Author's Commentary: The Toll Plaza Problem

Michael Tortorella
Managing Director
Assured Networks, LLC
Middletown, NJ
assurenet@verizon.net

Introduction

Entrants choosing the 2017 Toll Plaza Problem presented judges with an interesting conundrum. While some papers were quite good, none of the papers reaching final judging contained all of the features that are expected of Outstanding papers.

Every paper had something missing, and different papers were missing different things. Some papers considered the required extensions but did not include any sensitivity analysis. A few papers incorporated a sensitivity analysis but did not include a realistic assessment of strengths and weaknesses.

As a rule, the first thing judges look for is whether a paper satisfies all the clearly-stated requirements of the problem. The problem-setting committee felt that this year's problem statement was especially clear about required elements; indeed, considerable pains were taken by the committee to make sure that the problem statement was clear and simple.

It was surprising, then, that so many papers failed to address the requirements. Many papers did address most of the requirements, but none addressed all of the requirements.

Remember, also, that there are also general MCM requirements of all entrants. These are not usually stated again in the problem statement; but good solutions will always address sensitivity analysis, assessment of model strengths and weaknesses, and explicit model construction and execution.

We urge coaches and faculty advisers to train teams to list for themselves all the requirements that can be identified from a problem statement, as

The UMAP Journal 38 (3) (2018) 319–322.

well as the general MCM requirements, and from time to time during the solution process to re-examine the list to make sure everything is being considered appropriately.

Read the Problem Statement!

There are also specific issues concerning this particular problem. A prominent one is that many teams did not read the problem statement carefully. They appeared to think that the problem asked them to solve for the proper number B of tollbooths. This was in fact an MCM problem in 2005. This year's problem statement clearly indicates that B (and the number L of highway lanes) are given, and the solution (the optimal merge-area design) is to be expressed as a function of B and L. There is no need to determine B (indeed, there is not enough information given in the problem to determine B), and many teams—even some who went on to deal with the real problem at hand—wasted time on this.

The takeaway for coaches and faculty advisers is to train their teams to read the problem carefully and thoroughly so that mistakes like this do not happen.

Similarly, even though the problem statement explicitly cautions against this, many teams engaged in extensive performance analysis of existing merge-area designs when the problem instead calls for design improvements. Indeed, this is fundamentally a multi-criterion optimization problem: Given B, L, and the various other constraints, what is the best merge-area shape and size, considering safety, throughput, and cost?

Perhaps it is too much to expect that undergraduates provide a full solution to this optimization in detail. However, a crucial point for the teams is that they need to penetrate the problem enough to be able to determine what the linchpin of the problem is—in this case, the multi-criterion optimization.

A major purpose of the MCM is to foster model-building skill in the participants. One of the key components of that skill is the ability to discern what approaches are germane to the problem and may lead to fruitful results, and what approaches are simply not aligned with the problem needs and should be avoided.

Cite References Appropriately

Teams need to learn to cite references appropriately. The judges found many reports that used the same illustration (an aerial photo of a toll plaza on the Garden State Parkway) without any attribution. Similarly, many reports included pieces of mathematics that were drawn from other sources without citation of those sources.

There is no shame in acknowledging in print that you are using the work of others to build your own solution on. Indeed, the opposite is true: It is unethical to take the work of others and incorporate it into your own work without giving proper credit through a formal and focused citation.

Write a Succinct Summary

The summary or abstract must contain a capsule summary of the results. Most teams use the summary as a brief description of the work that they did or the methods that they used. *This is not satisfactory*, and reports whose summary did not include any results were downgraded.

The purpose of the summary is to entice the reader into wanting to read more of your report. This is better done by including some results from the solution, because doing so arouses the reader's curiosity: How did they get that? Does this make sense? A summary with some results and no description of method is far superior to a summary that describes methods used but no results obtained. Coaches and faculty advisors, please take note of this important point. The vast majority of papers did not ever get to the point of saying what their solution was in specific, numerical terms: "This is the shape of the merge area, and it is so many meters long and so many meters wide" (or other appropriate size description). Someone who is going to build a merge area based on the recommendations in a report needs these dimensions; without them, no building can take place.

Coaches and faculty advisers are urged to train their teams to ask what a solution should look like. In this case, specific numbers are required. Many teams used a cellular automaton (CA) model. However, a CA model necessarily operates on one pair (B, L) at a time. Solution of this problem required consideration of a whole space of possible (B, L) values to find the best merging area shape and size for each B and L.

Indeed, a significant challenge was contained in the fact that once a merge-area shape and size is chosen for a particular B and L, the merge area must remain suitable under changing conditions of traffic flow, weather, and other variables—here is where sensitivity analysis comes into play. It's not clear how you would perform this kind of analysis with a single CA solution. Many teams considered one or two of the three performance criteria (throughput, safety, and cost); but very few considered all three together, as was required. The problem statement was explicit about this, as well as about the need to avoid simply doing a performance analysis of an existing merge-area shape, and rather determining a better or best shape considering the requirements. We again urge coaches and faculty advisers to train their teams to read problem statements closely and summarize the conditions and requirements for themselves, as an aid to staying on track during the solution and report-writing processes.

Smaller Issues

Some smaller issues include:

- Sloppy notation: Many teams used L for length (of something). But L is specified in the problem statement as the number of travel lanes exiting the merge area—you can't use it for anything else.
- Too few teams used diagrams appropriately in their report. A solution to the problem can be described without a diagram, but it is immeasurably easier for a reader if some simple diagram is included. Teams are advised to make liberal use of diagrams not only for their own understanding but also for clearer communication in their reports.

Conclusion

The Toll Plaza Problem posed interesting challenges for teams and judges alike. I hope that coaches and faculty advisers can take away some useful advice from this summary, while judges continue to concentrate on fairness and the difficult task of discerning the true meaning of reports written by nonprofessionals under time pressure.

About the Author

Mike Tortorella is Managing Director of Assured Networks, LLC, and Adjunct Professor of Systems Engineering at Stevens Institute of Technology. He retired from Bell Laboratories as a Distinguished Member of the Technical Staff after 26 years of service. He holds the Ph.D. degree in mathematics from Purdue University. His current interests include stochastic flow networks, network resiliency and critical infrastructure protection, and stochastic processes in reliability and performance modeling. Mike has been a judge at the MCM since 1993 and particularly enjoys the MCM problems that have a practical flavor of mathematical analysis of social policy. Mike enjoys amateur radio, playing the piano, and cycling.