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Master's Programme in Data Science

Differentially Private Markov Chain Monte Carlo

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2. Background

2.1 Differential Privacy

Differential privacy [2] is a property of an algorithm that quantifies the amount of information about private data an adversary can gain from the publication of the algorithm's output. The most commonly used definition uses two real numbers, ϵ and δ the quantify the information gain, or, from the perspective of the data subjects, the privacy loss of the algorithm.

The most common definition is called (ϵ, δ)-DP, approximate DP or ADP [2]. The case where $\delta = 0$ is commonly called ϵ-DP or pure DP.

Definition 1. *An algorithm $\mathcal{M} \colon \mathcal{X} \to \mathcal{U}$ is (ϵ, δ)-ADP if for all neighbouring inputs $x \in \mathcal{X}$ and $x' \in \mathcal{X}$ and all measurable sets $S \subset \mathcal{U}$*

$$P(\mathcal{M}(x) \in S) \leq e^{\epsilon} P(\mathcal{M}(x') \in S) + \delta$$

The neighbourhood relation in the definition is domain specific. With tabular data the most common definitions are the add/remove neighbourhood and substitute neighbourhood.

Definition 2. *Two tabular datasets are said to be add/remove neighbours if they are equal after adding or removing at most one row to or from one of them. The datasets are said to be in substitute neighbours if they are equal after changing at most one row in one of them.*

The neighbourhood relation is denoted by $\sim$. The definitions and theorems of this chapter are valid for all neighbourhood relations.

There many other definitions of differential privacy that are mostly used to compute (ϵ, δ)-bounds for ADP. This thesis uses two of them: Rényi-DP (RDP) [3] and zero-concentrated differential privacy (zCDP) [1]. Both are based on Rényi divergence [3], which is a particular way of measuring the difference between random variables.

Definition 3. *For random variables with density or probability mass functions P and*

Q the Rényi divergence of order $1 < \alpha < \infty$ is

$$D_\alpha(P \,||\, Q) = \frac{1}{\alpha - 1} \ln E_{x\sim Q}\left(\frac{P(x)}{Q(y)}\right)$$

Orders $\alpha = 1$ and $\alpha = \infty$ are defined by continuity:

$$D_1(P \,||\, Q) = \lim_{\alpha\to 1-} D_\alpha(P \,||\, Q)$$

$$D_\infty(P \,||\, Q) = \lim_{\alpha\to\infty} D_\alpha(P \,||\, Q)$$

Both Rényi-DP and zCDP can be expressed as bounds on the Rényi divergence between the outputs of an algorithm with neighbouring inputs:

Definition 4. *An algorithm $\mathcal{M}$ is (α, ϵ)-Rényi DP if for all $x \sim x'$*

$$D_\alpha(\mathcal{M}(x) \,||\, \mathcal{M}(x')) \leq \epsilon$$

$\mathcal{M}$ is ρ-zCDP if for all $\alpha > 1$ and all $x \sim x'$

$$D_\alpha(\mathcal{M}(x) \,||\, \mathcal{M}(x')) \leq \rho\alpha$$

There are many different DP algorithms that are commonly used, which are also called mechanisms [2]. This thesis only requires the most commonly used one: the Gaussian mechanism.

Definition 5. *The Gaussian mechanism with parameter σ^2 is an algorithm that, with input x, outputs a sample from $\mathcal{N}(x, \sigma^2)$, where $\mathcal{N}$ denotes the normal distribution.*

The most common use case for the Gaussian mechanism is first computing a function $f\colon \mathcal{X} \to \mathbb{R}$ for private data and feeding the result into the Gaussian mechanism to achieve DP. This leads into the definition of sensitivity of a function

Definition 6. *The l_p-sensitivity Δ_p, with neighbourhood relation $\sim$, of a function $f\colon \mathcal{X} \to \mathbb{R}^n$ is*

$$\Delta_p f = \sup_{x\sim x'} ||f(x) - f(x')||_p$$

2.2 Bayesian Inference and MCMC

3. Differentially Private MCMC

4. Conclusions

Bibliography

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