(x_2, y_2)

$\boxed{\text{overflow Check}}$

$x_0 y_0 \quad x_0 y_{in} \quad x_0 y_{mint}$

$x_{in} y_0 \quad\quad x_{in} y_{mint}$

$x_{mint} y_0 \quad x_{mint} y_{in} \quad x_{mint} y_{mint}$

$$(x_0 + x_{in} + x_{mint})(y_0 + y_{in} + y_{mint}) - x_{in} y_{in} \le MaxTotalSup^2$$

$$P = \frac{y_0 + y_{in} + y_{mint} + y_{in}}{x_0 + x_{in} + x_{mint} + x_{in}} \le 2\,MaxTotalSup$$

$$(x_{out}, y_{out}) \le 2\,MaxTotalSup^2$$

Mint

$M \times M^2 \times 4$

$4M^3 \le 2^{32}$

$M \le 2^{10}$

✓ $\quad 2 \quad \boxed{x_0 y_{mint}}$

$2 \quad\quad 2x_{in} y_{mint}$

$x_{mint} y_0 \quad 2x_{mint} y_{in} \quad 2x_{mint} y_{mint}$

$\le 2M^2$

$$y_{mint}(x_0 + 2x_{in} + x_{mint}) + x_{mint}(y_0 + 2y_{in} + y_{mint})$$

Article

- Intro
  - ↳ Necessity of Dark Pool
  - ↳ Necessity of "Decentralized" DP.
  - ↳ Other on-going Attempts
- How to Implement
  - ↳ How to Match?
    - ↳ LOB (C/P) (x : Complexity)
    - ↳ Volume Matching (x : need oracle)
    - ↳ AMM. (✓)
  - ↳ How to hide order info?
    - ↳ Batch. / Uniform Clearing
      - ↳ NE & PoA of Batching AMM. (*)
    - ↳ add noise
- Conclusion
- Further Improvements
- Appendix
  - ↳ Proof of (*)

Paper 2

Liquidity Taking Game in (FM)AMM.

↳ NE.

↳ PoA.

$$\frac{y_0 + 2N y_{in}}{x_0 + 2N x_{in}} = GM\left(P_{ext}, \frac{y_0 + 2(N-1) y_{in}}{x_0 + 2(N-1) x_{in}} \right)$$

Dutch Auction for closing batch.

userToPoolX $= \gamma_t x_0 + x_{in}$
userToPoolY $= \gamma_t y_0 + y_{in}$

poolToUserX $= x_{out}$
poolToUserY $= y_{out}$

---

if userToPoolX $\geq$ poolToUserX :
    transferFrom
else :
    transfer.

(same for y)

(Cont'd, Dutch Auction Batch Closing Fee)

$$ARB = -\gamma \cdot \left(\frac{L}{\sqrt{p}} \cdot p_{ext} + L\sqrt{p} \right)$$
$$+ (1+\gamma) \cdot \frac{L\sqrt{p}}{2} \left(\sqrt{\frac{p_{ext}}{p}} - 1 \right)^2$$

$$= -\gamma \cdot \frac{L\sqrt{p}}{2} \left(\frac{p_{ext}}{p} + 1 \right) \cdot 2$$
$$+ \frac{L\sqrt{p}}{2} \left(\sqrt{\frac{p_{ext}}{p}} - 1 \right)^2 + \gamma \cdot \frac{L\sqrt{p}}{2} \left(\sqrt{\frac{p_{ext}}{p}} - 1 \right)^2$$

$$= \frac{L\sqrt{p}}{2} \left(\sqrt{\frac{p_{ext}}{p}} - 1 \right)^2 - \gamma \cdot \frac{L\sqrt{p}}{2} \left(\sqrt{\frac{p_{ext}}{p}} + 1 \right)^2$$

$ARB > 0$

$$\Longleftrightarrow \gamma < \left(\frac{\sqrt{\frac{p_{ext}}{p}} - 1}{\sqrt{\frac{p_{ext}}{p}} + 1} \right)^2 \approx \frac{1}{4} \left(\sqrt{\frac{p_{ext}}{p}} - 1 \right)^2$$

$$\frac{p_{ext}}{p} \longleftarrow 1.01$$

$$\Rightarrow \gamma < \frac{1}{4} \times (1.005 - 1)^2 = \frac{1}{4} \times \frac{1}{200} \times \frac{1}{200}$$
$$= \frac{1}{16} \times \frac{1}{10^4}$$

$$\log_2(16000) \approx 17.3$$
$$\Rightarrow \gamma_N = 1 / 2^{N+16}$$

NE, PoA w/ fee rate r.

ARB

$$= -(1+r)(px+y) + (1-r)\left(p \cdot \frac{y}{p_c} + p_c x\right)$$

$$= -r\left(px + y + \frac{p}{p_c} y + p_c x\right)$$

$$+ \left(\frac{p}{p_c} y + p_c x - px - y\right)$$

$$= (y - p_c x)\left(\frac{p}{p_c} - 1\right) - r(y + p_c x)\left(1 + \frac{p}{p_c}\right)$$

$$\frac{Y+2y}{X+2x} = \frac{Y+2y'}{X+2x'} = p_c$$

WLOG $y' > y$. (i.e., $x' > x$).

$$\Rightarrow y' - y = p_c (x' - x).$$

$$(y' - p_c x')\left(\frac{p}{p_c} - 1\right) - r(y' + p_c x')\left(1 + \frac{p}{p_c}\right)$$

$$PnL' - PnL = -r\left((y'-y) + p_c(x'-x)\right) \cdot \left(1 + \frac{p}{p_c}\right) < 0.$$

$\Rightarrow$ optimal response is either $(x, 0)$ or $(0, y)$.

i) $P_{-i} \in \left(\frac{1-r}{1+r} P, \frac{1+r}{1-r} P \right)$

Player i does not participate.

ii) $P_{-i} < \frac{1-r}{1+r} \cdot P.$

$$P_f = \sqrt{\frac{1-r}{1+r} \cdot P \cdot P_{-i}} \quad \forall i \in [N].$$

$\Rightarrow \quad P_{-i} = P_{-j} \quad \forall i \neq j$

$\Leftrightarrow \quad y_i = y_j$

$$\Rightarrow \quad \frac{Y + 2N y_{eq}}{X} = \sqrt{P \cdot \frac{Y + 2(N-1) y_{eq}}{X}}$$

iii) $P_{-i} > \frac{1+r}{1-r} P.$

vice versa.

$$\frac{Y}{X + 2N x_{eq}} = \sqrt{P \cdot \frac{Y}{X + 2(N-1) x_{eq}}}$$

LVR estimation

i) $P > \frac{1+r}{1-r} \cdot \frac{Y}{X}$

We have

$$P_c = \frac{Y + 2Ny}{X} = \sqrt{P \cdot \frac{Y + 2(N-1)y}{X}}$$

Thus

$$LVR\Big|_{r=0} = N \cdot (y - P_c x) \left(\frac{P}{P_c} - 1 \right)$$

$$= \left(P_c \cdot \frac{X}{Y + 2(N-1)y} - 1 \right) Ny$$

$$= \left(\frac{Y + 2Ny}{Y + 2(N-1)y} - 1 \right) \times Ny$$

$$= \frac{2Ny^2}{Y + 2(N-1)y}$$

$$= pX \cdot \frac{2Ny^2}{(Y + 2Ny)^2}$$

$$= \frac{1}{2N} pX \cdot \frac{1}{\left(1 + \frac{Y}{2Ny}\right)^2} \leq \frac{1}{2N} \cdot pX$$

ii) $P < \frac{1-\gamma}{1+\gamma} \cdot \frac{Y}{X}$

We have

$$P_c = \frac{Y}{X+2Nx} = \sqrt{P \cdot \frac{Y}{X+2(N-1)x}}$$

Again,

$$LVR\Big|_{\gamma=0} = N \cdot ARB$$

$$= N \cdot (y - P_c x)\left(\frac{P}{P_c} - 1\right)$$

$$= pNx\left(\frac{P_c}{P} - 1\right)$$

$$= pNx \cdot \frac{2x}{X+2(N-1)x}$$

$$= \frac{P}{2N} \cdot 4N^2x^2 \cdot \left(\frac{Y}{X+2Nx}\right)^2 \cdot \frac{1}{PY}$$

$$= \frac{Y}{2N} \cdot \frac{4N^2x^2}{(X+2Nx)^2}$$

$$= \frac{Y}{2N} \cdot \frac{1}{\left(1 + \frac{X}{2Nx}\right)^2} \leq \frac{Y}{2N}$$

$$\Rightarrow LVR = O\left(\frac{1}{N}\right).$$

similar result for $\gamma > 0$.

$$p_c = \frac{Y+2Ny}{X} = \sqrt{\frac{1-r}{1+r}\, p \cdot \frac{Y+2(N-1)y}{X}}$$

ARB

$$= -(1+r)(px+y) + (1-r)\left(p \cdot \frac{y}{p_c} + p_c x\right)$$

$$= -(1+r)\,y + (1-r)\cdot \frac{p}{p_c}\cdot y$$

$$= y(1+r)\left(\frac{1-r}{1+r}\cdot\frac{p}{p_c} - 1\right)$$

$$= y(1+r)\left(\frac{X}{Y+2(N-1)y}\cdot p_c - 1\right)$$

$$= y(1+r)\cdot\frac{2y}{Y+2(N-1)y}$$

$$= 2y^2\cdot(1-r)p\cdot\frac{X}{(Y+2Ny)^2}$$

$$= \frac{(1-r)pX}{2N^2}\cdot\frac{4N^2y^2}{(Y+2Ny)^2}$$

$$= \frac{(1-r)pX}{2N^2}\cdot\frac{1}{\left(1+\frac{Y}{2Ny}\right)^2} \geq c \text{ (gas Fee)}$$

$$\frac{(1-r)\,pX}{c} \geq 2N^2 \cdot \left(1 + \frac{Y}{2Ny}\right)^2$$

$$= 2\left(N + \frac{Y}{2y}\right)^2$$

$$= \frac{1}{2y^2}\left(Y + 2Ny\right)^2$$

$$\frac{1-r}{c} \cdot p \cdot 2y^2 \geq \frac{(Y+2Ny)^2}{X}$$

$$= \frac{1-r}{1+r}\, p \cdot \left(Y + 2(N-1)y\right).$$

$$\Longleftrightarrow \begin{cases} (1+r) \cdot 2y^2 \geq c\left(Y + 2(N-1)y\right). & (1) \\ \dfrac{Y+2Ny}{X} = \sqrt{\dfrac{1-r}{1+r}\, p \cdot \dfrac{Y+2(N-1)y}{X}} & (2) \end{cases}$$

$$f_1(N) = y = f_2(N)$$

$$\Rightarrow \quad N = g(c) \qquad \& \quad LVR = g(C)$$

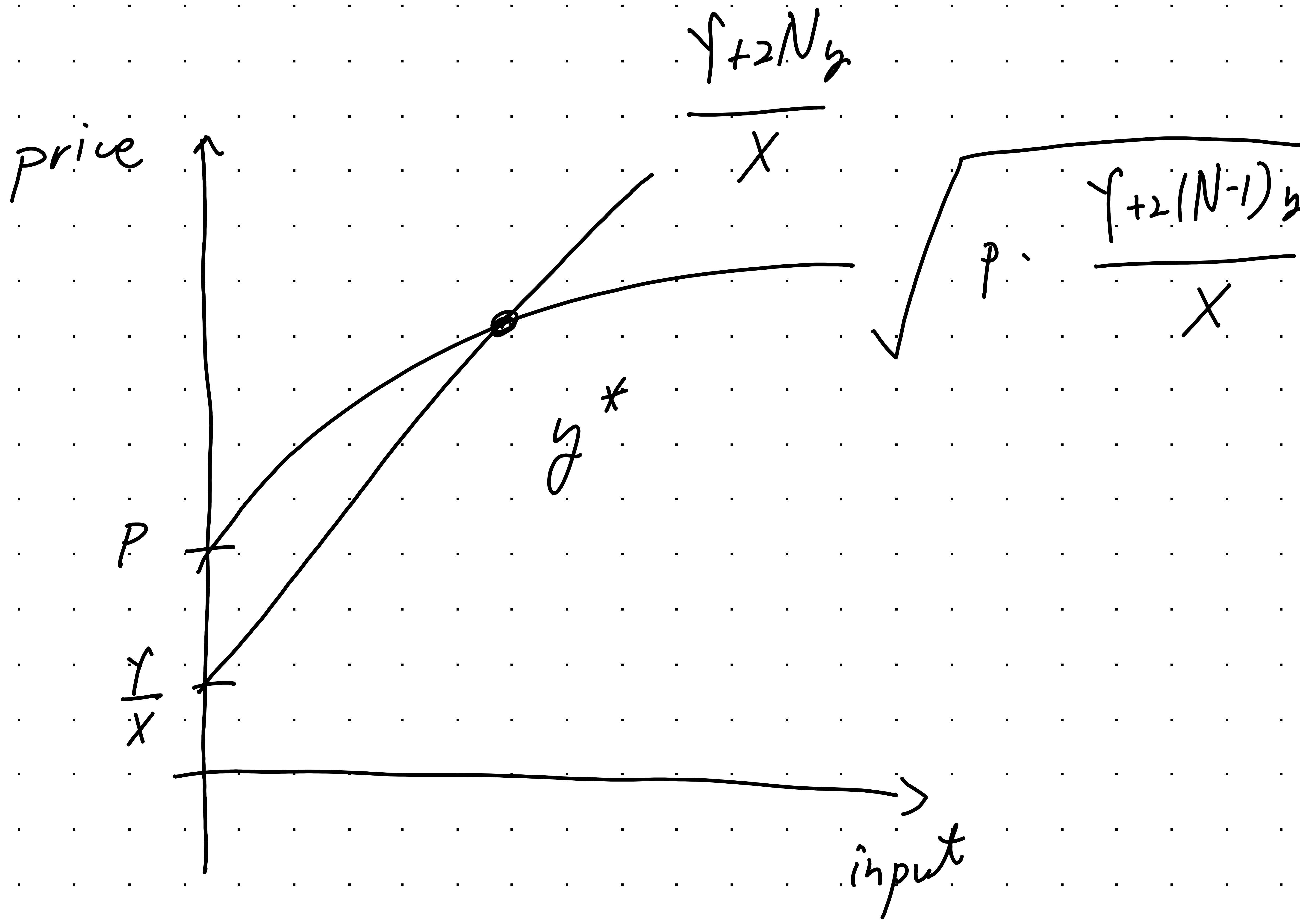
$$\frac{Y+2Ny}{X} = \sqrt{p \cdot \frac{Y+2(N-1)y}{X}}$$

i) $y \leftarrow 0.$

$$\frac{Y}{X} < \sqrt{p \cdot \frac{Y}{X}}$$

ii) $y \leftarrow \frac{1}{2N}\left(pX - Y\right) = \frac{X}{2N}\left(p - \frac{Y}{X}\right)$

$$p > \sqrt{p \cdot \frac{Y+2(N-1)y}{X}}$$



Structure of Article.

I. Introduction to FM-AMM.

II. Introdution to the Game.

lemma.

lemma.

Theorem (N is Exogenous)

Theorem (N is endogenous w.r.t cost c)

III. Stochastic Variant

simulation.

IV. References

i) FM-AMM Paper

ii) Cost of P/L LPing

iii) FBA

iv) Market fix itself?

v) LVR 2022

vi) LVR 2023

Account

address → address → euint

(owner) (token)

Pool

byte32 → PoolStruct

(PoolId) ↳

tokenX
tokenY

reserveX
reserveY

protocolX
protocolY
lpTotalSupply
lpBalanceOf

epoch

uint32 → batchStruct

Batch — order

- totalSwapX
- totalSwapY
- totalMintX
- totalMintY
- address (trader) → Order struct (order)
  - swapX
  - swapY
  - mintX
  - mintY
  - Claimed

result Summary

- initialReserveX
- initialReserveY
- FinalReserveX
- FinalReserveY
- mintedLPTokenAmount

$$\underbrace{\boxed{1}\boxed{1}\boxed{\quad 15 \quad}\boxed{\quad 15 \quad}}_{32}$$

$$\underbrace{111 \cdots 1}_{15}\ \underbrace{00 \cdots 0}_{15}$$