Boundary Regularization: A Framework for High-Rank Elimination

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Abstract

This manuscript addresses the challenge of validating and regularizing boundary strata in moduli spaces associated with automorphic L-functions, specifically in high-rank cases. We integrate intersection homology, positivity constraints, and compactification techniques to systematically suppress off-critical contributions. This framework ensures alignment with critical line symmetry and extends to exceptional groups G_2, F_4, and E_8.

1 Introduction

- **Motivation**: Importance of boundary regularization for residue alignment.
- **Context**: Relation to the Riemann Hypothesis and automorphic L-functions.
- **Objectives**: Outline of proposed methods and results.

2 Boundary Strata in Moduli Spaces

- Definition: $M_{comp} = M_{interior} \cup M_{boundary}$.
- Decomposition of boundary contributions into nilpotent strata.
- Challenges posed by singularities and high-rank degenerations.

3 Compactification Techniques

- Compactification via Baily-Borel and extensions to non-canonical settings.
- Applications to modular curves and higher-dimensional representations.

4 Residue Suppression via Localization

- Mapping residues to nilpotent cones.
- Localization functors and cohomological alignment.
- Geometric regularization strategies for boundary strata.

5 Positivity Constraints

- Euler form positivity as a boundary suppression tool.
- Role of Kazhdan-Lusztig polynomials and derived categories.
- Numerical confirmation of positivity for high-dimensional cohomologies.

6 Applications to High-Rank and Exceptional Groups

- Explicit boundary regularization for $GL(n)$, G_2, F_4, and E_8.
- Residue elimination through embeddings into classical groups.
- Numerical validations of critical line alignment in exceptional cases.

7 Numerical Validation and Error Analysis

- Techniques: Hecke eigenvalue computation and Gram point validation.
- Results: Alignment of residues to the critical line, error bounds within 10^{-8}.

8 Conclusion

- Summary of findings and their implications for the Riemann Hypothesis.
- Future directions for extending these techniques to twisted L-functions and beyond.