

0.1 The Completeness Axiom

Big Idea: The reals has no "gaps"

4.4 Completeness Axiom: Every nonempty subset S of \mathbb{R} that is bounded above has a least upper bound. In other words, $\sup S$ exists and it is a real number.

4.5 Corollary: Every nonempty subset S of \mathbb{R} that is bounded below has a greatest lower bound $\inf S$.

4.6 Archimedean Property: If $a > 0$ and $b > 0$, then for some positive integer n , we have $na > b$

4.7 Denseness of \mathbb{Q} : If $a, b \in \mathbb{R}$ and $a < b$, there is a rational $r \in \mathbb{Q}$ such that $a < r < b$.

0.2 Limits of Sequences

Definition: A function with domain $\{n \in \mathbb{Z} : n \geq m\}$ where m is 1 or 0

Notation: $(s_n)_{n=m}^{\infty}$ or $(s_n)_n \in \mathbb{N}$ and sequence noted by parantheses while set is defined by curly brackets

Defintion: values need to close to limit for all large n

Definition 7.1: For each $\epsilon > 0$ there exists a number N s.t. $n > N$ implies $|s_n - s| < \epsilon$