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A methodology for control-relevant nonlinear system identification using restricted complexity models

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Abstract

A broadly-applicable, control-relevant system identification methodology for nonlinear restricted complexity models (RCMs) is presented. Control design based on RCMs often leads to controllers which are easy to interpret and implement in real-time. A control-relevant identification method is developed to minimize the degradation in closed-loop performance as a result of RCM approximation error. A two-stage identification procedure is presented. First, a nonlinear ARX model is estimated from plant data using an orthogonal least squares algorithm; a Volterra series model is then generated from the nonlinear ARX model. In the second stage, a RCM with the desired structure is estimated from the Volterra series model through a model reduction algorithm that takes into account closed-loop performance requirements. The effectiveness of the proposed method is illustrated using two chemical reactor examples.

Keywords: Control relevant modeling; system identification; Nonlinear systems; Volterra series; Reduced order models

1. Introduction

Real-life process systems are inherently nonlinear. Traditionally, control systems for these processes have been designed under the assumption that the system under control will fluctuate only in a small neighborhood from the desired operation point. Thus, a nonlinear system can be reasonably approximated with a linearized approximation of the true physical system. Yet it has been realized for a long time that due to today's highly competitive market, an ideal control system should not just hold fast a process plant to a certain fixed state. Instead, modern control systems must be able to keep a plant operating at the most profitable state according to market information [29]. This requires control systems that operate over a large operating regime. The larger the operating regime, the greater the need for nonlinear control design methods. On the other hand, many published nonlinear control design techniques have found few applications in the process industries. Among the various reasons for this phenomenon include the fact that many nonlinear control design methods involve sophisticated mathematics and result in controllers that demand large amounts of computation. The latter consideration makes real-time implementation, online tuning and controller maintenance very difficult in a plant environment.

A possible solution to these problems is to rely on control systems designed on the basis of restricted complexity models (RCMs). Fig. 1 shows some typical examples of RCMs [12]. A model structure consisting of a linear dynamic block trailed by a zero-memory nonlinear block which is then followed by another linear dynamic block is called a cascade model. Without the first dynamic block a cascade model is called a Hammerstein model; similarly, without the trailing dynamic block it is called a Wiener model. If a system can be represented by a Wiener model, the system nonlinearity can then be effectively eliminated by properly transforming the output variable before the output is fed back to the controller (Fig. 2). This is the familiar variable transformation method [24]. Similar methods have been widely accepted in process industries. For example, it is a common practice in control of high purity distillation columns to use the logarithmic transformation of the concentration measurement as the controlled variable. The control scheme in Fig. 2 can be regarded a generalization of this and other ad hoc methods. Similarly, if a system can be

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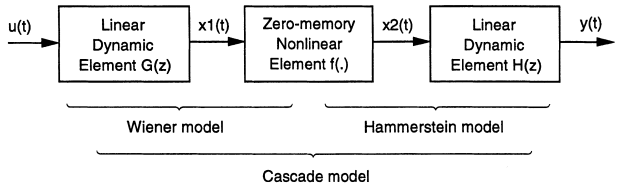


Fig. 1. Typical restricted complexity models.

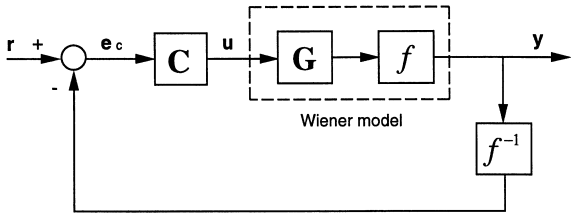


Fig. 2. Variable transformation method of the Wiener system.

represented as a Hammerstein model, then system nonlinearity can be effectively eliminated through a gain scheduling scheme (Fig. 3). Thus, control design based on RCMs often leads to control schemes which retain the merits of a linear controller, i.e. ease of interpretation and few real-time computations, while providing improved performance. Yet despite their attractiveness, real-life systems are usually much more complicated than RCM structures. A natural question is then whether control performance must be sacrificed significantly in exchange for RCM approximations, and how should a RCM be estimated such that it retains the important control-relevant information of the true system. An answer to these questions lies in control-relevant identification.

Control relevant identification considers how to integrate the performance requirements of the control system into the task of system identification. Intuitively, when a restricted complexity model is used to describe a complex nonlinear system, it is expected that the model can only capture a portion of the system's dynamic characteristics. For a given restricted complexity model structure, the system characteristics that will be captured in the model are solely determined by the identification procedure. Hence, if the estimated RCM is to be used for control design purposes, it is important that control design requirements are directly accounted for in the determination of design variables in the identification step. For linear systems, the benefits of using a control-relevant technique have been demonstrated by

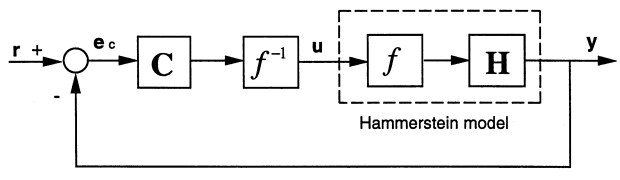


Fig. 3. Gain scheduling method of the Hammerstein system.

many authors (e.g. [33,36,42]). In the nonlinear case, Zheng and Zafiriou [43] proposed a control-relevant identification approach based on a local robust stability condition. In their formulation, a model uncertainty coefficient must be estimated explicitly. Hence only the noise-free case is considered. By extending the algorithm of Rivera and Morari [31], Ling and Rivera [20] propose a control-relevant model reduction method for Volterra series models.

Clearly, a restricted complexity model can also be estimated through traditional methods. The problem has been studied by many authors [11,25,38,41]. For these techniques the structural characteristics of the restricted complexity model are exploited to form a simple least squares or instrumental variable type estimation problem. The model performance is often evaluated by its "open-loop" mean square prediction error. On the other hand, since the RCM is only a much simplified description of the true system, two restricted complexity models may give a very similar mean square prediction error, yet display quite different behaviors [19]. This in turn leads to a bewildering situation: a small change in identification design variables such as the input design may lead to a very different model. A similar situation is also observed when a linear model is used to approximate a highly nonlinear system [6].

The objective of this paper is to present a systematic control-relevant identification procedure using nonlinear restricted complexity models estimated from noisy input/output plant measurements. The paper is organized as follows. In Section 2, admissible restricted complexity model structures are defined. In Section 3, an overview of the proposed method is presented. The method contains two major stages. The first stage involves the estimation of a nonlinear ARX (NARX) and a Volterra series models. In the second stage, the restricted complexity model is estimated from the Volterra series model through a control-relevant model reduction algorithm. The estimation procedure for the NARX and Volterra series models is described in Section 4. Section 5 focuses on the control-relevant model reduction problem, while Section 6 considers the model validation issue. Some practical guidelines for identification experimental design are given in Section 7. Two illustrative examples involving a CSTR and a polymerization reactor are given in Sections 8 and 9. Finally, Section 10 provides a brief summary and some conclusions.

2. Admissible restricted complexity models

Many simple or restricted complexity nonlinear model structures have been proposed in the literature [13,25,39]. In this paper, a restricted complexity model is defined as any subset models of the following general Volterra series model [34,35]:

$$\Psi:\ y(t)=\sum_{i=1}^{K}\sum_{n_1=1}^{J}\cdots\sum_{n_i=n_{i-1}}^{J}h_i(n_1\cdots,n_i)\prod_{j=1}^{i}u(t-n_j)+d(t) \tag{1}$$

where $y(t)\in R$ is the system output, $u(t)\in R$ is the system input, $d(t)\in R$ is an additive disturbance term, and h_i denotes the ith order Volterra kernel. Typical examples of restricted complexity models satisfying this definition include various block-structured models whose memoryless nonlinear elements are analytic. By expanding the memoryless nonlinear elements as a Taylor series and representing the linear dynamic elements by their impulse responses, it is not difficult to show that a block-structured model can be transformed into a Volterra series representation. Consider, for example, the case of Wiener model (Fig. 1). If the linear dynamic block $G(z)$ has the impulse response

$$G(z)=g_1z^{-1}+g_2z^{-2}+\cdots$$

and the memoryless nonlinear block $f(x_1)$ is characterized by

$$f(x_1)=x_1+\gamma_3[x_1]^2+\gamma_3[x_1]^3+\cdots,$$

its Volterra series representation is then given by

$$\begin{aligned}x_2(t)&=x_1(t)+\gamma_2x_1(t)^2+\gamma_3x_1(t)^3+\cdots\\&=\sum_{i=1}^{\infty}g_iu(t-i)+\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\gamma_2g_ig_ju(t-i)u(t-j)\\&\quad+\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\gamma_3g_ig_jg_ku(t-i)u(t-j)u(t-k)+\cdots\end{aligned}$$

Using a restricted complexity model for control design often leads to a nonlinear control scheme familiar to process engineers, as discussed in the Introduction. Specifically, consider the Internal Model Control (IMC) design of a Hammerstein model shown in Fig. 4, where the controller q consists of the inverse Hammerstein model preceeded by a filter F, and F is assumed to be a first order linear filter. Assume that G has been limited to a first order system. The system in Fig. 4 can then be put into the traditional form shown in Fig. 5, where the controller is the familiar PI controller followed by a nonlinear gain scheduling block f^{-1}. If F and G are denoted by

$$F(z)=\frac{(1-\delta)z^{-1}}{1-\delta z^{-1}}\qquad G(z)=\frac{bz^{-1}}{1-az^{-1}},$$

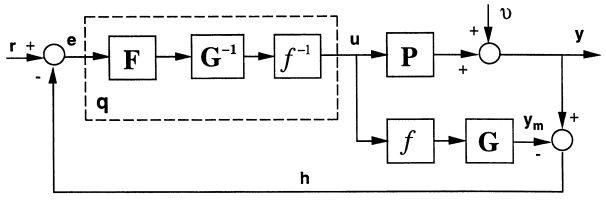


Fig. 4. IMC design of the Hammerstein model.

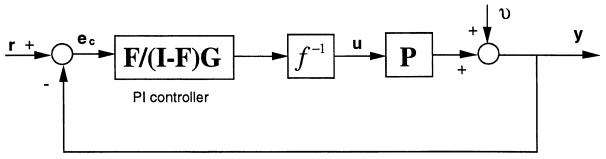


Fig. 5. PI with gain scheduling control of the Hammerstein model.

then the PI controller parameters can then simply computed by

$$P=(1-\delta)/b,\quad I=a.$$

When a control-relevant approach is used, our experience has shown that is usually enough to restrict the nonlinear block f to a polynomial of degree less than 4. The inverse f^{-1} can then be computed analytically. Hence, the controller according to Fig. 5 can be implemented in real time without involving any iterative computation.

By selecting a restricted complexity model for identification, it is possible to obtain a simple nonlinear controller that is easy to implement. Correspondingly, we develop a control relevant identification method to avoid significant closed-loop performance degradation due to the RCM approximation error.

3. Control-relevant identification of nonlinear systems: a two-stage approach

Consider the feedback system in Fig. 6 where p is a nonlinear plant, $\tilde{p}\in\Psi$ represents a restricted complexity model of p, and q is a controller designed on the IMC principle [10,22]. For the system identification problem under study, the plant p is given as a set of plant input/

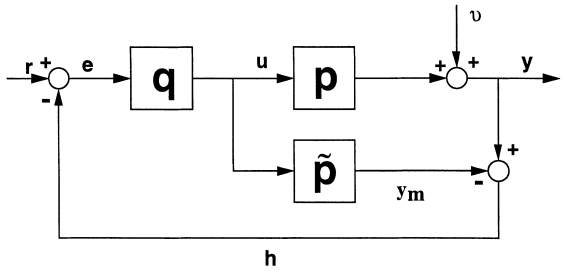


Fig. 6. A feedback system: relating control design to system identification.

output measurements. The purpose of control relevant identification is to find a restricted complexity model $\tilde{p}$ such that once a controller q is designed according to $\tilde{p}$, certain performance of the closed-loop system in Fig. 6 is optimized. If $\|e_c\|_2$, the Euclidean norm of set point tracking error $e_c = r - y$, is used as the criterion of closed-loop performance, a control-relevant identification problem is then formulated as

$$\tilde{p} = \arg \min_{\tilde{p}\in\Psi} \|e_c\|_2 \tag{2}$$

In the traditional identification problem, one seeks a model such that the weighted prediction error is minimized,

$$\tilde{p} = \arg \min_{\tilde{p}\in\Psi} \left\| W(y - \hat{y}) \right\|_2 \tag{3}$$

where $\hat{y}$ represents a prediction of the system output y, and W is a weighting function. The difference between two problem formulations is clear: in the control relevant case the control design requirements are integrated into the identification procedure such that closed-loop performance dominates the identification problem, while in the traditional problem formulation the closed-loop performance issue is essentially ignored.

The issue then is how to solve the minimization problem posed in (2). In this paper, we propose a two-stage solution approach:

- First, we estimate a Nonlinear ARX (NARX) model from the input/output measurements, and apply the method of Diaz and Desrochers [7] to approximate the NARX model with a truncated Volterra series model.
- Secondly, using a control-relevant model reduction algorithm, a restricted complexity model is computed from the truncated Volterra series model.

This approach is philosophically similar to the method used for linear systems shown in Fig. 7 [33,40,44]. In the linear case one first estimates a high order FIR or ARX model from process measurements.

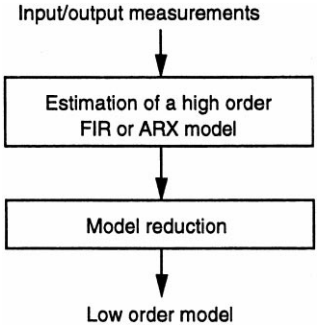


Fig. 7. The two-step estimation procedure for linear systems.

From the high order model, a model reduction algorithm is used to compute a low order model. The high order estimate is regarded to be asymptotically unbiased [21], i.e. the estimation step of high order model involves only the variance error. The model reduction step is performed in a deterministic setting, only bias errors are considered. The motivation for the nonlinear case here is similar: we use the first stage as a means to "clean out" the noise/disturbance effect on the process measurements. The control-relevant approximation problem can then be investigated in the second stage in a noise/disturbance-free setting.

The Volterra series model is selected in the first stage because of its amenity to control design and analysis [34,35] and its suitability for control-relevant model reduction [20]. The Internal Model Control (IMC) design of a Volterra series model has been studied by Doyle et al. [9]. Certainly, a Volterra series model itself represents a very general nonlinear model structure which can describe a broad class of nonlinear systems [4,27]. Yet an issue that arises here is that for many problems in practice, the huge number of model parameters makes it unrealistic to estimate the Volterra series model directly from the process data. To overcome this situation, it is necessary to decompose the full order Volterra model estimation stage into another two steps. First, we estimate a parsimonious NARX model from the process measurements. Estimating the NARX model using an orthogonal least squares method is a relatively mature technique ([15,5]). Using the NARX model structure along with the orthogonal least squares estimation method, most systems can be described by a model with less than 20 parameters. From the NARX model, a truncated Volterra series model can then be computed using the algorithm proposed by Diaz and Desrochers [7]. Fig. 8 is an overview of this two-stage

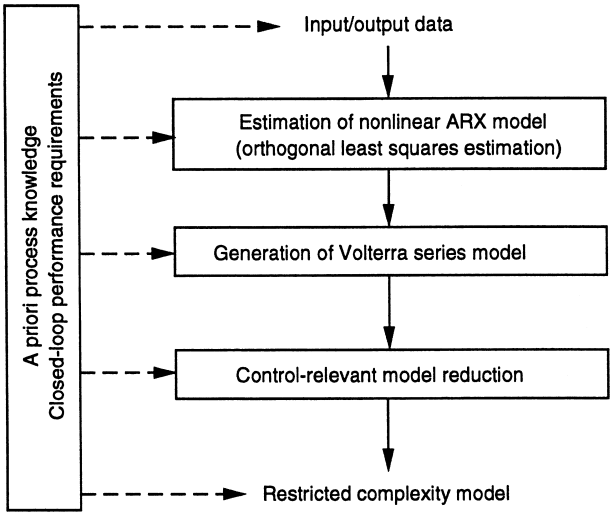


Fig. 8. Control-relevant identification of nonlinear systems: a two-stage approach.

solution scheme. In the following sections, we shall elucidate each step with more details.

4. Estimation of NARX and Volterra series models

A finite Volterra series model has a form previously shown in Eq. (1)

$$\begin{aligned} y(t) &= p_1[u] + p_2[u] + \cdots + p_K[u] + d(t) \\ &= \Sigma_{i=1}^{K} \Sigma_{n_1=1}^{J} \cdots \Sigma_{n_i=n_{i-1}}^{J} h_i(n_1, \cdots, n_i) \Pi_{j=1}^{i} u(t - n_j) \\ &\quad + d(t) \end{aligned} \tag{4}$$

where $p_i[u]$ denotes the ith order Volterra kernel. Two issues prevent one from estimating the Volterra series model directly from system input/output measurements. First, the Volterra series model contains a large number of model parameters. To form a least squares type parameter estimation problem, a huge amount of input/output data is needed, which implies a long identification experiment time in practice. Secondly, even though the process model in (4) itself is linear-in-parameters, a rigorous treatment of the disturbance term $d(t)$ will lead to a model which is no longer linear-in-parameters. For example, $d(t)$ can be characterized as [2]:

$$d(t) = G^e[e(t)] + G^{ue}[u(t), e(t)] \tag{5}$$

where $e(t)$ is a zero mean white noise sequence with some unknown probability density function, $G^e[e(t)]$ represents a function of $e(t-i))i = 1, 2, \cdots)$ and $G^{ue}[u(t), e(t)]$ is a function containing cross terms of $u(t-i)$ and $e(t-i)$ $(i = 1, 2, \cdots)$. When the disturbance model (5) is used, the prediction error estimation problem of model (4) must be solved by a nonlinear programming algorithm. Considering the number of model parameters involved, the solution to the nonlinear programming problem can be quite challenging. Hence, instead of estimating the full order Volterra model directly from the system measurements, we consider a two-step estimation procedure:

4.1. Step one

We first estimate a polynomial NARX model from the process measurements. A polynomial NARX model has a form:

$$y(t) = \theta_0 + \Sigma_{i=1}^{l} \Sigma_{i_1=1}^{M} \cdots \Sigma_{i_l=i_{l-1}}^{M} \theta_{i_1} \cdots i_l \Pi_{k=1}^{l} x_{i_k}(t) + e(t) \tag{6}$$

where

$$x_1(t) = y(t-1), \quad x_2(t) = y(t-2), \ldots,$$

$$x_{n_y}(t) = y(t - n_y), \; x_{n_y+1}(t) = u(t-1),$$

$$x_{n_y+2}(t) = u(t-2), \quad \ldots, \quad x_{n_y+n_u}(t) = u(t-n_u),$$

and

$$M = n_y + n_u.$$

The polynomial NARX model forms a linear-in-parameters model. Compared to the more general NARMAX model [16], model (6) is regarded to give a better balance between the model's ability to describe a complicated system and the complexity of the associated parameter estimation problem [37]. Expressed in vector form, we have

$$y(t) = V(t)\theta + e(t) \tag{7}$$

where $V(t)$ is the measurement vector

$$\begin{aligned} V(t) = [&x_1(t), \quad x_2(t), \quad \cdots, \quad x_M(t), \quad x_1(t)x_1(t), \\ &x_1(t)x_2(t), \quad \cdots, \quad x_M^l(t)] \end{aligned}$$

and θ is the model parameter vector

$$\theta = [\theta_1, \quad \theta_2, \quad \cdots, \quad \theta_M, \theta_{11}, \quad \theta_{12}, \quad bn9, \quad \theta_{M\cdots M}]^T$$

Let Y denote the output measurement sequence

$$Y = [y(0), \quad y(1), \quad \cdots, \quad y(N-1)]^T.$$

where N is the length of the measurement sequence. The following equation set can then be established.

$$Y = A\theta + \Gamma \tag{8}$$

with

$$A = \begin{bmatrix} V(0) \\ V(1) \\ \cdots \\ V(N-1) \end{bmatrix}.$$

and

$$\Gamma = [e(0) \quad e(1) \cdots e(N-1)]^T,$$

The least squares solution to (8) is a well established technique. It satisfies the normal equation

$$A^T A\theta = A^T Y \tag{9}$$

where $A^T A$ is assumed to be full rank, but it can be ill-conditioned. Many approaches can be considered to compute θ, such as Gaussian elimination, Singular Value Decomposition, the orthogonal decomposition, etc. A numerically superior method is the one based on the orthogonal decomposition of $A^T A$, which has been studied by many authors [5,15,28,30]. Another advantage

of the orthogonal decomposition method is that a procedure can be established to pick up those model terms from the full-order model (6), which are most significant to describing the system dynamics. Using this procedure, most process systems can be described by a parsimonious NARX model with less than 20 model parameters. Since the method has been extensively reported in the literature, we omit its details here.

4.2. Step two

From the NARX model, a Volterra series model which is more amenable to control-relevant analysis can be generated. The algorithm proposed by Diaz and Desrochers [7] describes how to transform polynomial difference equation models into corresponding kernels of a truncated Volterra series model. It is briefly summarized below as follows:

In order to simplify the notation, we first define vector $Z(t)$ and the Kronecker symbol $Z^i(t)$ as:

$$Z(t) = \left[y(t-1), \cdots, y\left(t-n_y\right), u(t-1), \cdots, u(t-n_u)\right]^T \tag{10}$$

$$Z^i(t) = Z(t) \otimes Z(t) \cdots \otimes Z(t) \qquad \text{(i times)} \tag{11}$$

where $\otimes$ denotes the Kronecker product. Using these notations, the estimated NARX model can then be expressed as

$$y(t) = F_1 Z(t) + F_2 Z^{(2)}(t) + \cdots + F_l Z^{(l)}(t) \tag{12}$$

where parameter vectors F_i $(i = 1, 2, \cdots, l)$ are known constant vectors. Furthermore, we define

$$Z_1(t) = \left[y_1(t-1), \cdots, y_1\left(t-n_y\right), u(t-1), \cdots, u(t-n_u)\right]^T \tag{13}$$

$$Z_i(t) = \left[y_i(t-1), \cdots, y_i\left(t-n_y\right), 0, \cdots, 0\right]^T, \quad i = 2, 3, \cdots \tag{14}$$

with $y_i(t)$ being a function of time, yet to be determined. The Kronecker symbol of $Z_i(t)$, $Z_i^{(j)}(t)$, is defined similarly to (11). According to Diaz and Desrochers, a Volterra series model can then be computed by solving a series of linear difference equations:

$$y_1(t) = F_1 Z_1(t) \tag{15}$$

$$y_2(t) = F_1 Z_2(t) + F_2 Z_1^{(2)}(t) \tag{16}$$

$$y_3(t) = F_1 Z_3(t) + F_2(Z_1 \otimes Z_2 + Z_2 \otimes Z_1) + F_3 Z_1^{(3)}(t) \tag{17}$$

Eq. (15) is a linear difference equation of y_1. Its solution gives the first order Volterra kernel p_1:

$$y_1(t) = p_1[u] \tag{18}$$

Substituting (18) into (16) leads to a linear difference equation of y_2. Solving this equation, the second order Volterra kernel p^2 is obtained,

$$y_2(t) = p_2[u] \tag{19}$$

Similarly, the higher order Volterra kernels can be computed. While the computations involved in generating the higher order kernels can be tedious, the calculations involve straightforward algebraic manipulation and no iterative procedures are required. Furthermore, explicit expressions for p_1 to p_3 have been given in Diaz and Desrochers [7].

The two-step identification procedure presented here is useful but care must be taken in its application to general nonlinear systems. Particularly, if the system under study displays nonlinear behavior such as chaotic dynamics, output multiplicity or subharmonic generation, a truncated Volterra model may not represent a suitable approximation. As noted in Pearson and Ogunnaike [27], finite Volterra models can be used to describe fading memory nonlinear systems over an operating range around a single steady-state; if the problem under consideration meets this criteria then the method will be adequate.

5. Control relevant estimation of restricted complexity models

Having obtained a Volterra series model from data, the second stage is concerned with the estimation of the restricted complexity models through control relevant model reduction. In this section, we present a control relevant model reduction algorithm which reduces the Volterra series model to a restricted complexity model defined earlier. A more detailed treatment of the control-relevant model reduction problem can be found in Ling [18] and Ling and Rivera [20].

To analyze a model's control-relevance, consider the closed-loop system in Fig. 9 [9], where q, the controller, consists of the inverse model $\tilde{p}^{-1}$ preceeded by a filter F. For a Volterra series model, its inverse can be represented as a feedback structure with the inverse of the linear kernel in the feedforward path and all the nonlinear kernels in the feedback path [34], as shown in Fig. 9. The filter F is assumed to be linear. For the control-relevant model reduction problem under consideration, p represents the full order Volterra series model and $\tilde{p} \in \Psi$ is a restricted complexity model. The control relevant model reduction problem is posed as

$$\tilde{p} = \arg \min_{\tilde{p} \in \Psi} \|e_c\|_2 \tag{20}$$

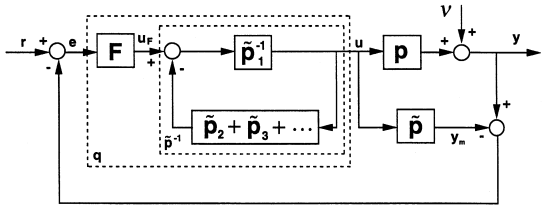


Fig. 9. IMC design of Volterra series models.

where e_c represents the tracking error $e_c(t) = r(t) - y(t)$. Direct solution of (20) is generally difficult. The key issue is thus to simplify (20) into a form which can be solved without overwhelming effort.

From Fig. 9, we have

$$e_c = (I - \tilde{p} * q) * (I + (p - \tilde{p}) * q)^{-1}[r - d] \tag{21}$$

where '$*$' denotes the composition of related operators and I denotes the identity operator. In (21), p is a Volterra series model. By definition, $\tilde{p}$ can also be represented as a Volterra series model. It can then be shown that e_c can be represented as another Volterra series [20]:

$$e_c = (\eta * w_1 + \eta * w_2 + \cdots)[r - d] \tag{22}$$

where $\eta = I - F$, and $w = \sum w_i$ is the Volterra series representation of the operator $(I + (p - \tilde{p}) * q^{-1}$.

$$\begin{aligned} w_1 &= s_1^{-1} = I + \left[-\left(p^1 - \tilde{p}_1\right)q_1\right][1 + (p_1 - \tilde{p}_1)q_1]^{-1} \\ &= I + w_1' \\ w_2 &= -s_1^{-1} * s_2 * w_1 \\ w_3 &= -s_1^{-1} * (s_2 * (w_1 + w_2) - s_2 * w_1 - s_2 * w_2 + s_3 * s_1) \\ &\vdots \end{aligned} \tag{23}$$

with

$$\begin{aligned} s_1 &= I + (p - \tilde{p}_1)q_1 \\ s_2 &= (p_1 - \tilde{p}_1) * q_2 + (p_2 - \tilde{p}_2) * q_1 \\ s_3 &= (p_1 - \tilde{p}_1) * q_3 + (p_2 - \tilde{p}_2) * (q_1 + q_2) \\ &\quad - (p_2 - \tilde{p}_2) * q_1 - (p_2 - \tilde{p}_2) * q_2 + (p_3 - \tilde{p}_3) * q_1 \\ &\vdots \end{aligned} \tag{24}$$

and

$$\begin{aligned} q_1 &= \tilde{p}_1^{-1} F \\ q_2 &= -\tilde{p}_1^{-1} * \tilde{p}_2 * q_1 \\ q_3 &= -\tilde{p}_1^{-1} * (\tilde{p}_2 * (q_1 + q_2) - \\ &\quad \tilde{p}_2 * q_1 - \tilde{p}_2 * q_2 + \tilde{p}_3 * q_1) \\ &\vdots \end{aligned} \tag{25}$$

Kernels of the Volterra series (22) have an important pattern which can be directly observed from (23), (24) and (25): the first order kernel $\eta * w_1$ is only a function of the first order kernel $\tilde{p}_1$, the second order kernel $\eta^* w_2$ is only a function of the first two kernels $\tilde{p}_1$ and $\tilde{p}_2$, and so on. Equivalently, (22) can be written as

$$e_c = \left(\eta + \eta * w_1' + \eta * w_2 + \cdots\right)[r - d] \tag{26}$$

In Eq. (26), we have decomposed the closed-loop control error e_c into different terms. The first term η is the nominal performance term and is not directly related to the modeling error. It comes from the fact that perfect control was detuned in the NIMC design through the filter F. The second term $\eta * w_1'$ comes from the open-loop modeling error in the first order kernel, $(p_1 - \tilde{p}_1)$. The third term $\eta * w_2$ can be attributed to the open-loop modeling error in the first two kernels, $(p_1 - \tilde{p}_1)$ and $(p_2 - \tilde{p}_2)$, and so on. Since η does not depend on $\tilde{p}$, the control-relevant model reduction problem can thus be written as:

$$\tilde{p} = \arg\min_{\tilde{p} \in \Psi} \left\| \left(\eta * w_1' + \eta * w_2 + \cdots\right)[r - d] \right\|_2 \tag{27}$$

The decomposition of the control error e_c in Eq. (26) also suggests a way to simplify the model reduction problem posed in (27). In the simplified problem formulation, we compute the restricted complexity model through a sequential minimization problems:

$$\tilde{p}_1 = \min_{\tilde{p}_1} \left\| \eta w_1'[r - d] \right\|_2 \tag{28}$$

$$\begin{aligned} \tilde{p}_2 &= \min_{\tilde{p}_2} \left\| \eta w_2[r - d] \right\|_2 \\ &\vdots \end{aligned} \tag{29}$$

$$\tilde{p}_n = \min_{\tilde{p}_n} \left\| \eta w_n[r - d] \right\|_2 \tag{30}$$

Clearly, Eq. (28)–(30) imply that the control error which arises from the modeling error in each kernel has to be minimized, and not just the overall control error. Consequently, since w_1 is a function of only $\tilde{p}_1$, and w_2 is a function of only $\tilde{p}_1$ and $\tilde{p}_2$, and so on, the computation of each kernel is now decoupled. Solving the minimization problem (28) gives the linear kernel $\tilde{p}_1$; having obtained $\tilde{p}_1$, the minimization problem in (29) can now be solved to give the second order kernel $\tilde{p}_2$, and so on. This decoupling scheme philosophically follows the ideas presented by Doyle et al. [9]. The benefit of using this decoupled approach is that solving the minimization problems posed in Eqs. (28)–(30) are generally easier than solving the original problem (27). The estimation problem according to (28) is identical to the

control relevant model reduction problem for linear systems formulated by Rivera and Morari [31], which can be solved effectively using prefiltered prediction-error estimation methods [32]. Furthermore, when $\tilde{p}$ is a block structured model, estimation problems for each of the nonlinear kernels $\tilde{p}_i$ $(i = 2, 3, \cdots)$ is only a single variable minimization problem, which contrasts the nonconvex multivariable nonlinear programming problem implied by Eq. (27).

6. Model validation

In the previous sections, a sequential procedure for restricted complexity modeling was presented. Correspondingly, a multiple step model validation procedure must be implemented to ensure that each step has been executed properly. Adequacy of the NARX model can be examined via classical validation methods. For example, the number of model terms included in the final NARX model can be determined by an information criterion such as Akaike's Information Criterion [1,28], combined with the cross validation. Residual analysis [3] can also be used in this step; however, residual analysis of nonlinear systems always requires some high order statistics. In order for the residual analysis to give meaningful conclusions, the measurement sequence must be of sufficient duration.

Computing the Volterra series model from the NARX model is performed in a completely deterministic setting. As noted in Section 4, the NARX model may exhibit dynamic behavior that cannot be adequately captured by a truncated Volterra model, so some prior judgement must be carried out when applying the method to general systems. If the problem under study conforms to the underlying assumptions for a finite Volterra series model (i.e. it is a fading memory nonlinear system over an operating range around a single steady-state) then the truncation error in the Volterra series model represents the only source of modeling error in this step. Clearly, the truncation error will decrease as the maximum lag J of the Volterra series model increases. For a given J, if the computed Volterra kernels $h(n_1, n_2, \cdots, n_i)$, for $i = 1, 2, \cdots, K$ vanish as $n_i \rightarrow J, \forall i$, it is an indication of sufficient J. If necessary, the computed Volterra series model can be further validated by crossvalidation against simulated data generated from the estimated NARX model.

The final step in model validation concerns the closed-loop performance obtained from the restricted complexity model. From a practical standpoint, closed-loop performance of the restricted complexity model can always be assessed through simulations at various operation conditions. Notice that at this stage a validated NARX model is already available which provides a reasonable approximation of the true plant; this model can be considered as the "truth model" in a simulation environment.

7. Guidelines for input perturbation design

In order for the restricted complexity model to capture important control-relevant system information, the estimated NARX model must correctly describe the important dynamics of the system under study. This requirement can be satisfied through (1) a carefully-designed identification experiment design which ensures that the process measurements are informative, and (2) validation of the NARX model. In this section, we establish some practical guidelines for the input perturbation signal design to insure the first requirement.

The input design problem for nonlinear system identification has been studied by many researchers ([8,14,17,26]. Generally speaking, the question how an input perturbation design is related to the model performance in nonlinear system identification is still not well understood. Yet based on information theory, Leontaritis and Billings [17] argue that for a completely unknown nonlinear system, the optimum input is a Gaussian white noise sequence when there is a power constraint on the input, or a uniformly distributed white noise sequence when there is an amplitude constraint on the input. For most process industry applications, the amplitude of an input sequence must always be restricted in order to limit the effects on normal plant operation and for safety considerations (i.e. "plant-friendliness"). Hence it is reasonable to select a uniformly distributed white noise sequence as the input signal. Ling and Rivera [19] showed that the switching time (the minimum time interval in which the signal switches from one level to another) may significantly influence the ability of a model to capture the process steady-state nonlinearity. Based on simulation results on a distillation column, it is suggested that a switching time not lower than the nominal process time constant be used. Based on these thoughts, we suggest the following input design guidelines:

i. Use a white noise input sequence uniformly distributed between $[a_l\ a_u]$, where the lower and upper limits (a_l and a_u, respectively) can be determined according to safety considerations and the operation condition under which the estimated controller is eventually to be used.
ii. Set the switching time of the white noise sequence to nT_s, where T_s is the sampling time and n is a positive integer, such that nT_s is approximately equal to the nominal process time constant around the nominal operation point. This leads to a random step input where the step amplitude is white noise uniformly distributed between $[a_l\ a_u]$.
iii. In some cases, using an input with certain probability distribution may improve parameter estimation (see Pearson and Doyle [26]). The required probability distribution can be obtained by adjusting the switching time randomly according

to a probability density function (PDF). Hence, if necessary, the switching time can also be selected as a random variable, e.g. uniformly distributed in $[(n-i)T_s\ (n+i)T_s]$ with $i < n$ being an integer.

8. A CSTR example

In this section, we apply the proposed method to a continuous stirred tank reactor (CSTR) control problem. A reversible exothermic reaction occurs in the reactor: $A \Longleftrightarrow R$. The control objective is to regulate the reactor inlet stream temperature such that the maximum concentration of product R is obtained in the outlet stream. The dynamics of the reactor are described by the following equations:

$$\begin{aligned}\dot{x}_1 &= 1 - x_1 - a_1 x_1 e^{-b_1/x_3} + a_2 x_2 e^{-b_2/x_3}\\ \dot{x}_2 &= -x_2 + a_1 x_1 e^{-b_1/x_3} - a_2 x_2 e^{-b_2/x_3}\\ \dot{x}_3 &= u - x_3 - 0.05\left(a_1 e^{-b_1/x_3} x_1 - a_2 x_2 \mathrm{e}^{b_2/x_3}\right)\\ y &= x_2\end{aligned} \tag{31}$$

$$a_1 = 3\times 10^5,\ a_2 = 6\times 10^7,\ b_1 = 50.3271,\ b_2 = 75.4907$$

with nominal operation conditions given by: $x_{10} = 0.509262$, $x_{20} = 0.490738$, $x_{30} = 4.194537$, $u_0 = 4.17$.

A salient feature of the above reactor is that its steady-state gain may change sign as the operation condition changes (Fig. 10). It is generally difficult to obtain satisfactory performance using linear techniques. Nonlinear identification of the system has been investigated by several authors [14] but usually a NARMAX or Volterra series model is considered. In this study, by applying the proposed control-relevant approach, we approximate the reactor dynamics with a simple Wiener model.

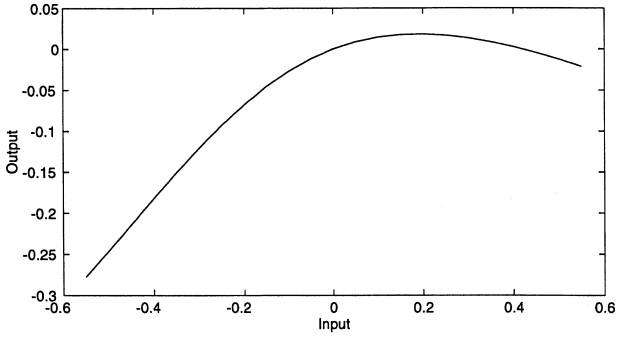


Fig. 10. Steady-state response of the continuous stirred tank reactor.

Using the input design guidelines proposed in Section 7, a random step sequence with a switching time of 2 min and a white noise step amplitude uniformly distributed between [−0.4 0.6] is generated (Fig. 11). The system is assumed to have a disturbance at the input side which consists of a zero-mean Gaussian white noise sequence with variance 0.004, colored by a first order linear system $H(z) - = 0.3z^{-1}/\left(1 - 0.7z^{-1}\right)$. In this setting, the signal/noise ratio of the measurements is about 28. With a sampling time of 0.5 min, 600 pairs of input/output data are collected for model parameter estimation and another \$200\$ data pairs are collected for cross model validation.

Using the modified Gram-Schmidt orthogonalization procedure, the estimated NARX model is obtained which contains 14 model terms (Table 1). The number of model terms included in the NARX model is determined according to the plot in Fig. 12. From the NARX model, a third order Volterra model is then generated. Cross validation results for these models are given in Fig. 13.

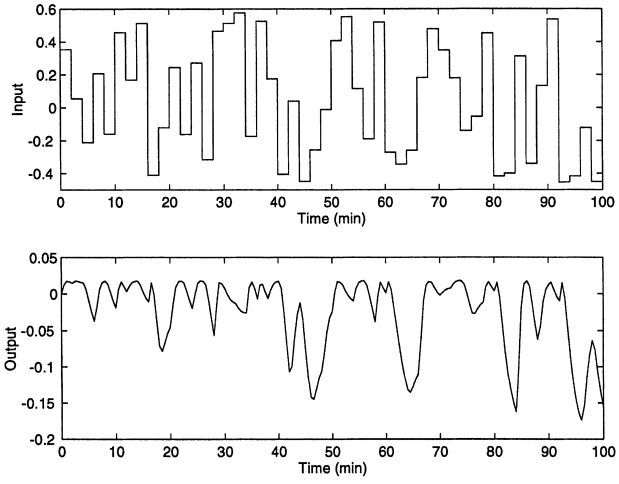


Fig. 11. The input/output measurement sequences for the CSTR example.

Table 1
NARX model parameters, CSTR example

Term	θ_i	Term	θ_i	Term	θ_i
$x(t-1)$	0.0431	$x_2(t-1)$	−0.0234	$x(t-3)y(t-2)$	0.3935
$x(t-2)$	−0.0036	$x^2(t-2)$	0.0247	$x^3(t-2)$	0.0930
$y(t-1)$	1.2159	$y^2(t-1)$	−1.0535	$x(t-1)x^2(t-2)$	−0.1273
$y(t-2)$	−0.5456	$x(t-)1 - x(t-2)$	−0.0723	$x(t-1)y^2(t-1)$	−1.5736
$y(t-3)$	0.0880	$x(t-1)y(t-1)$	−0.5094		

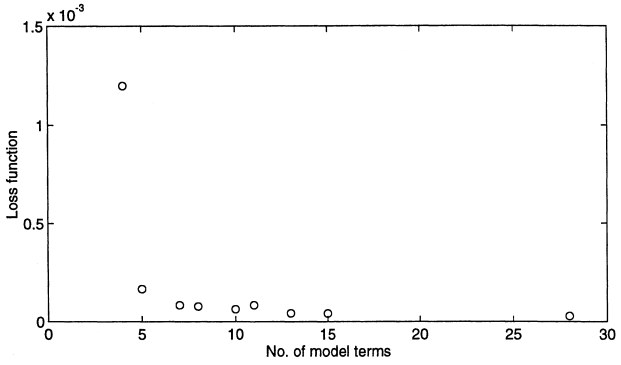


Fig. 12. Plot of loss function vs. number of NARX model terms.

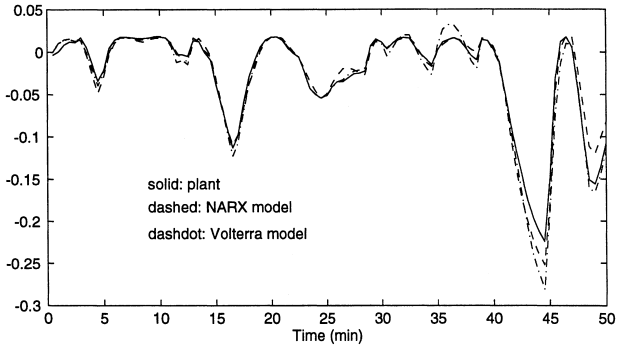


Fig. 13. Cross model validation results, CSTR example.

Our goal is to approximate the reactor dynamics with a simple Wiener model, whose linear dynamic element is a first order system

$$G(z) = \frac{b_1}{1 + a_1 z} \tag{32}$$

and memoryless nonlinear element has a form

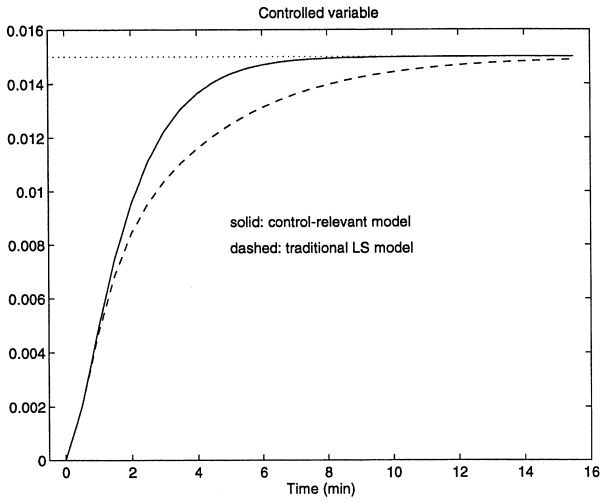
$$f[v(t)] = v(t) + \gamma_1 v^2(t) + \gamma_2 v_3(t) \tag{33}$$

The control objective is assumed to follow a step set point change according to

$$r(s) = \left(-0.2 + 0.215e^{-10s} - 0.015\mathrm{e}^{-20s}\right)/s \tag{34}$$

The control-relevant Wiener model assuming the following filter in the IMC design,

$$F(z) = \left(1 - e^{0.5/2.5}\right)z^{-1}/1 - \mathrm{e}^{-0.5/2.5}z^{-1} \tag{35}$$



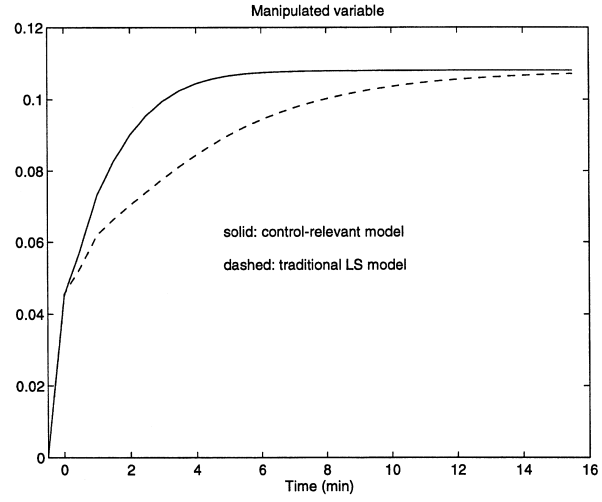


Fig. 14. CSTR example. Comparison of closed-loop responses between the control-relevant and the traditional least squares Wiener models: tracking of a 0.015 step set point change.

is given in Table 2. For the sake of comparison, a model estimated directly from the process measurements via the traditional least squares method is also given in Table 2.

Closed-loop responses generated from controllers designed from the two models are given in Figs. 14–16. Given a +0.015 step set point change, Fig. 14 shows that the control relevant model clearly gives a much smaller settling time. In Fig. 15, for a step set point in the negative direction (−0.2), a similar result is

Table 2
Wiener model parameters, CSTR example

	a_1	b_1	γ_1	γ_{22}	MSPE$\times 10^4$
Control-relevant model	−0.6211	0.0636	−17.6184	−38.7376	4.0212
Traditional LS model	−0.7208	0.0622	−13.8809	24.6015	1.6743

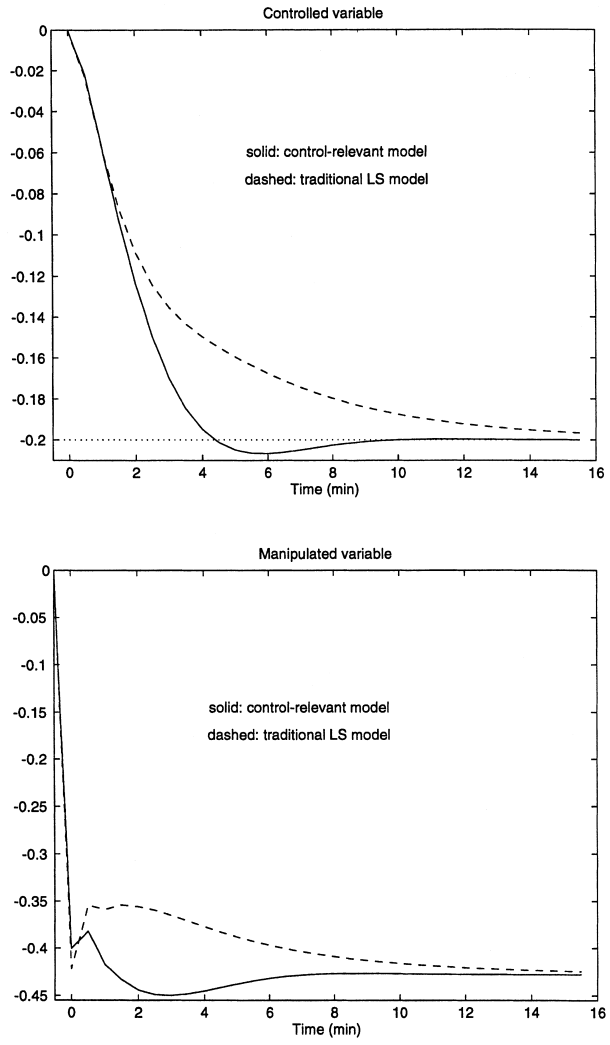


Fig. 15. CSTR Example. Comparison of closed-loop responses between the control-relevant and the traditional least squares Wiener models: tracking of a −0.2 step set point change.

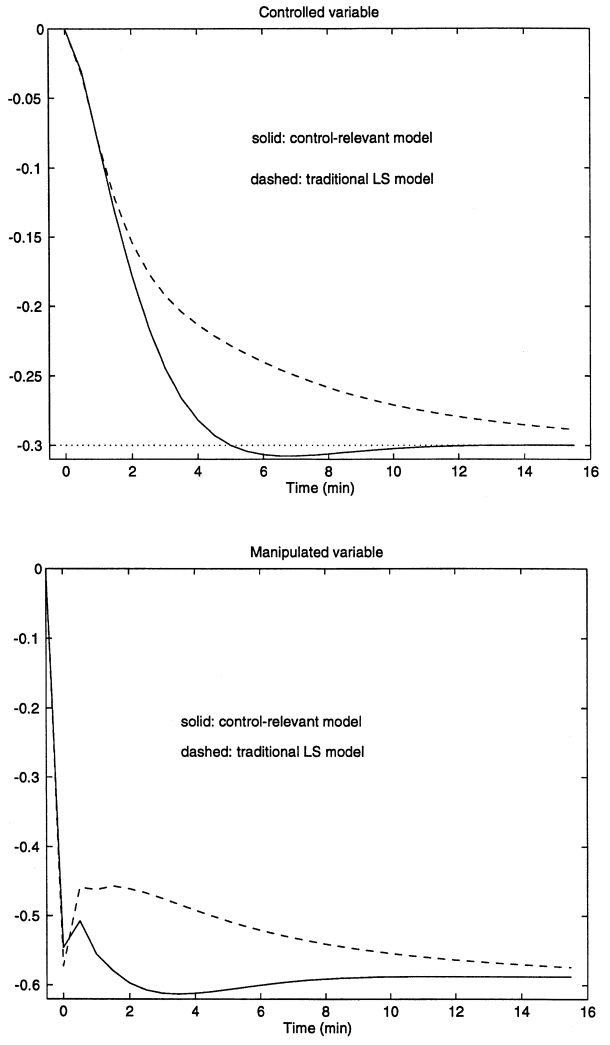


Fig. 16. CSTR example. Comparison of closed-loop responses between the control-relevant and the traditional least squares Wiener models: tracking of a −0.3 step set point change.

obtained. Further increasing the magnitude of setpoint change in the negative direction (50% higher than specified in the control-relevant estimation problem), the closed-loop performance difference between the two models becomes even larger, as evidenced in Fig. 16.

The open-loop mean square prediction error (MSPE) of the two models based on the cross validation data set is given in Table 2. Despite its superior closed-loop performance, the control-relevant model clearly displays the worse open loop response in terms of MSPE. This result shows another aspect of model control relevance: when a restricted complexity model is used in the identification procedure, an estimated model can represent the true system only to a certain extent. A model which gives a smaller open-loop MSPE does not necessarily lead to the better closed-loop performance. These results are consistent with well-known control-relevant modeling results for linear systems (Rivera and Gaikwad [33]).

9. A polymerization reactor example

The second application example we consider is a polymerization reactor (Doyle et al. 1995; [9]). Free-radical polymerization of methyl methacrylate occurs in the reactor with azo-bis-isobutyronitrile as initiator and toluene as solvent. The control objective is to regulate the product number average molecular weight (NAMW) by manipulating the flow rate of the initiator. Under certain assumptions, dynamics of the reactor can be described by following model equations:

$$\begin{aligned}
\dot{x}_1 &= 10(6 - x_1) - 2.4568 x_1 \sqrt{x_2} \\
\dot{x}_2 &= 80u - 10.1022 x_2 \\
\dot{x}_3 &= 0.002412 x_1 \sqrt{x_2} + 0.112191 x_2 - 10 x_3 \\
\dot{x}x_4 &= 245.978 x_1 \sqrt{x_2} - 10 x_4 \\
y &= x_4 / x_3
\end{aligned} \tag{36}$$

Steady-state operation conditions are given by: input $u_0 = 0.016783$ m^3 h^{-1}, output $y_0 = 25000.5$ NAMW and the corresponding state variables $x_{10} = 5.50677$, $x_{20} = 0.132906$, $x_{30} = 0.0019752$, $x_{40} = 49.3818$. The input and output variables are normalized using $v = (u - u_0)/u_0$ and $w = (y - y_0)/y_0$.

Input/output data are generated by exciting the system with white noise uniformly distributed between -0.5 to 0.5. Using a sampling time of 0.05 h, 1000 input/output data are collected, among which the first 800 data are used for parameter estimation and the other 200 data are used for cross validation. No artificial noise or disturbances are introduced to the measurements in this example. Since this is a noise-free case, we use the orthogonal least squares method directly to estimate the Volterra series model. Starting with a model structure $J = 15$ and $K = 4$ [cf. Eq. (1)], the modified Gram-Schmidt orthogonal least squares method picks up 387 model terms.

We approximate the system with a Hammerstein model whose zero-memory nonlinear element is a polynomial of degree 3

$$f(v) = v + r_2 v^2 + r_3 v^3 \tag{37}$$

and the dynamic linear element is a first order system

$$G(z) = \frac{bz^{-1}}{1 + az^{-1}}. \tag{38}$$

The control problem is assumed to track a step set point change represented in the Laplace domain as

$$r(s) = \left(-0.1 + 0.7e^{-5s} - 0.6\mathrm{e}^{-10s}\right)/s \tag{39}$$

Using the following filter in IMC design,

$$F(z) = \frac{(1-\delta)z^{-1}}{1 - \delta z^{-1}}, \qquad \delta = \mathrm{e}^{-0.05/0.3} \tag{40}$$

model parameters estimated via the control-relevant model reduction are given in Table 3. For the sake of comparison, a model directly estimated from the input/output data via the least squares method (Narendra and Gallman [23]) is also given in Table 3. Notice that for the Hammerstein model in Eq. (37) and (38), the resulting controller can be implemented as a *PI* controller followed by a simple variable transformation block f^{-1}. For the control-relevant case, PI controller parameters are given by: the gain $K_c = -0.194$ and the integration constant $\tau_I = 0.089$.

The model performance is evaluated by the closed-loop responses given in Fig. 17 and 18. Again, the superiority of the control-relevant model can be clearly observed. The open-loop mean square prediction error based on the cross validation data set is given in Table 3. As with the previous example, despite its superior closed-loop performance, the control relevant model displays the worse open-loop response in terms of MSPE.

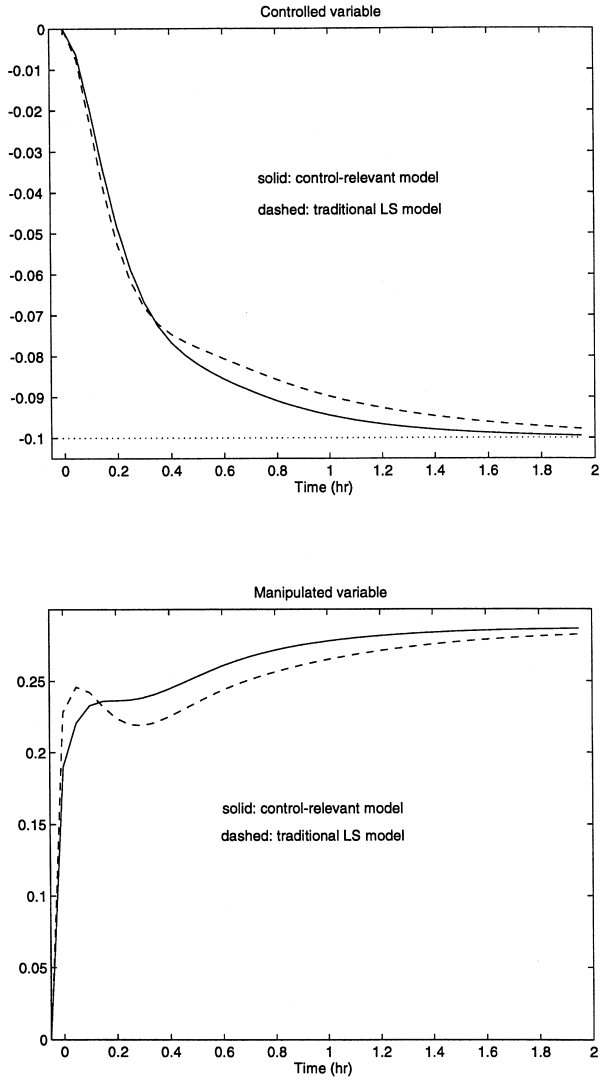


Fig. 17. Polymerization reactor: closed-loop response to a -0.1 step set point change.

Table 3
Hammerstein model parameters

Model	a	b	r_2	r_3	MSPE$\times 10^4$
Least squares model	−0.846	−0.068	−0.104	0.265	2.6238
Control relevant model	−0.793	−0.089	0.541	−0.240	−6.7522

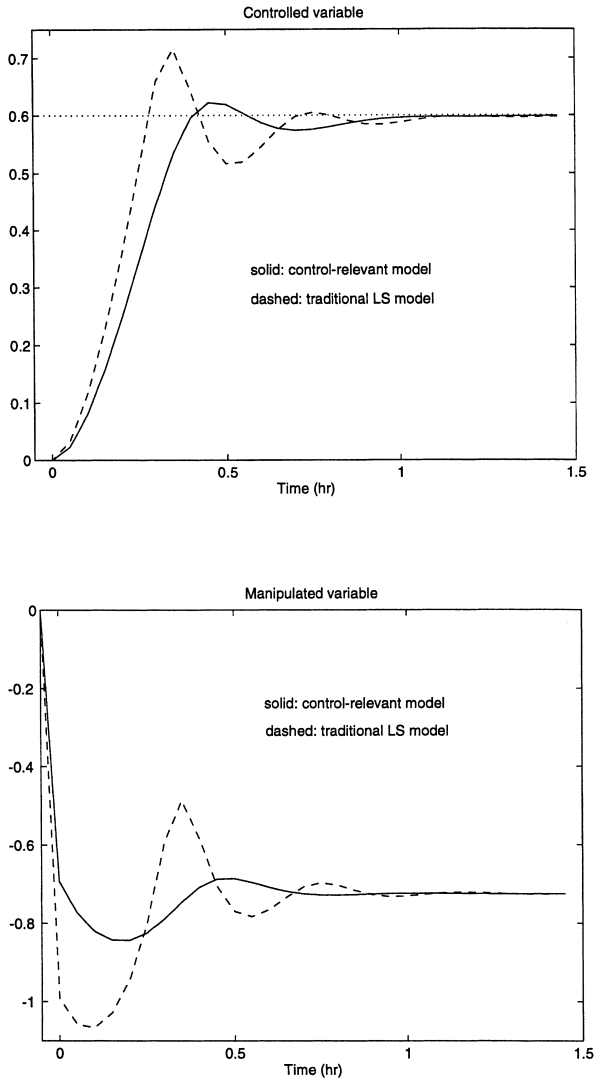


Fig. 18. Polymerization reactor: closed-loop responses to a +0.6 step set point change.

10. Summary and conclusions

Motivated by the need for simple nonlinear control strategies meaningful to process industries, a control-relevant nonlinear system identification method for restricted complexity models has been presented. The method parallels the widely used two-stage identification method for linear systems. In the first stage, a NARX and a Volterra series model are estimated from the process measurements. The desired restricted complexity model is then estimated through a control-relevant model reduction algorithm. A corresponding model validation procedure is implemented to ensure that each stage is executed properly. Some practical guidelines for identification experiment design are also provided. The effectiveness of the proposed method was confirmed on two simulated reactor examples.

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