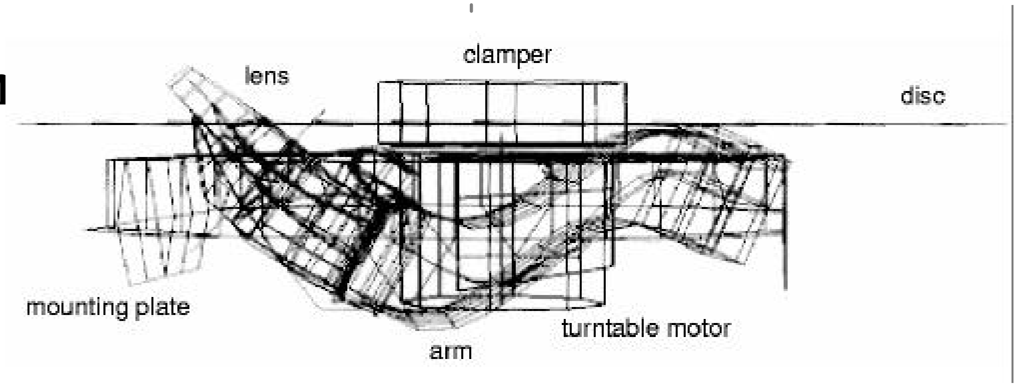
order reduction not goal in itself

Part of design

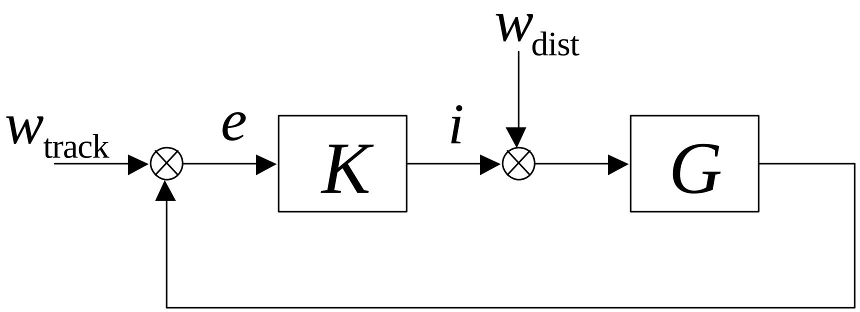
Introduction of CD-player mechanism

- Rotating compact disc
- swing arm for radial tracking
- with optical pick-up unit (vertically suspended lens) for focussing
- mounting plate, disc, arm not infinitely stiff
- 2x2 servo control design problem

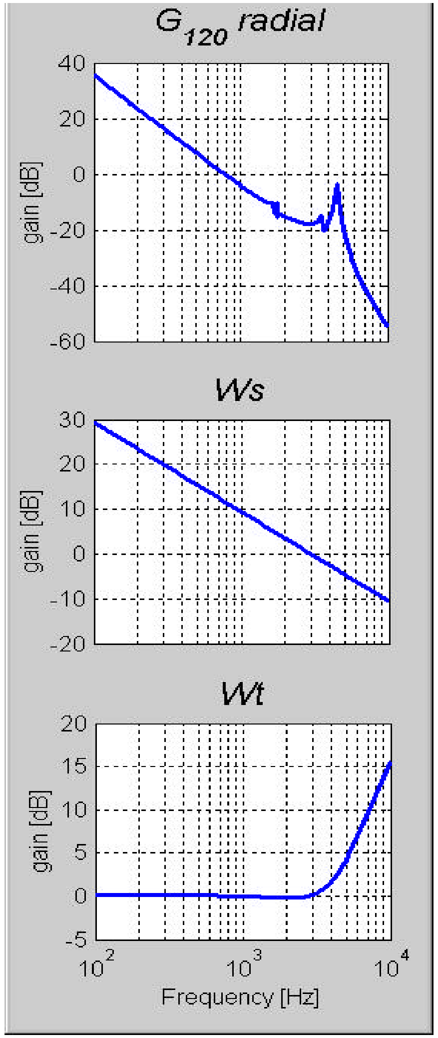




Radial control loop



- **No weights on input signals**
- **output weights**
  - **Ws to penalize low-frequent error e**
  - **Wt to penalize high-frequent current i**



Standard reduction techniques:

- **Modal** **A diagonal**
  - + **invariant for input & output**
  - + **direct**
  - + **no stability constraint**
- **balanced** **Gramians P & Q diag**
  - + **input-output significant**

Both suitable for truncation and singular perturbation

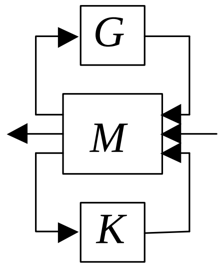
Not optimal, how about closed-loop?

Closed-loop balancing

Transform

- **model state**
- **controller state**

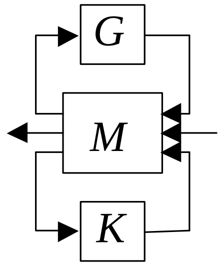
$$\begin{bmatrix} \breve{T}_G & & \\ & I_M & \\ & & \breve{T}_K \end{bmatrix}$$



to get closed-loop system Gramians with

- **closed-loop G-HSV**
- **closed-loop K-HSV**

$$\begin{bmatrix} \Sigma_G & \times & \times \\ \times & \times & \times \\ \times & \times & \Sigma_K \end{bmatrix}$$



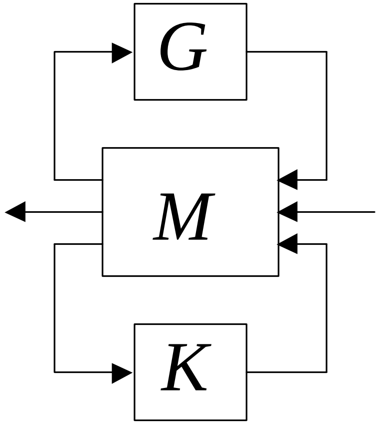
Closed-loop balanced reduction

- **Solve P, Q from Lyapunov equations of l (G,M,K)**
- **extract G-part:** P_G & Q_G
- **find balancing transformation:**

$$T^{*}Q_G T = T^{-1}P_G T^{-*} = \mathrm{diag}(\,\mathrm{HSV}_G\,)$$

- **apply T to G:** $G_{bal} = (T^{-1}A_G T,\ T^{-1}B_G,\ C_G T,\ D_G)$
- **truncate G_{bal}**

Reduction in twin feedback configuration

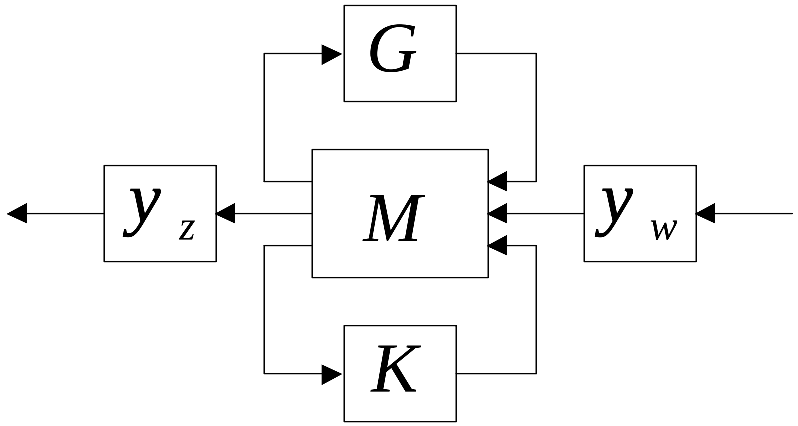


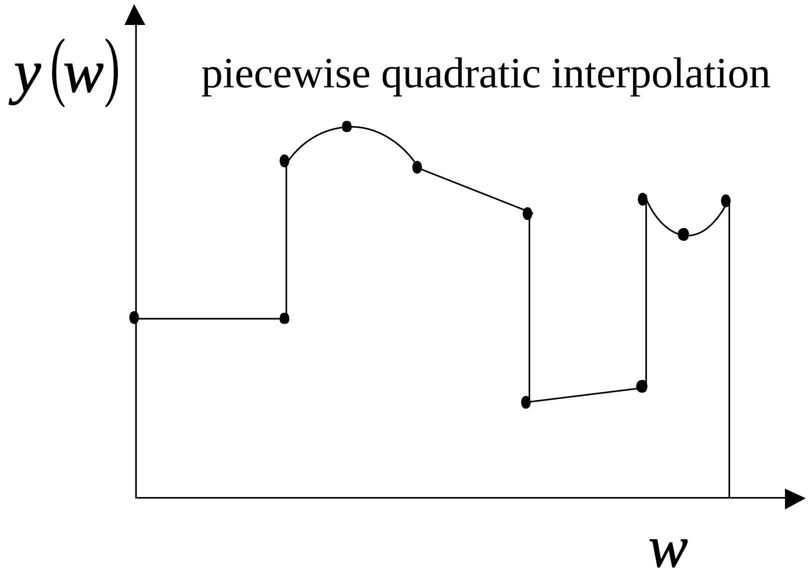
$$\breve{G}_m = \mathrm{bal}R_m\Big(\,\mathrm{l}\,(\underline{G_h}, M, K)\Big)$$

Tuning freedom:

- selection of order
- controller or model

Separate weighting in the form of power spectra





$$\breve{G}_m = \mathrm{bal}R_m\left(y_z \cdot \mathrm{I}\,(\underline{G_h}, M, K)\cdot y_w\right)$$

Iterative low-order control design

Available:

- **High order model G**
- **Weights in M**
- **First guess controller K**

Model reduction

$$d = \frac{\left\| \mathrm{I}(G_h, M, K) - \mathrm{I}(G_m, M, K) \right\|}{\left\| \mathrm{I}(G_h, M, K) \right\|}$$

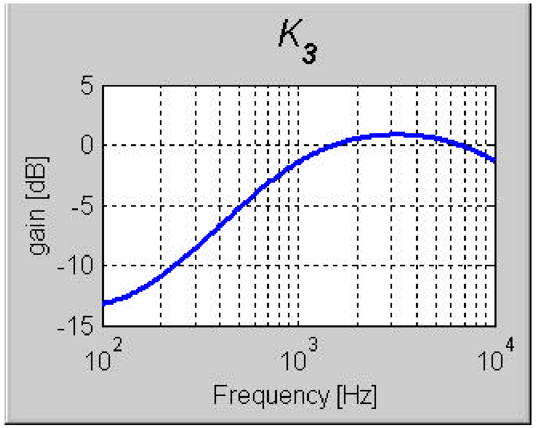
Optimal control

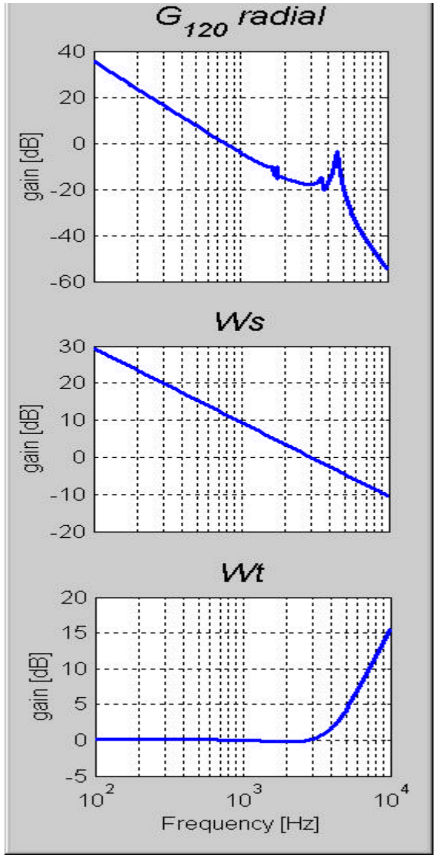
Controller reduction

$$g = \left\| \mathrm{I}(G\ , M, K_r\ ) \right\|$$

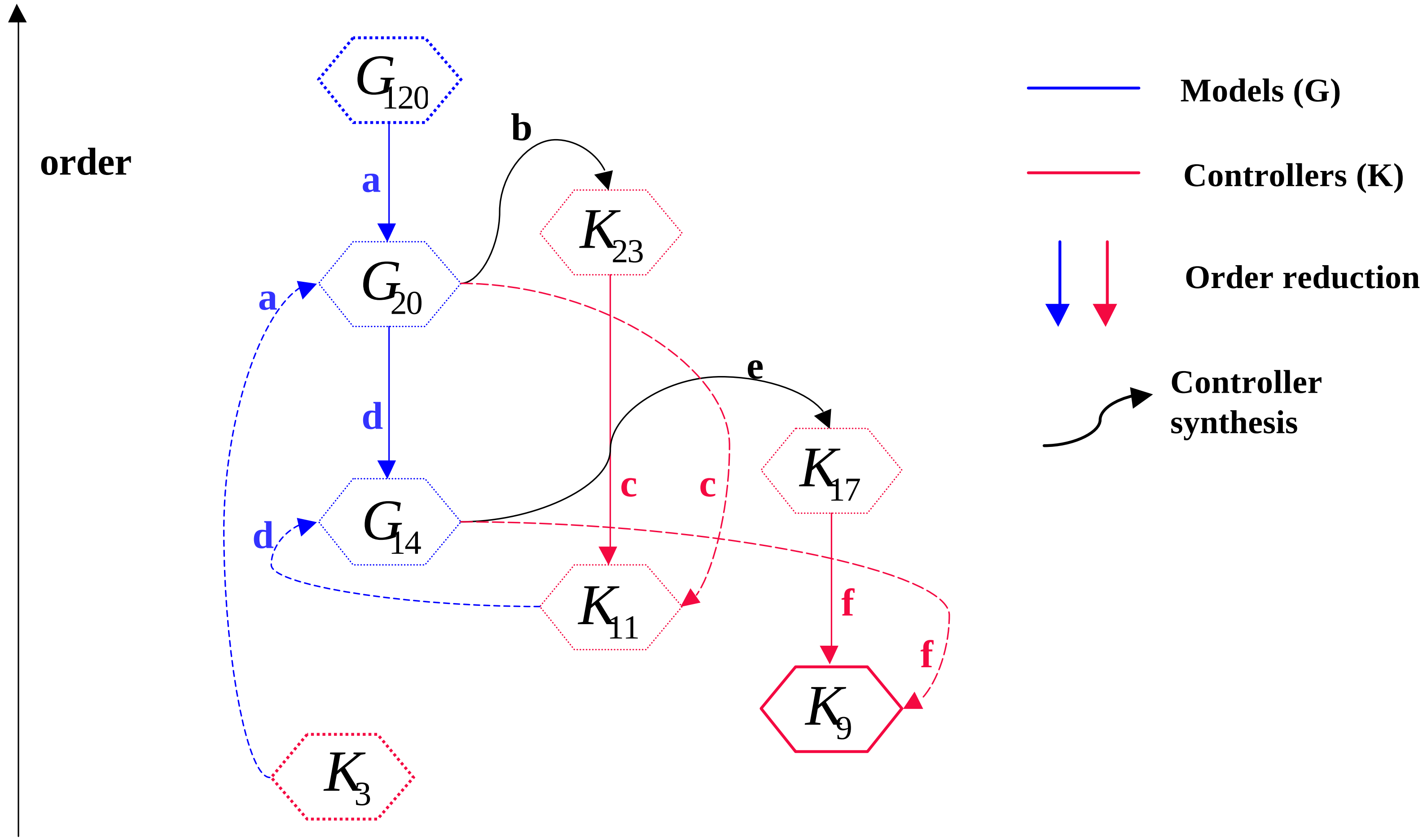
CD-player example

- **Model G120**
- **Ws order 1, Wt order 2**
- **First guess**
  order-3 controller:





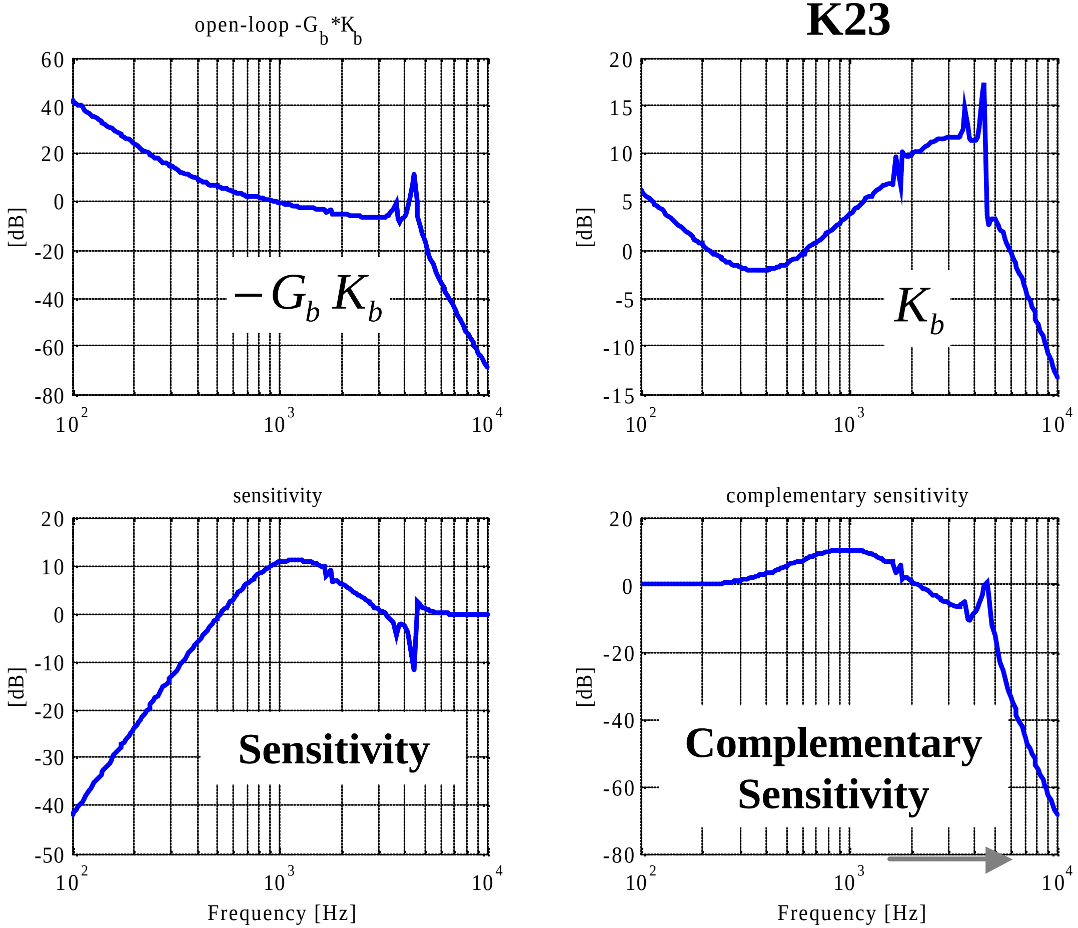
Sequence of model and controller reduction



Closed-loop evaluation

$g_{23} = 913.76$ | $g_{17} = 913.77$

K23

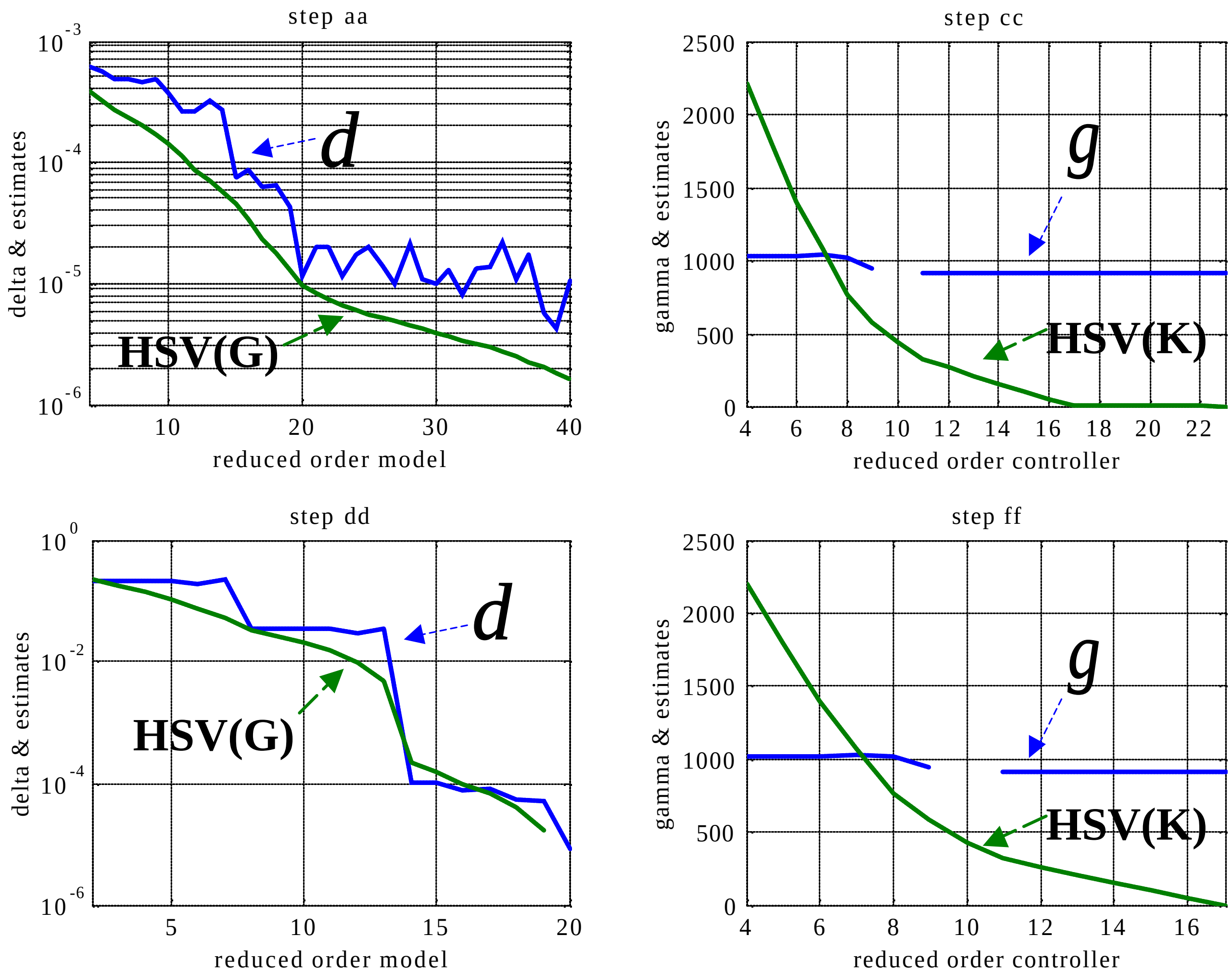


K17

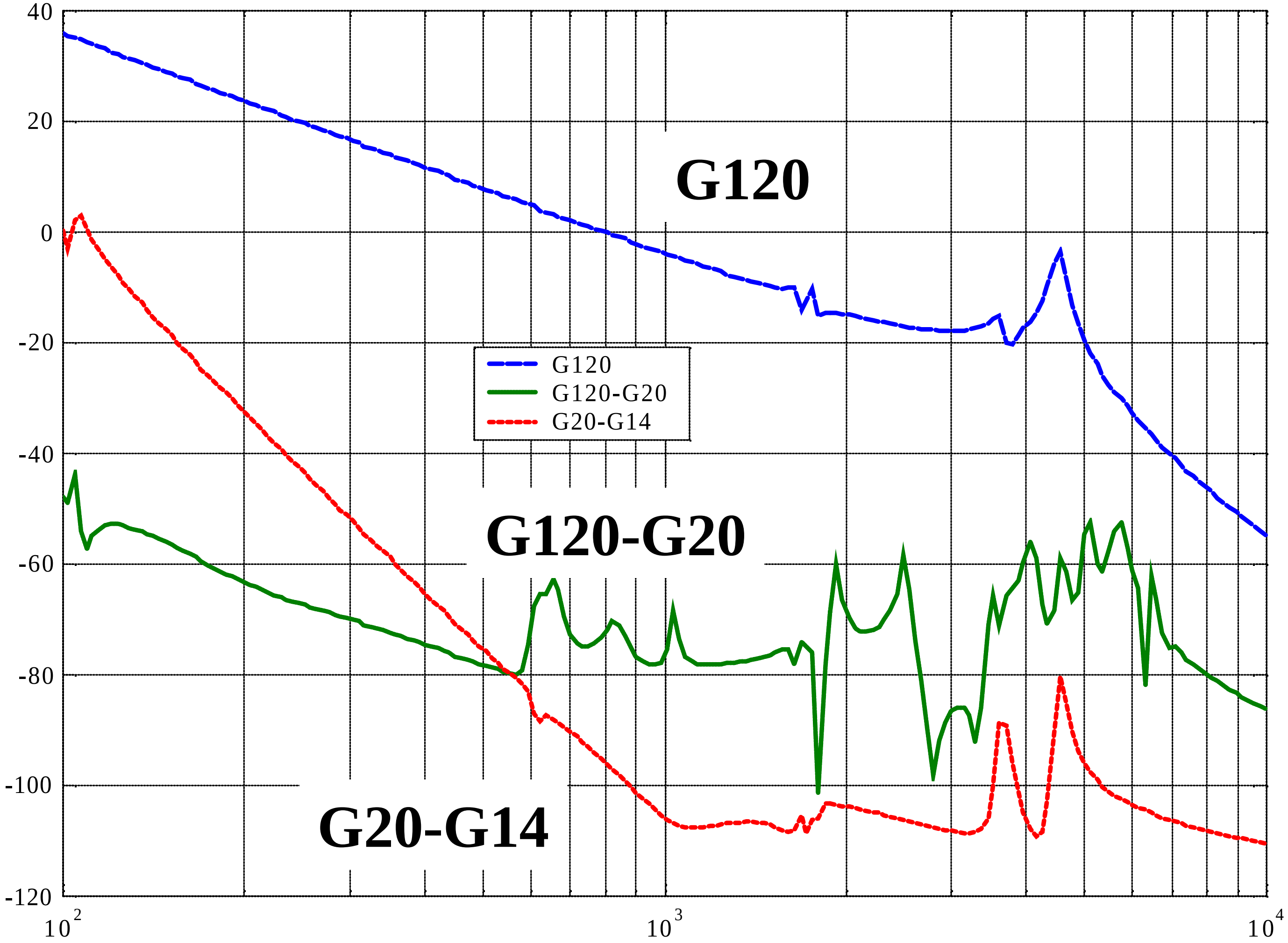
No visual difference

Model reduction OK

HSV and performance measures

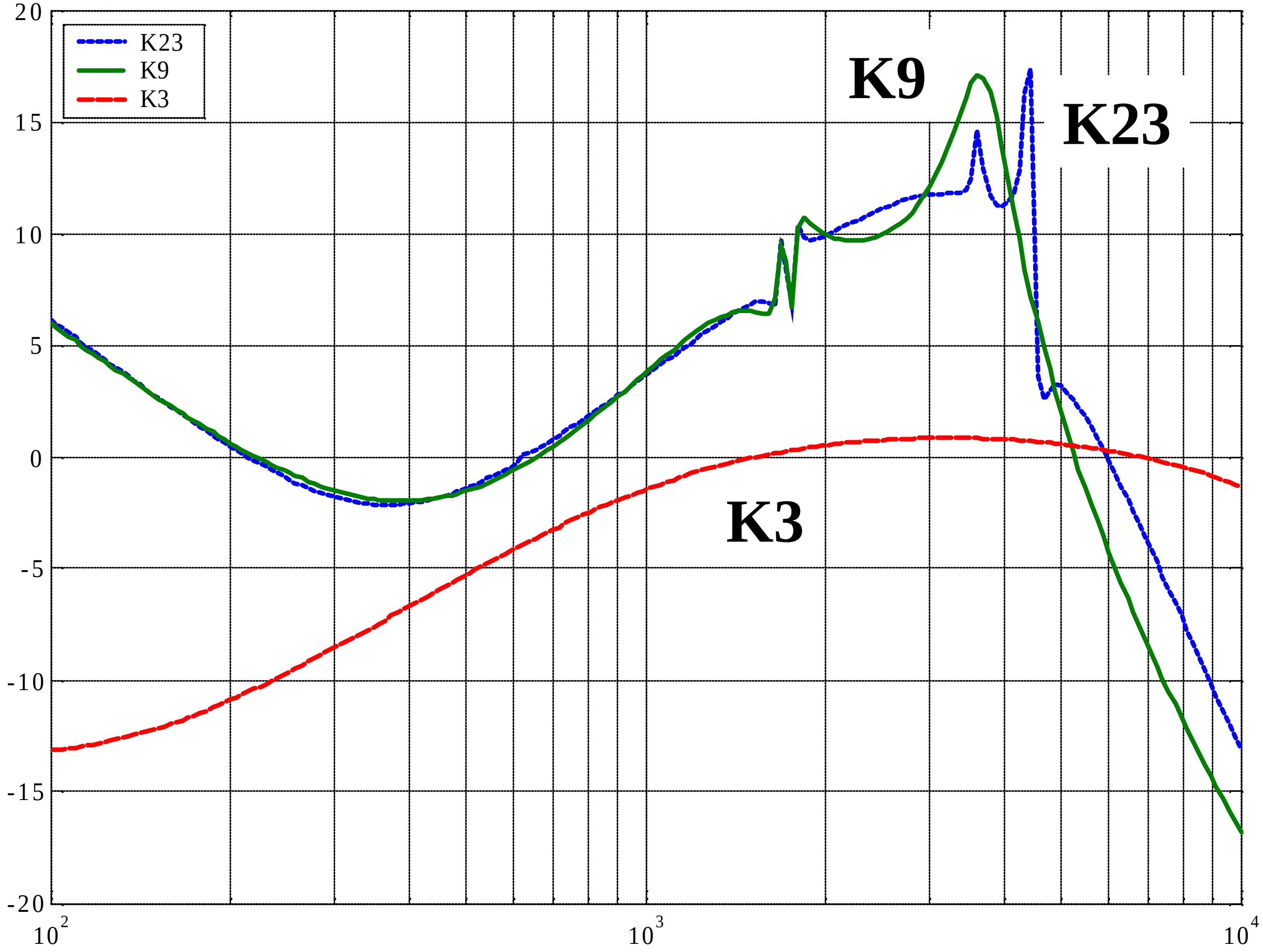


Models



40
20
0
-20
-40
-60
-80
-100
-120
10^2
10^3
10^4
G120
G120
G120-G20
G20-G14
G120-G20
G20-G14

Controllers



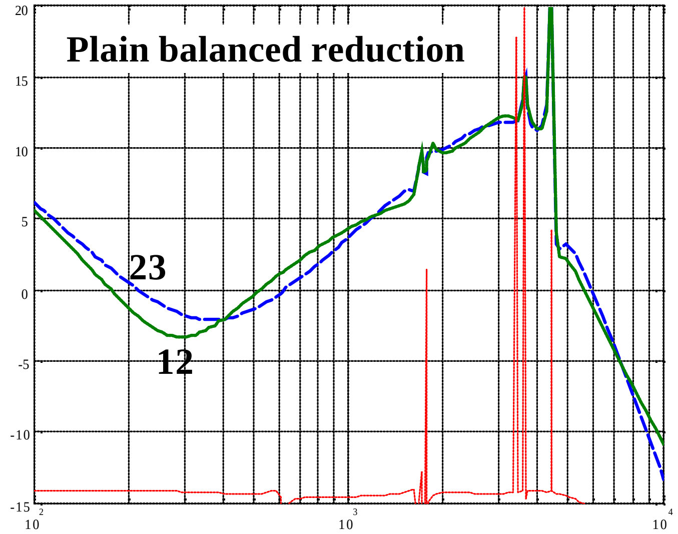
Direct controller reduction with manually shaped frequency weights

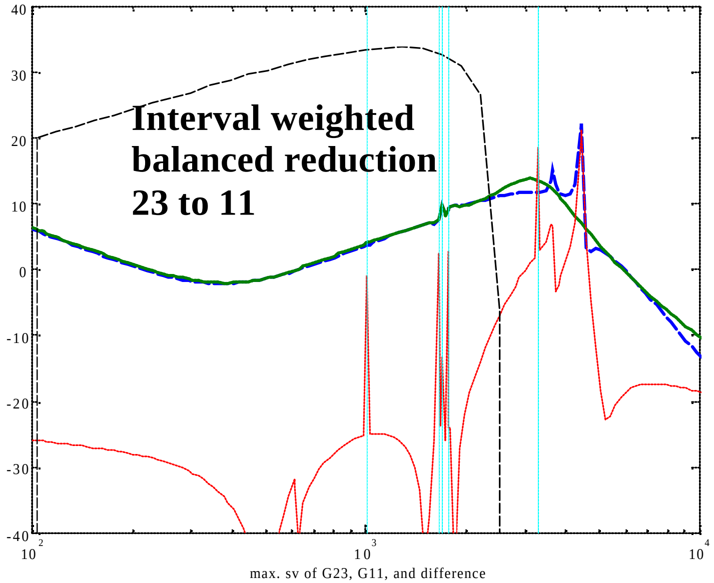
K23

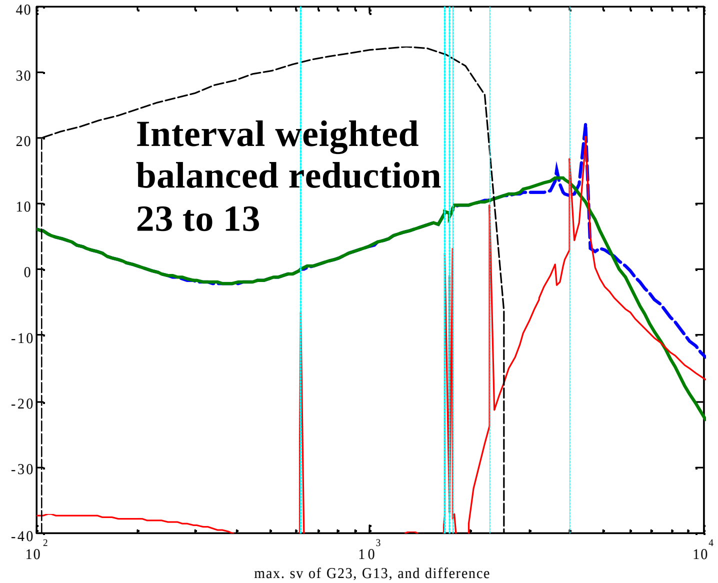
Kreduced

error

weighting







Recommendations from Experience:

- **See order reduction as a tool for control design**
- **Use robust and fast algorithms**
- **Exploit specific structure of model**
- **Apply iterative scheme (with frequency tuning)**
- **Evaluate results in more than one measure (graphics)**
- **Choose odd *or* even order**
- **Do not rely on "a priori" HSV-based error bounds**
- **Customize and automate in later stage**

Further reading:

P.M.R. Wortelboer, M. Steinbuch and O.H. Bosgra
Iterative Model and Controller Reduction using Closed-loop Balancing, with application to a Compact Disc Mechanism
Int. J. Robust Nonlinear Control 9, 123-142 (1999)