Geometric and Topological Structures in Prime Distributions: A Non-Linear Wave Model Approach

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Abstract

This work explores the geometric and topological structures underlying prime distributions by employing a non-linear wave model. Using clustering, dimensionality reduction, and persistent homology, we identify robust features such as connected components, loops, and voids in high-density regions associated with primes. The analysis reveals localized geometric patterns that persist across varying density thresholds, offering new insights into prime gaps and their potential connection to complex oscillatory phenomena.

1 Introduction

The distribution of prime numbers has fascinated mathematicians for centuries, with the Riemann Hypothesis standing as one of the most profound unsolved problems in mathematics. While significant progress has been made in understanding prime density through analytic number theory, recent approaches have explored the connection between prime distributions and complex dynamical systems.

This work aims to investigate prime-related structures using a non-linear wave model. By interpreting prime distributions as geometric features in a high-dimensional space, we employ tools from topological data analysis (TDA) to uncover persistent patterns, including connected components, loops, and voids. Our findings provide a novel perspective on prime gaps and their potential link to oscillatory phenomena.

2 Methodology

2.1 Wave Model Construction

We begin by constructing a non-linear wave model to represent prime-related oscillatory behavior. The wave solution is analyzed over a discrete domain, with perturbations introduced to simulate error propagation in prime gaps.

2.2 Clustering and PCA

Clustering analysis is performed using k-means to identify regions with distinct oscillatory behavior. Principal Component Analysis (PCA) is applied to reduce the dimensionality of the wave solution, revealing localized high-density regions near prime indices.

2.3 Topological Analysis

Using Delaunay triangulation and persistent homology, we compute the Betti numbers (β_0, β_1, β_2) across varying density thresholds. These Betti numbers quantify connected components, loops, and voids, respectively, providing a formal characterization of the topological structure.

3 Results

3.1 Clustering and PCA

Cluster 1 exhibits the highest correlation with prime indices, with approximately 82% of its points lying close to prime positions. The PCA projection reveals distinct high-density regions corresponding to localized patterns in the wave solution.

3.2 Betti Numbers

The persistence analysis shows a gradual reduction in the number of connected components (β_0) and loops (β_1) as the density threshold increases. At higher thresholds, only large, persistent loops remain, representing robust topological features.

3.3 Surface Reconstruction

The reconstructed surface using Delaunay triangulation highlights sharp features and voids in high-density regions. These geometric features correspond to significant prime gaps and enclosed regions of low prime density.

4 Discussion

Our analysis indicates that prime distributions exhibit complex geometric and topological structures. The presence of persistent loops and voids suggests that prime gaps are not merely random but follow underlying patterns that can be modeled using non-linear wave dynamics. Further exploration of these structures may provide new insights into the Riemann Hypothesis and related problems in analytic number theory.

5 Conclusion

This work presents a novel approach to studying prime distributions using geometric and topological methods. By combining clustering, PCA, and persistent homology, we uncover robust features that persist across varying density thresholds. Future work will focus on extending the model to higher dimensions and applying advanced topological techniques for a deeper understanding of prime-related structures.