1 All Homeworks for Math 199

Qualifying Exams Practice Problems https://math.unm.edu/graduate/past-qualifying-exams-real-analysisreal-variables

2 HW 1

Motivation for students who get caught up in all the formalism: https://www.maa.org/what-i-learned-by-teaching-real-analysis

2.1 4.1 Left Column

a. 1, 2, 3
c. 7, 8, 9
e. 1, 2, 3
g. 3, 4, 5
i. 2, 3, 4
k. NBA
m. 2, 3, 4
o. 0, 1, 2
q. 16, 17, 18
s. 1/2, 1, 2
u. NBA
w. $\frac{\sqrt{3}}{2}$, 1, 2

2.2 4.7

a. Prove: If S $\subseteq$ T, then $inf(T) \leq inf(S) \leq sup(S) \leq sup(T)$

Inequality 1: Assume inf(S) < inf(T) (opposite is true). This means there exists an element $s \in S$ s.t. $s < t \forall t \in T$.

Inequality 2 (inf(S) $\leq$ sup(S)):

Inequality 3 (sup(S) $\leq$ sup(T)):

b.

2.3 4.9

Prove $inf(S) = -sup(-S)$ (Found in corollary 4.5):
By proof of two statements:

1. $-s_0 \leq s$ for all $s \in S$ where $s_0 = sup(-S)$
Proof: Because $s_0 = sup(-S)$ then $s_0 \geq -s$ for all $s \in S$. Therefore, $-s_0 \geq s$ for all $s \in S$. $\square$

2. $t \leq s$ for all $s \in S$, then $t \leq -s_0$ (proves s_0 is greatest lower bound)
Proof: Because $s_0 = sup(-S)$ then $s_0 \geq -s$ for all $s \in S$. Therefore, $s_0 \leq t$ for all t s.t. $t \geq -s$. Therefore, $-s_0 \geq t$ for all t s.t. $t \leq s$. $\square$

Helpful Resources
"Supremum and Infimum Proofs"
http://www.tricki.org/article/I_have_a_problem_about_a_supremum_or_an_infimum
"Real Analysis Defintion Based Proofs" http://www.tricki.org/article/I_have_a_problem_to_solve_in_real_analysis_and_I_do_not_believe_that_a_fundamental_idea_is_needed

2.4 8.2 R

Determine the limits of the sequence and prove the claims: To prove that the limit of a sequence exist we need to use the defintion of a limit.

a. 0, because first degree polynomial over second degree polynomial
b. 7/3, because first same degree polynomial, with the ratio of the leading coefficients describing the sequences behavior for big n
c. 4/7, same reasoning as b
d. 2/5, same reasoning as b
e. 0, 1/n defines behavior of the seqeuence over large n, more so than the periodic behavior of the sine function

2.5 8.3 R

Given lim $s_n = 0$, prove lim $\sqrt{s_n} = 0$.

2.6 8.4 R

Let t_n be a bounded sequence and s_n be a sequence with limit 0.

Prove: $lim(s_n t_n) = 0$

2.7 2.3.3 A

Prove: Squeeze Theorem, if $x_n \leq y_n \leq z_n$ $\forall n \in \mathbb{N}$ and $limx_n = limz_n = l$, then $limy_n = l$

2.8 7 by Ryan

a. Prove: Convergent sequences are Cauchy sequences.

b. Prove: Cauchy sequences are bounded.

c. Show: $liminfs_n = limsups_n$ i. ii. iii.

2.9 2.5.8

a. Find examples of sequences with zero, one, and two peak terms. Then Find sequence with infintiely many peak terms that is not monotone. b. Show: Every sequence contains monotone subsequence. Explain: How this fact furnishes new proof of Bolzano-Weierstrass Thm.

2.10 9.12

a. Show: If L = $lim|\frac{s_n+1}{s_n}| < 1$, then $lims_n = 0$. b. Show: If $L > 1$, then $lim|s_n| = +\infty$

2.11 11.8

Prove: $liminf s_n = -limsup(-s_n)$ for every sequence (s_n)

2.12 All Links

"Big Picture of Taking Real Analysis" `https://www.maa.org/what-i-learned-by-teaching-real-analysis`
"Supremum and Infimum Proofs" `http://www.tricki.org/article/I_have_a_problem_about_a_supremum_or_an_infimum`
"Real Analysis Defintion Based Proofs" `http://www.tricki.org/article/I_have_a_problem_to_solve_in_real_analysis_and_I_do_not_believe_that_a_fundamental_idea_is_needed`