A self-power-supplied snowboard course

Abstract

Vertical air, air-time (AT) and degrees of rotation, et al are most important index in snowboarding. It is meaningful in designing a practical course to help the snowboarder improving their performance. In traditional half-pipe skateboarding the power provided to the athletes is just come from the slope and the initial kinetic energy. To overcome these weaknesses we design a new half-pipe in which the snowboarder can provide power for their own. In our model only the physical factors, air resistance, contact friction, et al are considered. We do not interest in the professional level or the psychological state of the athletes. We focus on how to improve the vertical air and the AT or the length of the path through which the snowboarder twisting in the air. To achieve these, we design an energy and velocity vector conversion plat (EVVCP) in the flat bottom of the half-pipe. It is a 'W' shape design. We conclude that the improving of the vertical air and the AT is mainly depends on the vertical jump capacity of the athletes. Generally, 40cm for males and 35 cm for females higher than their currently scores are achieved. And also the inadequacy of our design is discussed.

Keywords: Snowboarding, Vertical air, Resistance, Energy conversion

Contents

1. Introduction

1.1 Restatement of the Problem

Half-pipe snowboarding, a challenging, passionate and also aesthetic sport, has captured our sight recent years. The snowboard competitors are required to perform an acrobatic routine when they above the half-pipe (Figure 1). The sport is focused upon the aesthetic where the method by which athletes achieve the competitive purpose is of utmost importance. Until now, the sport has recently received little attention from sport science and has had little to do with scientific enquiry into elite performance. Although half-pipe snowboarding is judged by subjective measures, it has been documented that a strong relationship exists between a number of objective key performance variables such as air-time and degrees of rotation. Intuitively, the higher athlete leaving the edge the longer time they will have for performing an acrobatic routine such as rotation. It is of most importance to design a snowboard course to help the snowboarders improving their competitive level.

Figure 1[1]. Australian half-pipe snowboard athlete is in training. Breckenridge Colorado USA 2008. Image: Ben Alexander.

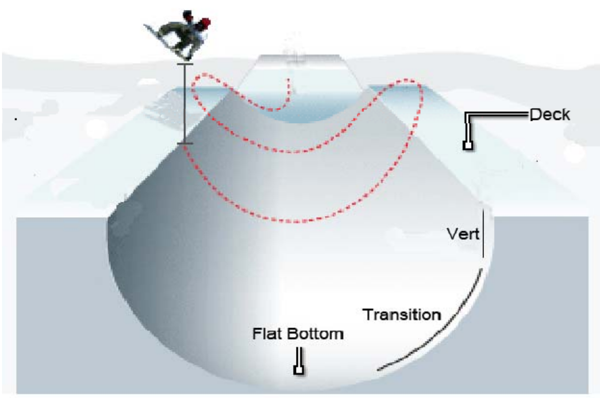

Figure 2. Schematic drawing of the half-pipe

A traditional half-pipe is shown in Figure 2. Usually, it has seven components: two decks, two verts, two transitions and a flat bottom. To overcome the resistance resulting from air and contact friction between snowboard and the surface of the half-pipe, the half-pipe has an inclination angle of 16º recommended by 47th international ski congress[2]. All the power provided to the athlete twisting in the air is produced in the inclination and the initial kinetic energy of the snowboarder. When skiing in the half-pipe the athletes could not accelerate themselves. Considering this weakness of the current half-pipe, we now focus on how to improve the snow course which will allow the athletes to accelerate themselves.

1.2 General Description of Our Idea

Our idea is schematic shown in Figure 3. A ball will stop on the collision point, when it dropping on the flat ground. All the gravitational potential energy (GPE) will convert into thermal energy, but without velocity vector conversion (Figure 3.a). When the same ball dropping on a concavity, the GPE will convert into kinetic energy mostly and the velocity vector is also converted.

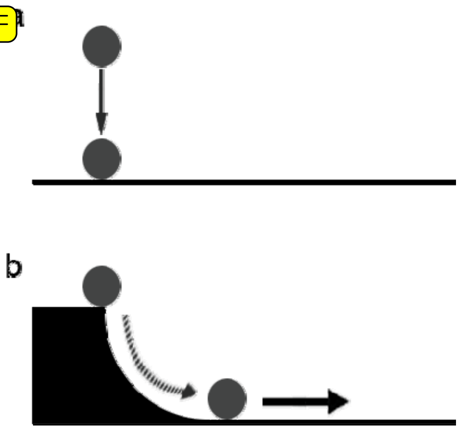

Figure 3. a. Only energy conversion, b. Both energy and velocity conversion

The same case could happen in snowboarding (Figure 4).

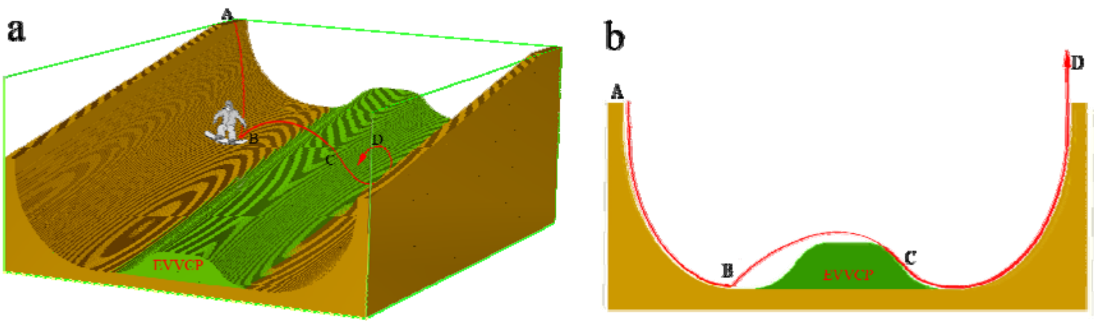

Figure 4. a. Three-dimensional image of our idea. b. One slice of figure 4.a in the Vertical direction

As is vividly depicted in Figure 4, the snowboarder enters the half-pipe at point A. Then he slides down along the transition part of the half-pipe. When he reaches the flat bottom (point B) of our new designed half-pipe, he could jump onto the EVVCP by which the inner energy of the athlete changing into kinetic energy. Because of the inclination angle of the half-pipe, the athlete could slide on the flat the EVVCP also. No matter he drops on the flat or the slope of the EVVCP (point C), he will get additional kinetic energy which would help him to reach a higher point D. When the athlete jumps into the air from point B, he will get more GPE. With the help of the EVVCP, some of the GPE convert into kinetic energy. Generally, the EVVCP helps the athlete accelerate himself.

2. Principles and Definitions

This section illustrates the basic mechanical equations and the global symbols used in this paper.

2.1 Some Physical Formulas Used in Our Model

The nature of the snowboarding dynamics[3] is a kinematics process obeying the Newtonian Mechanics. By analyzing the process (see section 4 for details), gravity, air resistance and contact friction dominant the kinematics. Respectively, the three force are described by

$$G = mg,$$

$$F_a = -kSv \text{ [4]},$$

$$F_\mu = -\mu F_N,$$

The meaning of each symbol is illustrated in 2.2.

2.2 Parameters Definition

All the parameters defined here will be used widely throughout this paper. Additional variables may be declared later, but will be confined to a particular section.

F refers to the net force acting on the snowboarder.

G refers to the gravity of the snowboarder.

m refers to the mass of the snowboarder.

g the acceleration of gravity. Obviously, $g = 9.8 m/s^2$.

R refers to all the resistance of the friction.

F_a refers to the air resistance.

F_μ refers to the contact friction between snowboard and the surface of the half-pipe.

Obviously, $\vec{R} = \vec{F}_a + \vec{F}_\mu$, and $\vec{F} = \vec{G} + \vec{F}_a + \vec{F}_\mu$.

k refers to the coefficient of air resistance, $k = 2.94 Ns/m^3$ [4].

S refers to the areas of windward side of the athlete. By actual measurement, $S \approx 0.6m^2$.

v refers to the velocity of the athlete.

μ refers to the friction coefficient of the contact surface between snowboard and the half-pipe. Approximately, μ is treated as an constant (see section 3 for detail) and $\mu \approx 0.04$ [5].

F_N refers to the pressure perpendicular to the surface of the half-pipe.

θ refers to the emergence angle between horizon and the velocity $\vec{v}$.

Throughout this paper, x is defined as the horizontal coordinate and y as the vertical one.

3. Assumptions

It is a reasonable assumption that the physical status of the half-pipe will not change no matter how the athlete slip on it. Further, it is a completely contact when the snowboard slip on the surface of the half-pipe. It is said that the snowboard is keeping deformation when it works. So the friction coefficient μ is assumed to be a constant.

We assume that the athletes are well skilled snowboarders. They can ensure completing all the actions we required.

Additional, the athletes are treated as rigid body. The areas of windward side of the athlete are constant.

4. Mechanical Analysis and Optimization

As described in section 1.2, the athlete sliding, jumping and twisting when performing. Three forces act on him[3]. They are gravity air resistance and contact friction. By analyzing the three forces, the differential equations of the kinematics will be obtained. Then the Trajectory equation will also be obtained. The force acting on the athlete is shown in Figure5.

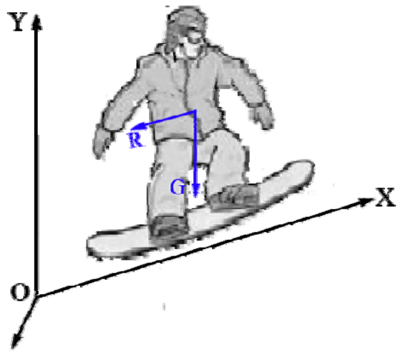

Figure 5. Force analyzing of the athlete.

The net force is

$$\vec{F} = \vec{G} + \vec{R} \\ = -mg\vec{j} - kS\vec{v} - \mu F_N \vec{e}_{N_\perp} \tag{1}$$

The $\rightarrow$ denotes the direction of each force.

Simple as the equation it is difficult to solve it in algebra, because of the complex and changeable force F_N. Fortunately, the force F_N is eliminated, when the athlete jumping into the air at point B and twisting in the air. Then, the net force become

$$\vec{F} = -mg\vec{j} - kS\vec{v} \tag{2}$$

Solving this equation, the parameters of the EVVCP will be optimized.

4.1 Projectile Motion with Air Resistance

The athletes are treated as rigid body. It is just like a projectile with air resistance [6, 7], when they moving in the air. So that, the net force is

$$\vec{F} = -mg\vec{j} - kS\vec{v} \tag{3}$$

Working with horizontal and vertical accelerations x and y respectively, we get the simple uncoupled pair of differential equations:

$$\ddot{x} = -kS\dot{x} \tag{4}$$

$$\ddot{y} = -g - kS\dot{y} \tag{5}$$

Integrating twice and noting that $x(0) = 0$, $y(0) = 0$, $x'(0) = v_0 \cos\theta$ and $y'(0) = v_0 \sin\theta$ we get

$$x(t) = \frac{1}{kS}\left[v_0 \cos\theta(1 - e^{-kt})\right] \tag{6}$$

$$y(t)=\frac{1}{(kS)^2}\left[-kStg+g+kSv_0\sin\theta-e^{-kSt}\left(g+kSv\ \sin\theta\right)\right] \tag{7}$$

The trajectory equation will be obtained when time (t) is eliminated between the $x(t)$ and $y(t)$ giving

$$y=x\tan\theta+\frac{g}{kS}\frac{x}{v_0\cos\theta}+\frac{g}{(kS)^2}\ln\left(1-\frac{kSx}{v_0\cos\theta}\right) \tag{8}$$

Obviously, two parameters affect the movement. By adjusting the variables we could find the optimized way by which the athlete emerge into the air, and also this will help us to optimize the snowboard course.

4.2 Optimize parameters of the EVVCP

As is depicted in Figure 6, the athlete jumps up between point B and B' and he drops on the EVVCP between point C and C'. Only in this way can he accelerate himself.

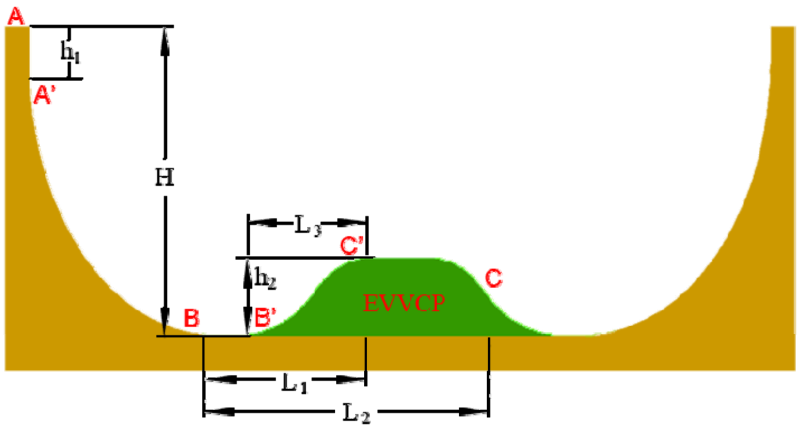

Figure 6. Parameters of the half-pipe

One rule must be obeyed when the athletes jumping is that the athletes jump between point B and B' and the dropping point must between C and C'. This rule determines the size of the EVVCP. In order to optimize L_1 and L_2, two parameters in equation (8) must be solved first, v_0 and θ. In fact, θ is determined by two the components of v_0. So it is of most importance to solve the parameter v_0.

We solve it by two steps.

4.2.1 Vertical component

In Figure 6, h_2 depends on the vertical jump capacity of the athletes. The table below provides a ranking scale for adult athletes[8].

Table 1. Vertical jump capacity of adult athletes

Rating	**Males (cm)**	**Females (cm)**
excellent	> 70	> 60
very good	61-70	51-60
above average	51-60	41-50
average	41-50	31-40
below average	31-40	21-30
poor	21-30	11-20
very poor	< 21	< 11

For skilled snowboarders, we choose 60cm for males and 50cm for females. Using the data illustrated in table 1, we can get the vertical jumping velocity of the athletes.

Using energy conservation

$$\frac{1}{2}mv_{0\perp}^2 = mgh_{\max} + \int_0^{h_{\max}} kSv(h)dh \tag{9}$$

and the first-order differential of $y(t)$

$$\dot{y} = \frac{1}{kS}\left[\left(g + kSv_{0\perp}\right)e^{-kSt} - g\right] \tag{10}$$

We find that

$$\frac{1}{2}mv_{0\perp}^2 = mgh_{\max} + \frac{S}{2k^2}\left[kv_{0\perp}\left(kv_{0\perp} - 2g\right) + 2g^2\ln\frac{g + kv_{0\perp}}{g}\right] \tag{11}$$

It is a coupling equation, because it has a form of $v_{\perp 0} \sim \ln v_{\perp 0}$. There is no algebraic solution of this equation. Now, taking $m = 75kg$, $g = 9.8m/s^2$, $k = 2.94Ns/m^3$ and $S = 0.6m^2$ into (11) , we get

$$h_{\max} = 0.00909\ln\frac{1}{3.33674 + v_{0\perp}} + 0.05061v_{0\perp}^2 + 0.00272v_{0\perp} + 0.01095 \tag{12}$$

The numerical solution of this equation is shown in Figure 7.

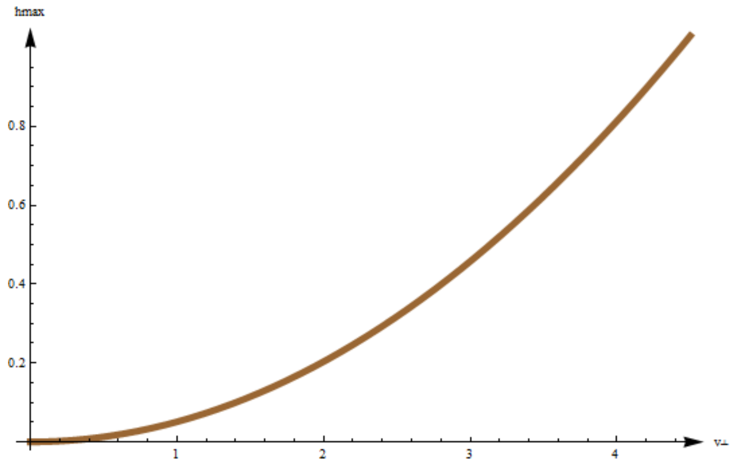

Figure 7. Numerical solution of this equation (12)

Reading from Figure 7 we get $v_{0\perp} = 3.44m/s$ corresponding to $h_{\max} = 60cm$ for males.

4.2.2 Horizontal component

In order to get an accurate solution of the horizontal component of the velocity, numerical calculation of equation (1) must be operated. There is also an approximate method to solve this problem. Let's consider two limitations of the horizontal component. If the inclination angle of the half-pipe is 0º and zero initial kinetic energy, the athlete will get minimum velocity.

$$\min\left(v_{0\parallel}\right) = \sqrt{2gH} \tag{13}$$

By similar analysis we get

$$\max\left(v_{0\parallel}\right) = \sqrt{2g\left(H + L\sin\alpha\right)} \tag{14}$$

L is the length of the half-pipe and α is the inclination angle of the half-pipe. The recommend value[2] is 120m and 16 º respectively.

And also using the data[2] recommend by 47th international ski congress. We get

$$\min(v_{0\parallel}) = 8.85m/s \tag{15}$$

And

$$\max\left(v_{0\parallel}\right) = 26.96m/s \tag{16}$$

Taking $v_{0\perp} = 3.44m/s$ (males), (15) and (16) into (8), note that $\theta = \tan v_{0\perp}/v_{0\parallel}$, we show the result in Figure 8.

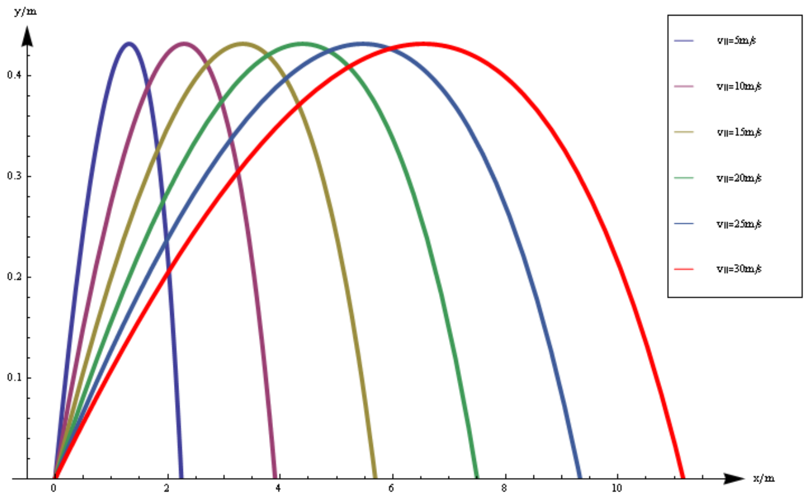

Figure 8. The path along which the athlete flying in the air (males).

The height of the peak is almost the same, though the horizontal velocity changing a lot. So it is reasonable that the height of the EVVCP keeping 40cm, $h_2 = 40cm$. Higher h_2 is, more difficulty will appear. Lower h_2 is, the athlete will get less kinetic energy. Figure 8 also gives the information that athletes must jump near point B' if they have less horizontal velocity on the bottom of the half-pipe and vice versa. Reading from Figure 8, the length of L_3 should not longer than 1m, corresponding to $v_{0||} = 5m/s$, $L_3 = 1m$. Otherwise, the athlete may hit the B'C' slope. Considering $v_{0||} = 30m/s$, the longest red line, in this case the athletes must jump from point B. Or they may hit the B'C' slope or jumping over point C. So, the length of L_1 should not longer than 7m, $L_1 = 7m$. Also from this line, we find that L_2 should not less than 11m, $L_2 = 11m$. Considering L_1, L_2 and L_3, the width of the flat top of the EVVCP is deduced to be 2.5m.

Another conclusion we conclude is that the increasing of vertical air of each male athletes is about 40cm.

The same calculation is also carried out for female athletes (Figure 9), $h_{max} = 50cm$ in equation (12).

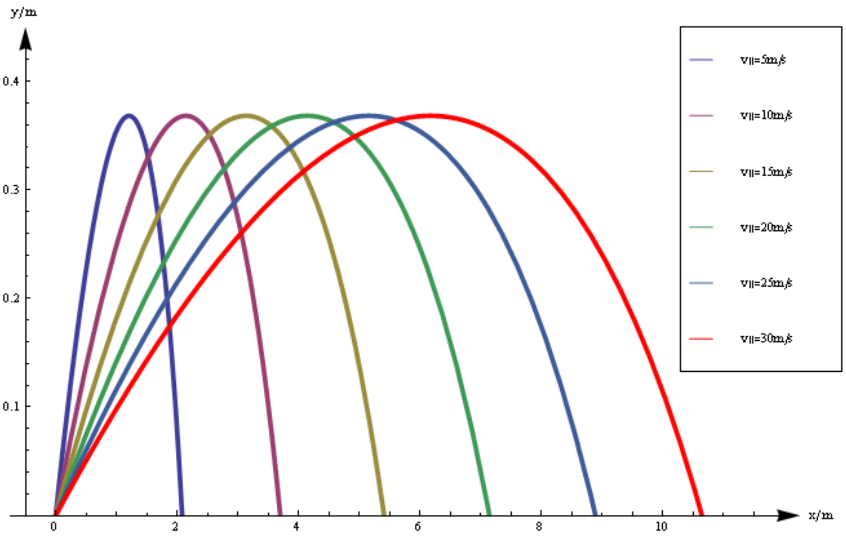

Figure 9. The path along which the athlete flying in the air (females)

Compare with Figure 8, we conclude that the height of the EVVCP for females will not higher than 35cm. And the other size of it will be the same as males.

4.3 Optimize parameters of the other part of the half-pipe

Although the EVVCP helps athletes accelerate themselves, other part of the half-pipe has room to be improved.

As discussed above, the half-pipe must be adaption to the EVVCP. There must be enough space at the bottom of the half-pipe. We conclude that the bottom of it should be 16.5m which agree with [2].

When the athletes twisting in the air, considering equation (8), the angle θ is an independent variable affects the motion.

Now, we keep the velocity being a constant. It is shown in Figure 10 that how the angle θ affecting on the motion.

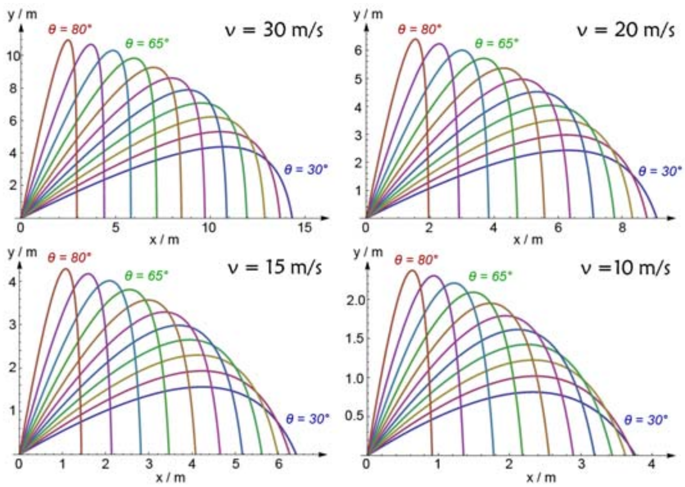

Figure 10. Relationship between the trace and angle θ

We can see that there is a specific value of the angle, $\theta \approx 65^{\circ}$, which optimizing the motion. However, when the athletes keep sliding down along the inclination of the half-pipe, the velocity parallel to the inclination is also keeping increasing. So the athletes must keep adjusting their emerging attitude to approach this optimized emerging angle.

5. Further discussion

Some further discussion illustrated in this section.

5.1 Influence of the Resistance

We define the influence function:

$$\phi = \phi_a + \phi_\mu$$

ϕ_a refers to the air resistance. We can estimate it with the help of Figure 8.

In Figure 8, when the athlete jumping vertically the height of the peak is 0.6m. Because of the horizontal component, the path of the athlete in the air enlarged. The air resistance consumes more kinetic energy. So the height of the peaks in Figure 8 is smaller than 0.6m. So

$$\phi_a = \frac{0.6-0.4}{0.6} \approx 33.3\%$$

ϕ_μ refers to contact friction between snowboard and the surface of the half-pipe.

With the recommended data of [2], we calculate that the maximum velocity if about 30m/s, without considering all the resistances. See Figure 10 for details. The height of the vertex is about 10m with $v = 30m/s$. Actually, the highest scores achieved is about 5m[9]. So

$$\phi = \frac{10-5}{10} = 50\%$$

Then

$$\phi_\mu = \phi - \phi_a = 16.7\%$$

The results show that the air resistances play main role in snowboarding.

5.2 Other Notes in Our Design

Though we discussed how the athletes jumping in section 4, there is no strict rule actually. To say the least, it doesn't matter that the athletes drops between point B' and point C'. They will also slip upon the EVVCP, however, collision loss happened in this case.

Also, we would like point out that why we let the athletes jumping rather than slipping onto the EVVCP. When slipping onto it, all the resistance acts on the people. When jumping, just air resistance affects the movement.

5.3 Disadvantages of Our Model

Some disadvantages still appeared, though higher scores will be achieved when the athletes using our model.

On the one hand, the jumping point is difficult to choose. It needs much more training in practical. In fact, jumping before point B' and then dropping on point C is the optimized path.

On the other hand, two different EVVCPs (difference in height) for males and females respectively are designed in this paper. This also increases the difficulties in practical. This question will be eliminated when we choose the same height of the EVVCPs.

Thirdly, the EVVCP may increase the possibility of injury. For example, when the athletes collide with the point C' with a badly angle, most of the kinetic energy will loss. This is a hard collision from which the injury happened.

Fortunately, all the three problems would be conquered through professional training. One of the most important indexes of the athletes in our model is the vertical jumping capacity.

6. Conclusion

We have designed a new 'W' shape half-pipe to help the snowboarders improving their professional capacity. We conclude that the improvements are mainly depends on the vertical jump capacity of the athletes. The higher they jump the more scores they will receive. Generally, 40cm for males and 35 cm for females higher than their currently scores are achieved. It is said that our model helps the athletes getting about more 10% scores than their currently performance.

7. References

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