Error Robustness: Strengthening Numerical Methods with Redundancy Checks for $GL(n)$, $n > 3$

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Abstract

This manuscript develops a robust framework for enhancing the numerical stability of computations involving automorphic L-functions, particularly for $GL(n)$ with $n > 3$. We propose a series of redundancy checks, including cross-validation with alternative spectral decompositions, precision augmentation strategies, and error-bounded iterative methods. These techniques ensure the alignment of critical residues while mitigating computational inaccuracies in high-dimensional settings.

1 Introduction

- **Motivation**: Importance of numerical robustness for automorphic L-functions in high dimensions.
- **Context**: Challenges in computing residues and eigenvalues for $GL(n)$, $n > 3$, under finite precision.
- **Objectives**: Development of redundancy checks and error correction techniques for numerical computations.

2 Challenges in High-Dimensional Computations

- Increasing numerical instability as n grows due to larger data sets and denser spectra.
- Sensitivity of eigenvalue computations to rounding errors and truncation effects.
- Importance of maintaining residue alignment with critical line symmetry.

3 Proposed Redundancy Checks

- **Cross-Validation Techniques**: Use of multiple spectral decomposition methods to validate computed eigenvalues.
- **Alternative Representations**: Comparison of results from Hecke operators and Langlands functoriality to ensure consistency.
- **Gram Point Validation**: Enhanced use of Gram points for zero-location accuracy.

4 Precision Augmentation Strategies

- Employing arbitrary-precision arithmetic libraries for improved computational accuracy.
- Iterative refinement of computed residues using Newton's method with higher-order corrections.
- Adaptive truncation in the Riemann-Siegel formula to minimize truncation errors.

5 Error-Bounded Iterative Methods

- Design of iterative algorithms that incorporate error bounds at each step.
- Monitoring convergence criteria to detect and correct numerical instabilities.
- Verification through independent numerical simulations.

6 Applications to $GL(n)$, $n > 3$

- Implementation of redundancy checks in computations for $GL(4)$, $GL(5)$, and beyond.
- Numerical validation of critical line symmetry with error bounds below 10^{-12}.
- Case studies demonstrating the efficacy of proposed methods for $GL(4)$ symmetric power L-functions.

7 Numerical Results

- Results of enhanced precision methods for $GL(n)$, $n > 3$, showing significant error reduction.
- Comparisons of residue alignment with and without redundancy checks.
- Error analysis demonstrating robustness against rounding and truncation effects.

8 Conclusion

- Summary of the proposed redundancy and precision enhancement strategies.
- Implications for future computations involving high-rank automorphic L-functions.
- Future work: Extending methods to twisted and higher-dimensional L-functions.