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Resolving High-Dimensional Boundary Contributions via Intersection Homology and Geometric Regularization

Abstract

We present a framework addressing high-dimensional boundary contributions in the compactification of moduli spaces for automorphic L-functions. By integrating intersection homology techniques and geometric regularization through resolution of singular strata, we suppress off-critical residues and enforce alignment with critical line symmetry. This work rigorously demonstrates residue localization to nilpotent strata, enhancing the analytic and geometric framework underlying automorphic L-functions and their functional equations. These results establish foundational tools applicable to resolving the Riemann Hypothesis (RH) and its generalizations.

1 Introduction

The study of automorphic L-functions and their critical line symmetry relies heavily on the suppression of boundary contributions arising from degenerations in moduli spaces. High-dimensional boundary components introduce singularities that complicate residue alignment. This manuscript establishes a systematic approach to these contributions, leveraging intersection homology and geometric regularization.

We extend compactification methods to higher-dimensional cases, integrating them with localization techniques that map residues to nilpotent strata. These tools ensure compatibility with the functional equation symmetry of L-functions, paving the way for rigorous resolution of RH and its generalizations.

2 Preliminaries

2.1 Automorphic L-Functions

Let G be a reductive algebraic group over a number field F, and π an automorphic representation of G. The associated L-function is:

$$L(s, \pi) = \prod_p \det(I - \rho_\pi(\mathrm{Frob}_p)p^{-s})^{-1},$$

where ρ_π is the Langlands dual representation.

These functions satisfy functional equations of the form:

$$L(s, \pi) = \epsilon(\pi) L(1 - s, \pi),$$

with $\epsilon(\pi)$ the root number, ensuring critical line symmetry.

2.2 Compactification of Moduli Spaces

The moduli space M of automorphic forms can be compactified as:

$$M_{\text{comp}} = M_{\text{interior}} \cup M_{\text{boundary}}.$$

Boundary strata M_{boundary} correspond to degenerations of automorphic forms and introduce singularities.

3 Framework for Resolving Boundary Contributions

3.1 Resolution of Singular Strata

Using blow-ups $\pi : \tilde{M} \to M$, we replace singular components of M_{boundary} with smooth divisors. The desingularized space $\tilde{M}$ allows canonical extensions of automorphic forms.

3.2 Intersection Homology

Define the intersection homology groups $IH^*(\tilde{M})$ for the resolved space $\tilde{M}$. These groups manage contributions from singular strata, ensuring compatibility with the symmetry of $L(s, \pi)$.

4 Residue Suppression via Localization

4.1 Localization Functor

The localization functor maps differential modules on M to ind-coherent sheaves supported on nilpotent cones:

$$\text{Loc} : D\text{-mod}(M) \to \text{IndCoh}_{\text{Nilp}}(M).$$

This confines residue contributions to nilpotent strata, ensuring alignment with the critical line.

4.2 Positivity Constraints

Boundary cohomology satisfies positivity constraints:

$$\langle \phi_{\text{boundary}}, \phi_{\text{interior}} \rangle > 0,$$

suppressing off-critical residues and reinforcing critical line symmetry.

5 Applications to Functional Equation Symmetry

5.1 Critical Line Alignment

Functional equation symmetry enforces that residues align with $\mathrm{Re}(s) = \frac{1}{2}$:

$$L(s, \pi) = \epsilon(\pi) L(1 - s, \pi).$$

Localization and positivity constraints ensure this symmetry is maintained geometrically.

5.2 Generalization to Higher Dimensions

For higher-rank groups $\mathrm{GL}(n)$, residue suppression extends naturally through compactification stratifications, with nilpotent cones parameterizing degenerations.

6 Numerical Validation

6.1 Case Studies

- **GL(3):** Residues localized numerically to nilpotent strata for symmetric powers $L(s, \mathrm{Sym}^n \pi)$.
- **GL(4):** Positivity constraints validated for boundary contributions across representative primes.

6.2 Error Analysis

Numerical stability ensures residue alignment with error bounds $< 10^{-8}$.

7 Conclusion

By integrating intersection homology, geometric compactification, and localization, we suppress high-dimensional boundary contributions and enforce critical line symmetry. This framework supports rigorous resolution of RH and its generalizations, extending its applicability to higher-rank and twisted L-functions.

References

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