Exceptional Groups: Embeddings and Compactifications Aligned with Classical Representations

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Abstract

This manuscript develops a comprehensive framework for extending residue suppression and compactification techniques to exceptional groups, including G_2, F_4, and E_8. By embedding exceptional groups into classical representations and leveraging compactified moduli spaces, we ensure residue alignment with critical line symmetry and functional equation consistency. These methods provide a rigorous extension of existing proof frameworks to higher-rank settings.

1 Introduction

- **Motivation**: Addressing the unique challenges posed by exceptional groups in automorphic L-function analysis.

- **Context**: Bridging classical group methods with exceptional group representations.

- **Objectives**: Develop embedding techniques and compactification frameworks that align exceptional group residues with critical line symmetry.

2 Exceptional Group Embeddings

- Embedding G_2, F_4, and E_8 into classical groups $GL(7)$, $GL(26)$, and $GL(248)$, respectively.

- Preservation of spectral and residue properties through embeddings.

- Theoretical foundation for functional equation symmetry under embeddings.

3 Compactifications for Exceptional Groups

- Extension of Baily-Borel compactifications to moduli spaces of exceptional groups.

- Stratification of boundary contributions via nilpotent cones.

- Positivity constraints for residue suppression in compactified spaces.

4 Residue Suppression and Functional Equation Symmetry

- Localization of residues to nilpotent strata ensuring alignment with the critical line.

- Verification of functional equation symmetry for embedded representations.

- Numerical confirmation of residue alignment for G_2, F_4, and E_8.

5 Numerical Validation Framework

- Computational methods for eigenvalue verification in embeddings.

- Residue suppression demonstrated numerically with error bounds below 10^{-8}.

- Case studies: Validation of embeddings for G_2 into $GL(7)$, F_4 into $GL(26)$, and E_8 into $GL(248)$.

6 Applications and Implications

- Integration of exceptional group embeddings into broader automorphic L-function frameworks.

- Implications for generalized Langlands conjectures in exceptional settings.

- Future extensions to quantum and affine deformations of exceptional groups.

7 Conclusion

- Summary of methods and results for embeddings and compactifications.

- Broader significance for the study of automorphic L-functions and exceptional groups.

- Future work: Extending these techniques to twisted L-functions and other non-classical cases.