Team Control Number

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Problem Chosen

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**2017**

**MCM/ICM**

# Safety , Efficiency and Satisfaction


**Summary Sheet**

In order to improve passenger throughput and reduce variance in wait time, we search for bottlenecks, optimize the process of security check and evaluate the impact made by cultural differences on our model through sensitivity analysis.

To figure out bottlenecks in security check process, from literature [1], we introduce the average number of tokens, which can represent the level of blocking. Then, according to the security check process, we establish a **Generalized Stochastic Petri Net Model** to calculate out the average number of tokens. In order to figure out the passenger flow volume and transitions in Generalized Stochastic Petri Net Model, we build a **Queue model**, which analyzes the queues in front of ID check entrance and the queues waiting to enter the security check channel. Combining with the data in table given by the problem, we figure out transitions. Then, we get that passenger flow volume is **6.58 per minute** through the queue model. By creating a corresponding **MC network**, we can figure out the average number of tokens in each place. By comparing the average number of tokens, we find that bottlenecks are **the process of preparing for check, process of open-box inspection and the process of waiting for baggage**.

According to the solving for bottlenecks in task A, we figure out four improvements: **strengthening the training for security agents, adding X-ray scanners in each channel, adding baggage sorting workers and adding pre-check channels.** Thus, we can solve problem of the passenger flow volume and the variance in waiting time. In order to verify the effectiveness of these improvements and get concrete values, we build a **multi-objective programming model.** To begin with, we determine three indexes: variance of waiting time $D$, bearable passenger flow volume $Q$ and average waiting time $T$. We use $Q$ and $T$ to measure passenger throughput and use $D$ to measure the difference of passenger waiting time. Because of the independence among each index, we use the method of traversing search to solve for four groups of transition $t_i$ in the condition of meeting the optimization of four indexes. Then, we determine the optimal group of $t_i$ through AHP. The specific improvements are **doubling the training for security agents, adding one X-ray scanner in each channel, adding two baggage sorting workers and adding two pre-check channels.**

Then, we take the values of $t_i$ we get from the multi-objective programming model into the Generalized Stochastic Petri Net Model for checking. We get the average number of tokens in each place and find these numbers decrease obviously, which means our model is well checked.

Later, by analyzing the influence made by cultural differences, we find that difference in safety consciousness, privacy and individual efficiency may have impacts on the probability of carrying banned items, the process of pat-down inspection and preparing time for security check respectively. Then, we make sensitivity analyses on the multi-objective programming model with the change of each index. We find that the probability of baggage carrying banned items caused by lack of safety consciousness has the greatest impact on our models.

Finally, we write a recommendation for security managers on the basis of our models, including some globally applicable suggestions and suggestions for specific cultures.



**Keywords: Queue model  Petri Net multi-objective programming  cultural differences**

# Contents

# I. Introduction

## 1.1 Background

The past 20 years witnessed the booming terrorism, due to which the security standards of international airports have been upgraded obviously. As a result, it would take passengers more time to pass through airport security check. The issues of long queues appeared throughout the world

Recently, the U.S. Transportation Security Agency (TSA) has come up with some methods such as hiring new agents, deferring non-essential agent training and encouraging people to sign up for Pre-Check. These methods could ease queues to a great extent. However, the extra cost of implementing the new measures is unclear. What's more, with the rise in the number of people paying for TSA Pre-Check, some Pre-Check passengers complain that the Pre-Check lines become longer and longer. These methods can't solve the problem fundamentally. In addition, the high variance in checkpoint lines is also a big issue all around the world. Because of this, passengers can't make a proper plan before their arrival at airports. Hence, it's urgent for security managers to think of methods to minimize inconvenience to passengers in the basis of keeping the high security standard.

## 1.2 Our Work

- Analyze the process of security check in airport to determine the method for studying security process.
- Analyze the data given in the table and clear the meaning of each data.
- Build a queuing theory model to study the queue of the ID check entrance and security check channel.
- Build a network model to allow us to study the whole process of security check channel.
- Apply the data given in the table in to the model and get the solution of the passenger flow volume and bottleneck.
- Identify relevant indexes, build the model and propose some recommendations to improve airport security.
- Analyze the impact of cultural differences on the model, find the influence factor and analyze their sensitivity to obtain their influence on the model.
- Evaluate our model and extend our model to airports in various regions.

Here is a **flow chart** to demonstrate our thinking in the paper:

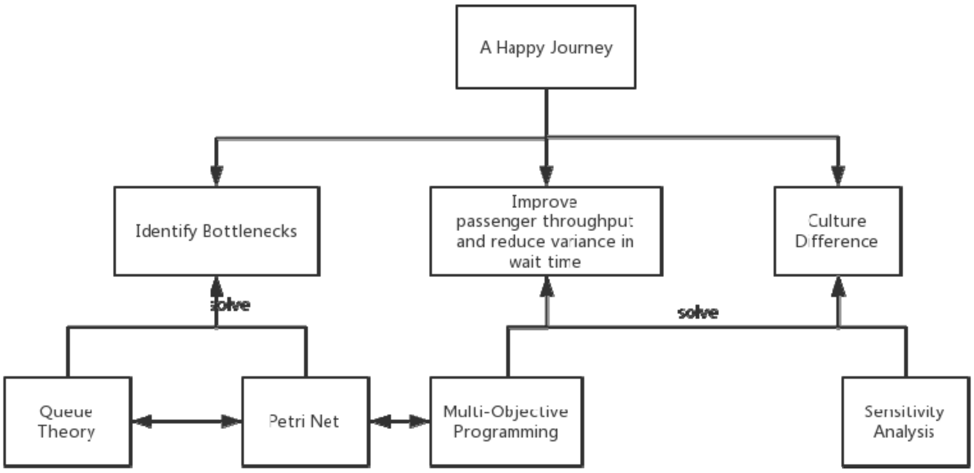


**Figure 1.Structure Of Our Work**

# II. Symbols and Definitions

| Symbols | Meanings |
|---|---|
| $Q$ | passenger throughput |
| $s_i$ | The number of tokens |
| $t_i$ | time of transition |
| $v_i$ | rate of initiation |
| $\lambda$ | average arrival rate of passengers |
| $\mu$ | average arrival rate of passengers |
| T | The average waiting time |
| D | The variance of waiting time |
| $\beta_1$ | Improve the service level |
| $m_1$ | Increase the number of baggages sorting staffs |
| $m_2$ | Increase the number of security agents and X-ray machines |
| $n$ | Increase the number of pre-check entrance |

# III. Assumptions

To simplify our model, we make the following basic assumptions:

## 3.1 Basic Assumptions

- After ID check, passenger choose the shortest line to queue up.
- No one jump a queue.
- The number of passengers arriving at ID check entrance in unit time is subject to poisson distribution
- The equipments of security check operate normally

## 3.2 Assumptions for the Given Table

- The service time of Milimeter Wave Scanner is equal to the service time of metal detector
- The **ID check time for TSA pre-check is column C in the given table**
- The **ID check time for reqular check is column D in the given table**

# IV. Model

## 4.1 Model and Solution for Task 1

### 4.1.1 The M/M/C Multi-Server Queue System

In the M/M/C multi-server queue system, we use $c$ to represent the number of servers, $n$ to represent the number of passengers, and $\mu$ to represent the average arrival rate of passengers. Besides, we assume that the average service rate of servers $\mu$ is same and each server is independent of the others.

As for the whole system, when $n$ is less than $c$, the new average service rate we can get is $n\mu$ .And, when n is equal or greater than $c$, the new average service rate we can get is $c\mu$ .

When the system is stable, we can assume that the switching-in rate is equal to the switching-out rate. Then, we can get the state transfer equations[7]:

$$\begin{cases} \lambda P_0 = \mu P_1 \\ \lambda P_{n-1} + (n+1)\mu P_{n+1} = (\lambda + n\mu)P_n & 1 \le n < c \\ c\mu P_{n+1} + \lambda P_{n-1} = (\lambda + c\mu)P_n & n \ge c \end{cases}$$

Then we can derive $p_n$ via the equations as follows:

$$P_0 = \left[ \sum_{k=0}^{c-1} \frac{1}{k!} \left( \frac{\lambda}{\mu} \right)^k + \frac{1}{c!} \cdot \frac{1}{1-\rho} \cdot \left( \frac{\lambda}{\mu} \right)^c \right]^{-1} \quad \textbf{(1)}$$

$$P_n = \begin{cases} \frac{1}{n!}(\frac{\lambda}{\mu})^n * P_0 & 1 \le n < c \\ \frac{1}{c!c^{n-c}}(\frac{\lambda}{\mu})^n * P_0 & n \ge c \end{cases} \tag{2}$$

Besides, the length of the queue can be expressed as:

$$L_q = \sum_{n=0}^{\infty}(n-c)P_n = \frac{(c\rho)^c \rho}{c!(1-\rho)^2}P_0 \tag{3}$$

As for the Multi-Server Queue System, we use $c_1$ to represent the number of the Regular checkpoints and use $c_1^{'}$ to represent the number of the Pre-Check checkpoints. Besides, we use $n_1$ and $n_1^{'}$ to represent the number of passengers. We use $\lambda_1$ and $\lambda_1^{'}$ to represent the average arrival rate of passengers. We use $\mu_1$ and $\mu_1^{'}$ to represent the average service rate.

Then, we can get the model of passenger flow volume:

$$Q = \frac{M}{T} \tag{4}$$

$$M = Lq_1 + Lq_2 + L;$$

Where, M represents the number of passengers in security check area;T represents the average waiting time.

There may be congestion at the ID check entrance, or at the security check entrance, and vice versa, since the queues at the front of the ID check entrance and the security check entrance are different. So in this paper, we set up two queuing sequences. We consider the queuing process of passengers at ID check entrance be queuing sequence 1 and the queuing process of passengers at security check entrance be queuing sequence 2. The analysis is shown in the part of model solution.

### 4.1.2 Generalized Stochastic Petri Net Model

A Petri net (also known as a place/transition net) is one of several mathematical modeling languages for the description of distributed systems.

Passengers are classified into two groups: Pre-check passengers and regular passengers. Pre-Check passengers and their bags go through the same screening process. The only difference is that a few modifications are designed to speed up the screening process. Therefore, we can think that each lane in Zone B is same except for some parameters. Thus, we focus on a lane in Zone B first.

Based on the theory of Petri net, we could establish a model for the process of security checks. With the help of this Petri net, we could figure out the average number of **token** and the **rate of transition** in each place. Therefore, we could determine the cause of bottlenecks.

By introducing two parameters, $t$ : time of transition and $v$ : rate of initiation, we can get the Generalized Stochastic Petri Net Model, which is as follow :

$$GSPN = (S, T, F, M_0, v)$$

Where, $S$ is a finite set of places; $T$ is a finite set of transitions; $F$ is a set of directed arcs; $M$ is the network marks; $v$ is the parameter of initiation rate.

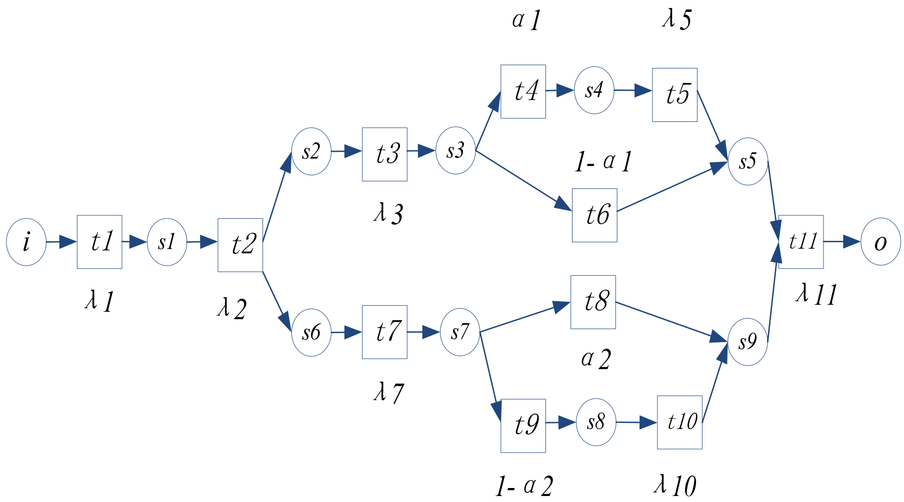


**Figure 2.Generalized Stochastic Petri Net**

In this **figure 1**, *i* represents the input place; o represents the output place; $s_i$ represents places for $i = 1, 2, ..., 9$ ; $t_i$ represents transitions for $i = 1, 2, ..., 11$ .

More specifically， $s_1$ represents the passengers who is preparing to enter Zone B; $s_2$ represents the belongings waiting for X-ray screening; $s_6$ represents the belongings which is under X-ray screening; $s_4$ represents the suspicious belongings ; $s_5$ represents the belongings waiting to be taken; $s_6$ represents the passengers waiting to process through a millimeter wave scanner or metal detector; $s_7$ represents the passengers who has finished the process of scanning by a millimeter wave scanner or metal detector; $s_8$ represents passengers who need to receive a pat-down inspection by a security officer in Zone D; $s_9$ represents the passengers waiting for their belongings;

$t_1$ represents the process of inspecting identification and boarding documents; $t_2$ represents the process of preparing for the security check; $t_3$ represents the process of passengers' belongings being screened by X-ray machines; $t_4$ represents that the belongings is found suspicious; $t_5$ represents the process of the open-baggage inspection ; $t_6$ represents that the belongings is unsuspicious; $t_7$ represents the process of passenger passing through a millimeter wave scanner ; $t_8$ represents that the millimeter wave scanner does not have an alarm; $t_9$ represents that the millimeter wave scanner has an alarm; $t_{10}$ represents the process of pat-down inspection; $t_{11}$ represents the process of passenger walking towards Zone C ;

We can isomorphism the **Markov Chain Net**:

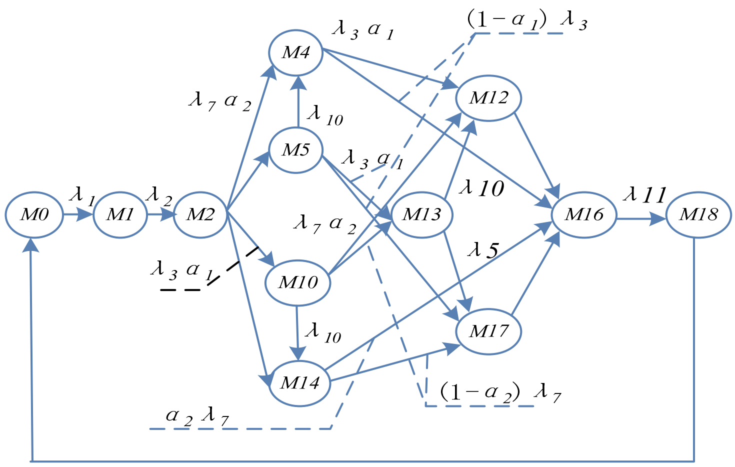


**Figure 3. Markov Chain Net**

In this MC Net Model, the set of places can be expressed as:

$$S=\{s_1,s_2,...,s_9\}$$

Similarly, the set of places transitions can be expressed as:

$$T=\{t_1,t_2,...,t_{11}\}$$

Then, we determine rate of initiations according to transitions. The set of the rate of initiation can be expressed as:

$$V=\{v_1,v_2,...,v_{11}\}$$

Since $t_4$, $t_6$, $t_8$ and $t_9$ are judgment processes, we can regard these processes as transient processes. Thus, we get the set of transient transitions:

$$T_b=\{t_4,t_6,t_8,t_9\}$$

From the figure, we can assume that the probability of $t_4$ happening is $\alpha_1$ and the probability of $t_9$ happening is $\alpha_2$. According to the Probability Equation of Complementary events, the probability of $t_6$ happening is $1-\alpha_1$ and the probability of $t_8$ happening is $1-\alpha_2$.

Because some transient transitions is instant, we can get substantial existence in MC net: $M_i=\{M_0,M_1,M_2,M_4,M_5,M_{10},M_{12},M_{13},M_{14},M_{16},M_{17},M_{18}\}$

According to the probability of each state $X=(x_0,x_1,\cdots,x_{18})$ and Density matrix $Q$, we can get equations:

$$\begin{cases}\overline{X}\cdot\overline{Q}=\overline{0}\\ \sum x_i=1\end{cases} \quad \textbf{(5)}$$

From the probability of $Mi$, we can get the **average number of token** $\overline{u}(s)$ and **the rate of transition** $U(t)$:

$$\overline{u}(s)=\sum_j j\cdot P[M(s)=j] \quad \textbf{(6)}$$

$$U(t)=\sum_{M\in E} P[M] \quad \textbf{(7)}$$

Finally, we can identify bottlenecks via these models.

### 4.1.3 The Solution of Task 1

**I. The solution of passenger flow volume**

According to **Eq.(4)**, we can get the passenger flow volume:

$Q$=6.58 per min

**II. Calculation for transition time and initiation rate**

◆Calculation for $t_1$ and $t_2$

In this section, we only focus on Regular-Check checkpoints. We would figure out $t_1$ and $t_2$ on the basis of queuing theory.

First, we make an analysis on the queue waiting for the inspection of identification and boarding documents. Then, we get an M/M/2 queuing theory model. From Column *A* and *B* in table, we get $\frac{1}{\lambda_1}=12.6s$, which represents the average time interval between the arrival time of two passengers. Then, we get that $\lambda_1$ is equal to 0.0794; From the Column *C* and *D* in table, we obtain $\frac{1}{\mu_1}=12.9s$, which represents the average check time. Then, we get that $\mu_1$ is equal to 0.0980; According to the Equation in **Generalized Stochastic Petri Net Model**, we obtain $L_q=1.23$, $W_q=11.32$, and $Ws=20.52$ .Because $t_1$ is equal to $Ws$, we get $t_1=20.52$.

Then, we make an analysis on the queue waiting for security checks. First, we simplify this problem. Assuming that the number of security checkpoints is $C_2$, we have:

$$C_2\lambda_2=q_1 \tag{8}$$

Where, $q_1$ represents the number of passengers flowing out ID check in unit time.

When the queue in front of ID check entrance reaches equilibrium states, $q_1$ is equal to $\lambda_1$

The system for the queue in front of the ID check is a M/M/3 system. Then, we can get $\lambda_2$ for security checkpoint. If the number of security checkpoints is 3, $\lambda_2$ is equal to 0.0794. The calculation of $\mu_2$ is following:

$$\mu_2=\frac{1}{t_7+t_{pre}} \tag{9}$$

$t_{pre}$ is the process of people's Preparation, Here $t_{pre}$ equals 25s

$$t_2=w_q+t_{pre} \tag{10}$$

From the Column E in table, we get that $\mu_2$ is equal to 0.028.According to the Equation in **Queuing theory model**, we obtain $L_q=1.36$ and $W_q=17.1$. Besides, $t_2$ represents the sum of waiting time $W_q$ and preparing time. We obtain $t_2=42.1s$.

Calculation for regular checkpoints is same, and specific data is listed in Appendix.

◆Calculation for the rest time

According to Column F and G in the excel table given by the problem, we can calculate out the average X-ray inspection time for Pre-check checkpoints and regular checkpoints respectively. In addition, we can calculate out the time that the passengers spend on millimeter wave scanning according to Column E in the table given by the problem; Finally, according to Column H, we find that the maximum time that from passengers' putting baggage on the X-ray machine to take away it is 68 seconds. So we come into conclusion that the time should be about 68 seconds, if need to check out the box. Here we take the value as 70s. Then, we can get a table about the performance of the security check:

**Table 1:Statistics for security checks**

| Process | $t_i$ | time/s | $v_i$ | Value of $\lambda$ |
|---|---|---|---|---|
| ID Check | $t_1$ | 22.7 | $v_1$ | 2.64 |
| Preparation for security check | $t_2$ | 42.1 | $v_2$ | 1.43 |
| Screening for belongings | $t_3$ | 3.7 | $v_3$ | 16.22 |
| Find of suspicious belongings | $t_4$ | ------ | $\alpha_1$ | 0.1 |
| Open-baggage inspection | $t_5$ | 70 | $v_5$ | 0.86 |
| No find of suspicious belongings | $t_6$ | ------ | 1-$\alpha_1$ | 0.90 |
| Millimeter wave scanning | $t_7$ | 11.6 | $v_7$ | 5.17 |
| No alarm | $t_8$ | ------ | 1-$\alpha_2$ | 0.90 |
| Having alarm | $t_9$ | ------ | $\alpha_2$ | 0.05 |
| Pat-down inspection | $t_{10}$ | 30 | $v_{10}$ | 2 |
| Walking towards Zone C | $t_{11}$ | 10 | $v_{11}$ | 6 |

**III. Calculation for Petri net**

After converting the Petri net to the Markov network, we can get the probability of each state is $X=\left(x_0,x_1,\cdots,x_{18}\right)$.

According to the Density matrix $Q$, we can get:

$$\begin{cases}\vec{X}\cdot\vec{Q}=\vec{0}\\ \sum x_i=1\end{cases}$$

Then, we obtain the equations as follow:

$$\begin{cases}2.64x_0-6x_{18}=0\\ 2.64x_0-1.43x_1=0\\ 1.43x_1-16.22x_2=0\\ 0.0517x_2-8.45x_4+2x_5=0\\ 5.12x_2-10.45x_5=0\\ 0.42x_2-6.03x_{10}=0\\ 8.03x_2+0.86x_{10}-5.17x_{14}=0\\ 0.42x_4+0.0517x_{10}+2x_{13}-0.86x_{12}=0\\ 0.42x_5+5.12x_{10}-2.86x_{13}=0\\ 8.03x_5+0.86x_{13}+5.12x_{14}-2x_{17}=0\\ 8.03x_4-0.86x_{12}+0.0517x_{14}-6x_{16}+2x_{17}=0\\ \sum x_i=1\end{cases}$$

Then, we use MATLAB to figure out the probability of each state, average number of token in each plane and the rate of transition. The results are following in **Table 2:**

**Table 2:probability、average number of token and rate of transition**

| Probability $x_i$ | $x_i$ | Place | Average number of token | Transition | Rate of transition |
|---|---|---|---|---|---|
| $x_0$ | 0.18060 | $i$ | 0.18060 | $t_1$ | 0.18060 |
| $x_1$ | 0.30961 | $s_1$ | 0.30961 | $t_2$ | 0.30961 |
| $x_2$ | 0.03501 | $s_2$ | 0.03501 | $t_3$ | 0.03501 |
| $x_3$ | 0.00427 | $s_3$ | ------ | $t_5$ | 0.10201 |
| $x_4$ | 0.01715 | $s_4$ | 0.01360 | $t_7$ | 0.03501 |
| $x_5$ | 0.00244 | $s_5$ | 0.10201 | $t_{10}$ | 0.07881 |
| $x_6$ | 0.01824 | $s_6$ | 0.03501 | $t_{11}$ | 0.34122 |
| $x_7$ | 0.00688 | $s_7$ | ------ | | |
| $x_8$ | 0.05478 | $s_8$ | 0.07881 | | |
| $x_9$ | 0.07949 | $s_9$ | 0.34633 | | |
| $x_{10}$ | 0.21206 | $o$ | 0.07947 | | |
| $x_{11}$ | 0.07947 | | | | |

**IV. Analysis**

From the table above, we can know that the average numbers of token in place $s_1$, $s_5$, $s_9$ are relatively large. In this case, the average number of token can be viewed as the degree of obstruction. That is to say, it is easier to form bottlenecks in the process of preparing to enter Zone B, the process of open-baggage inspection and the process that the passengers wait for their belongings.

**Here are the problems we find in the process of security check:**

◆The lack of security checkpoints leads to passengers preparing and waiting for a long time before the security, making the rate security checking slow and affecting traffic and security checking rate.

◆When baggage is found suspicious, the lack of security agents results in that time for checking baggage is too long. Besides, the lack of advanced equipment is also a cause.

◆Space for packaging after security checking is so small that passengers are so crowded with long line, thus results in low efficiency.

## 4.2 Model and Solution for Task 2

From task A, we know that it is easier to form bottlenecks in the **process of preparing to enter Zone B, the process of open-baggage inspection** and **the process that the passengers wait for their belongings**. In order to magnify the flow of passengers and reduce the variance of passengers' waiting time, we propose some improvements:

◆ Improve the service level of each security agent or the quality of the equipment to shorten the time-consuming difference passengers spent on millimeter-wave security inspection

◆ Increase the number of security agents and X-ray machines to improve the check efficiency of open-checking

◆ Increase the number of baggages sorting staffs to make sure that passengers can get back their baggages faster after checking and finish the security check earlier.

◆ Increase the number of pre-check entrance to reduce the waiting time for pre-check passengers.

Then, we model these improvements:The following table displays our model:

**Table 3:Model of Improvements**

| Method | Model |
|---|---|
| Improve the service level | $\beta_1$ |
| Increase the number of baggages sorting staffs | $m_1$ |
| Increase the number of security agents and X-ray machines | $m_2$ |
| Increase the number of pre-check entrance | $n$ |

In order to get the most optimal passenger flow volume and the minimum variance of waiting time, we need to optimize each stage of security check to reduce the time it cost and narrow down the difference. Then, we obtain the relations of time:

$$t_1^{'}=\frac{t_1}{\beta_1^2},\quad t_2^{'}=\frac{t_2}{2n}$$
$$t_5^{'}=\frac{t_5}{m_2},\quad t_{11}^{'}=\frac{t_{11}}{m_1} \tag{11}$$

Where, $t_1$ represents the process of ID check;$t_2$ represents the process of preparing for the security check;$t_5$ represents the process of the open-baggage inspection ;$t_{11}$ represents the process of passenger walking towards Zone C ;the rest of $t_i$ is same with $t_i$ in task A.

Since the optimizing of the variance of waiting time and the passenger flow volume is related to the proposal above, we build a Multi-objective nonlinear programming model. First, we determine the indexes: variance of waiting time $D$, bearable passenger flow volume $Q$ and average waiting time $T$. We use $Q$ and $T$ to measure passenger throughput and use $D$ to measure the difference of passenger waiting time.

▲**The variance of waiting time *D*:**

Generally, we use variance to measure the difference of waiting time. As for the whole waiting time, the Equation of variance is as follow:

$$D=\sum T_i^2 P_i - E(T)^2 \tag{12}$$

In the whole process of security check, for one passenger, $t_1,t_2,t_7,t_{10},t_{11}$ are independent of one another. For baggages, $t_1,t_2,t_3,t_5,t_{11}$ are independent of one another. Thus, we get:

$$D=\sum D(t_i) \tag{13}$$

By measuring all the processes from passenger's perspective, the variation of $t_1$ and $t_2$ is depended by numbers of passengers and the efficiency of airport security checkpoints; The variation of $t_5$ and $t_{10}$ is judged by whether passengers taking abnormal items or not. If so, the time consumed is also affected by efficiency of dealing with the emergencies.

Meanwhile, the other time is mainly decided by the airport work efficiency. In order to reduce differences of the congeners' waiting time, ensuring the stability of the security check must be the first step. Thus, under certain condition of the stability of security check's efficiency, divergences of waiting time are mainly resulted from the processes of $t_1$, $t_2$ , $t_3$ and $t_4$.

As for the processes of $t_5$ and $t_6$, the factor which affects the variance of waiting time could also be understood as whether passengers carry suspicious belongings or not. Since they are small probability events, the variance of waiting time in these two processes can be expressed as:

$$D(t_5,t_6)=t_5^{\,2}\cdot\alpha_1+(1-\alpha_1)\cdot\alpha_2\cdot t_6^{\,2}-\left[t_5\cdot\alpha_1+(1-\alpha_1)\cdot\alpha_2\cdot t_6\right]^2 \qquad \textbf{(14)}$$

The number of passengers is an uncontrollable factor. While, the processes of $t_1$ and $t_2$ are both related to the number of passengers. Thus, we apply queuing theory to the analysis of this problem:

$$D(t_1)=\sum_{k=m}\left[\left(k-c_1+1\right)\cdot\frac{1}{\mu_1}\right]^2\cdot P_k-\overline{w_1}^{\,2}$$

$$D(t_2)=\sum_{k=m}\left[\left(k-c_2+1\right)\cdot\frac{1}{\mu_2}\right]^2\cdot P_k-\overline{w_2}^{\,2} \qquad \textbf{(15)}$$

$$D=D(t_1)+D(t_2)+D\left(t_5,t_6\right)$$

**▲The bearable passenger flow volume *Q***

The definition of passenger flow volume is the number of passengers who enter or exit one place in unit time. In this case, if the number of passengers is greater than the bearable passenger flow volume *Q*, the security checkpoint will be crowded. In order to make the flow smooth, for any queuing system, the number of passengers,$\lambda$, that arrival at the queue should be less than the service rate, which can be expressed as:

$$\lambda\le c\cdot\mu$$

That is to say, the service rate of the entrance and the security check channel should meet this requirement. Thus， we try our best to satisfy the expression:

$$c_1\mu_1=c_2\mu_2$$

At the same time, The bearable passenger flow volume *Q* can be expressed as:

$$Q=(c_1\mu_1,c_2\mu_2) \qquad \textbf{(16)}$$

**▲The average waiting time *T***

According to Petri net figure， *T* represents the time of the process of security check. Unless any emergency happening, the passing time between passengers and baggage is almost equal. By calculating the mean value, we can get the Equation of average time *T*:

$$T=\alpha_1(t_1+t_2+t_3+t_5+t_{11})+(\alpha_2-\alpha_1\alpha_2)(t_1+t_2+t_7+t_{10}+t_{11})+\frac{2t_1+2t_2+t_3+t_7+2t_{11}}{2}(1-\alpha_1-\alpha_2+\alpha_1\alpha_2) \qquad \textbf{(17)}$$

**▲Determination of resource constraints**

We can assume that the estimated cost is *R*, the cost of opening ID check channel is $R_1$, and the number of channels we want to open is $\Delta c_1$. Besides, we assume that the cost of opening security check channel is $R_2$, and the number of channels we want to open is $\Delta c_2$. Besides, we set the cost of training a staff to $r_1$, the ratio of improved efficiency to $\beta_1$ and the number of staff to $m_1$. In addition, we assume that the average cost of updating equipments is $r_2$, the number of equipments is $m_2$ and the ratio of improved efficiency is $\beta_2$. Then, we can get the boundary conditions.

Since the maximum time between getting boarding check and completion of security check is about two hours, we can assume that the maximum value of $t_i$ is 7200s.

According to the normalized four indexes and the time $t_i$ for each stage, we can get a **multi-objective nonlinear programming model** :

$$
\begin{aligned}
&\min D \\
&\min Q^{\circ} \\
&\min T \\
&s.t.\begin{cases}
D, Q^{\circ}, T, \eta, t_i > 0 \\
D = \sum_i D(t_i) \\
Q = q(c, \mu) \\
T = \sum_j f(t_j) \\
t_1 + t_2 = \dfrac{k \cdot \rho \cdot (c\rho)^5}{c!(1-\rho)^2} \\
\Delta c_1 R_1 + \Delta c_2 R_2 + r_1 m_1 + r_2 m_2 - R \le 0 \\
t_i < 7200 \\
c < 10
\end{cases}
\end{aligned}
\qquad \textbf{(18)}
$$

Where, the value of $\rho = \dfrac{\lambda}{c \cdot \mu}$ can be obtained from the **Queuing theory model** in task *A*. As a matter of convenience, we use the minimum value of the reciprocal of passenger flow volume $Q^{\circ}$ to represent the maximum value of passenger flow volume *Q*.

We use $\Delta c_1$ and $\Delta c_2$ to represent the number of servers which is supposed to open. By solving this multi-objective programming model, we could find proper $t_1$, $t_2$, $t_5$ and $t_{11}$. Then, we can figure out the number of security agents needed to add and the service level needed to be raised.

**Solving for task B**

From task A we have already known the value of each $t_i$ in both Pre-check channel and regular channel, which make no influence on passenger flow volume and the variance of waiting time. The data are as follows:

**Table 4:Data of Waiting Time**

| Process | TSA pre-check | | Regular check | |
|---|---|---|---|---|
| Screening for belongings | $t_3$ | 3.7 | $t_3$ | 7.1 |
| Find of suspicious belongings | $t_4$ | 0 | $t_4$ | 0 |
| No find of suspicious belongings | $t_6$ | 0 | $t_6$ | 0 |
| Millimeter wave scanning | $t_7$ | 11.6 | $t_7$ | 11.6 |
| No alarm | $t_8$ | 0 | $t_8$ | 0 |
| Having alarm | $t_9$ | 0 | $t_9$ | 0 |
| Pat-down inspection | $t_{10}$ | 30 | $t_{10}$ | 30 |

First, we take regular check as an example. Since the indexes are independent in this problem, we can use the method of multi objective programming to solve it hierarchically. By using traversing search algorithm, we search $t_1$, $t_2$, $t_5$, $t_{11}$ for the optimal solution of objective. Specific steps are as follows:

**Traversing search algorithm:**

**Step1**: Because the number of added servers and cost for equipments and staffs are constant, we search for the solution of $t_1$ and $t_2$ , under the circumstance where $Q$ is the maximum, in units of 0.1s.

**Step2**: On the basis of step 1, we can get the minimum solutions of the variance of waiting time by searching $t_5$ and $t_{11}$.

**Step3**: From former steps, we can determine the range of $t_1$, $t_2$, $t_5$ and $t_{11}$. Then, we can search the solution which has the minimum time and the maximum efficiency in this range.

**Step4**: By using AHP, we can figure out the weight of each index. Then, bring $t_i$ ,which meet the condition, into score equation. We can get the solution group which scores the fewest points.

First, through traversing search algorithm, we find that the range of $t_1$, $t_2$, $t_5$, $t_{11}$ are same within the margin of error. Four groups of solution are as follows:

**Table 5: Four groups of solution**

| *solution* | $t_1$ | $t_2$ | $t_5$ | $t_{11}$ |
|---|---|---|---|---|
| $group_1$ | 16.6 | 29.7 | 50.3 | 7.5 |
| $group_2$ | 17.7 | 27.3 | 49.1 | 7.9 |
| $group_3$ | 17.2 | 27.8 | 51.3 | 7.7 |
| $group_4$ | 18.6 | 26.8 | 50.6 | 8.7 |

According to AHP, we figure out the weight of four transitions $t_1$, $t_2$, $t_5$, $t_{11}$ for the four indexes:

**Table 6: the weight of four transitions**

Relations of weight factors (C to D)

| *D* | $t_1$ | $t_2$ | $t_5$ | $t_{11}$ |
|---|---|---|---|---|
| $t_1$ | 1 | 1/4 | 1/2 | 1/5 |
| $t_2$ | 4 | 1 | 7 | 8 |
| $t_5$ | 2 | 1/7 | 1 | 9 |
| $t_{11}$ | 5 | 1/8 | 1/9 | 1 |

Relations of weight factors (C to Q)

| *Q* | $t_1$ | $t_2$ | $t_5$ | $t_{11}$ |
|---|---|---|---|---|
| $t_1$ | 1 | 1/4 | 1/5 | 1/8 |
| $t_2$ | 4 | 1 | 6 | 9 |
| $t_5$ | 5 | 1/6 | 1 | 3 |
| $t_{11}$ | 8 | 1/9 | 1/3 | 1 |

Relations of weight factors (C to T)

| *T* | $t_1$ | $t_2$ | $t_5$ | $t_{11}$ |
|---|---|---|---|---|
| $t_1$ | 1 | 1/8 | 1/2 | 1/5 |
| $t_2$ | 8 | 1 | 6 | 7 |
| $t_5$ | 2 | 1/6 | 1 | 4 |
| $t_{11}$ | 5 | 1/7 | 1/4 | 1 |

According to the weight from schematic layer to criterion layer, we can get the weight matrix for each part. Then, by using MATLAB, we get the weight matrix after sorting:

$$A=\begin{bmatrix}0.2710\\0.2135\\0.3253\\0.1902\end{bmatrix}$$

Thus, we can get the **score equation**:

$$v = 0.2710t_1 + 0.2135t_2 + 0.3253t_5 + 0.1902t_{11} \quad \textbf{(19)}$$

Then, we find that group2 has the highest score. Hence, the solution in group2 $t_i = \{17.7, 27.3, 49.1, 7.9\}$ is the optimal solution. We can get ***D*$_{min}$=253.54, *T*$_{min}$=85.43*s* ,*Q*$_{max}$=55 per minute** for regular channel and ***D*$_{min}$=120.99, *T*$_{min}$=66.56s** , ***Q*$_{max}$=49 per minute** for pre-check channel.

According to ***Eq*.(11)**, we can determine the service level should be increased by double. To be specific, we need to **strengthen the training for security agents**. Second, we need **two more pre-check channels**. Third, we need **add 2 baggage sorting workers** to each X-ray scanner. Besides, we should **add one X-ray scanner** in each channel.

## 4.3 The Checking of Model

According to the solution of this model, we can get optimized $t_i$ in each stage. First, we calculate the average number of tokens and the rate of transition for regular check channel. Combining with the result we get in task *A*, we can obtain a figure as follows:

**Table 7:Data of Checking**

| Probability $x_i$ | $x_i$ | Place | Average number of token | Transition | Rate of transition |
|---|---|---|---|---|---|
| $x_0$ | 0.13052 | $i$ | 0.23052 | $t_1$ | 0.23052 |
| $x_1$ | 0.20112 | $s_1$ | 0.10112 | $t_2$ | 0.10112 |
| $x_2$ | 0.02069 | $s_2$ | 0.02069 | $t_3$ | 0.02069 |
| $x_3$ | 0.00331 | $s_3$ | ------ | $t_5$ | 0.10485 |
| $x_4$ | 0.02418 | $s_4$ | 0.02872 | $t_7$ | 0.02069 |
| $x_5$ | 0.00524 | $s_5$ | 0.14485 | $t_{10}$ | 0.10400 |
| $x_6$ | 0.03857 | $s_6$ | 0.02069 | $t_{11}$ | 0.54329 |
| $x_7$ | 0.02016 | $s_7$ | ------ | | |
| $x_8$ | 0.05965 | $s_8$ | 0.10400 | | |
| $x_9$ | 0.10297 | $s_9$ | 0.16262 | | |
| $x_{10}$ | 0.33528 | $o$ | 0.49654 | | |
| $x_{11}$ | 0.05830 | | | | |

From this table, the maximum value of average number of token and rate of transition are **0.23052** and **0.49654** respectively. From the theory of Petri net, we can know that the numbers of tokens are small in each place $p_i$. That is to say, our model succeeds in improving passenger throughput and reducing variance in wait time. Thus, we can regard our model reasonable. The same conclusion we can draw for TSA pre-check channel. Tables are shown in **Appendix.**

# V. Sensitivity Analysis

Different parts of the world have their own cultural norms. In this section, we do sensitivity analysis to analyze how these factors have impact on our model. First, we need to determine the main factors :

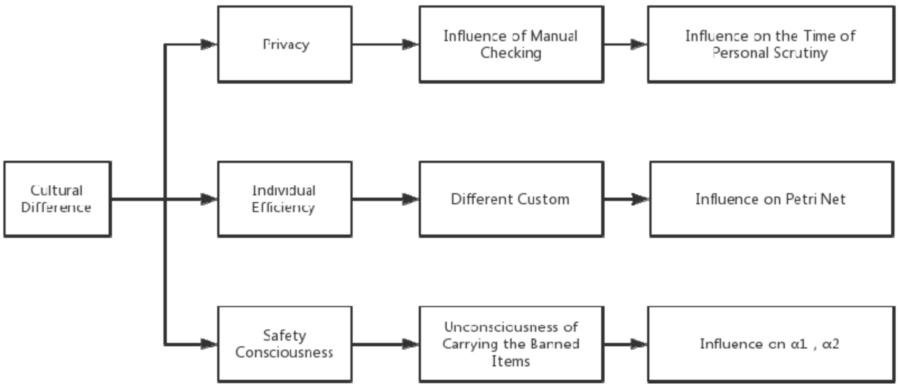


**Figure 4.Simple Analysis of Cultural Difference**

**●Culture difference in consciousness**

In some areas, there are chances that passengers carry banned items unconsciously, which lead to the difference of $\alpha_1$ and $\alpha_2$. Analyze the sensitivity in this case, with $\alpha_2$ equal to 0.01, and we can get:

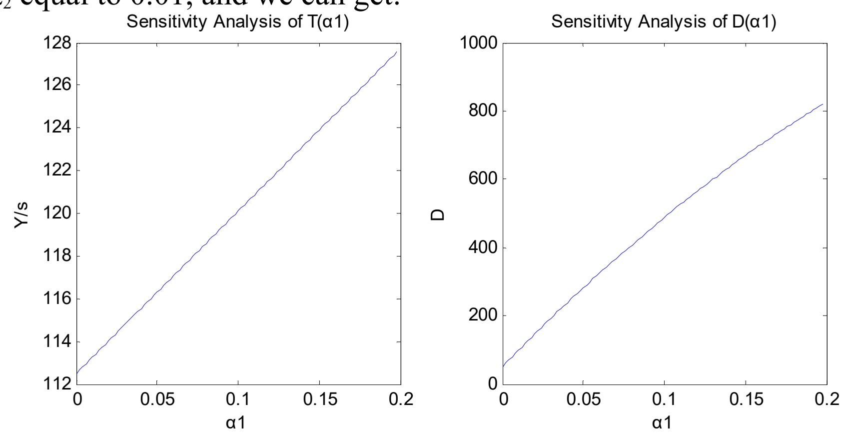


**Figure 5.Sensitivity Analysis of T/D about $\alpha_1$**

**Table 8:Data of Sensitivity Analysis about $\alpha_1$**

| $\alpha_1$ | 0.05 | 0.075 | 0.1 | 0.125 |
|---|---|---|---|---|
| $T$ | 116.3071 | 118.282 | 120.106 | 122.082 |
| $D$ | 282.2 | 392.3776 | 488.2 | 585.6376 |

From Figure 3 , we can get the relationship between $T$ ,D and $\alpha_1$ are almost linear respectively. However, the rate of change for D is higher and, as $\alpha_1$ increases, the rate of rise becomes slower. From **Table 8** , in the condition that initial $\alpha_1$ is equal to 0.05,we can conclude that $T$ is increased by 2.9%、5.3%、7.6% every time $\alpha_1$ is increased by 50%. However, $D$ is increased by 39%、73%、107% every time $\alpha_1$ is increased by 50%.Thus, we can draw a conclusion that $\alpha_1$ has a positive correlation with the variance.

In the condition that $\alpha_1$ is equal to 0.05, the sensitivity analysis for $\alpha_2$ is as follows:

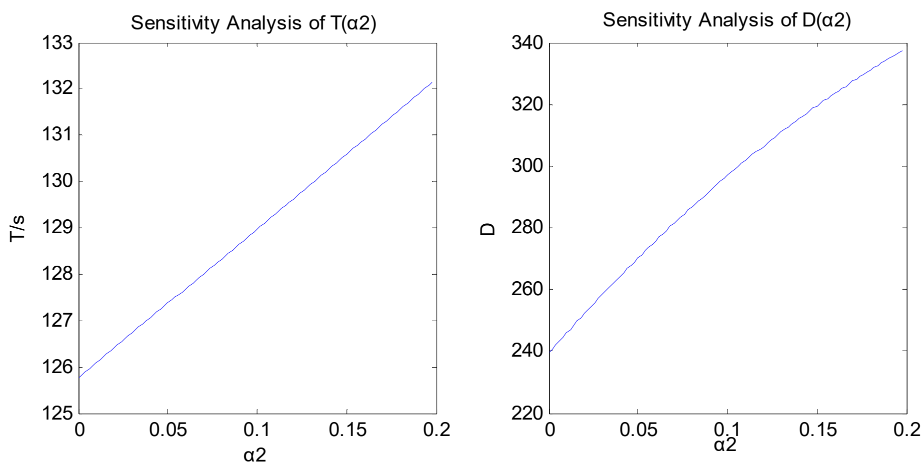


**Figure 6.Sensitivity Analysis of T/D about $\alpha_2$**

**Table 9:Data of Sensitivity Analysis about $\alpha_2$**

| $\alpha_1$ | 0.01 | 0. 02 | 0. 03 | 0.04 |
|---|---|---|---|---|
| $T$ | 126.075 | 126.397 | 126.720 | 127.043 |
| $D$ | 245.924 | 252.235 | 258.384 | 264.370 |

From **Figure 6** , we can get the relationship between $T$ ,D and $\alpha_2$ are almost linear respectively. However, the rate of change for D is higher and, as $\alpha_1$ increases, the rate of rise becomes slower. From **Table 9** , in the condition that initial $\alpha_2$ is equal to 0.01,we can conclude that $T$ is increased by 0.3%、0.5%、0.79% every time $\alpha_1$ is increased by 100%. However, D is increased by 2.4%、5.1%%、7.9% every time $\alpha_1$ is increased by 100%.Thus, there is a positive correlation between them and the rate of change is higher for *D. Compared to* $\alpha_1$ , the change is relatively small.

Hence, considering the affects made by suspicious belongings for our model, the main factor is $\alpha_1$

- **Culture difference in individual efficiency:**

Comparing to other passengers, Chinese passengers are known for prioritizing individual efficiency. In our model, the preparing time would be less in $t_2$ stage, which leads to the rise of security channel service rate. Analyze the sensitivity in this case, by changing parameter $t_{pre}$ and we can get:

**Table 10:Data of Sensitivity Analysis about $t_{pre}$**

| $t_{pre}$ | 25 | 20 | 15 |
|---|---|---|---|
| $T$ | 116.307125 | 91.949 | 71.938 |
| $D$ | 282.2 | 274.32 | 262.12 |

The initial $t_{pre}$ is 25s and it is decreased by 20% every time. As a result, $T$ is decreased by 20.81%、38% and $D$ is decreased by 2.8%、7.0%.There is a positive correlation between them and $t_{pre}$ has greater impact on $T$, which is appropriate for the situation.

- **Cultural difference in privacy**

Under different cultural background, passengers have different attitude towards privacy. The passengers who value privacy will increase difficulty in pat-down inspection. Thus, we do sensitivity analysis for $t_{10}$:

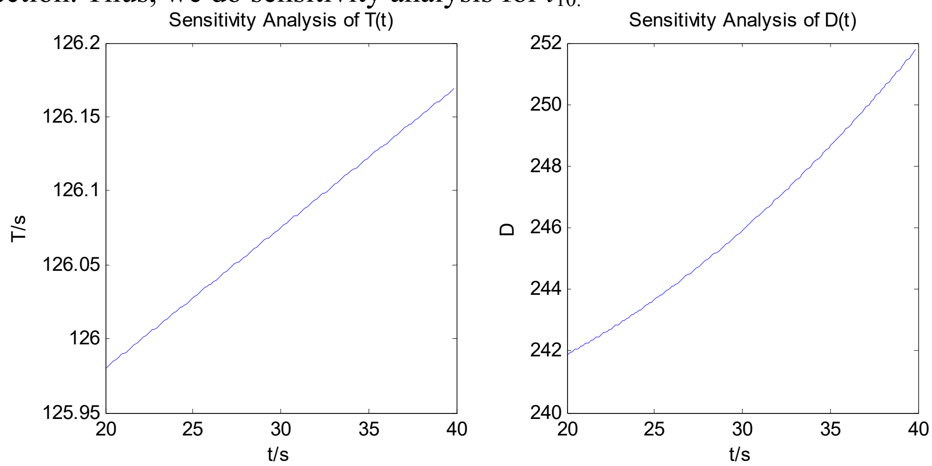


**Figure 7.Sensitivity Analysis of T/D about $t_{10}$**

**Table 11:Data of Sensitivity Analysis about $t_{10}$**

| $t_{10}$ | 20 | 25 | 30 | 35 | 40 |
|---|---|---|---|---|---|
| T | 125.98 | 126.03 | 126.08 | 126.12 | 126.17 |
| D | 241.88 | 243.67 | 245.92 | 248.65 | 251.78 |

From **Figure 7**, we can see that $T$ and $t_{10}$ are linear. And, there is a positive correlation between them. Besides, D and t10 are also has a positive correlation. From the **Table 11**, we can conclude that the change of $T$ and $D$ is small. That is to say, this difference has little impact on our model.

From the analysis above， We find that the probability of baggage carrying banned items caused by lack of safety consciousness has the greatest impact on our model. As for this problem, the mainly method of improvement is to reduce the time for open-baggage inspection. We can solve this problem by raising the service level of security check agents and adding the number of baggage checkpoints.

# VI. Strengths and Weaknesses

## 6.1 Strengths

- We take the synchronism of the security check process into account and build a Petri net model which can accurately find the bottlenecks.
- The results of the simulations according to our model are well reliable and accurate. It is possible to us to apply our model to many airport security ports.
- We model the method of improvements and combine it with the time for each stage of the security check process, which can get the specific plan.
- We take different cultural difference into account.

## 6.2 Weaknesses

- The optimization algorithm employing in our model has bad portability.
- Our model has some limitations. For example, we don't consider the influence of the sudden emergencies.

# VII. Suggestions

## Recommendations paper

Dear sir/Madam:

Along with the improvement of requirements for airport security check, the time passengers cost becomes longer and longer. Because of the uncertainty of baggage amount and cultural differences, the waiting time would be increasingly hard to manage, which would result in bad experiences. Hence, in order to figure out the bottlenecks in the process of security check, we build a model, which is globally applicable, to improve the process of security check. Now, we propose policy and procedural recommendations for your reference:

First, we analyze the table and the figure which describes the process of security check. Then, we build a Petri net model based on queuing theory to figure out bottlenecks. We find that bottlenecks are the process of preparing for check, process of open box inspection and the process of waiting for baggage. According to our model, recommendations are as follows:

◆ Improve the service level of each security agent or the quality of the equipment to shorten the time-consuming difference passengers spent on millimeter-wave security inspection

◆ Increase the number of security agents and X-ray machines to improve the check efficiency of open-checking

◆ Increase the number of baggages sorting staffs to make sure that passengers can get back their baggages faster after checking and finish the security check earlier.

◆ Increase the number of pre-check entrance to reduce the waiting time for pre-check passengers.

Then, we found that cultural difference has a large impact on the process of security check according to our research. For example, millimeter wave scanners are wildly used in security check in Western countries. While, in Eastern countries, manual security check is the main way. In order to increase efficiency, we need to respect cultural differences to avoid conflicts. The recommendations are as follows:

◆Set up special channels for passengers from different areas to select.

◆Make brochures in different languages to guide the passengers from different areas.

All of above are the bottlenecks and solutions we find through our model. We are trying our best to provide some suggestions for improving the method of security check, which can be globally applicable. However，due to time constraints, our research has its limits. But we really hope our recommendations can help you in some way and we also hope you can get a more efficient and secure environment in your airport.

Thanks for your attention

# VIII. References

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# IX. Appendix

## ●The Data of TSA Pre-check

**Table 12: Data1 of TSA Pre-check**

| Process | $t_i$ | time/s | $v_i$ | Value of $\lambda$ |
|---|---|---|---|---|
| ID Check | $t_1$ | 22.7 | $v_1$ | 2.64 |
| Preparation for security check | $t_2$ | 42.1 | $v_2$ | 1.43 |
| Screening for belongings | $t_3$ | 3.7 | $v_3$ | 16.22 |
| Find of suspicious belongings | $t_4$ | ---- | $\alpha_1$ | 0.1 |
| Open-baggage inspection | $t_5$ | 70 | $v_5$ | 0.86 |
| No find of suspicious belongings | $t_6$ | ---- | $1-\alpha_1$ | 0.90 |
| Millimeter wave scanning | $t_7$ | 11.6 | $v_7$ | 5.17 |
| No alarm | $t_8$ | ---- | $\alpha_2$ | 0.05 |
| Having alarm | $t_9$ | ---- | $1-\alpha_2$ | 0.95 |
| Pat-down inspection | $t_{10}$ | 30 | $v_{10}$ | 2 |
| Walking towards Zone C | $t_{11}$ | 10 | $v_{11}$ | 6 |

**Table 13:Data2 of TSA Pre-check**

| Probability $x_i$ | $x_i$ | Place | Average number of token | Transition | Rate of transition |
|---|---|---|---|---|---|
| $x_0$ | 0.17562 | $i$ | 0.17562 | $t_1$ | 0.17562 |
| $x_1$ | 0.32422 | $s_1$ | 0.32422 | $t_2$ | 0.32422 |
| $x_2$ | 0.02168 | $s_2$ | 0.02168 | $t_3$ | 0.02168 |
| $x_3$ | 0.00107 | $s_3$ | ------ | $t_5$ | 0.11306 |
| $x_4$ | 0.00584 | $s_4$ | 0.02020 | $t_7$ | 0.02168 |
| $x_5$ | 0.00582 | $s_5$ | 0.11306 | $t_{10}$ | 0.08133 |
| $x_6$ | 0.03472 | $s_6$ | 0.02168 | $t_{11}$ | 0.34046 |
| $x_7$ | 0.01331 | $s_7$ | ------ | | |
| $x_8$ | 0.06218 | $s_8$ | 0.08133 | | |
| $x_9$ | 0.07727 | $s_9$ | 0.34046 | | |
| $x_{10}$ | 0.20101 | $o$ | 0.07727 | | |
| $x_{11}$ | 0.07727 | | | | |