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2017
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Summary Sheet

Application of Multi-hop HF Radio Propagation Models in signal detection

The development of HF propagation using the ionosphere is likely to be an important part of radio communication technology. In this paper, a series of models are developed to study the propagation of multi-hop HF radio.

Firstly, we set up **the signal propagation model in the case of single hop**. Here, a geometric model of a typical single-hop process is built firstly. Based on it, we analyze radio transmission process. Then, we study the effects of different ocean surfaces on signal reflections. Next, the **transmission loss model** is established to analyze the various losses that may occur. Finally, build up the **signal intensity model**. With it, we can analyze the reflected electric field strength. For a 100 W HF constant carrier signal with a transmission frequency less than MUF, the field strength, after the calm sea surface reflection, is always higher than the field strength after the turbulent sea surface.

Secondly, we established **a multi-hop signal propagation model**. At the beginning, we analyze the relationship between the number of hops and the launch elevation angle. Then, the source of noise is analyzed as well, and we establish the **atmospheric noise prediction model**. Finally, we can calculate the signal to noise ratio and the maximum number of hops. As a result, when the launch angles are 20 °, 40 °, 60 °, the maximum number of hops are 1 times, 2 times and 2 times, one by one. Here, the frequency is fixed in 10MHz.

Thirdly, we build the **ground reflection loss model** to study the radio signal reflection in different terrain. The loss in the rugged mountains is greater than that in the flat mountains. When in the same roughness, the mountains surface lost more than that in the sea surface. Besides, we assume the transmission frequency is 10Hz, and the launch angle is 45 °. As a result, the calm sea surface reflects more times, 2 times, while the flat mountain surface reflects 1 times, as many as all rough surfaces.

Fourthly, we establish **signal reception model**, taking the turbulence on the ocean into account. In this part, we analyze the signal propagation process firstly, considering the mobile shipboard receiver. Next, we build the **MUF predictive model** to study MUF trends under different weather conditions. Then, we establish the **communication coverage model**. In this model, we measure the coverage ability of the HF signal, keeping the same transmission path. At last, the signal reception model of a vessel on a turbulent ocean is established, based on **Rayleigh distribution of internal noise prediction model**. This model is used to study the SRN and time of the receiver.

In order to simulate the real marine environment, we select the launch tower locates in 22.20N, 113.55E. Besides, the selected ship departs from Macao to Long Beach, which is about to encounter a turbulent current. Therefore, we simulate the change of MUF and signal coverage. Then, we simulate the 24-hour shipboard receiver's acceptance noise with **Monta Carlo**. Finally, we get the received SNR of the day. Comparing it with the lowest recoverable SNR, we know the vessel cannot receive the available signal from 14 to 22.

Last but not least, we make sensitivity analysis of the angle and frequency. Besides, we do error analysis on wave height.

Key Words: Signal Propagation Model, Atmospheric Noise Prediction Model, MUF Predictive Model, Rayleigh distribution, Monta Carlo simulation

Content

1. Introduction

1.1. Background

High-frequency (HF) radio waves are radio waves with a frequency of 3-30 Mhz. It can achieve long-distance signal transmission, through multiple reflections from the ionosphere and off the ground. When HF radio waves below the maximum available frequency (MUF), it also enables "multi-hop propagation". Since, in this case, the signal can go through multiple reflections. Once the signals have returned to Earth from the ionosphere, they are reflected back up by the Earth's surface. At the same time, it can again pass through another "reflection" of the ionosphere. High-frequency radio waves follow each successive jump.

Naturally, the intensity of the signal at each "reflection" decreases. It also found that different areas of the earth reflect different ways of radio signals. The nature of the reflective surface determines the intensity of the reflected wave, the same with the distance the signal eventually propagates, while maintaining the useful signal integrity. It is expected that the sea is a very good reflection surface, while the desert area is poor. In addition, the maximum available frequency, MUF, varies with season, current time and solar energy conditions. The frequencies above MUF are not reflected or refracted. It passes through the ionosphere into space.

1.2. Our Tasks

Since the ocean area occupies 70% of the earth's surface, reflections on the surface of the ocean are important, when considering radio-wave reflections. According to experience, reflections on a turbulent ocean are weaker than on a calm ocean. Ocean turbulence will affect the electromagnetic gradient of seawater, changing the local permittivity and permeability of seawater, while it also changes the height and angle of the reflecting surface. Rough seas are oceans, whose wave height, shape and frequency are rapidly changing. Besides, the direction of signal propagation may also change. Here are our tasks:

- Build ocean signal reflection model. Compare the strength of a constant carrier signal, the frequency is lower than the MUF, a HF power of 100 W, in two cases: the first reflection on the turbulent ocean surface and the first reflection on the calm ocean surface.
- Calculate the maximum number of hops for this signal, before the signal strength is 10 decibels below the available signal-to-noise (SNR) threshold. On condition that additional reflections occur on a calm ocean surface
- Compare the results of a reflection of task 1, with mountainous or rugged terrain, and flat terrain.
- Change the model to accommodate ship-borne receivers, moving on a turbulent ocean. Calculate the time, the ship can hold using the same multi-hop path.

2. General Assumptions

We make the following assumptions to complete our model through this paper. Further improvements of these simplified assumptions will be achieved later with more reliable data.

- **Neglect the velocity distribution of electrons and ions**. According to the electromagnetic ion theory, ignoring the velocity distribution of electrons and ions, the electron average drift velocity is used to describe.
- **Not consider the impact of natural disasters**.
- **Ignore tower height.**
- **Ignore the effects of path bending during electromagnetic wave propagation**. The degree of ionization is different, although these changes usually occur more slowly, but effective. In addition, there is also the movement of ionospheric winds or air. As the degree of ionization is not constant, any air movement will result in a change in the electron density distribution in the ionosphere. This in turn affects the path length.
- **Suppose the earth is a regular sphere**.

3. Notions and Symbol Description

We will define the following variables here as they are widely used throughout our paper. Additional variables may be defined later, but will be confined to a particular section.

SYMBOL	DEFINITION	SYMBOL	DEFINITION
Δ	Launching elevation	$\mathbf{c}$	The radiator between transceivers.
α	Incident angle, the angle of radio waves into the ionosphere	$H_{1/3}$	Effective wave height
ρ	Electron density in the ionosphere.	U	Wind speed
L_i	Ionospheric absorption loss	ρ_h	Average square wave height
f_H	Magnetic frequency, measuring from 1.2 to 1.5MHz	ρ_0	Fresnel reflection coefficient
I_j	Ionospheric absorption coefficient	λ	Carrier wave length
L_P	Propagation loss	ε	Sea surface dielectric constant
r	Sky-wave propagation distance	ε_r	Dielectric constant
D	Big circle distance	σ_e	Sea surface conductivity
P	Transmitter transmit power	ρ_s	Mirror scattering factor
L_o	Ocean reflection loss	τ	Rough surface roughness factor
Y	Extra system loss	β	The diffuse reflectance of a rough surface
E	Short wave communication field	ρ_d	Diffuse reflectance
E_n	The noise of the atmosphere	δ	Sea surface reflection coefficient
F_{am}	The median value of the atmospheric radio noise figure	F_a	Atmospheric radio effective noise figure

4. Analysis of Signal Propagation in Single Hop

Sky wave is the most important form of HF radio transmission, which leaves the transmitting antenna in a straight line and returns to the earth through ionosphere. When the HF radio frequency is less than MUF, multi-hop propagation process is shown in Figure 1.

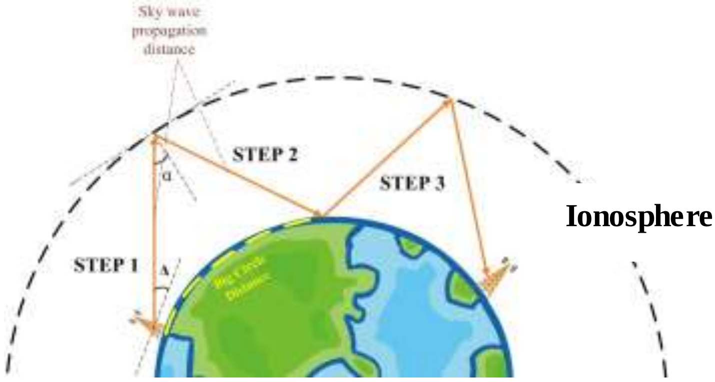


Figure 1 Relevant factors of costs and benefits

Here, we divide the process of communication into three stages, which specifically analyze the transmission of signal.

In **Step1**, signal sent from the transmitting end to the ionosphere, as shown in Figure 1.The incident angle alpha has a great influence on the communication distance. For a longer distance, a larger angle of incidence is used, and a smaller angle of incidence is used. However, if the incident angle is too small, the signal will pass through the ionosphere and do not have to fold back to the ground. If the incident angle is too large, the signal will be absorbed before it reaches the ionosphere. Therefore, it is very important to choose the range of incidence angle.

In **Step2**, signal sent from the ionosphere to the sea surface. The oblique distance is related to many factors, such as the mode of the radio wave transmission, the number of reflections, the incident angle, the distance of the great circle and the height of the ionosphere. According to the geometric relationship, we obtain the following equations.[1]

$$r = \frac{2R\sin\frac{D}{2R}}{\cos(\Delta + \frac{D}{2R})}$$

$$\Delta = 90 - (\alpha - \frac{D}{2R})\frac{180}{\pi}$$

$$\alpha = \arctan\left(\frac{\sin\frac{D}{2R}}{1 - \cos\frac{D}{2R} - \frac{h}{R}}\right)$$

Where, r refers to the oblique distance after a reflection from the ionosphere, while h denotes the reflectance point where the ionosphere height. Owing that HF radio waves are usually reflected in the E, F_1, F_2 ionosphere, h usually equals to 110km when reflection occurs at layer E. Besides, F_1 exists in the daytime, its height measures 170-220km.

D is the big circle distance, namely the distance, traveled by the sky wave, along the surface of the earth. From the cosine theorem of the spherical trigonometry, we can get the distance of the big circle between any two points on the earth.

$$D = 2\pi R \times c / 360^0$$

$$c = \arccos(\sin A_{lat} \sin B_{lat} + \cos A_{lat} \cos B_{lat} \cos A_{long} - B_{long})$$

Where, R is the Earth's average radius. We assume the Earth is a spherical, where R is fixed value equaling to 6370km. c is the radiator between transceivers. And $A_{lat}, A_{long}, B_{lat}, B_{long}$, represent latitude and longitude between transceivers respectively. Here, east longitude and north latitude are positive, west and south latitude are negative.

In **Step3**, the signal is reflected to the ionosphere. Under the same conditions, after the first two steps, the energy loss of the communication signal is consistent, as well as the intensity. However, when the signal strikes to the sea and then reflects, the signal energy will have a significant difference.

5. Establish Ocean Reflection Model

As mentioned above, when the signal is launched into the sea, the loss is complicated, due to the different roughness of the sea surface. We divide the model into three cases: completely still ocean surface, calm ocean surface and turbulent ocean surface. They correspond to smooth, relatively smooth and rough surface respectively. The sea surface reflection can be shown in Figure 2.

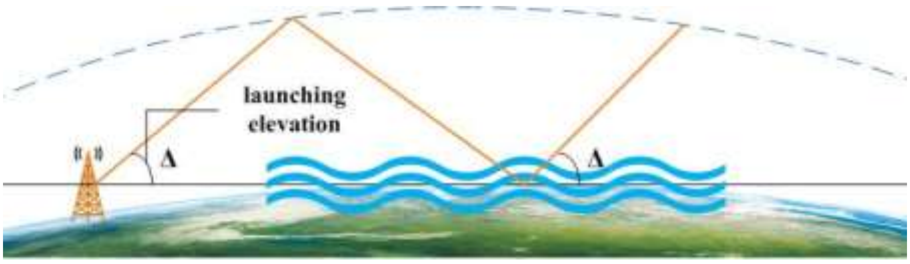


Figure 2 Sea surface reflection

Before we analyze the three situation, we first study the effect of wind speed on wave height. Different wind speeds are corresponded to different wave heights. For non-calm sea surface, irregular wave effective height is usually expressed as $H_{1/3}$. Related research shows that the effective wave height $H_{1/3}$ can reflect the wave growth.[2]We use $H_{1/3} = 0.0214U^2$ to describe the relationship between wind speed and effective wave height. Where, U denotes wind speed at sea, while average square wave height ρ_h is the mean square root of sea surface wave height. value of sea surface wave height. The relationship between them is shown as $H_{1/3} = 4\rho_h$.

Different ocean surface roughness affect the ocean reflection coefficient. In this way, we discuss three cases of complete still ocean surface, calm ocean surface and turbulent ocean surface.

5.1. Reflection on a still ocean

Assume that the sea is a perfectly smooth interface, which is an ideal situation. At this time, surface reflection coefficient ρ equals to Fresnel reflection coefficient ρ_0, namely, $\rho = \rho_0$. The formula of Fresnel reflection coefficient[3] is shown below.

Reflection coefficient in the case of vertically polarized waves, that is, the electric field vector is parallel to the ground.

$$\rho_0 = \begin{cases} \exp[-2(2\pi\tau)^2] & 0 < \tau < 0.1 \\ \dfrac{0.812537}{1 + 2(2\pi\tau)^2} & \tau \geq 0.1 \end{cases}$$

Reflection coefficient in case of horizontal polarized waves, that is, the electric field vector is perpendicular to the ground

$$\rho_0 = \begin{cases} \sqrt{2}|\rho_0|3.68\tau & 0 < \tau < 0.1 \\ \sqrt{2}|\rho_0|(0.454 - 0.858\tau) & 0.1 < \tau < 0.5 \\ \sqrt{2}|\rho_0|0.025 & \tau \geq 0.5 \end{cases}$$

Where, ε refers to sea surface dielectric constant, which is a function of carrier wave length λ, sea surface conductivity σ_e and dielectric constant ε_r. Here, $\varepsilon = \varepsilon_r - j60\sigma_e$.

Where, λ can be calculated by $c = \lambda f$. f refers to radio frequency, HF measures 3-30MHz. and c is the speed of light.

5.2. Reflection on a calm ocean

In the previous section, we calculate the reflection coefficient for a perfectly smooth interface with Fresnel formula. However, under the actual situation, it is impossible for the sea surface to fluctuate completely and smoothly. If the reflective surface has a certain degree of roughness, but relatively flat, in other words, it meets the Rayleigh criterion, the reflection coefficient can be expressed as:

$$\rho = \rho_0 \rho_s$$

Where, ρ_s refers to mirror scattering factor.

$$\rho_s = \begin{cases} \exp[-2(2\pi\tau)^2] & 0 \prec \tau \prec 0.1 \\ \dfrac{0.812537}{1+2(2\pi\tau)^2} & \tau \succ 0.1 \end{cases}$$

This formula shows that the roughness of the reflecting surface attenuates the amplitude of the specular reflection. [4]

τ is the rough surface roughness factor, described as $\tau = \rho_h \sin\Delta / \lambda$.

5.3. Reflection on a turbulent ocean surface

Rayleigh's criterion cannot be met when the surface of the ocean is rough. At this point, specular reflection decreases, while diffuse reflection increases with increasing roughness. The energy, reflected from the sea to the antenna, is mainly due to diffuse reflection. It occurs in a much larger area than the first Fresnel zone, regardless of its phase, with large amplitude fluctuations. The theory of geometrical optics does not satisfy diffuse reflection, which means that the incident angle is not equal to the reflection angle. We cannot establish the geometric relation with the direct wave. Thus, the diffuse reflectance of a rough surface β needs to be multiplied by a diffuse reflectance.

$$\beta=\rho_0\rho_d$$

ρ_d denotes diffuse reflectance. It is formed by the launch elevation angle Δ, square wave height ρ_h and the impact of electromagnetic wavelength λ. Through theoretical research, we get the diffuse reflection coefficient expressed as following. [5]

$$\rho_d=\begin{cases}\sqrt{2}\left|\rho_0\right|3.68\tau & 0\prec\tau\prec 0.1\\ \sqrt{2}\left|\rho_0\right|(0.454-0.858\tau) & 0.1\prec\tau\prec 0.5\\ \sqrt{2}\left|\rho_0\right|0.025 & \tau\geq 0.5\end{cases}$$

6. HF radio field intensity estimated in both cases

6.1. The total transmission loss model

The total transmission loss include ionospheric absorption loss L_i, propagation path loss L_p , ocean reflection loss L_o and extra system loss Y during multi-hop transmission. The loss of each part is described as follows.

1、 Ionospheric absorption loss

When signals are injected into the ionosphere, a certain amount of ionospheric absorption loss L_i occurs, due to absorption of a part of energy by the D and E layers. Ionospheric absorption loss can be classified as two forms, non-offset absorption and offset absorption. Since the offset absorption is generally less than 1 dB, we neglect it. Non-offset absorption calculation is very complex. In fact, it is often used semi-empirical formula:

$$L_i=\frac{667.2\sec\alpha}{(f+f_H)^{1.98}+10.2}\sum_{j=1}^{n}I_j$$

Where, f_H is magnetic frequency, measuring from 1.2 to 1.5MHz. Other variables are the same as above. I_j refers to the ionospheric absorption coefficient, obviously affected by the sun's motion . It is calculated according to the following general formula.

$$I_j=(1+0.0037\overline{R_{12}})(\cos 0.881\gamma_j)^{1.3}$$

Where, $\overline{R_{12}}$ represents the average of 12 months of sunspots, while γ_j is the sun zenith angle. It can be obtained by referring to the related literature of sky wave frequency prediction.

2、 Propagation path loss

Through step1 and step2, the signal finishes a complete jump. Propagation loss increases by the increase of dissemination distance. Therefore, propagation loss is also called propagation path loss L_P . The formula is shown below.

$$L_P = 32.42 + 40\lg f + 20\lg r$$

Where, f is the radio frequency, while r refers to sky-wave propagation distance, which is the distance, traveled from ground to the ionosphere, then reach ground again by reflecting. It is also known as the distance or oblique distance on the radio wave propagation path.[6]

3、 Oceanic surface loss

According to the sea surface reflection coefficient in different situations, we obtain the received power of the reflected wave L_0. We know that the ratio of the reflected power is equal to the square of the reflection coefficient. The received power of the reflected wave from the sea, relative to the direct wave, can be calculated.

$$L_o = 10\lg \delta^2 (dB)$$

Where, δ refers to the sea surface reflection coefficient. When specular reflection dominates, $\delta = \rho$. Otherwise, $\delta = \beta$.

4、 Extra system loss

Extra system loss is a general loss in addition to other losses of the three, such as ionospheric spherical focusing, multipath interference, polarization losses and other yet undefined losses. Y is an unstable parameter. It is related to geomagnetic latitude, season, local time, path length and other factors. Accurate calculation of its value is very difficult.

In mid-latitudes, additional system losses can be approximated as a function of local time. We can look at the ITU Report 533 to estimate additional losses.

5、 Total loss

Therefore, according to the formation of transmission loss, HF radio wave transmission loss can be expressed as:

$$L = L_{fr} + L_i + L_o + Y$$

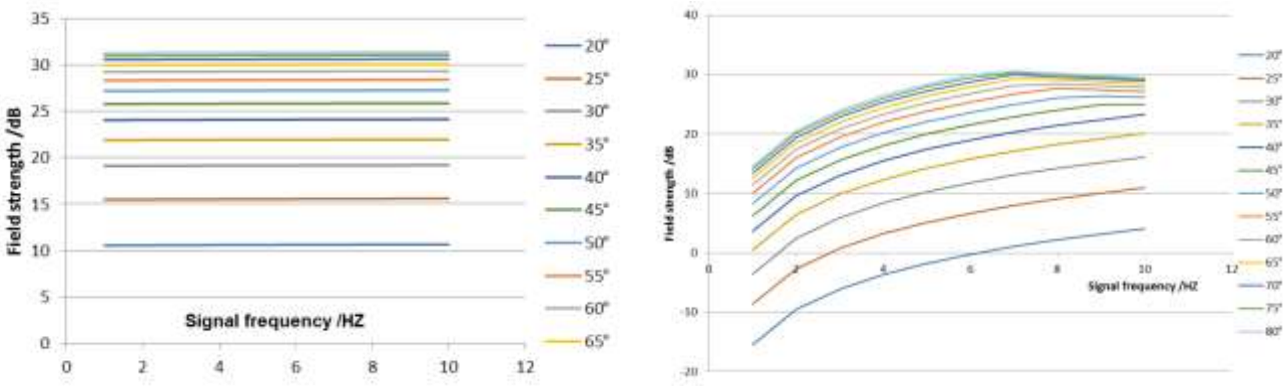
6.2. Calculate short-wave communication field strength

The calculation formula of short wave communication field E can be expressed as:

$$E = 137.2 + 20\lg f + 10\lg P - L$$

Where, P is the transmitter transmit power. The transmitter antenna radiation gain value is ignored. Here, we do not consider antenna gain. The final expression of shortwave communication field.

Here, we take a calm sea wind and turbulent sea for the 1m/s and 17m/s. $\epsilon_r = 80$, $\sigma_e = 5$. Draw the relationship between the frequency of the signal and the field intensity after the first reflection at different emission angles.



Calm sea surface Turbulent ocean surface

Figure 3 the relationship between frequency and the field intensity

It can be seen from the Figure 3, in a calm sea, the signal strength is greatly influenced by the emission angle rather than the frequency. It is decided by the emission angle. The greater the angle, the greater the field strength is. However, in the turbulent ocean surface, with the same frequency, the greater the angle, the greater the field strength is.

In general, no matter how the sea surface roughness, with the increase of the angle, its influence on the intensity gradually weakened. When the firing angle is less than 45 degrees, the impact angle of the field strength was greater than 45 degrees, smaller impact angle.

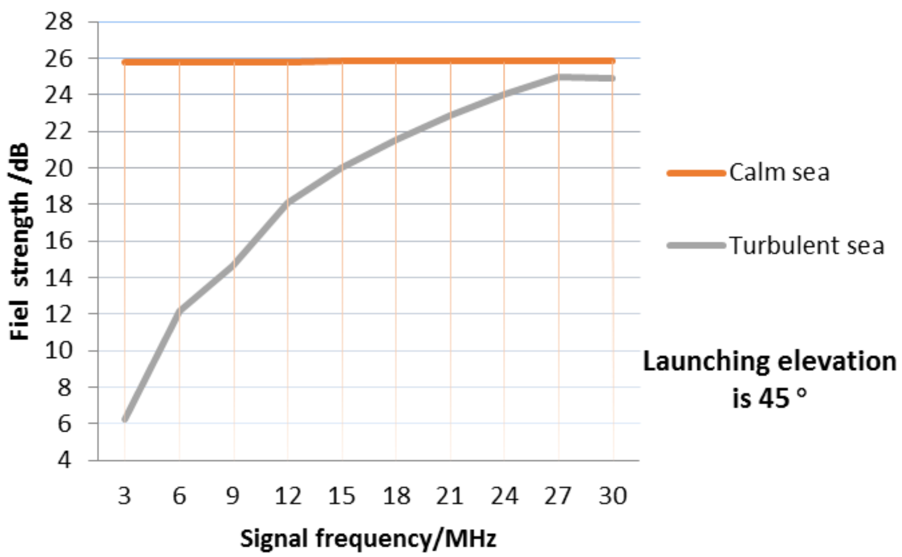


Figure 4 short-wave communication field strength in 45 degree

From the Figure 4, the same frequency signal, through the field after the reflection of calm sea surface, is always higher than the sea after rushing through the field. Signal strength after cutting through the calm sea less, and. At the same time, it almost not affected by their own frequency. However, the strength of rushing in low frequency is very small. With the increase of the signal frequency, the intensity gradually increased. The influence of signal frequency in low frequency is obvious.

7. How many times can it jump?

The signal can experience bouncing many times, before the signal strength drops to 10 decibels available SNR (SIGNAL-NOISE RATIO) threshold. Define the ionosphere reflex once as a jump. In this section, we will discuss the influence factors and determinants of the number of jumps.

7.1. Relationship hops and launch elevation angle

In the multi-hop propagation, the size of the launch angle Δ has an important impact on the number of hops.

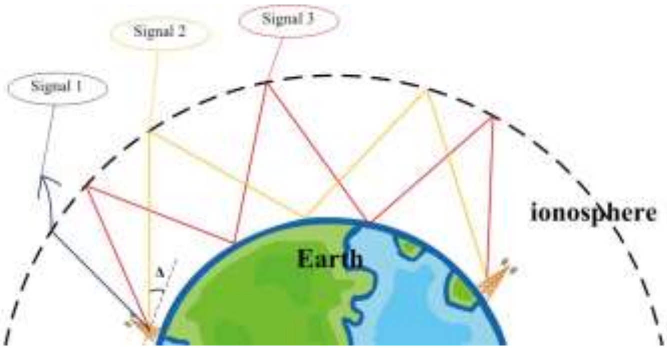


Figure 5 Relationship hops and launch elevation angle

In a certain range, the larger the angel, the more hops. As shown in the Figure 5, in order to reach the same distance, the second signal jumps 2 times and the third signal jumps 3 times. However, the launch angle cannot be too large. If the elevation angle of launch too large, the signal will pass through the ionosphere, without folding back to the ground. Conversely, if it is too small, the signal will be absorbed, before it reaches the higher ionosphere with a higher ionization density.

7.2. Atmospheric noise prediction model

Atmospheric noise intensity varies with frequency, time, season and geographical position. Based on empirical model, atmospheric radio noise field strength is calculated as follows:

$$E_n = F_a + 20\lg f - 96.8(dB\mu V/m)$$

Where, F_a is atmosphere radio effective noise figure, which can be found from the world radio noise profile given in CCIR-322[7][8].

Since the propagation of noise is affected by the ionosphere, its intensity varies with frequency, time, season, geographical location and climate. Therefore, we cannot obtain a fixed coefficient of atmospheric noise intensity. However, in order to quantify our next measurement process, here, we intend to select a location, fitting an accurate coefficient to represent the overall atmospheric noise intensity coefficient. In order to ensure the reliability of the communications band and facilitate the calculation, we see the more complicated weather in the southeast coast of China as the main analysis object. We select summer when atmospheric noise is strong. The curve fitting result is as follows. Here, we obtain the function expression of atmospheric noise field E_n and working frequency f.[9]

$$E_n = -0.0019 f^4 + 0.098 f^3 - 1.8 f^2 + 13 f - 13(dB)$$

7.3. Calculate SNR

The quality of HF reception depends not on the absolute value of the signal strength but on SNR. It is the difference between the strength of the electric field at the receiving point E and the noise. The noise is generated by the electronic environment and the physical and chemical environment. However, maritime HF communications are relatively less subject to industrial interference. Similarly, the noise figure of cosmic noise is usually lower. As a result, they are not considered here. Atmospheric noise E_n is therefore the main source of external noise for maritime HF communications. So, SNR can be expressed is as follows:

$$SNR = E - E_n (dB)$$

7.4. Calculate the number of hops

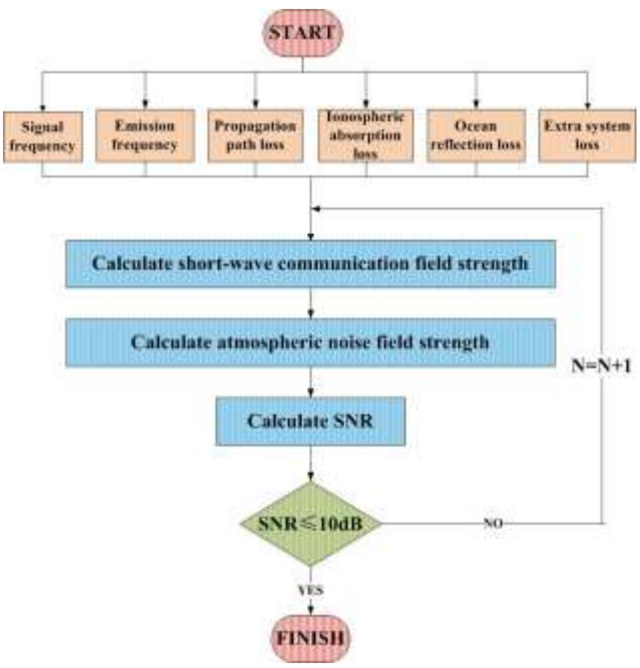


Figure 6 Calculate the number of hops

Step1: Calculate ionospheric absorption loss L_i, propagation path loss L_p and ocean reflection loss L_o, there is also extra system loss Y during multi-hop transmission.

Step2: Calculate the communication field strength in the receiving point.

Step3: Calculate the atmospheric noise interference field strength.

Step4: Calculate SNR.

Step5: Determine the relation between SNR and 10dB. If the SNR is greater than 10dB, then the number of hops add 1 (N = N +1), after that, return to Step 1. Otherwise, the end of the process to calculate the final number of hops. In the beginning, $N_0 = 0$.

7.5. Result Analysis

Table 1 Hops in different angles

Hops	Angle	20^o	40^o	60^o
1	E	36	49.52	54.8
	SNR	18.00	31.52	36.80
2	E	7.33	28.32	36.02
	SNR	-10.67	10.32	18.02
3	E		24.39	22.43
	SNR		6.39	4.43

By the Table 1, the frequency for 10MHz, when the angle is 20 °, signal that can reflect 2 times, while jump 1 time. In angle of 40 °, the signal reflects 3 times, namely jump 2 times. When the angle increases to 60 °, the signal reflects 5 times, in that, it jumps2 times.

As the Angle of launch increases, the number of reflections increases, which leads to the increase of the number of bounces.

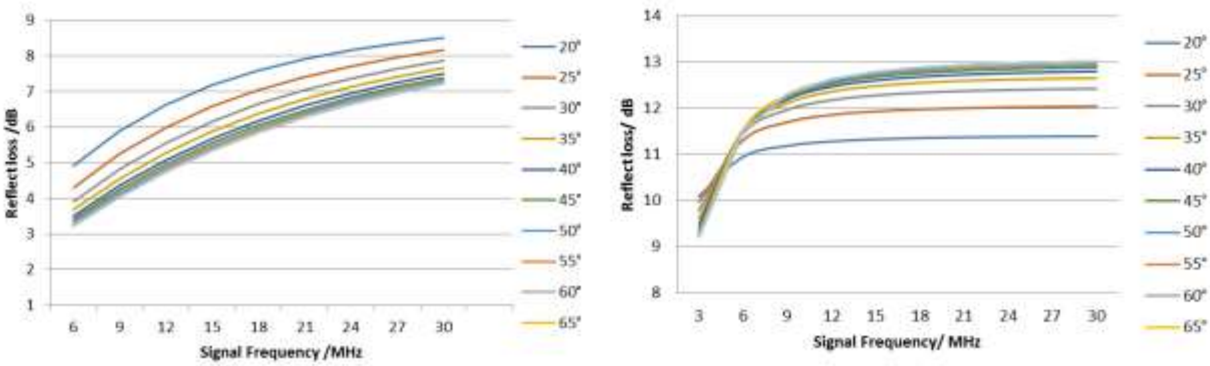
8. Establish ground reflection loss model

Ground reflection loss is similar to the ocean reflection loss. In the multi-hop space-wave propagation mode, when the reflection medium is ground, the propagation loss should not only consider the radio waves entry into the ionosphere twice, but also the loss of ground reflection. The formula for estimating this loss is as follows:

$$L_g = (N-1)\times 10\lg\left(\frac{|\rho_0|^2 + |\rho_0^{'}|^2}{2}\right)^2 (dB)$$

Where, $\varepsilon = \varepsilon_r - j60\lambda\sigma$, σ denotes surface conduction. Several classic forms of ε_r and σ can be obtained by referring to CCIR Rec.832

- **Comparison of strength**



Flat mountain surface Rugged mountain surface

Figure 7 The relationship between signal frequency and intensity (reflection loss) in rugged and flat terrain.

We firstly compare the relationship between signal frequency and intensity in mountainous conditions. As we can see in Figure 7. In the two cases, the first time the reflection loss will increase with the increase of signal frequency. However, the effects of angles on the frequency in flat and rugged mountain are completely different. With the increase of the angle, the first flat mountain reflection loss is reduced, and the reflection loss of rugged mountain areas increased.

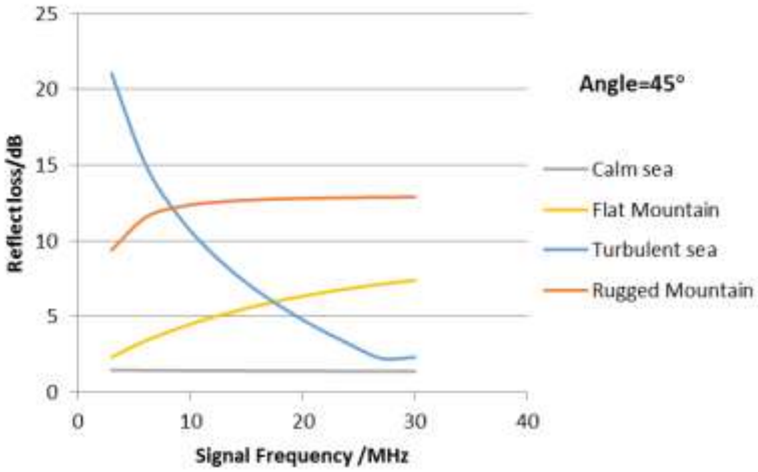


Figure 8 When both are rough and smooth surfaces, the relationship between signal frequency and intensity on land and sea.

As shown in figure 8. Different from the ocean surface smooth, the first reflection loss of the flat mountain surface increases, with the signal frequency increasing. When the two surface are all roughness, the increase of signal frequency will increase the loss. The same as the ocean, reflecting surface loss of rugged mountain is higher than that of the flat surface.

- **Comparison of hop count (number of reflection)**

Table 2 Sea and land in the rough and smooth, the reflection times and hop (10MHz, 45 °)

	1	2	3
Calm sea surface.	25.80166576	12.92661914	8.991543599
Flat mountain surface	22.74308949	9.868042865	-
Rough sea surface	18.54274449	5.667697868	-
Rugged mountain surface	14.87468547	1.999638845	-

From Table 2, we know that when the emission frequency is fixed at 10Hz and the emission angle is 45° , the number of calm sea surface reflection is highest, for the 2 time. Flat mountain surface and all rough surface reflection times are 1 times. The same surface roughness, mountain loss is more than sea reflection loss. Similarly, rough surface also costs the mountain reflection a lot.

9. How long can it communicate in turbulence?

If we want to know how long the ship is communicating in the turbulent ocean, we first need to understand the communication process in the ocean. The process can be divided into three stages.

In stage one, after the signal shots from the transmitter, in the actual situation, the communication frequency should be as close as possible to MUF, for obtaining the best communication effect. Therefore, we establish the MUF prediction model and select the optimal working frequency.

In the second stage, the signal is transmitted from the transmitter to the ship. We've been studying this process before. Next, we use the environmental signal-to-noise ratio as a medium to draw a ring of coverage maps of the oceans.

In the third stage, the signal arrives at the receiver (ship). However, there is also internal noise inside the receiver, which affects the signal-to-noise ratio. Therefore, we use Monte Carlo method to simulate the internal noise. Then determine whether the ship can receive the signal or not. The entire process is shown in Figure 9.

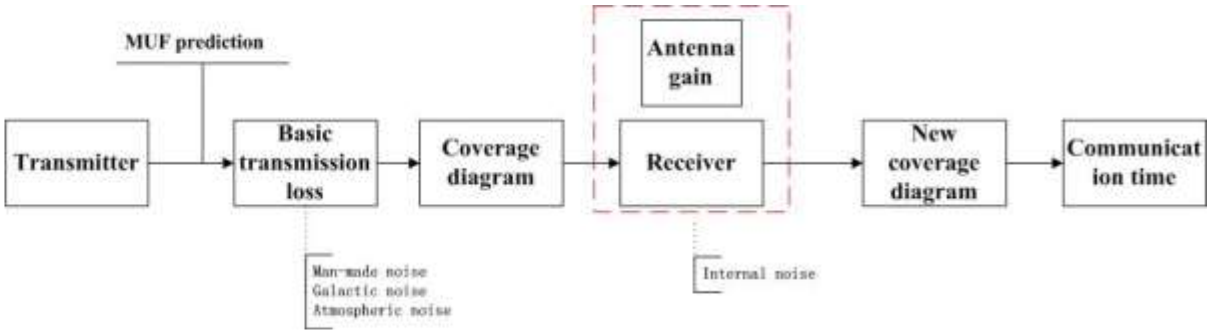


Figure 9 Communication process

9.1. MUF Prediction Model

MUF is related to the electron density of ionosphere and the incidence angle of electromagnetic waves entering the atmosphere. The greater the electron density, the greater the MUF value is. When the electron density changes with time (day, night, season, year) and geographical location, MUF changes with these factors as well. In actual communication, the frequency of communication should be as close to MUF as possible, due to obtain the best communication effect.

MUF expression has been given in the relevant research.[10]

$$MUF = A_2(SSN).A_3(mounth).A_4(time).M.f_0F_2$$

Where, M denotes inclination, which reflects the dependence of MUF on the length of transmission path. f_0F_2 refers to the critical or penetrating frequency of the electromagnetic wave to the F_2. A_2, A_3, A_4 respectively represents the dependence of M on sunspots, seasons and time.

9.2. Internal noise prediction model based on Rayleigh distribution

We know that signal-to-noise ratio is an important performance metric for radar receivers. However, when the signal arrives at the receiver, the SNR of the received signal will be lower. Due to the rapid fluctuation of internal noise, it is statistically independent [11]. Therefore, simulating the generation of internal noise is easier than simulating the generation of rain, snow and sea clutter. We will use Monte Carlo method to simulate the generation of internal noise[12].

$$P(E) = \int_0^{2\varphi} \frac{E}{2\pi\sigma^2} \exp[-E^2 / 2\sigma^2] d\varphi = \frac{E}{\sigma^2} \exp[-E^2 / 2\sigma^2]$$

The formula[13] illustrates the distribution of noise amplitude. Among them, E is a random variable that can be used as the instantaneous amplitude of noise. σ^2 is RMS, as noise intensity. Here, we take $\sigma^2 = 2$.

9.3. A signal reception model for a ship in a turbulent ocean

Based on the signal transmission process we can find the signal power at the receiving point P_R. The signal is transmitted to generate transmit power P_T. Signal propagation in the air must exist in the path loss L_P. There is a fixed receiving antenna gain when the receiving point receives the signal. Here we define it as a fixed value g.

$$P_R = P_T + g - L_P$$

In order to ensure effective communication, the received SNR is greater than the minimum SNR K required by the system itself.

$$E - E_n \geq 10 \lg K (dB)$$

It is widely accepted that SNR not less than 0dB.

10. Simulation of ship signal receiving on turbulent ocean

To get closer to the actual situation, we choose ships and transmitter in real circumstances. We choose a route travelling in the Pacific, the specific information shown in Figure10 and Table3. The chart shows that the voyage takes 13 days on the sea. It, February 13th, is the seventh day of the vessel departure. It is similar with general ships. At this time, the voyage is probably in the middle of the Pacific vessel position. Therefore, the research results of the vessel more applicable. In addition, the route is close to a straight line (as the route marker map shows). Thus, forecast future travel is easier for voyage and the follow-up study will be easy to do. When it comes to the transmitter, we choose a transmitter, the Pacific XX9 Macau, located in Macao China, mainly because of its location in the course of the vessel within the scope of our research needs. Since the multi hop path is unchanged, at this point, we assume that the tower launch angle remains unchanged.

Table 3 Basic information of the ship

Name	Route	Depart Time	Arrive Time
MSC SAVONA	FL804N	2018/02/07（WED）	2018/02/20（TUE）

Figure 10 The lane from America to Macao, China

The nature plays a vital role to the communication quality. Therefore, it is necessary to understand the weather conditions on the Pacific navigation process simulation. Figure11 is the 24 hour wave direction prediction that NOAA 's National Weather Service announced. The wave height is the average height of the highest 1/3 ($H_{1/3}$).

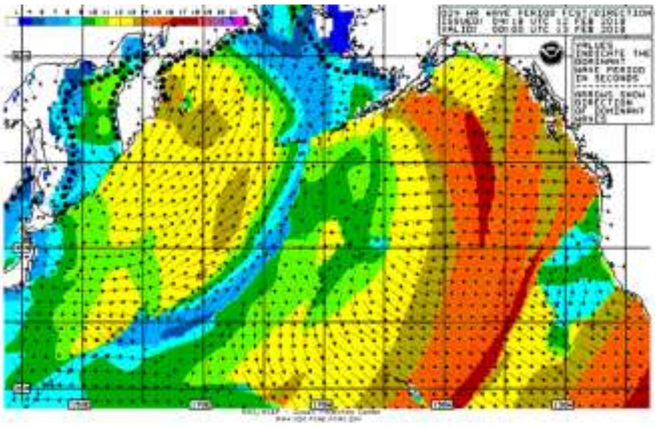


Figure 11 24 hour wave direction prediction

（http://www.opc.ncep.noaa.gov/shtml/P_024hrwper_mobilecolor.gif）

With the above conditions, we can simulate the ship communication on the ocean. Obviously, from the ocean forecast figure, we know, on February 13th, weather condition in Pacific is not good. Thus, the communication of ships is predicted more necessary.

10.1.Simulation of MUF Distribution

Since the short wave is a time-varying channel, the channel characteristic changes with frequency changing. Therefore, in HF communication, working frequency cannot be chosen arbitrarily. Otherwise, we cannot establish a reliable communication. In this case, we simulate the prediction of MUF. The hardware simulation environment: Intel (R) Core (TM) i5 -4210M, 2. 30GHz CPU, 8.0GB of memory. Software environment: Windows 10 operating system and Matlab R2017a simulation software

Before simulating, we set up a series of parameters, shown in Table 4.

Table 4 Simulation parameters of MUF

Time	**13/2/2018**
Groups	Month.Day=6.00 SSN=100 Qindex=0.000
Transmitter	22.20N 113.55E MACAU, China
Path	Short Distance: 6192km 3343nmi 3848mi Azimuth: 74.8deg
Freq(MHz)	6.078 7.200 9.700 11.850 13.700 15.350 17.725 21.650 25.885
System	Noise Min Angle Req.Rei. Req SNR Multi Tol Multi Del Absorp 145(-dBw) 0.10deg 90% 75dB 3.00dB 0.10msec Normal
Fprob	1.00*foE 1.00*foE1 1.00*foE2 1.00*foE3

Finally, the results of simulation prediction are shown in figure 12.

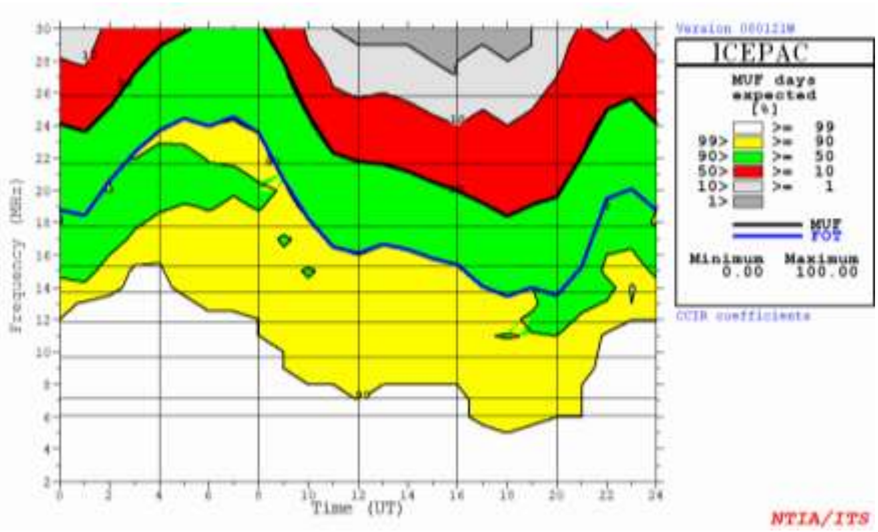


Figure 12 The results of simulation prediction

Figure12 reflects the different frequency sky wave communication, from the transmitting station, in 24 hours. Green represents good communications. Each session has a good performance of communication band. If the time the frequency can reach the ideal sky wave communication, for example, in 12, selecting the sky-wave frequency between 16-22MHz, we can maintain good communication quality in this period of time at sea.

10.2.Simulation of the Coverage Intensity

We adopt ITS HF Propagation wave propagation analysis software calculation. The simulation parameters are as follows.

Table 5 Short-wave communication link for long-term predictive simulation parameters

Time	**13/2/2018**
Period	24 hrs
Name	XX9 MACAU
Latitude	22.20N
Longitude	113.55E
TX antenna	Dipole@10M(33ft)

TX power	100W
TX mode	CW
Band	20M(14.1MHz)
Great-circle path	Long-path

Figure 13 is the image for the next 24 hours of maritime signal strength prediction. We get interception of a picture every four hours. The red part is the strongest signal area. Yellow, green, blue signals areas are very weak, poor communication.

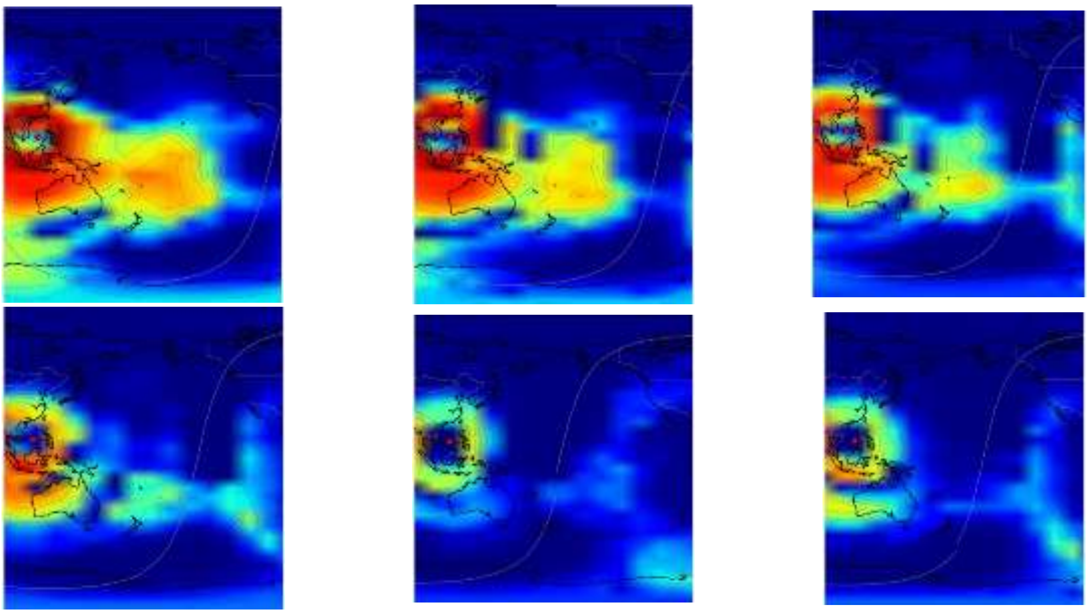


Figure 13 24 hours forecast result

Deep layer low pressure persisting west of the area near 31N144W continues to support widely scattered showers and isolated thunderstorms north of the convergence zone to around 26N and west of 128W. The pressure gradient between the low pressure and strong high pressure well north of the area is maintaining east to southeast moderate to fresh winds across this same area. It results in the poor communication time, which correspond with the distribution of the SNR of the signal received by the receiver in 24 hours.

10.3. Simulation of internal noise by Monte Carlo

In the course of navigation , the noise distribution and probability density histogram are shown below.

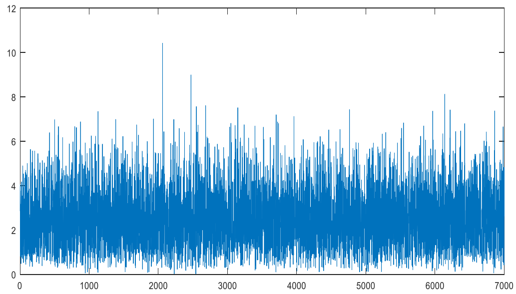


Figure 14 Noise distribution

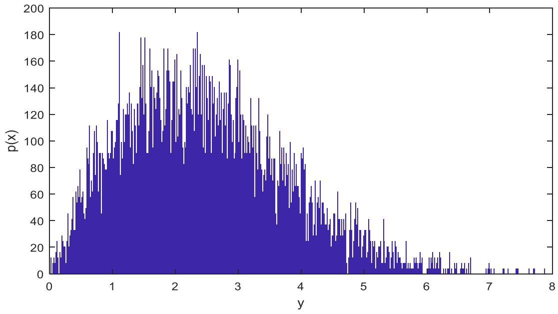


Figure 15 Noise probability density histogram

Noise is mainly distributed in the 2-4dB between, at some times the noise reached 11dB.

Most of the noise is between 0-6dB. The probability of noise falling at an intensity of 2 dB is the largest.

10.4. Signal reception of Shipborne receiver

In February 13th, the SNR of the signal received by the shipborne receiver is shown in Figure 16.

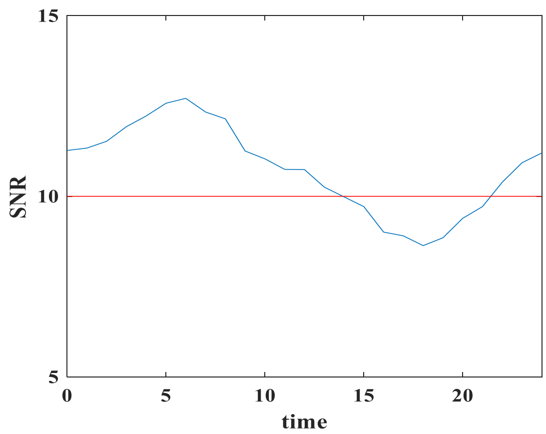


Figure 16 SNR of the signal received by the receiver in 24 hours

As a result of the weather, the SNR, between 14.92 and -22.44 hours, is below 10 dB and the tower signal cannot be received. From this, we can calculate the time when no signal from the tower was received

$$14.92 - 22.44 = 7.52 \text{ hours}$$

Fortunately, the ship can choose to receive signals from other towers. although it cannot receive the signal from the tower. Now the signal of towers basically covers all the oceans, to ensure the safety of sailing people

11. Error Analysis

According to the waves - wind speed formula, we calculate the root mean square wave height of the waves. Using **Matlab software** to draw the wave simulation image.

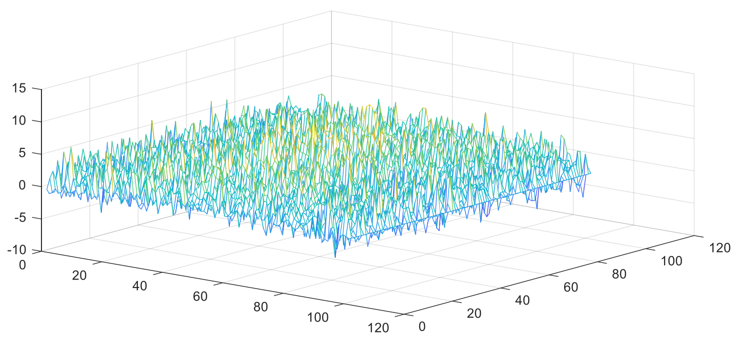


Figure 17 Wave simulation image Speed=13m/s

Regardless of the wind speed, the error between the simulated and calculated wave height does not exceed 5%, indicating that the model can better reflect the relationship between the sea wave and the wind speed.

Table 6 the relationship between the sea wave and the wind speed.

Speed	1	4	7	10	13	16	19
Calculated Value	0.00535	0.0856	0.26215	0.535	0.90415	1.3696	1.93135
Analog value	0.00555	0.0816	0.24945	0.5561	0.86645	1.4373	1.97485
Error rate	3.70%	4.60%	4.80%	3.90%	4.10%	4.90%	2.30%

12. Sensitivity Analysis

In this part, based on the actual data, we use 5 degrees as a step to change the angle, analysis of changes in sea surface reflection loss. Besides, we analyze the sensitivity of frequency in the certain angle.

12.1. Sensitivity of Angle

Table 7 Sensitivity of the angle (Calm ocean surface)

	30°	35°	40°	45°	50°	55°	60°
Reflection loss	1.416699	1.422959	1.427024	1.429676	1.431391	1.432476	1.433137
The rate of change	0.71%	0.45%	0.29%	0.19%	0.12%	0.08%	0.05%

Table 8 Sensitivity of the angle (Turbulent ocean surface)

	30°	35°	40°	45°	50°	55°	60°
Reflection loss	12.200308	10.98283	9.9770519	9.138090	8.435847	7.849294	7.36332
The rate of change	9.61%	7.80%	6.45%	5.38%	4.50%	3.76%	3.11%

We can see that the reflection loss is very sensitive to the angle. It proves that our model is suitable for various angles.

12.2. Sensitivity to Outburst Flood

Table 9 Sensitivity of the frequency (Calm ocean surface)

	9MHz	12MHz	15MHz	18MHz	21MHz	24MHz	27MHz
Reflection loss	1.439856	1.429676	1.4206768	1.412513	1.4049799	1.3979445	1.3913142
The rate of change	0.82%	0.69%	0.61%	0.56%	0.51%	0.48%	0.45%

Table 10 Sensitivity of the frequency (Turbulent ocean surface)

	9MHz	12MHz	15MHz	18MHz	21MHz	24MHz	27MHz
Reflection loss	11.59615	9.13809	7.235879	5.68489	4.376082	3.24437	2.2478
The rate of change	16.52%	11.69%	9.05%	7.38%	6.22%	5.38%	4.74%

We can see that the reflection loss is very sensitive to the angle. It proves that our model is suitable for various frequencies.

13. Strengths and Weaknesses

13.1. Strengths

- **We reasonably simplify complex scenes**. Whether in turbulent waves or steep mountains, their shapes are irregular and indefinable. If microscopic considerations apply to a wave signal striking different waves or slopes, the slope of the plane is different. It will lead to different angles after reflection. Therefore, when we choose the macroscopic angle, we can regard the steep mountains as the planes with different degrees of roughness in the turbulent waves, making the problem easier to analyze.
- **We use a variety of simulation models**. In order to get the result closer to the actual situation, we use the simulation method to get the required value.
- **Our models solve the problem in reality**. In order to better study the signal acceptance of vessels on turbulent ocean, we have selected certain flights and routes and made reasonable prediction on the signal reception of future flights, which is of practical significance.

13.2. Weaknesses

- **The empirical model can be inaccurate with reality**. Due to the fact that we do not understand, we use more empirical models in the loss model. This behavior will result in our final error.
- **We neglect the effects of extreme weather**. The problems we solve do not necessarily apply to extremely bad weather, such as hurricanes, tsunamis, etc.

14. Future Work

The effect of antenna gain cannot be ignored until the receiver receives the signal. In the previous model, we consider the antenna gain to be 5 dB. However, in a complex sea condition, more noise is generated, just because of the effect of greatly dampening the antenna gain, due to the violent rocking of the ship. Therefore, in the future work we will study the receiver antenna gain characteristics.

The defined gain P_0 is the ratio of the power $P(\alpha,\beta)$ received by the load when accepted by $G(\alpha,\beta)$ from this direction to the power accepted by the other ideal antenna from that direction.

$$G(\alpha,\beta) = \frac{P(\alpha,\beta)}{P_0}$$

Where, α, β are the signal and the horizontal direction of the angle and the angle of the antenna and the horizontal direction.

Received power can be obtained from two aspects. On the one hand, we use numerical statistical analysis to detect the signal received power, under the actual situation. Then, we can obtain empirical distribution of values. On the other hand, we can analyze the situation of ship sway and use the knowledge of physics to get the model.

In the future work, we will start from these two aspects.

15. Conclusion

In this paper, we build a series of models to study the propagation of multi-hop HF radio. Firstly, we set up the signal propagation model in the case of single hop. It is aimed to study the intensity of the signal, after a sea surface reflection in different situations. Secondly, we established a multi-hop signal propagation model. It can reflect the maximum hops in the calm sea signal. Thirdly, we build a model of ground reflection loss. The model researches the multi-hop high-frequency radio signal reflection in different terrain. Fourthly, we establish signal reception model, taking the turbulence on the ocean into account. In order to simulate the real marine environment, we chose real flights and ports for simulation experiments.

16. Synopsis of results

An important part of radio communication technology is the use of HF propagation in the ionosphere

In **Part1**, we have set up a signal propagation model, which is used to study the intensity and the maximum number of the signals after reflected in different conditions. For high frequency constant carrier signals, whose transmit power is lower than MUF and power P is 100W. The field strength, after calm sea surface reflection, is always higher than that experiences turbulent surface. After calm sea, the signal intensity reduces less. It is hardly affected by its own frequency. However, the turbulent sea surface is less intense at low frequencies. As the signal frequency increases, the intensity becomes larger.

When it comes to calculating the maximum number of calm sea, we assume the transmission frequency is fixed at 10Hz， and the launch angle is 45 °. The calm sea surface reflects more times, 2 times, while the flat mountain surface reflects 1 times, as many as all rough surfaces. On the surface with the same roughness, the mountain behaves worse than the sea. The mountain lost more than the ocean reflection loss. It is the same to the mountain surface.

In **Part2**, we establish the ground reflection loss model, studying the reflections of multi-hop HF radio signals under different terrains. First of all, we compare the signal frequency and intensity in a rugged and mountainous condition from the aspect of intensity. As seen in the figure below. In either case, the first reflection loss increases as the signal frequency increases. However, the impact of launch angle on the flat and rugged mountain is completely different. With the angle increasing, the first reflection loss in flat mountains decreases, while the reflection losses in the rugged mountains increase.

Unlike a smooth ocean surface, the first reflection loss on a flat land surface increases with increasing signal frequency. When both surfaces are rough, an increase in the signal frequency will result in an increase in loss. As with the ocean, the rough mountain surface has a higher loss of reflection than a flat mountain surface.

In **Part3**, we have established a signal reception model that considers vessels on turbulent oceans. It is used to calculate the time, when the receiver receives the signal with multiple paths unchanged. In order to simulate the real marine environment, we select the launch tower locates in 22.20N, 113.55E. Besides, the selected ship departs on February 7 from America to Macao, China. On February 13, the fish is located in the middle of the ocean. Namely, the coming bad whether makes it trapped in a turbulent current. Therefore, we use Matlab R 2011a to simulate the signal coverage. Then, we simulate the 24-hour shipboard receiver's acceptance noise with Monta Carlo. Finally, we get the SNR of the day the ship received it. Comparing it with the lowest recoverable SNR, we know the vessel cannot receive the available signal from 14 to 22.

In conclusion, our results will be summarized as follows：

- The field strength after calm sea surface reflection is always higher than that after the turbulent sea surface;
- Take the frequency as 10MHz. When the launch angles are 20 °, 40 °, 60 °, the maximum number of hops are 1 times, 2 times and 2 times, one by one.
- Loss in the mountains is greater than in the flatlands. On the surface with the same roughness, the mountains reflect more than the ocean.
- We assume the transmission frequency is fixed at 10Hz， and the launch angle is 45 °. The calm sea surface reflects more times, 3 times, while the flat mountain surface reflects 2 times, as many as all rough surfaces.
- The vessels, depart from the America to Macao, China on February 7, are unable to receive the signal on February 13 from 14-22.

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Appendices

PROGRAM

MATLAB code to calculate waves

```
function temp_Es = solve_1(f, delta)
%prob_1
%f = 10;
%delta = pi/4;
lambda = 3 * 100000000/(f * 1000000);
v = 1;
H13 = 0.0214 * v^2;
sigma_h =  H13 / 4;
R = 6370000;
h = 110000;
D = double(solve(['x/15460 + pi/2 + atan(sin(x/15460)/(1 - cos(x/15460) -
110/6370)) == ' num2str(delta)],'x'));
r = R*sin(D/(2*R))/cos(delta + D/(2*R));
Lbf = sum([32.44 20*log10(f) 20*log10(2*r)]);
disp(Lbf);
%get Li
I = (1 + 0.0037 * 90) * power(cos(0.881 * 5/180 * pi), 1.3);
i100 = asin(0.985 * cos(delta));
Li = I * 677.2 * sec(i100) / (power(10 + 1.4, 1.98) + 10.2);
disp(Li);

%get Lg
epsilon_c = 80 - 60 * lambda * 5 * i;
Gamma = sigma_h * sin(delta) / lambda;

if Gamma > 0 && Gamma < 0.1
    rho_s = sqrt(exp(-2 * power(2 * pi * Gamma, 2)));
elseif Gamma > 0.1
    rho_s = sqrt(0.812537 / (1 + 2 * power(2 * pi * Gamma, 2)));
end

rho_0v = abs((epsilon_c * sin(delta) - sqrt(epsilon_c - power(delta, 2))) /
(epsilon_c * sin(delta) + sqrt(epsilon_c - power(delta, 2))));
rho_0h = abs((sin(delta) - sqrt(epsilon_c - power(delta, 2))) / (sin(delta) +
sqrt(epsilon_c - power(delta, 2))));
```

```
rho_0 = sqrt(rho_0v.^2 + rho_0h.^2);

if Gamma > 0 && Gamma < 0.1
    rho_d = sqrt(2) * abs(rho_0) * 3.68 * Gamma;
elseif Gamma > 0.1 && Gamma < 0.5
    rho_d = sqrt(2) * abs(rho_0) * (0.454 - 0.858 * Gamma);
elseif Gamma >= 0.5
    rho_d = sqrt(2) * abs(rho_0) * 0.025;
end

Alpha = rho_0 * rho_s;
Beta = rho_0 * rho_d;

Lg1 = 10 * log10(power(Alpha, 1));
Lg2 = 10 * log10(power(Beta, 2));

disp(Lg1);
disp(Lg2);

%calculate E
E = 88.36;
temp_Es = [];
for index = 1:1:100
    if mod(index,2) == 1
        temp_E = E - 20 * log10(index * r) - (index + 1) / 2 * Li - (index - 1)
/ 2 * Lg1;
    else
        temp_E = E - 20 * log10(index * r) - index / 2 * Li - index / 2 * Lg1;
    end
    temp_Es(index) = temp_E;
    if temp_E < 10
        reflects = index;
        SNR = temp_E;
        disp(index);
        break;
    end
end

Lg = -Lg2 + 2;
E = 88.36 - 20 * log10(2 * r) - Li + Lg2 - 2;
```

```
End
```

MATLAB code to calculate land

```
function temp_Es = solve_2(f, delta)
%prob_2
%f = 10;
%delta = pi/4;

lambda = 3 * 100000000/(f * 1000000);
v = 17;
H13 = 0.0214 * v^2;
sigma_h =  H13 / 4;

%get_Lbf
R = 6370000;
h = 110000;
D = double(solve(['x/15460 + pi/2 + atan(sin(x/15460)/(1 - cos(x/15460) -
110/6370)) == ' num2str(delta)],'x'));
r = R*sin(D/(2*R))/cos(delta + D/(2*R));
Lbf = sum([32.44 20*log10(f) 20*log10(r)]);
disp(Lbf);

%get Li
I = (1 + 0.0037 * 90) * power(cos(0.881 * 5/180 * pi), 1.3);
i100 = asin(0.985 * cos(delta));
Li = I * 677.2 * sec(i100) / (power(10 + 1.4, 1.98) + 10.2);
disp(Li);

%get Lg
%epsilon depends on land type
epsilon_ = 10 - i*60*lambda*0.002;
Rv = (epsilon_ * sin(delta) - sqrt(epsilon_ - cos(delta).^2)) / (epsilon_ *
sin(delta) + sqrt(epsilon_ - cos(delta).^2));
Rh = (sin(delta) - sqrt(epsilon_ - cos(delta).^2)) / (sin(delta) + sqrt(epsilon_
- cos(delta).^2));
Lg = 10 * log(abs((Rh.^2 + Rv.^2) / 2));
disp(Lg);

%calculate E
E = 88.36;
temp_Es = [];
```

```
for index = 1:1:100
    if mod(index,2) == 1
        temp_E = E - 20 * log10(index * r) - (index + 1) / 2 * Li + (index - 1)
/ 2 * Lg;
    else
        temp_E = E - 20 * log10(index * r) - index / 2 * Li + index / 2 * Lg;
    end
    temp_Es(index) = temp_E;
    if temp_E < 10
        SNR = temp_E;
        reflects = index;
        disp(index);
        break;
    end
end

Lg = -Lg;
E = 88.36 - 20 * log10(2 * r) - Li - Lg;

end
```

MATLAB code to simulate wave

```
n=100;
w=(pi/30:(2*pi-pi/30)/199:2*pi);
an=randn(1,100);
rad=(-pi:2*pi/99:pi);
g=9.8;
%h=zeros(201,201);
h=zeros(101,101);
j=1;
k=1;
t=0;
%[x,y]=meshgrid(-10:10,-10:10)
x=(-50:50);
y=(-50:50);
%x=(-50:0.5:50);
%y=(-50:0.5:50);
for m=1:101
    for n=1:151
```

```
    %   for t=0:0.1:30
          for i=1:100
              e=2*pi*rand();
              a=(w(i)^2)/g;

h(j,k)=h(j,k)+an(i)*cos(a*x(j)*cos(rad(i))+a*y(k)*cos(rad(i)-w(i)*t+e));
          end
          k=k+1;
          if k==101
              k=1;
          end
    end
    j=j+1;
end
mesh(x,y,h);
%surf(x,y,h)
wave_x = x;
wave_y = y;
wave_h = h;
save('wave.mat');
```

MATLAB code to simulate noise

```
x=rand(1,7000);
y=sqrt(-8.*log(1-x));
M=500;
[N,Y]=hist(y,M);
N=N/1000/((max(y)-min(y))/M);
bar(Y,N*65);
ylabel('p(x)');
xlabel('y');
t = 1:1:7000;
figure;
plot(t,y);
```