Team Control Number

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2017 ICM Summary Sheet

Shared Mobility: The Research on Automatic – Manually Traffic Flow

Abstract

Recently, cooperating cars has been proposed as a solution to release congestion. However, the behavior of these cars is not well understood. Thus, we are aimed to construct several models to analyze the effects of automatic – manually traffic flow.

Firstly, we establish the **Throughput Model based on Bayesian Formula** to study the change of the traffic flow roughly. We analyze the cooperation pattern between self-driving vehicles and regular vehicles. Then, we establish the throughput model based on Bayesian Formula to study the change of the traffic flow. The outcome shows that there is a positive correlation between them.

Secondly, we propose **a lane change model for mixed traffic flow based on CA**. We build **a lateral flow model** to studied the lane change of a single traffic flow in the first place. Then we propose **a lane change model for mixed traffic flow based on CA** to study the different behavior patterns of regular vehicle and self-driving vehicle lane change. **The mixed traffic flow algorithm** is designed accordingly. The flow changes with the change of the density, different percentage of the self-driving performing different. Equalization exists when the density float from 0.4-0.7. Tipping points exist when the density is 0.2 and 0.8.

Thirdly, we build **a single-objective optimization model** to find conditions under which lanes should be dedicated to these cars. The single-objective optimization model is set up with the goal of congestion time. The outcome shows that it is necessary to set a self-driving lane, only one, which makes the be effect maximize.According to the above models, we make several suggestions to the authority of the state of Washington.

The sensitivity analysis of our model has pointed out that traffic flow is very sensitive to the ratio of the self-driving vehicles. It proves that the model we establish is suitable for different ratios of the self-driving vehicles.

Key words: cooperating cars, traffic flow Bayesian Formula, Cellular automaton, single-objective optimization model

Contents

1 Introduction

In order to indicate the origin of optimization problem, the following background is worth mentioning.

1.1 Problem Background

Large traffic congestion is an important civilizational and commercial problem, especially in urban, densely inhabited areas. It causes delays in travel time, stress of drivers, noise, problems in organizing public transport and detours, larger air pollution, fuel and energy consumption etc.

For the sake of increase capacity of highways, without increasing number of lanes or roads. An innovative solution to do it is introducing autonomous (self-driving) and connected vehicles, capable of driving without any actions of human and communicating with each other (V2V – vehicle-to-vehicle communication) and with the infrastructure (V2I – vehicle-to-infrastructure communication), in order to ensure traffic safety and smoothness. We call them self-driving, cooperating cars, shown as Figure 1.

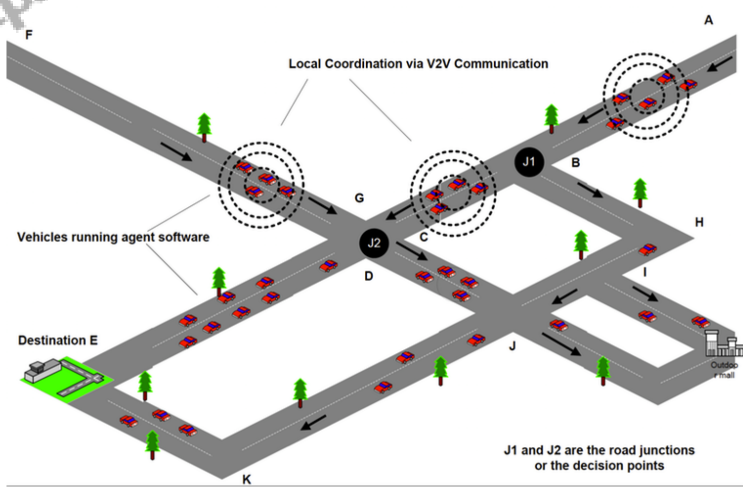


Figure 1. Self-driving, cooperating cars

https://theconversation.com/self-driving-cars-must-learn-trust-and-cooperation-79484

Though the self-driving, cooperating cars have been applied in several states in America, such as Nevada, California, we still not understand the behavior of these cars interaction well. The Governor of Washington state also desires to address the traffic congestion, particularly pronounced on Interstates 5, 90, and 405, as well as State Route 520, in this way. An analysis of the effects of allowing self-driving, cooperating cars on the roads is necessary.

1.2 Previous Research

In Automotive Engineering and Intelligent Transportation Systems, Connected Vehicle system is a multifaceted initiative targeted at improving passenger safety. In this system, vehicles are capable of communicating with other vehicles and with transportation infrastructures via wireless communications such as Dedicated Short Range Communications (DSRC). With this vehicle to vehicle (V2V) and vehicle to infrastructure (V2I) connectivity, information such as position, velocity, and acceleration of the other vehicles and traffic information can be made available to the communicating vehicles. [1] Apart from vehicle safety, this added information is useful in developing fuel economic control strategies, such as in Asadi and Vahidi (2011) [2]and HomChaudhuri et al. (2015), [3]and improvement of vehicle localization and mapping, such as in Parker and Valaee (2007) [4]and Adamey et al. (2013). [5]

A hierarchical model based fault diagnosis for electric power generation storage system is presented by Scacchioli et al. (2007) [6] while Pisu et al. (2006) [7] provided an adaptive threshold based diagnostics method for steer-by-wire systems. Zhang et al. (2009) [8] have shown a 'connected vehicle diagnostics' method for battery management systems. The connectivity presented Zhang et al.

1.3 Our tasks

For propose of analyzing the effects of allowing self-driving, cooperating cars on the roads, we need to propose a model of the effects on traffic flow. It should be able to solve the cooperation between self-driving cars, as well as the interaction between self-driving and non-self-driving vehicles. In order to achieve this goal, here, listing our tasks:

- Explore the relationship between the ratio of driverless cars and the traffic flow, when the proportion at the point of 10%, 50% and 90%.
- Find the equilibrium ratio of driverless cars, where the driverless cars perform best to ease the traffic. At the same time, seek out the critical point, where the performance changes significantly.
- Build a model to explore the conditions, when roads should be reserved for these driverless cars, if necessary.
- Propose some suggestions to the government owing to regulate driverless cars.
- Make analysis and assessment of our models, find the strength and weakness of them and make future predictions.

2 Symbols, Definitions and Assumptions

2.1 Symbols and Definitions

There are some symbols appear in the model. We show them below:

Table 1. Symbols and Definitions

SYMBOL	DEFINITION
a	the probability that a vehicle is self-driving or regular
A	self-driving car
M	regular car
H	headway
$\overline{\mathrm{H}}$	average headway of the mixed traffic
$\mathrm{P_t}$	throughput
i	segments
$f_{i,j,\bar{J}}(k)$	lateral flow, the traffic volume moving from lane j to lane $\bar{J}$
$r_{i,j}(k)$	on-ramp flow, the traffic volume entering from the on-ramp
$\gamma_{i,j}(k)$	off-ramp flow
j	lanes
T	time step
t=kT	simulation time
$\rho_{i,j}(k)$	density

2.2 General Assumption

In order to develop a mathematical model of traffic flow, we use an abstraction of the road network, the vehicles, the drivers and their behavior. Certain simplifying assumptions are therefore made:

- Road interference conditions, such as flatness, sufficient visibility, dry surface are consistent.
- With regard to drivers and vehicles, it is expected, among other things, that properties such as reaction time, willingness to engage in risks and technical proficiency follow an empirically proven statistical distribution.
- The properties of each car are identical with those of private cars.

Since the properties, such as length, footprint, of various of vehicles are difference, measuring the traffic flow while considering the length is complicated. We assume the all have the same properties to simplify the model.

- Assumed that drivers may consider a lane change when one of the adjacent lanes offers a higher speed or a lower traffic density.
- Ignore the terrain factor.
- The vehicles on the same lane not consider the lane change at the junction.
- Peak period lasts 2 hours, from 7:00 to 8:00 and 17:00 to18:00.
- The density remains unchanged, each road car distributed as evenly.

3 Throughput Model

3. 1 Cooperation pattern

Highway capacity significantly depends on the distance between cars. Since insufficient spacing usually causes rear-end collisions. Anyway, there is a minimum spacing which must be maintained during steady state traffic flow, if collision-free vehicle following must be guaranteed. In principle, the possibility of having a rear-end collision can be reduced by increasing the inter-vehicle spacing.

Thus, no matter the vehicle types are, when they have the same length, according to assumption 3, and the definite length, we can measure the traffic flow.

Due to the influence of subjective judgment, self-driving and regular cars have different spacing. We can recognize the length and spacing as a whole, shown in Figure 2.

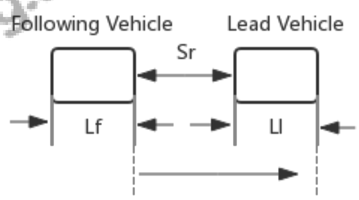


Figure 2. Vehicle Following[9]

Where we call spacing **Sr**, while the length of lead and following vehicle **Lf** and **Ll** separately. Thus, the length of a whole is the sum of **Sr** and **Ll**.

Therefore, the problem is converted into a problem of permutation and combination. Random sequencing of self-driving and regular vehicles in mixed traffic operations produce different combinations of pair of vehicles adjacent to each other. Two kinds of vehicles have four combinations, ***AA,AM,MA,MM***, (shown in Figure 3) where ***A*** refers to self-driving car and ***M*** represents regular car.

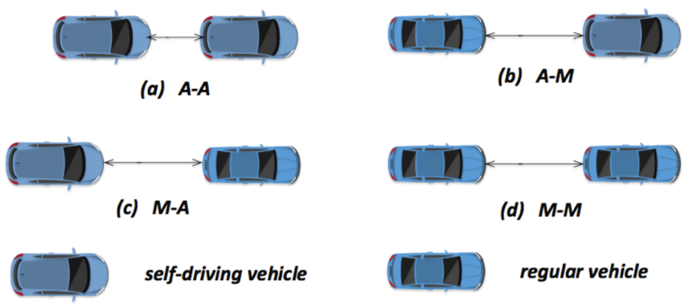


Figure 3. Four combinations

By attempting to communicate to the vehicle ahead, a self-driving vehicle will know whether the leader is self-driving or not. If the laggard is a self-driving vehicle, maintain the appropriate inter-vehicle spacing, between (b) and (c), since the self-driving vehicles have less time for reflect. If the laggard is a regular vehicle, keep the same distance no matter the leader is.

3.2 The Probability of Each Combination

When we call the radio of self-driving car **a**, the probability that a vehicle is self-driving or regular is given by $\boldsymbol{P(A) = a,\ P(M) = 1 - a.}$ Then, according to Bayes formula, the probability of these combinations can be represented as :

$$
\begin{gathered}
P(A|A) = a^2 \\
P(M|A) = a(1-a) \\
P(A|M) = (1-a)a \\
P(M|M) = (1-a)^2
\end{gathered}
$$

Where , $\boldsymbol{P(A|A)}$ refers to the probability that a self-driving vehicle is followed by a self-driving vehicle, while $\boldsymbol{P(M|A)}$ refers to the probability that a self-driving vehicle is followed by a regular vehicle and so on.

3.3 Define the Throughput

Since on the basis of assumption 3, the length of vehicle is assumed the same, we should pay more attention to the headway. We use $\boldsymbol{H}$ to represent headway, which represents the time required for the spacing.

Based on papers, [10] we know

$$\boldsymbol{H(A|A) = 0.31s,\ H(A|M) = 0.5s,\ H(M|A) = 1.8s,\ H(M|M) = 1.8s}$$

The average headway of the mixed traffic ($\overline{\mathbf{H}}$) can be measured by

$$\overline{H} = [P(A|A)H(A|A) + P(A|M)H(A|M) + P(M|A)H(M|A) + P(M|M)H(M|M)]$$

The final throughput($\mathbf{P_t}$) is calculated based on inter-vehicle data for the four possible outcomes. Thus, the throughput can be calculated as:

$$P_t = 3600/\overline{H}$$

3. 4 Solution and Result

When the lanes are covered, under the condition that the speed hasn't changed as usual. The basic model can be used to measure whether the radio of the self-driving cars will influence the traffic flow.

To solve the equations in our model, we use (the ODE45 numerical integrator) in MATLAB to find the results in Figure 4. From the figure, we know the throughput is curve relation with percentage of self-driving vehicles. It means there is a positive correlation between them, the increasing of the self-driving vehicles leading to the growth of traffic flow.

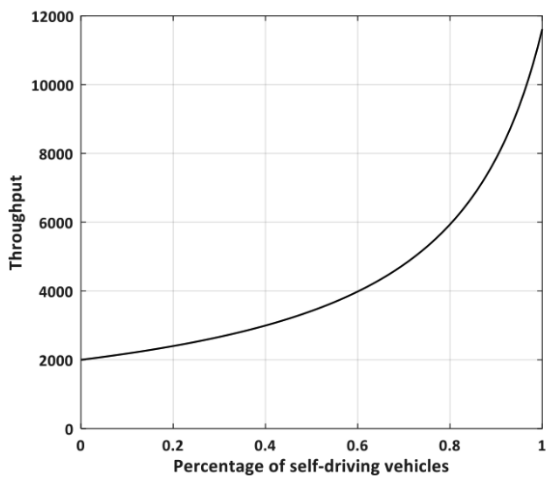


Figure 4. Results of the basic model

4 Lateral flow Model

In basic model, for the sake of simplicity, we ignore the lateral flow because the flow entering from an on-ramp has a priority. However, the lateral flow also has great influence on traffic actually. Here, we will consider it to improve the model.

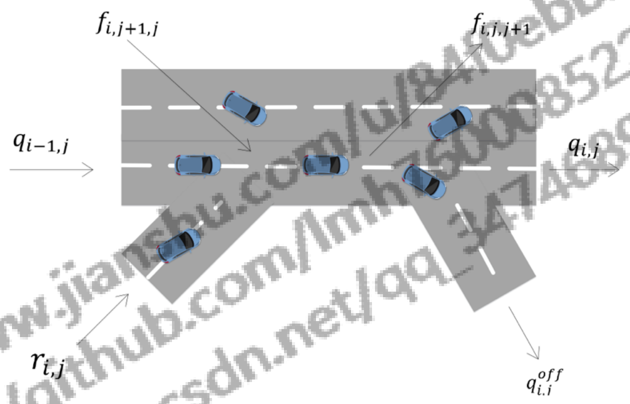


Figure 5. The segment-lane variables used in the model formulation

According to assumption one, assuming that drivers may consider a lane change when one of the adjacent lanes offers a higher speed or a lower traffic density, lanes always have lateral flow. Lateral flows, due to "natural" lane-changing are considered among adjacent lanes of the same segment, and corresponding rules must be defined in order to properly assign and bound the lateral flows. To start with, the maximum available flow for lateral movements is calculated based on the current amount of vehicles in the segment-lane:

$$\mathrm{F}_{i,j}(k) = \frac{L_i}{T}\rho_{i,j}(k)$$

The pursued approach is to compute the lateral flow from segment-lane(i,j) to (i, $\bar{J}$) as a rate $A_{i,j,\bar{J}} \in [0,1]$ of the value $F_{i,j}(k)$. We call this rate the attractiveness rate.

As a matter of fact, lane-changing behavior and flows are extremely hard to model accurately (both macroscopically and microscopically) because they depend on a very high number of partly interdependent factors. To start with, different vehicle types (e.g. cars and trucks) and related restrictions (e.g. regarding lane usage) may give rise to a variety of different lane-changing characteristics. Human driver behavior is another source of uncertainty and variation. Furthermore, lane-changing behavior is different in case of road curvature or grade, lane drop, as well as in tunnels, on bridges, etc.; it is also dependent on the number of lanes, environmental conditions (weather, lighting), traffic conditions (free-flow, dense, congested) and traffic signs. Last not least, lane-changing activity is quite particular in the vicinity of on-ramps and off-ramps, or at weaving sections. Given this diversity and complexity, it appears appropriate, in the present context, to provide for a simple basic lane-changing flow model, which would capture with some accuracy many "ordinary" situations; accompanied by a space–time dependent parameter that could be used to influence the model calculations appropriately whenever needed (e.g. near on- and off-ramps).

According to assumption 1, under "ordinary" conditions, the current attractiveness rate $A_{i,j,\bar{J}}$ may be deemed to increase proportionally to the current density difference $\rho_{i,j}(k) - \rho_{i,\bar{j}}(k)$ of adjacent lanes j and $\bar{j}$. This basic assumption, however, may be subject to variations due to various local effects, as mentioned earlier. For example, vehicles driving on the slow lane may consider a lane change upstream of on-ramps or off-ramps, to avoid interference with entering or exiting vehicles, respectively. To capture this variety of potential situations, the attractiveness rate is modelled to depend on the weighted density difference $P_{i,j,\bar{j}}(k)\rho_{i,j}(k) - \rho_{i,\bar{j}}(k)$, where the introduced factor $P_{i,j,\bar{j}}(k)$ is mostly equal to 1. However, it should be tuned to reflect particular location- dependent or time-dependent effects where needed. This factor considers the same difference in density between the adjacent lanes, irrespectively of the considered lane-changing direction. Thus, it respects the relation $P_{i,j,\bar{j}}(k) = \frac{1}{P_{i,j,\bar{j}}(k)}$ or all the adjacent segment-lane couples (i,j) and $(\mathrm{i},\bar{j})$.

Finally, the attractiveness rate $A_{i,j,\bar{j}}$ is computed as:

$$A_{i,j,\bar{j}}(\mathrm{k}) = \mu \max\left[0, \frac{P_{i,j,\bar{j}}(k)\rho_{i,j}(k) - \rho_{i,\bar{j}}(k)}{P_{i,j,\bar{j}}(k)\rho_{i,j}(k) + \rho_{i,\bar{j}}(k)}\right]$$

where the coefficient is a unique parameter in the range [0,1], reflecting a sort of "aggressiveness" in lane-changing. As mentioned earlier, we have $P_{i,j,\bar{j}}(k)$ for the vast majority of locations.

In conclusion, the lateral demand flow, in other words, the flow that will actually materialize if there is enough space in the target segment-lane, is assigned according to the following formula:

$$D_{i,j,\bar{j}}(\mathrm{k}) = \max\left[A_{i,j,\bar{j}}(\mathrm{k}) F_{i,j}(k) f_{i,j,\bar{j}}^{off}(k)\right]$$

$f_{i,j,\bar{j}}^{off}(k)$ refers to lateral flow, the traffic volume moving from lane j to lane $\bar{j}$.

Accounting that the vehicle can only move to the adjacent lane at once, thus, here $\bar{j} = j \pm 1$.

Although we know the maximum lateral demand flow, the capital of the lane may be not enough. In order to complete the lateral flow modelling development, we need to account for the space available in the segment-lane, which is receiving the vehicles moving laterally. To this end, the following function describing the available space in terms of flow acceptance is considered:

$$S_{i,\bar{j}}(\mathrm{k}) = \left[\rho_{\mathrm{i},\bar{j}}^{jam} - \rho_{i,\bar{j}}(k)\frac{L_i}{T}\right]$$

Since the available space may not be sufficient for accepting the lateral flow entering from both sides, the assigned quantity is proportionally distributed according to the following relation:

$$f_{i,j-1,j}(\mathrm{k}) = \min\left[1, \frac{S_{i,j}(\mathrm{k})}{D_{i,j-1,j}(\mathrm{k}) + D_{i,j+1,j}(\mathrm{k})}\right] D_{i,j-1,j}(\mathrm{k})$$

$$f_{i,j+1,j}(\mathrm{k}) = \min\left[1, \frac{S_{i,j}(\mathrm{k})}{D_{i,j-1,j}(\mathrm{k}) + D_{i,j+1,j}(\mathrm{k})}\right] D_{i,j+1,j}(\mathrm{k})$$

5 A multiple-lane traffic flow model

We have considered that, in case some vehicles are enabled to communicate with the infrastructure (V2I), it is possible to recommend to them variable speed limits, or even dictate them their maximum driving speed (in case they are also equipped with an ACC system), according to the real-time decisions of an external Decision Maker (DM). Similarly, V2I-equipped vehicles may receive from the DM lane-changing advices, so as to implement corresponding lateral flow decisions. To enable a rational design for these actions, an advanced traffic flow model is required, which considers the lane changes.

5.1 Basic model of CA

Traffic flow theory is the theory of using the laws of physics and mathematics to describe traffic characteristics. The classic traffic flow model includes probability and statistics model, car-following model, fluid dynamics model, cellular automaton model and vehicle queuing model.[12]

Since the cellular automaton (CA) model reproduces the complex traffic in a simple way and reflects the traffic flow characteristics, we choose this model as the basic model for our modeling.

CA divide rode into several sections of three cells. A cell corresponding to one or several cars, or a few cells corresponding to a car. The status of each cell empty or it contains the vehicle Speed. Each car at the same time in accordance with established rules of movement. These rules by the movement of vehicles to be followed by the rules of the movement and traffic rules, and includes driving behavior, external disturbances and other random change rules. Here, we expound the specific change rules.

- safety distance

In the beginning, to make the newly established model closer to the real traffic, we take the characteristics of the car's physical size, acceleration and deceleration performance and driver reaction time into account. According to the comparison between the distance gap_n and the safety distance $gap_{safe,n}$, the vehicle executes the corresponding speed and location evolution rules.

According to the calculation principle of safety distance, we can determine the safety distance $gap_{safe,n}$ and the safe speed $v_{safe,n}$ of driving. Safety distance refers to the safe distance, vehicles maintained with each other to avoid rear-end, when the current car emergency braking, which is with the car's braking ability, its own vehicle braking capacity and the driver's reaction time, as shown in Figure 6.

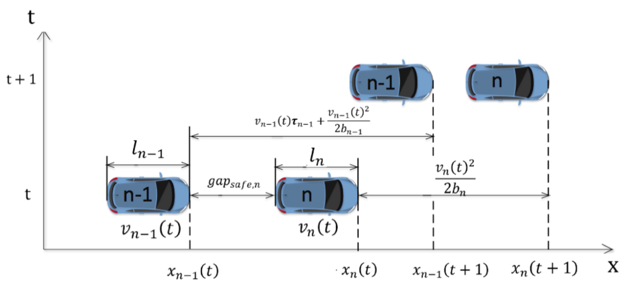


Figure 6. Safety distance

Based on the principle of calculating the safety distance, we have:

$$x_{n-1}(t) + \frac{v_{n-1}(t)^2}{2b_{n-1}} - l_{n-1} \geq x_n(t) + v_n(t)\boldsymbol{\tau}_n + \frac{v_n(t)^2}{2b_n}$$

$$gap_{safe,n} = x_{n-1}(t) - x_n(t) - l_{n-1} = v_n(t)\boldsymbol{\tau}_n + \frac{v_n(t)^2}{2b_n} - \frac{v_{n-1}(t)^2}{2b_{n-1}}$$

$$v_{safe,n} = -b_n\boldsymbol{\tau}_n + \sqrt{{b_n}^2{\boldsymbol{\tau}_n}^2 + b_n\left\{2[x_{n-1}(t) - x_n(t) - l_{n-1}] - \boldsymbol{\tau}_n v_n(t) + \frac{v_{n-1}(t)^2}{b_{n-1}}\right\}}$$

Where $gap_{safe,n}$ is the safety distance the n_{th} vehicle needed, while $x_n(t)$ and $x_{n-1}(t)$ refer to the location of the n_{th} and $(n-1)_{th}$, before the n_{th}, vehicle separately. l_{n-1} is the length of the forward vehicle and b_n is the maximum deceleration of the n_{th} vehicle. Besides, $\boldsymbol{\tau}_n$ and $v_n(t)$ separately present driver's reaction time and speed.

- **Acceleration**

In the driving process, when the safety distance, between n_{th} and $(n-1)_{th}$ vehicle, is greater than the spacing the n_{th} car needed, namely $gap_n > gap_{safe,n}$, the vehicle speed up to meet the driver's expectancy of higher speed:

$$v_n(t) \rightarrow \min\left(v_n(t) + a_n, V_{min}, v_{safe,n}(\mathrm{t}), gap_n(\mathrm{t})\right)$$

- Uniform velocity

When the safety distance, between n_{th} and $(n-1)_{th}$ vehicle, is the same with the spacing the n_{th} car needed, namely $gap_n = gap_{safe,n}$, the vehicle keep the speed to ensuring the safety of vehicle:

$$v_n(t) \rightarrow \min\left(v_n(t), gap_n(\mathrm{t})\right)$$

- **Reduction**

When the safety distance, between n_{th} and $(n-1)_{th}$ vehicle, is less than the spacing the n_{th} car needed, namely $gap_n < gap_{safe,n}$, the vehicle slow down to ensuring the safety. If $v_{n-1}(t) = 0$, namely The vehicle ahead keep the stationary state, making the spacing less than 0.5m.

$$v_n(t) \rightarrow \max\left\{\min\left(v_{safe,n}(\mathrm{t}), gap_n(\mathrm{t}) - 1\right), 0\right\}$$

Otherwise, observing the following formula：

$$v_n(t) \rightarrow \max\left\{\min\left(v_{safe,n}(\mathrm{t}), gap_n(\mathrm{t})\right), 0\right\}$$

- Randomly moderated

In regard to the uncertainty of the behavior, adding the random slowing probability R in the rules, where vehicles carry out in accordance with the random probability of slowing down speed. The following formula show the process:

$$v_n(t) \rightarrow \max\left(v_n(t) - b_n, 0\right)$$

- Location update

Based on the speed update, the vehicle position can be updated as shown in the following formula:

$$x_n(t+1) \rightarrow x_n(t) + v_n(t)$$

Where gap_n is the spacing between nth and (n-1)th vehicle and V_{max} represents the maximum speed. a_n and b_n refer to conventional acceleration and deceleration.

5.3 Basic Rules for changing lanes

Then, we numbered the lanes 1,2,...,M from the shoulder lane (close to the roadside), to the outer lane (close to the road median). Before analyzing the lane changing rules of a self-driving vehicle, we first analyze the lane changing rules of a regular vehicle. [13]

The regular vehicle will change lane or not according to the need. Vehicles should meet the overtaking principle and safety principles in the lane change process.

- The lane changing rules for vehicle on the outer lane.

When $gap_M^{(i)}$, the spacing in front of the i_{th} vehicle on the M_{th} lane, is less than V_{hope}, drivers' expectancy to the speed, they want to change the lane. If $gap_j^{(i)}$ is less than $fgap_{j-1,j}$, the spacing at the fore of the $(M-1)_{th}$ lane, while the $bgap_{M-1,M}$, the spacing behind the M_{th} lane, more than or equal to $bV_{M-1,M}$, the speed of nearest vehicle latter, the j_{th} vehicle move to the $(M-1)_{th}$ lane with a probability of $A_{M,M-1}$, keeping the speed consistent.

- The vehicles in M_{th} lane may turn left to the $(M-1)_{th}$ lane or turn right to the $(M+1)_{th}$ lane. Rules of changing lanes are shown as following:

①Turn left

When $gap_j^{(i)} < V_{hope}$, drivers have the demand to change lanes. If $gap_j^{(i)} < fgap_{j-1,j}$, $bgap_{j-1,j} \geq bV_{j-1,j}$, the j_{th} vehicle move to the $(j-1)_{th}$ lane with a probability of $A_{j,j-1}$, keeping the speed consistent.

①Turn right

When $gap_j^{(i)} \geq V_{hope}$, drivers have the demand to change lanes. If $gap_j^{(i)} > fgap_{j+1,j}$, $bgap_{j+1,j} \geq bV_{j+1,j}$, the j_{th} vehicle move to the $(j-1)_{th}$ lane with a probability of $A_{j,j-1}$, keeping the speed consistent.

- The lane changing rules for vehicle on the inter lane.

When $gap_1^{(i)} < V_{hope}$, drivers have the demand to change lanes. If $gap_1^{(i)} < fgap_{2,1}$, and $bgap_{2,1} \geq bV_{2,1}$, the j_{th} vehicle move to the $(j-1)_{th}$ lane with a probability of $A_{j,j-1}$, keeping the speed consistent.

5.3 Rules for changing lanes of Self-driving vehicles

Based on the change rules of the regular vehicles, we can analyze the rules of the self-driving ones. Since regular vehicles usually respect drivers' mind, the spacing between them is large when the traffic density is not dense. The driving mode of the regular vehicles is presented in Figure 7.

Figure 7. Driving mode of the regular vehicles

Compared with the regular vehicle, self-driving vehicle is able to obtain the information, which other self-driving vehicles send out during the driving process, including the location, road condition. Although the information of the regular ones is not available, they can be speculated by sensor. In regard to these information, self-driving vehicle can take the optimal strategy. They response faster than human beings based on the real-time information, sending by control center. Thus, they usually drive one after another, with tight junction as Figure 8. This kind of configuration can realize space saving, comparing with Figure 7.

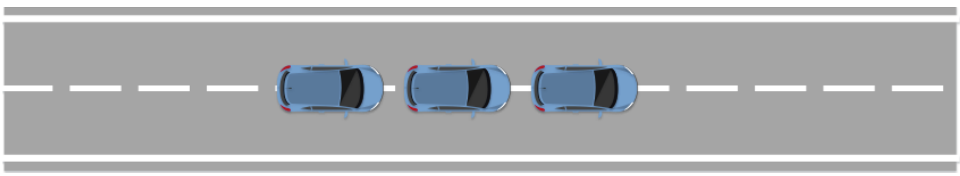


Figure 8. Driving mode of the self-driving vehicles

Since, there are both regular vehicles and self-driving vehicles existed on the lane, we can get a mixed traffic flow model, combining these two models above. Here, we look at three lanes, for example.

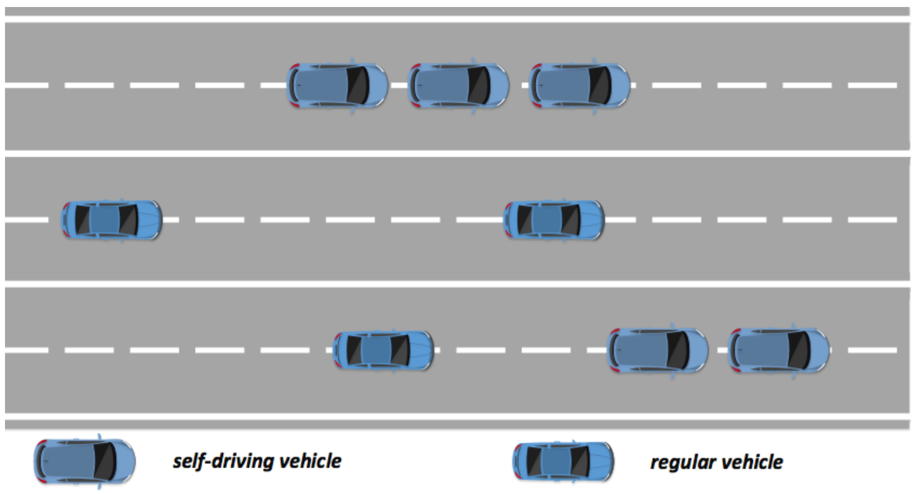


Figure 9. Mixed traffic flow model

Although there two vehicles following the same rules, as shown in the last section, their tendency to lane changing is different.

(1) The lane change rules for vehicles on the outermost lane:

If the $(i+1)_{th}$ vehicle in front of the i_{th} vehicle on the M_{th} lane is driverless, not change the lane. Otherwise, move to the $(M-1)_{th}$ lane, with the consistent speed.

(2) The lane change rules for vehicles on the innermost lane:

If the $(i+1)_{th}$ vehicle in front of the i_{th} vehicle on the 1_{st} lane is driverless, not change the lane. Otherwise, move to 2_{nd} lane, with the consistent speed.

The specific process above is shown in Appendix.

Table 2.
Lane change rules for vehicles on the middle lane

Input: Lane flow status
Output: the change lane of the i_{th} vehicle on the j_{th} lane

```
begin:
  if the (i+1)_th vehicle on the j_th lane is self-driving
      Do not change lanes
  else
      if the (i+1)_th vehicle on the (j-1)_th lane is self-driving
          if gap meet the conditions of lane change
              Transform to j-1 lane
          else
              Do not change lanes
          endif
      else
          if the ahead on the (j+1)_th lane is self-driving
              if gap meet the conditions of lane change
                  Transform to j+1 lane
              else
                  Do not change lanes
              endif
          else
              Do not change lanes
          endif
      endif
  endif
```

(3) The lane change rules for vehicles on the middle(j_{th}) lane:

① If the $(i+1)_{th}$ vehicle in front of the i_{th} vehicle on the j_{th} lane is driverless, not change the lane. Otherwise, following the rules to turn left.

② The rules to turn left：

If the $(i+1)_{th}$ vehicle in front of the i_{th} vehicle on the $(j-1)_{th}$ lane is driverless, turn to $(j-1)_{th}$ lane , with the consistent speed. Otherwise, following the rules to turn right.

③ The rules to turn right：

If the $(i+1)_{th}$ vehicle in front of the i_{th} vehicle on the $(j+1)_{th}$ lane is driverless, turn to $(j+1)_{th}$ lane , with the consistent speed.

6 Effects on Traffic Flow and Search for Special Point

According to the map, we know that Interstate 5 north and the south through the greater Seattle area with Interstate 90, 405 and State Route 520 crossing with it. Among them, Interstate 405 begins from the Interstate 5 and ends at the same interstate. Besides, State Route 520 and Interstate 90 almost parallel to each other crossing with Interstate 5.

In regard to Table in appendix, we calculate the average traffic flow of peak travel hours ($\mathrm{ave}P_t^{peak}$) and off-peak travel hours ($\mathrm{ave}P_t^{off}$) with the following formulas. Where $\mathrm{ave}P_{\mathrm{t}}$ refers to the average traffic flow per day. The results are XX and XX separately.

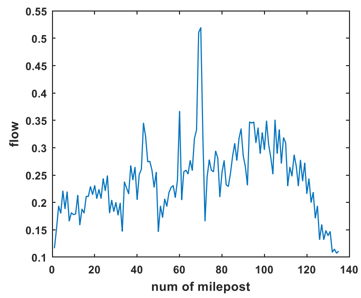
Since 8% of the daily traffic volume occurs during peak travel hours and the peak hours last 2 hours as we assumed before.

$$\mathrm{ave}P_t^{peak} = \frac{\mathrm{ave}P_{\mathrm{t}} \times 8\%}{2}$$

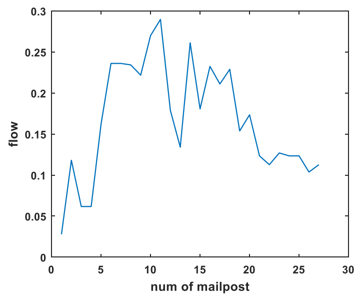
Thus, the rest 92% of traffic volume appears in the rest 22 off-peak travel hours.

$$\mathrm{ave}P_t^{off} = \frac{\mathrm{ave}P_{\mathrm{t}} \times 92\%}{22}$$

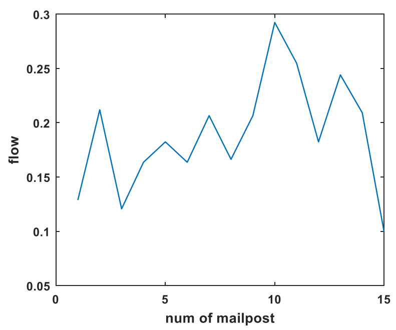
According to the excel, we fit the flow with the number of the milepost to show the road conditions of the four interstates.



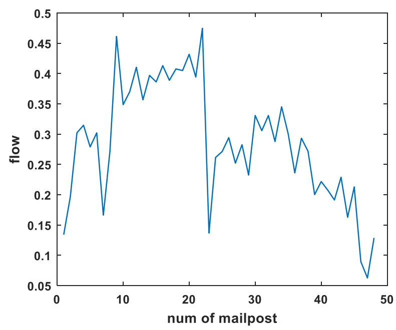
Interstate 5



Interstate 90



Interstate 405



State Route 520

Figure 10. Peak flow of the four Road

As we can see from the figure 10, on the location of 70th milepost on the Interstate 5, it reaches the peak flow. When it comes to Interstate 90, the first peak appears from 6th-12th milepost and the other appears from 15th-18th milepost. On the Interstate 405, the peak also appears in several locations, the 10th and 13th milepost. The State Route 520 has a long distance of congestion, from 9th to 22th milepost.

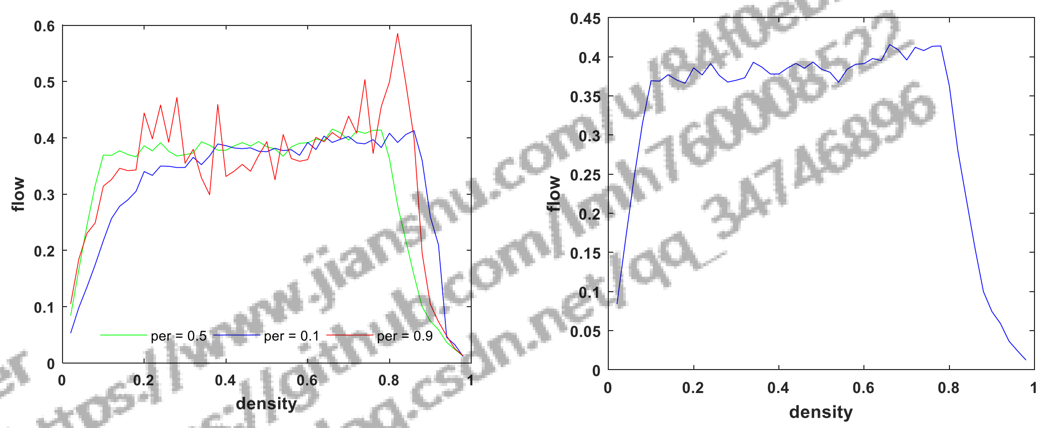


Figure 11. Flow-Density

As we can see from the left figure, the effect on traffic flow performs best when the density reaches over 0.8, namely in the peak period, no matter the percentage of the self-driving. When the density less than 0.2, the effect arrives best with the percentage is 0.5. As the density ranks from 0.2 to 0.4, higher the percentage, better the effect. While the density float from 0.4-0.7, the percentage of the self-driving does no influence on the effect.

From the figure left we know the flow changes with the change of the density, different percentage of the self-driving performing different. However, the tendencies of them are similar from 0.4-0.7. While as we can see in the right figure, the flow is stable when the density float from 0.16-0.8 at the ratio of the 10%. Thus, we recognize that the equalization exists when the density float from 0.4-0.7.

7 A Single-Objective Optimization Model

7.1 Build a Single-Objective Optimization Model

When the road is congested, self-driving and regular vehicles move on the road mixed. The advantages of self-driving vehicles are unable to make full use because of the extremely density. Thus, we decide to add self-driving vehicles to release the traffic jam. At this point, we stipulate that self-driving vehicles can only use the exclusive lane. Regular vehicles run on the road following the regular principles, not covering the exclusive lane. We assume that the rate of the self-driving vehicles is a_i. Besides, M refers to the numbers of lanes and m represents the numbers of exclusive lane. Thus, when $a_i < 100\%$, $\mathrm{m} \leq \mathrm{M} - 1$. We calculate the traffic flow during congestion period(Q):

$$\mathrm{Q} = Q_{self} + Q_{regu}$$

Where Q_{self} and Q_{self} represent the traffic flow of the self-driving exclusive lane and regular vehicles' lane separately.

According to the general formula of flow(q):

$$q = k \cdot v(k) = k \cdot v_{free} \cdot \left(1 - \frac{k}{k_{max}}\right) = v_{free} \cdot (k - \frac{k^2}{k_{max}})$$

We can get:

$$Q_{self} = n \cdot v_{free\_self} \cdot (k_{seif} - \frac{k_{self}^2}{k_{\max\_self}})$$

$$Q_{regu} = (M - n) \cdot v_{free\_regu} \cdot (k_{regu} - \frac{k_{regu}^2}{k_{\max\_regu}})$$

Putting these formulas together, we obtain:

$$Q = n \cdot v_{free\_self} \cdot \left(k_{seif} - \frac{k_{self}^2}{k_{\max\_self}}\right) + (M - n) \cdot v_{free\_regu} \cdot (k_{regu} - \frac{k_{regu}^2}{k_{\max\_regu}})$$

Where $t_{self} = \frac{a_i \cdot N}{Q_{self}}$ represents the total time self-driving vehicles needed to pass the road. Similarly, regular vehicles cost $t_{regu} = \frac{(1-a_i) \cdot N}{Q_{regu}}$. Besides, N refers to the number of the total vehicles passed. To maximize the efficiency, we should minimize the passing time. Then, we can get the passing time (T) of the traffic system:

$$T = \min\{\max(t_{self}, t_{regu})\}$$

7.2 Result: set a self-driving lane

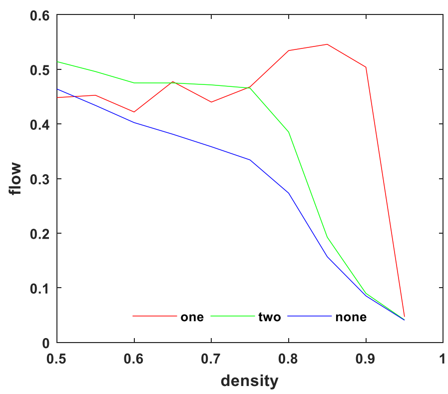


Figure 12. Optimization result

From the figure, we know set the self-driving lane can improve the traffic flow efficient. While one self-driving lane makes the best optimum efficiency, two lanes following it. Setting three lanes perform not as well as two lanes, even more one lane. It means it is necessary to set a self-driving lane, only one, which makes the be effect maximize.

8 Suggestions to Policy changes

According the results, we know the ratio of the self-driving has a positive effect on the traffic flow. However, it doesn't mean the more the better, in this case, constraint and support from policy is necessary to achieve the balance. There are four states and the District of Columbia, allowing the self-driving car, have passed legislation to specification put up related policies. We can learn something from them. In Nevada, a driver who obtains a technology certification from the facility will receive a special endorsement on his or her state driver's license. Colorado, for example, retains liability for damages with the driver who may or may not use autonomous "guidance technology". In Hawaii, absolve manufacturers of liability where a car has been retrofitted by a third party and operator (or driver) where there is no verifiable "recklessness" identified. South Carolina, like California, redefines "manufacturer" as whoever is responsible for installation of autonomous technology, either the original manufacturer or upfitter.

From the result above, we know the self-driving vehicles can release the traffic congestion when the density over 0.8, namely in the peak period. In space, it is necessary to set a self-driving lane, only one, which makes the be effect maximize. Thus, government should encourage the drivers to use self-driving vehicles by some methods. For example:

- When drivers buy designated self-driving cars, they will be subsidized with 10 percent of the cost.
- Build a dedicated lane for the driverless car and designate a dedicated time slot.

9 Sensitivity Analysis

In the process of building model based on CA model, some influencing factors of the traffic flow like the ratio of the self-driving vehicles, deceleration probability(p). They cause changes to the flow. Take the ratio of the self-driving vehicles as an example, and observe the effect on the traffic flow. Sensitivity of the ratio is obtained by calculating, as shown in the following figures.

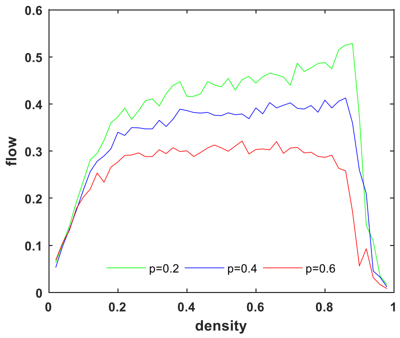


Figure 13. Accounting for 50%

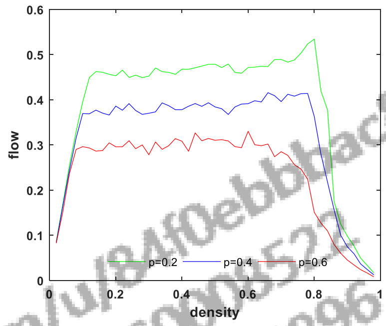


Figure 14. Accounting for 10%

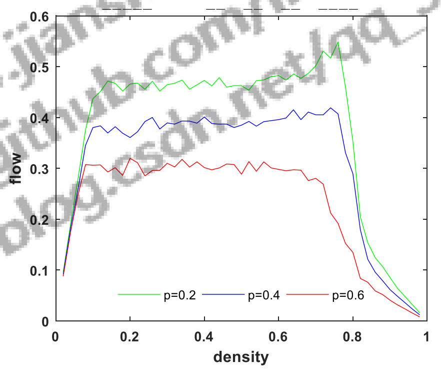


Figure 15. Accounting for 0%

We can see that the traffic flow is very sensitive to the ratio of the self-driving vehicles in the context of ±20%. It proves that the model we establish is suitable for different ratios of the self-driving vehicles.

Observation the influence of deceleration probability on the traffic flow. Change the deceleration probability, then taps out the sensitivity of the deceleration probability, as shown in the figures above.

10 Strengths and Weaknesses

Strengths

- Based on the traditional CA model, we propose the mixed traffic flow model, combining the driving mode of the self-driving and regular vehicles.
- We use CA model which is relatively simple but effective to solve the traffic flow problem. By examining the cell state changes, we can obtain the parameters such as the speed, displacement and headway of each vehicle at any moment, describing the micro-characteristics of the traffic flow. While the average velocity, density, and other parameters can be obtained, showing the macro-characteristics of traffic flow.
- Establish gradient models, where we build the original traffic flow model not considering the lane changes, then we propose a multiple-lane traffic flow model, adding more additional conditions.

Weaknesses

- We only consider the cars, but actually the traffic flow includes truck, minibus, etc.

- We only analyze the single road one by one, lacking comprehensive considers of four road.

11 Future Work: focus on longitudinal flows

In the previous model, we considered the lane change on a section of road. But there was a possibility of lane change before the section and the section. Therefore, in the future work, we defined the longitudinal flows and take this into consideration.

The longitudinal flows are defined as the flows going from a segment to the next downstream one, shown in Figure 14, while remaining in the same lane. Longitudinal flow is computed as the minimum between an upstream demand flow and a downstream supply flow. However, one important phenomenon that regularly appears in real traffic is the capacity drop phenomenon, namely, the reduction of discharge flow once queues start forming at a bottleneck location. The reasons for this phenomenon are not exactly clear. Here, we will based on the previous research, putting out our model, trying to figure out the problem.

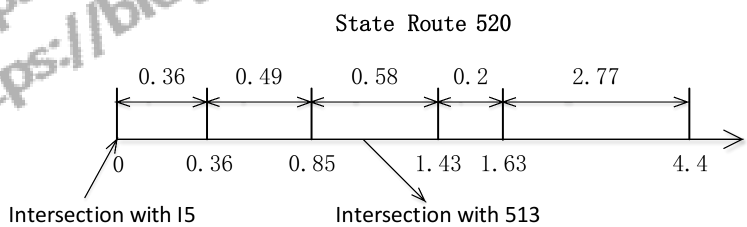


Figure 16 State Route 520

In our model, since we are interested in a linear formulation, the aforementioned capacity-drop approach is achieved by redefining the demand part of the FD in the following way:

In case of congestion, namely $\rho_{i,j} > \rho_{i,j}^{cr}$, where $\rho_{i,j}^{cr}$ is the critical density for the segment-lane (i, j), the flow is linearly decreased according to a fixed slope w^D. This can be seen in the sketched FD of Figure 17. Note that, in order to avoid interference with possible shockwaves caused by a back-spilling congestion, the relation $w^D < w^S$ must hold. This extension calls for the definition of an additional point in the FD, $q_{i,j}^{jam}$, namely, the flow that is allowed to leave a segment while it has entered a completely congested state, namely $\rho_{i,j} > \rho_{i,j}^{cr}$. Note that this simple modification of the demand function is not sufficient to create a capacity drop at the head of a congestion under all circumstances; this is achieved is here via the lateral and ramp flows, which act as sources for the LWR model (which are not considered explicitly in the demand and supply functions). It permits to obtain a density increase beyond the critical value in the segment-lane placed at the head of the congestion, which triggers the capacity drop. Other possibilities to account for capacity drop in a first-order model, maintaining linear constraints, are currently under investigation. Note that in the conventional model we have $q_{i,j}^{jam} = q_{i,j}^{max}$, namely no capacity drop at the head of congestion.

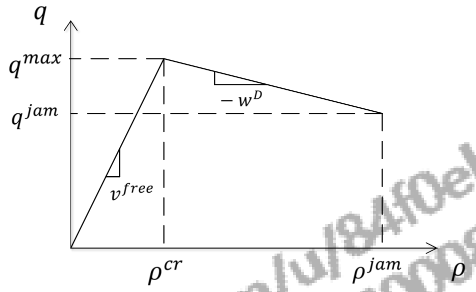


Figure 17. Relationship of density and flow

In conclusion, the demand part for the longitudinal flow calculation is computed as:

$$Q_{i,j}^{D}(k) = min\{v_{i,j}^{free}\rho_{i,j}(k), -w^{D}\rho_{i,j}(k) + g^{D}\}$$

Where

$$w^{D} = \frac{v_{i,j}^{free}\rho_{i,j}^{cr} - q_{i,j}^{jam}}{\rho_{i,j}^{jam} - \rho_{i,j}^{cr}}$$

$$g^{D} = \frac{\rho_{i,j}^{cr}(v_{i,j}^{free}\rho_{i,j}^{jam} - q_{i,j}^{jam})}{q_{i,j}^{jam} - \rho_{i,j}^{cr}}$$

12 Conclusions

Our paper provides several models on traffic flow. It is able to solve the cooperation between self-driving cars, as well as the interaction between self-driving and non-self-driving vehicles.

Firstly, we establish the Throughput Model. It is used to analyze the cooperation pattern between self-driving vehicles and regular vehicles. The model based on the Bayesian formula not consider the change lanes.

Secondly, we build a lateral flow model. In this part, we consider the effect of change lanes to a single traffic flow. Formulas are given orderly in this part. According to this model, we can get traffic on different lanes that have been converted to the next available lane.

Thirdly, we propose a lane change model for mixed traffic flow. CA is adopted in this part as the basic model. Here, the different behavior patterns of regular vehicle and self-driving vehicle lane change are studied and an algorithm is realized. After that, we find the equilibria and a tipping point.

Finally, we build a single-objective optimization model. In regard to it, we find conditions under which lanes should be dedicated to these cars.

Last but not least, sensitivity analysis and error analysis are attached to our model. We predict our future works and specifically state the merits and shortcomings.

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Letter

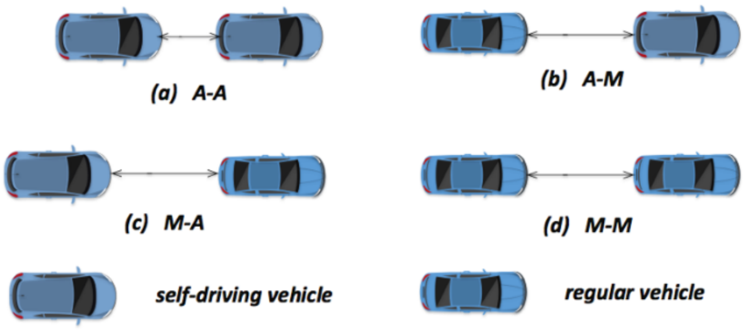
Dear Governor of Washington State:

It is my pleasure to give policy recommendation to Governor of Washington State. We have proposed a solution to increase capacity of highways without increasing number of lanes or roads. This solution will study the behavior of self-driving, cooperating cars interacting with the existing traffic flow and each other.

According to our information, traffic capacity is limited in many regions of the United States due to the number of lanes of roads. Especially, in the Greater Seattle area drivers experience long delays during peak traffic hours because the volume of traffic exceeds the designed capacity of the road networks. This is particularly pronounced on Interstates 5, 90, and 405 and State Route 520. Self-driving, cooperating cars have been proposed as a solution to the problem. However, you might not understand the behavior of these cars interaction well. Therefore, your problem might be: “How do the effects change as the percentage of self-driving cars increases?”, Do equilibria exist? Is there a tipping point where performance changes markedly?”, Under what conditions, if any, should lanes be dedicated to these cars?” and want us give you some other suggestions of the existing policy.

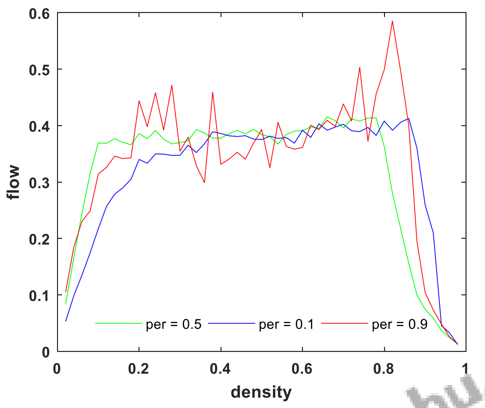
We propose several models of the effects on traffic flow. It is able to solve the cooperation between self-driving cars, as well as the interaction between self-driving and non-self-driving vehicles.

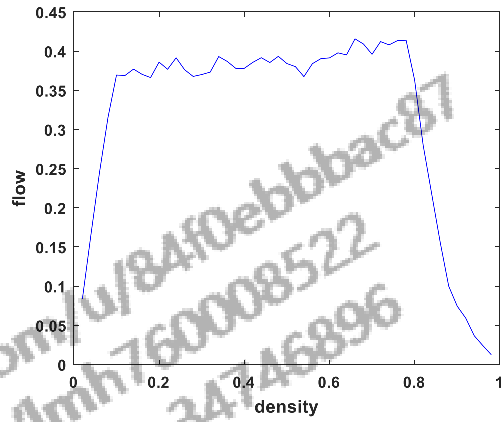
Firstly, before proposing a solution, we analyze the cooperation pattern between self-driving cars as well as self-driving and non-self-driving vehicles. By attempting to communicate to the vehicle ahead, a self-driving vehicle will know whether the leader is self-driving or not. If the laggard is a self-driving vehicle, maintain the appropriate inter-vehicle spacing, between (b) and (c), since the self-driving vehicles have less time for reflect. If the laggard is a regular vehicle, keep the same distance no matter the leader is. In order to make you understand the patterns, we make the following schematic diagram for you.



Secondly, we build a **lateral flow model** to studied the lane change of a single traffic flow. According to this model, we can get traffic on different lanes that have been converted to the next available lane.

Thirdly, we establish **a lane change model based on CA for mixed traffic flow** (*Cellular automaton*). The different behavior patterns of common car and automatic car lane change are studied and an algorithm is realized. After that, we find the equilibria and a tipping point. In order to give you a more intuitive view of the equilibria and a tipping point, we make a comparison diagram for you. The flow changes with the change of the density, different percentage of the self-driving performing different. Equalization exists when the density float from 0.4-0.7. Tipping points exist when the density is 0.2 and 0.8.





Finally, we build **a single-objective optimization model** to find conditions under which lanes should be dedicated to these cars. It is necessary to set a self-driving lane, only one, which makes the be effect maximize.

In accordance with the above research ideas, we suggest to make other policy changes. The first one is when drivers buy designated self-driving cars, they will be subsidized with 10 percent of the cost. The second aspect is building a dedicated lane for the driverless car and designate a dedicated time slot.

In summary, our model has developed a way to increase capacity of highways without increasing number of lanes or roads. Our model is feasible and reasonable and can be adjusted to various kinds of situation in reality. Besides, we use CA model which is relatively simple but effective to solve the traffic flow problem. To know how our model run in details, please check our paper.

Thank you again for taking the time to read our suggestions. We sincerely hope that our solution can solve the problem of congestion!

Best Regards,

Sincerely

30/1/2018

Appendix

1.PROGRAM

Results of the basic model

```
syms a
%
% pam=(1-a)*a;
% paa=a*a;
% pma=a*(1-a);
% pmm=(1-a)*(1-a);
%
% ham=0.3;
% haa=0.31;
% hma=1.8;
% hmm=1.8;
%
% av_head=pam*ham+paa*haa+pma*hma+pmm*hmm;
a=0:0.01:1;
through_out=3600./((1-a).*a.*0.3+a.*a.*0.31+a.*(1-a).*1.8+(1-a).*(1-a).*1.8);
  plot(a,through_out);
```


Simulate traffic flow based on CA

```
#from numba import jit
import time
import random
import copy
import json

def p_random(p):
    temp = random.random()
    if temp < p:
        return True
    else:
        return False
def get_gap(row, pos):
    gap = 1
    l = len(row)
    while 1:
```

```
        if row[(pos + gap) % l] is not None:
            return (gap - 1, row[(pos + gap) % l][0], row[(pos + gap) %
l][1])
        gap = gap + 1

def get_bgap(row, pos):
    bgap = 1
    l = len(row)
    while 1:
        if row[(pos - bgap) % l] is not None:
            return  (bgap  -  1,  row[(pos  -  bgap)  %  l][0],  row[(pos  -
bgap) % l][1])
        bgap = bgap + 1

def change_v(v, gap, vm, p):
    if v >= gap:
        v_last = gap
    elif v < vm:
        v_last = v + 1
    else:
        v_last = min(vm, gap)

    if p_random(p):
        v_last = v_last - 1
    v_last = max(v_last, 0)
    return v_last



def make(I, J):
    '''
    I: Lenth or N of the road
    J: N of lanes
    '''
    densities = []
    for i in range(1,20):
        densities.append(0.05*i)
    p_list = [0.2]
    speeds = [1, 2, 3, 4, 5]
    pers = [0.1, 0.5, 0.9]
    maps = {}



    for per in pers:
```

```
        result = {}
        for p in p_list:
            start = time.time()
            filename = 'result_less/per_{}_p_{}.txt'.format(per, p)
            for density in densities:
                print "solving condition with p = {}, density = {}, per
= {}".format(p, density, per)
                N = int(density * I * J)
                #generate map
                try:
                    mmap = copy.deepcopy(maps[(per,density)])
                except:
                    mmap = []
                    for j in range(J):
                        row = []
                        for i in range(I):
                            row.append(None)
                        mmap.append(row)
                    #generate cars
                    for i in range(int(per * N)):
                        while 1:
                            position = random.randint(0, I * J - 1)
                            j_pos = position / I
                            i_pos = position % I
                            if mmap[j_pos][i_pos] is not None:
                                continue
                            else:
                                mmap[j_pos][i_pos] = (3, 1)
                                break
                    for i in range(int((1 - per) * N)):
                        while 1:
                            position = random.randint(0, I * J - 1)
                            j_pos = position / I
                            i_pos = position % I
                            if mmap[j_pos][i_pos] is not None:
                                continue
                            else:
                                mmap[j_pos][i_pos] = (3, 0)
                                break
                    maps[(per,density)] = copy.deepcopy(mmap)
                #start
                v_ = [0]
                flow = [0]
                f_change = [0]
```

```
use_per = [0]
temp = []
for t in range(1,2001):
    sum_v_ = 0
    sum_f_change = 0
    temp = copy.deepcopy(mmap)
    for i in range(I):
        for j in range(J):
            if mmap[j][i] is None:
                continue
            v = mmap[j][i][0]
            car_type = mmap[j][i][1]
            sum_v_ = sum_v_ + v
            #mannual car
            if car_type == 0:
                '''
                #calculating v_hope
                gap = get_gap(mmap[j], i)[0]
                v_hope = min(5, v + 1)
                #don't need to shift
                if gap > v_hope:
                    v_last = v_hope
                    j_last = j
                    i_last = (i + v_last) % I
              temp[j_last][i_last] =(v_last, car_type)
                    temp[j][i] = None
                    continue
                '''
                v_hope = min(5, v + 1)
                if j - 1 >= 0:
        gapl, vl, car_typel = get_gap(mmap[j - 1], i)
                if j + 1 < J:
         gapr, vr, car_typer = get_gap(mmap[j + 1], i)
                gap = get_gap(mmap[j], i)[0]
                #try to shift
                if gap < v_hope:
                    #try left
            if j - 1 >= 0 and mmap[j-1][i] is None:
         #gapl, vl, car_typel = get_gap(mmap[j - 1], i)
      bgapl, bvl, bcar_typel = get_bgap(mmap[j - 1], i)
          if gapl > gap and bgapl > 0 and p_random(0.5):
                       v_last = change_v(v, gapl, 5, p)
                            j_last = j - 1
                            i_last = (i + v_last) % I
```

```
                    temp[j][i] = None
        temp[j_last][i_last] = (v_last, car_type)
              sum_f_change = sum_f_change + 1
                    continue
                #try right
      if j + 1 < J and mmap[j+1][i] is None:
     #gapr, vr, car_typer = get_gap(mmap[j + 1], i)
  bgapr, bvr, bcar_typer = get_bgap(mmap[j + 1], i)
  if gapr > gap and bgapr > 0 and p_random(0.5):
     v_last = change_v(v, gapr, 5, p)
                    j_last = j + 1
                    i_last = (i + v_last) % I

                    temp[j][i] = None
     temp[j_last][i_last] = (v_last, car_type)
           sum_f_change = sum_f_change + 1
                    continue
                #can't shift
                v_last = change_v(v, gap, 5, p)

                j_last = j
                i_last = (i + v_last) % I

                temp[j][i] = None
         temp[j_last][i_last] = (v_last, car_type)
                continue
            else:
                #don't shift
                v_last = change_v(v, gap, 5, p)

                j_last = j
                i_last = (i + v_last) % I

                temp[j][i] = None
        temp[j_last][i_last] = (v_last, car_type)
                continue

        else:
            #calculating v_hope
     gap, v, car_type_frot = get_gap(mmap[j], i)
            v_hope = min(7, v + 1)
            #don't shift
            if car_type_frot == 1:
```

```
                            v_last = change_v(v, gap, 7, p)
                            j_last = j
                            i_last = (i + v_last) % I

                            temp[j][i] = None
                     temp[j_last][i_last] = (v_last, car_type)
                            continue

                        if j - 1 >= 0:
                  gapl, vl, car_typel = get_gap(mmap[j - 1], i)
                        if j + 1 < J:
                  gapr, vr, car_typer = get_gap(mmap[j - 1], i)
                        #need to shift
                        if j - 1 >= 0 and mmap[j-1][i] is None:
                            #try left
                            if gap <= v_hope or car_typel == 1:
                 #gapl, vl, car_typel = get_gap(mmap[j - 1], i)
              bgapl, bvl, bcar_typel = get_bgap(mmap[j - 1], i)
             if gapl > gap and bgapl > bvl and p_random(0.5):
                               v_last = change_v(v, gapl, 7, p)
                                    j_last = j - 1
                                    i_last = (i + v_hope) % I

                                    temp[j][i] = None
                     temp[j_last][i_last] = (v_last, car_type)
                              sum_f_change = sum_f_change + 1
                                    continue
                        if j + 1 < J and mmap[j+1][i] is None:
                            #try right
                            if gap <= v_hope or car_typer == 1:
                 #gapr, vr, car_typer = get_gap(mmap[j + 1], i)
              bgapr, bvr, bcar_typer = get_bgap(mmap[j + 1], i)
                if gapr > gap and bgapr > bvr and p_random(0.5):
                              v_last = change_v(v, gapr, 7, p)
                                    j_last = j + 1
                                    i_last = (i + v_last) % I

                                    temp[j][i] = None
                     temp[j_last][i_last] = (v_last, car_type)
                               sum_f_change = sum_f_change + 1
                                    continue
                        #can't shift
                        v_last = change_v(v, gap, 7, p)
                        j_last = j
```

```
                                i_last = (i + v_last) % I

                                temp[j][i] = None
                           temp[j_last][i_last] = (v_last, car_type)
                                continue

                    mmap = copy.deepcopy(temp)
                    v_.append(sum_v_/float(N))
                    f_change.append(sum_f_change/float(N))
                    flow.append( sum_v_/float(N) * density)

                result[(density, p)] = {
                    'v_':sum(v_[-500:])/500.0,
                    'f_change':sum(f_change[-500:])/500.0,
                    'flow':sum(flow[-500:])/500.0,
                }
            flow = []
            v_ = []
            f_change = []

            for density in densities:
                flow.append(result[(density,p)]['flow'])
                v_.append(result[(density,p)]['v_'])
                f_change.append(result[(density,p)]['f_change'])
            with open(filename, 'w') as f:
                f.write(json.dumps(flow))
                f.write(json.dumps(v_))
                f.write(json.dumps(f_change))



            end = time.time()
            cost = end - start
            print 'cost {}s.'.format(cost)
    return result



if __name__ == '__main__':
    make(500, 3)
```

Lane changing rules for vehicles on the outermost lane

Input: Lane flow status
Output: the change lane of the i_{th} vehicle on the M_{th} lane
begin:
if the $(i+1)_{th}$ vehicle on the M_{th} lane is self-driving
  Do not change lanes
else
  if the $(i+1)_{th}$ vehicle on the $(M-1)_{th}$ lane is self-driving
    if *gap* meet the conditions of lane change
      Transform to M-1 lane
    else
      Do not change lanes
    endif
  else
    Do not change lanes
  endif
endif

Lane change rules for vehicles on the innermost lane

Input: Lane flow status
Output: the change lane of the i_{th} vehicle on the 1_{st} lane
begin:
if the $(i+1)_{th}$ vehicle on the 1_{st} lane is self-driving
  Do not change lanes
else
  if the $(i+1)_{th}$ vehicle on the 2_{nd} lane is self-driving
    if *gap* meet the conditions of lane change
      Transform to lane 2
    else
      Do not change lanes
    endif
  else
    Do not change lanes
  endif
endif

2.TABLE

Peak traffic flow				
2600	7160	8520	2480	6080
3400	8200	7920	520	6040
4320	7720	6920	1760	6440
4040	6800	7760	920	5880
5760	7600	7720	920	7080
4920	5480	7760	4840	4080
5720	7200	6920	5280	7800
4960	6440	7520	5280	7080
5400	7680	6480	5240	7680
4640	7200	7320	4960	5640
4000	8120	6640	6040	6320
4760	8480	7800	6480	5200
3560	8600	6840	4000	7400
4200	7800	6320	3000	6840
4040	8960	5640	5840	7400
4720	9560	7840	4040	6440
4720	6880	6480	5200	7720
5120	7640	7440	4720	6720
4800	7720	6080	5120	5280
5160	7520	7120	3440	6560
4640	8240	6920	3880	6080
5000	7720	5160	2760	4480
4640	9440	7880	2520	4960
5440	7440	6480	2840	4640
4960	9520	7480	2760	4280
5560	9680	6960	2760	5120
5400	5960	5160	2320	3640
6080	4960	6200	2520	4760
5480	7360	5360	3000	2000
5960	8280	6080	4400	1400
5280	7720	4840	6760	1920
5920	7640	5440	7040	3160
4400	8760	4480	6240	1800
7080	8400	4880	6760	2440
6760	6280	3840	3720	2720
6440	7600	4320	6080	2440
7960	8240	2960	6880	3080
7200	6920	3560	5200	2480

7880	6840	2960	5520	3080
5360	7560	3320	6120	4360
6560	6360	3120	5320	3800
6800	6880	3280	5920	2720
9000	6200	2440	5760	3640
8360	7080	2560	6160	3120
2600	7480	2400	5800	1480

Off-peak traffic flow				
2718.181818	8572.727273	8280	543.6363636	6314.545455
3554.545455	8070.909091	7234.545455	1840	6732.727273
4516.363636	7109.090909	8112.727273	961.8181818	6147.272727
4223.636364	7945.454545	8070.909091	961.8181818	7401.818182
6021.818182	5729.090909	8112.727273	5060	4265.454545
5143.636364	7527.272727	7234.545455	5520	8154.545455
5980	6732.727273	7861.818182	5520	7401.818182
5185.454545	8029.090909	6774.545455	5478.181818	8029.090909
5645.454545	7527.272727	7652.727273	5185.454545	5896.363636
4850.909091	8489.090909	6941.818182	6314.545455	6607.272727
4181.818182	8865.454545	8154.545455	6774.545455	5436.363636
4976.363636	8990.909091	7150.909091	4181.818182	7736.363636
3721.818182	8154.545455	6607.272727	3136.363636	7150.909091
4390.909091	9367.272727	5896.363636	6105.454545	7736.363636
4223.636364	9994.545455	8196.363636	4223.636364	6732.727273
4934.545455	7192.727273	6774.545455	5436.363636	8070.909091
4934.545455	7987.272727	7778.181818	4934.545455	7025.454545
5352.727273	8070.909091	6356.363636	5352.727273	5520
5018.181818	7861.818182	7443.636364	3596.363636	6858.181818
5394.545455	8614.545455	7234.545455	4056.363636	6356.363636
4850.909091	8070.909091	5394.545455	2885.454545	4683.636364
5227.272727	9869.090909	8238.181818	2634.545455	5185.454545
4850.909091	7778.181818	6774.545455	2969.090909	4850.909091
5687.272727	9952.727273	7820	2885.454545	4474.545455
5185.454545	10120	7276.363636	2885.454545	5352.727273
5812.727273	6230.909091	5394.545455	2425.454545	3805.454545
5645.454545	5185.454545	6481.818182	2634.545455	4976.363636
6356.363636	7694.545455	5603.636364	3136.363636	2090.909091
5729.090909	8656.363636	6356.363636	4600	1463.636364
6230.909091	8070.909091	5060	7067.272727	2007.272727
5520	7987.272727	5687.272727	7360	3303.636364
6189.090909	9158.181818	4683.636364	6523.636364	1881.818182
4600	8781.818182	5101.818182	7067.272727	2550.909091

7401.818182	6565.454545	4014.545455	3889.090909	2843.636364
7067.272727	7945.454545	4516.363636	6356.363636	2550.909091
6732.727273	8614.545455	3094.545455	7192.727273	3220
8321.818182	7234.545455	3721.818182	5436.363636	2592.727273
7527.272727	7150.909091	3094.545455	5770.909091	3220
8238.181818	7903.636364	3470.909091	6398.181818	4558.181818
5603.636364	6649.090909	3261.818182	5561.818182	3972.727273
6858.181818	7192.727273	3429.090909	6189.090909	2843.636364
7109.090909	6481.818182	2550.909091	6021.818182	3805.454545
9409.090909	7401.818182	2676.363636	6440	3261.818182
8740	7820	2509.090909	6063.636364	1547.272727
7485.454545	8907.272727	2592.727273	6356.363636	6314.545455