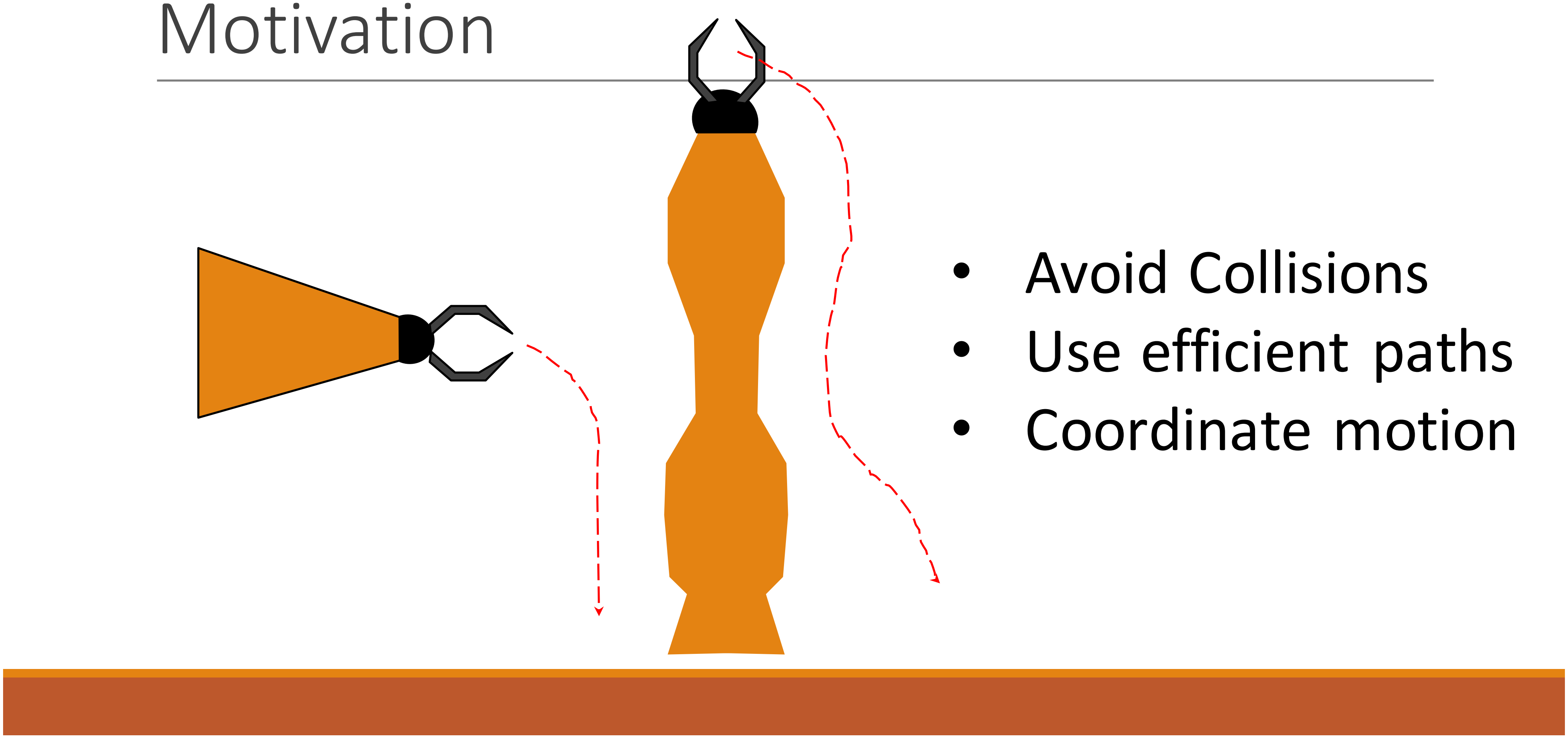
Laboratory 3
Motion Planning

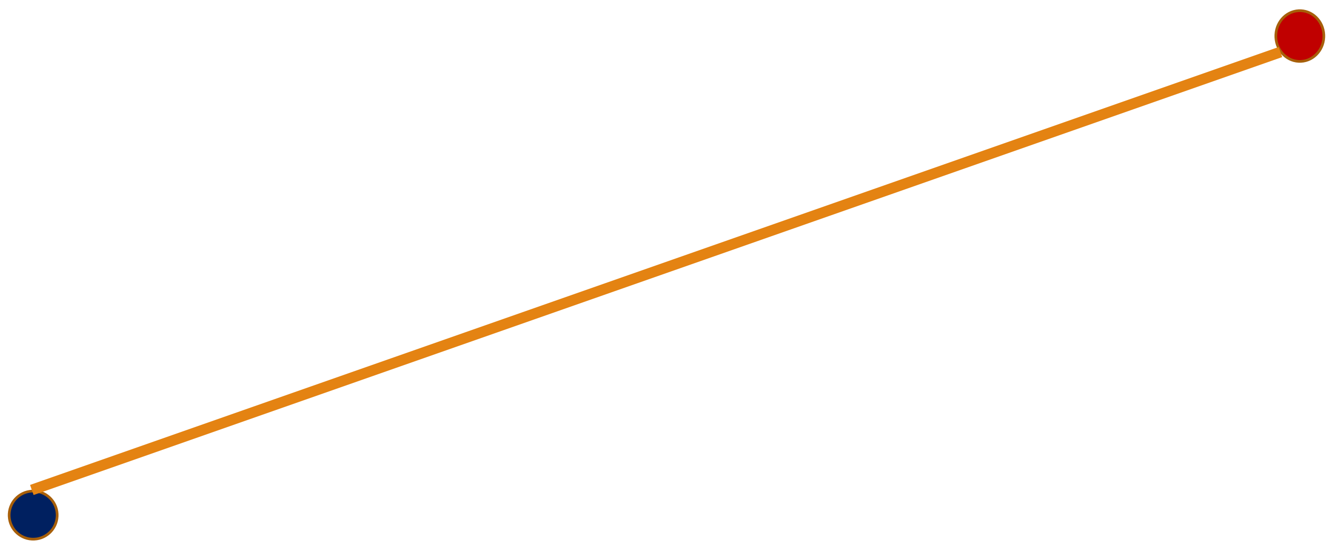
GLEBYS GONZALEZ

Motivation



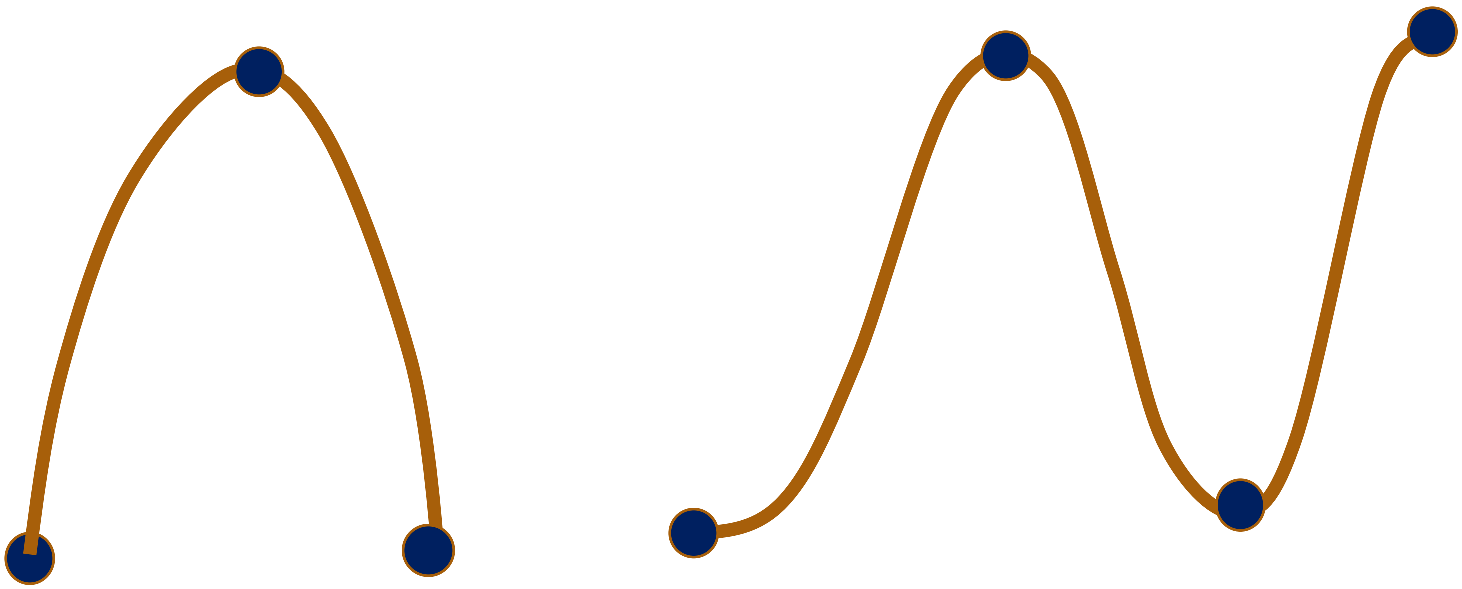
- Avoid Collisions
- Use efficient paths
- Coordinate motion

Interpolation



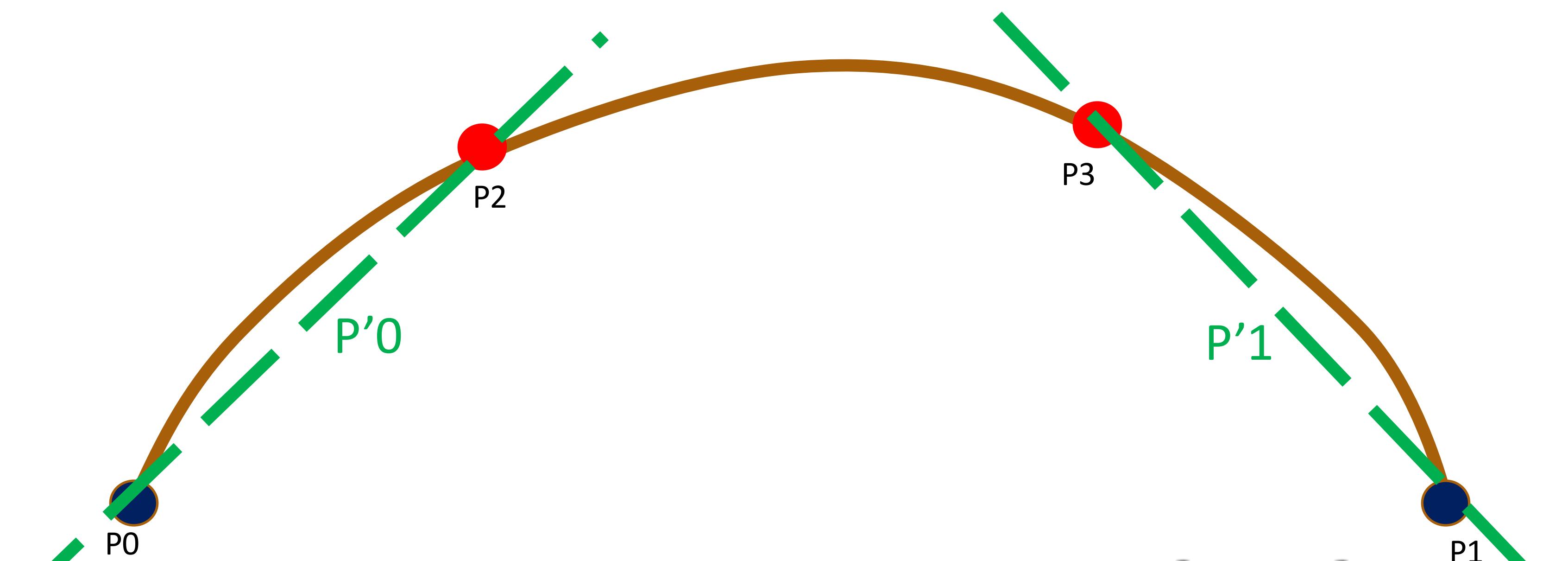
Straight Line: 2 points

Interpolation



N-Polynomial: needs at least n+1 points

Ferguson curves



Parametric curve: $\mathbf{P}(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$

Ferguson curves

1 $$\mathbf{P}(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

2 $$\left.\begin{aligned} \mathbf{P}(0) &= a_0 \\ \mathbf{P}(1) &= a_0 + a_1 + a_2 + a_3 \\ \mathbf{P}'(0) &= a_1 \\ \mathbf{P}'(1) &= a_1 + 2a_2 + 3a_3 \end{aligned}\right\} \text{Normalized}$$

Ferguson curves

3
$$\mathbf{P}(t) = (1 - 3t^2 + 2t^3)\mathbf{P}(0) \\ +(3t^2 - 2t^3)\mathbf{P}(1) \\ +(t - 2t^2 + t^3)\mathbf{P}'(0) \\ +(-t^2 + t^3)\mathbf{P}'(1)$$

4
$$\mathbf{P}(u) = \begin{bmatrix} 1 & t & t^2 & t^3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{P}(0) \\ \mathbf{P}(1) \\ \mathbf{P}'(0) \\ \mathbf{P}'(1) \end{bmatrix}$$

Ferguson curves example:

$$P_0 = (0,0,0)$$

$$P_1 = (8,-4,0)$$

$$P_2 = (12,3,0)$$

$$P_3 = (16,-2,0)$$

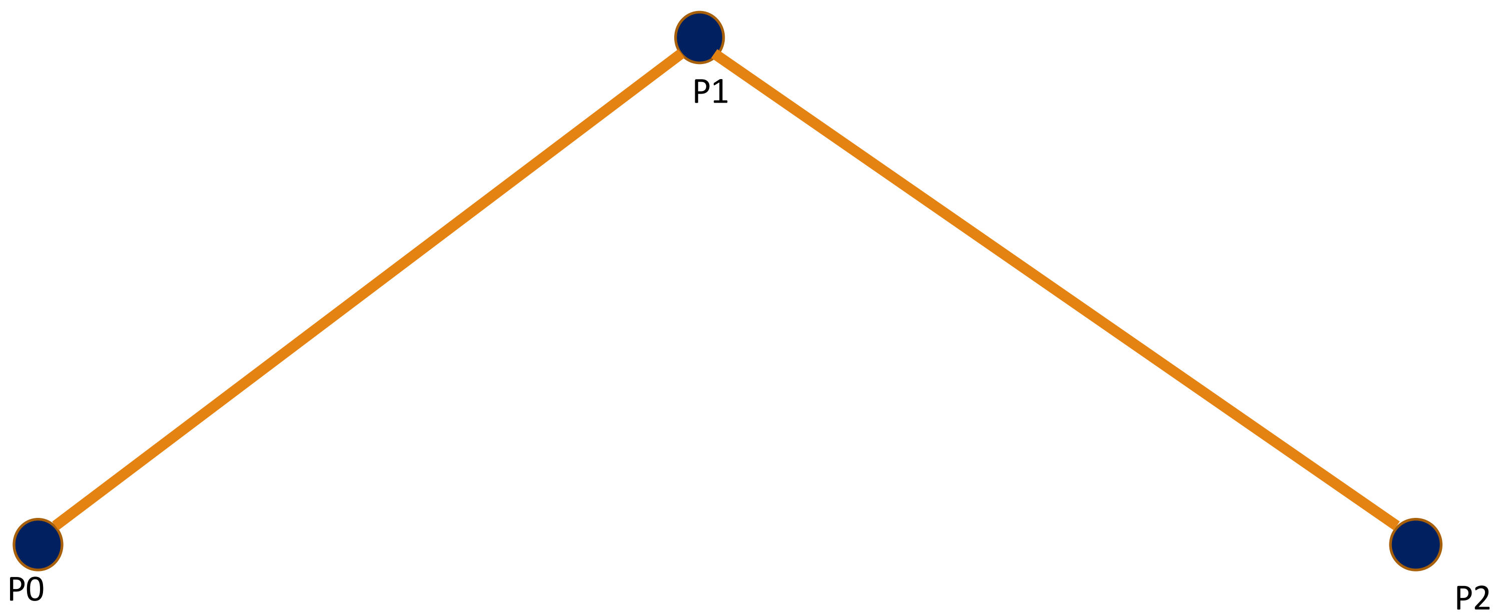
$$T_0 = P_1 - P_0 = (8,-4,0)$$

$$T_1 = P_3 - P_2 = (4,-5,0)$$

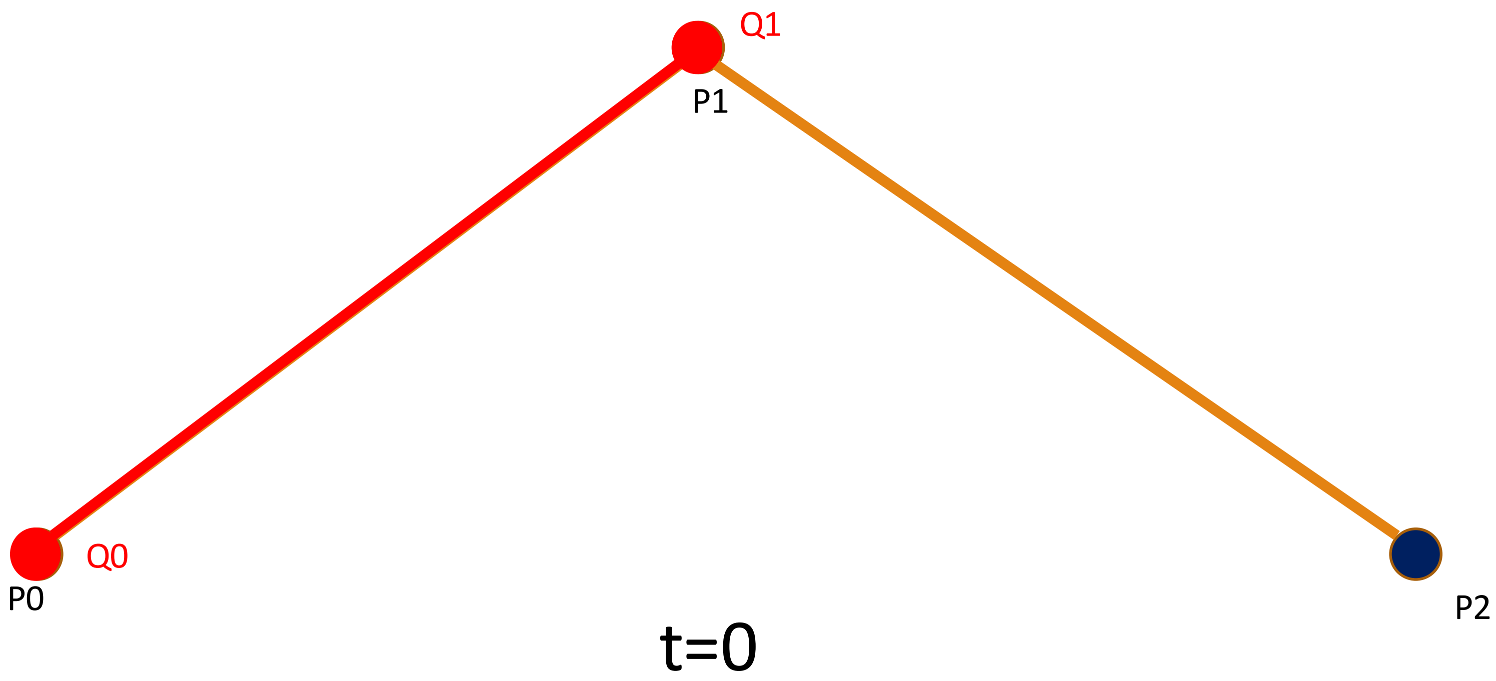
$$r(t) = \begin{bmatrix} 0 & 16 & 8 & 4 \\ 0 & -2 & -4 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} 2 & -3 & 0 & 1 \\ -2 & 3 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix} \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}$$

$$r(t) = \begin{bmatrix} -20t^3 + 28t^2 + 8t \\ -5t^3 + 7t^2 - 4t \\ 0 \end{bmatrix} \quad (0 \le t \le 1)$$

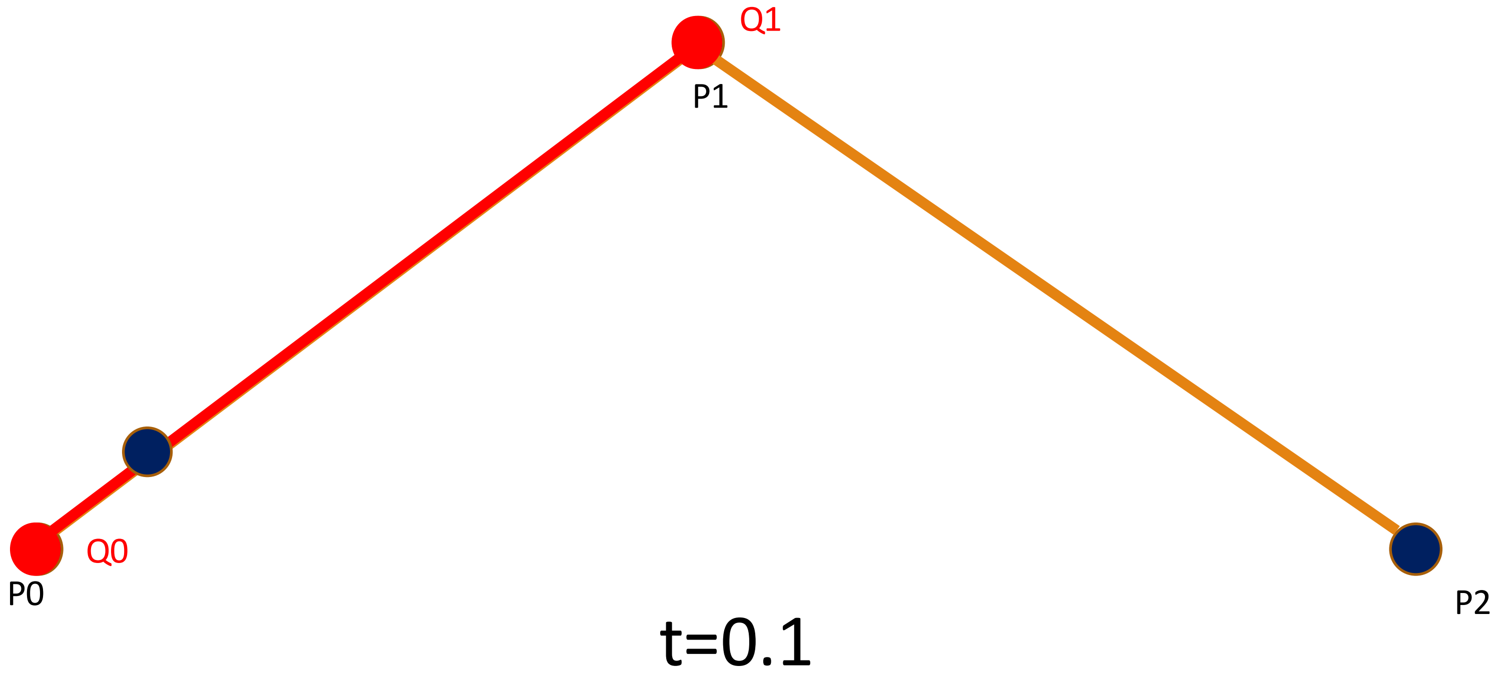
Bezier curves(Cuadratic)



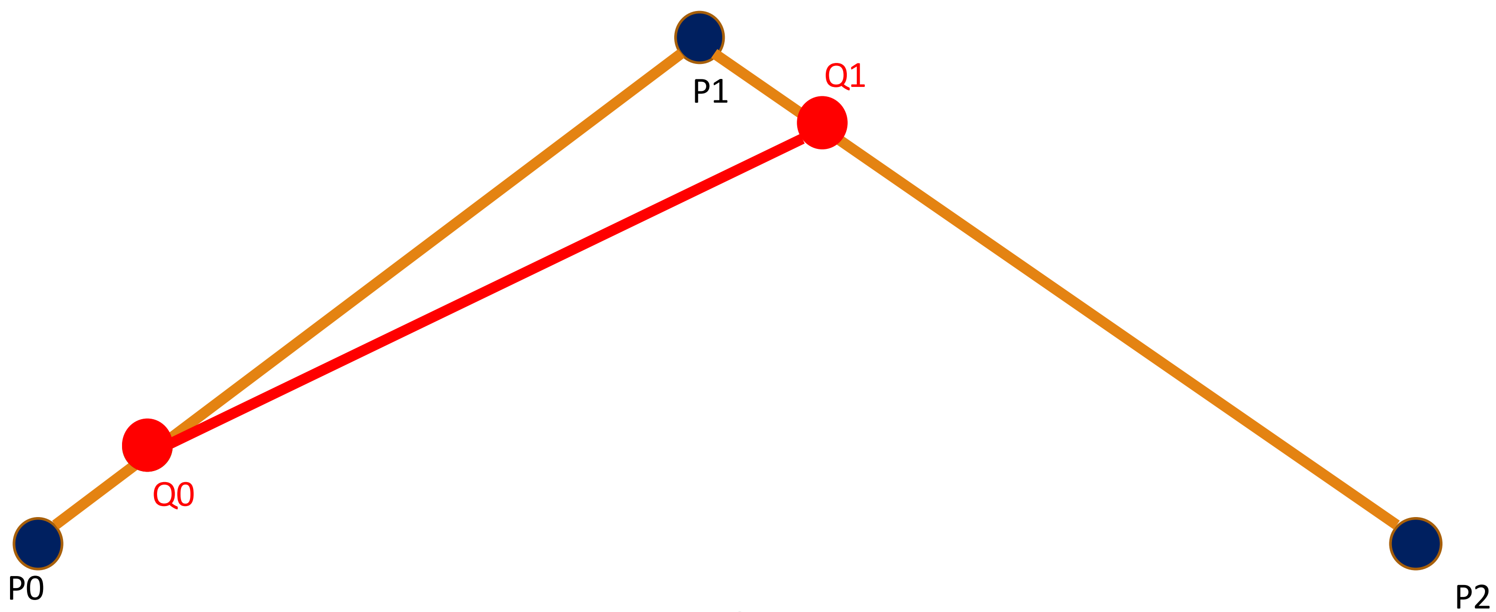
Bezier curves(Cuadratic)



Bezier curves(Cuadratic)

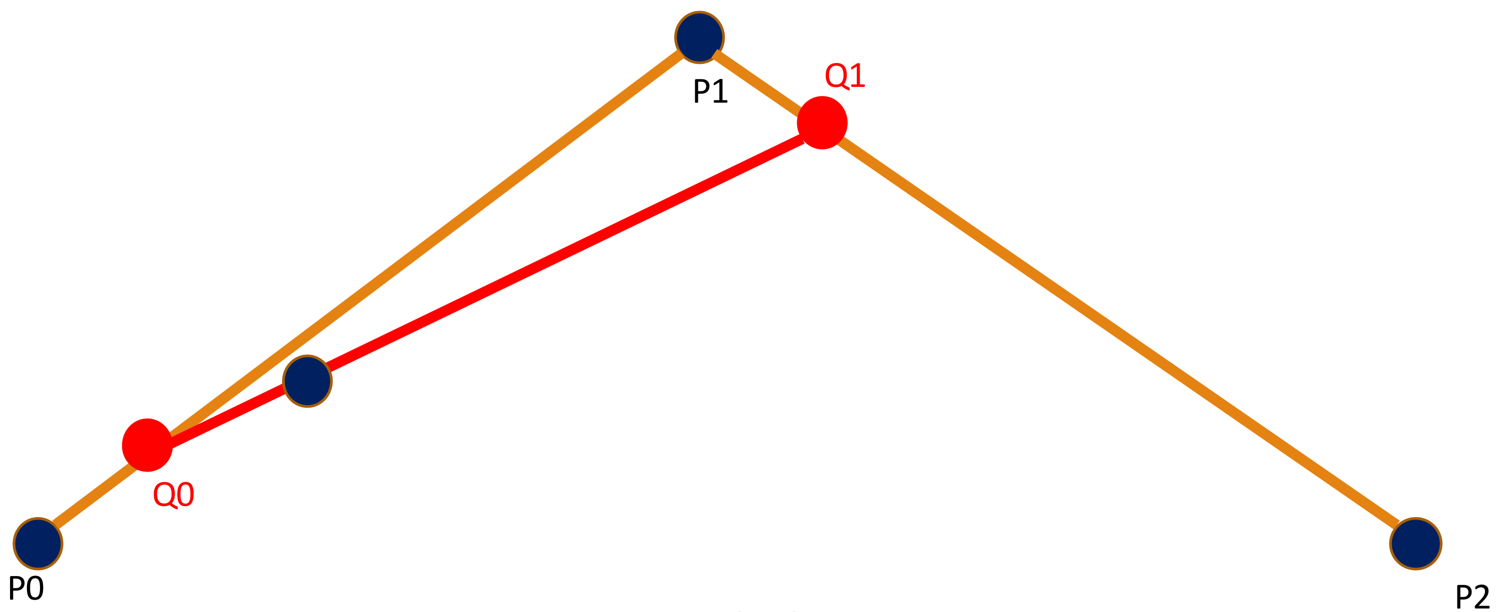


Bezier curves(Cuadratic)



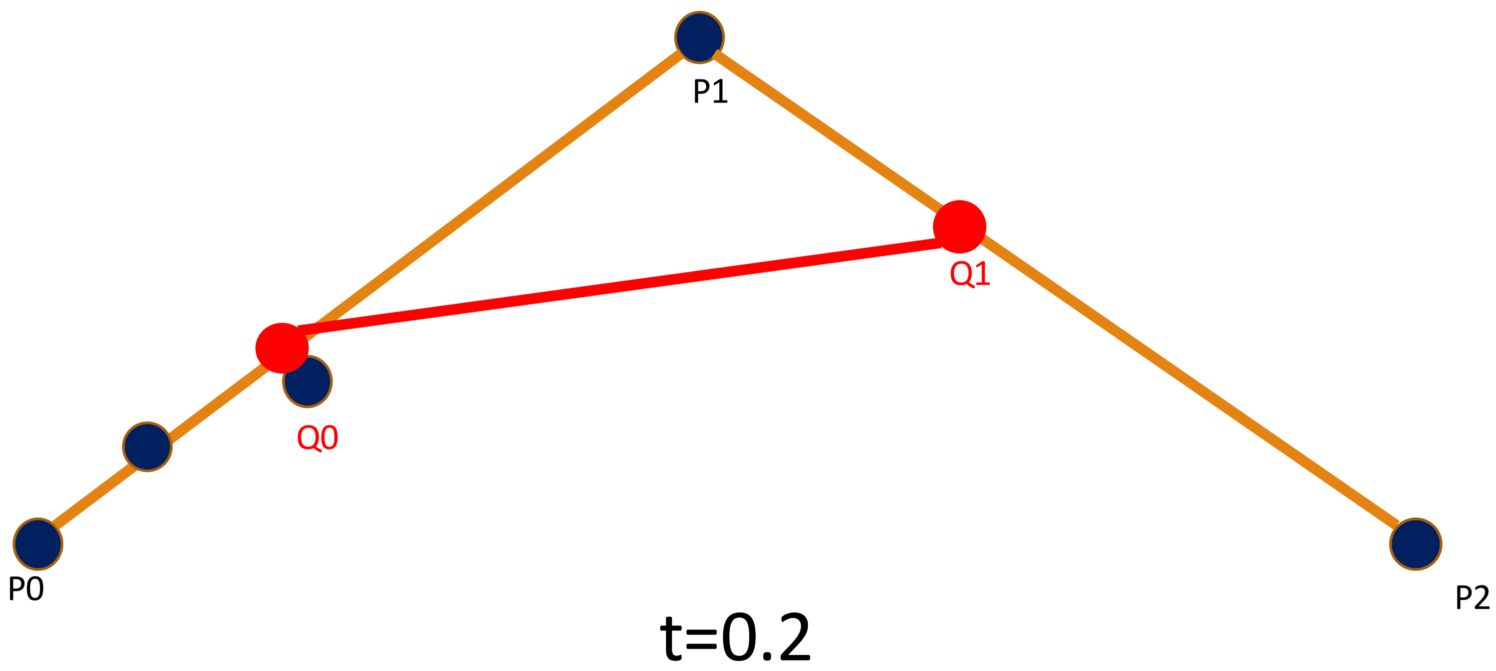
t=0.1

Bezier curves(Cuadratic)

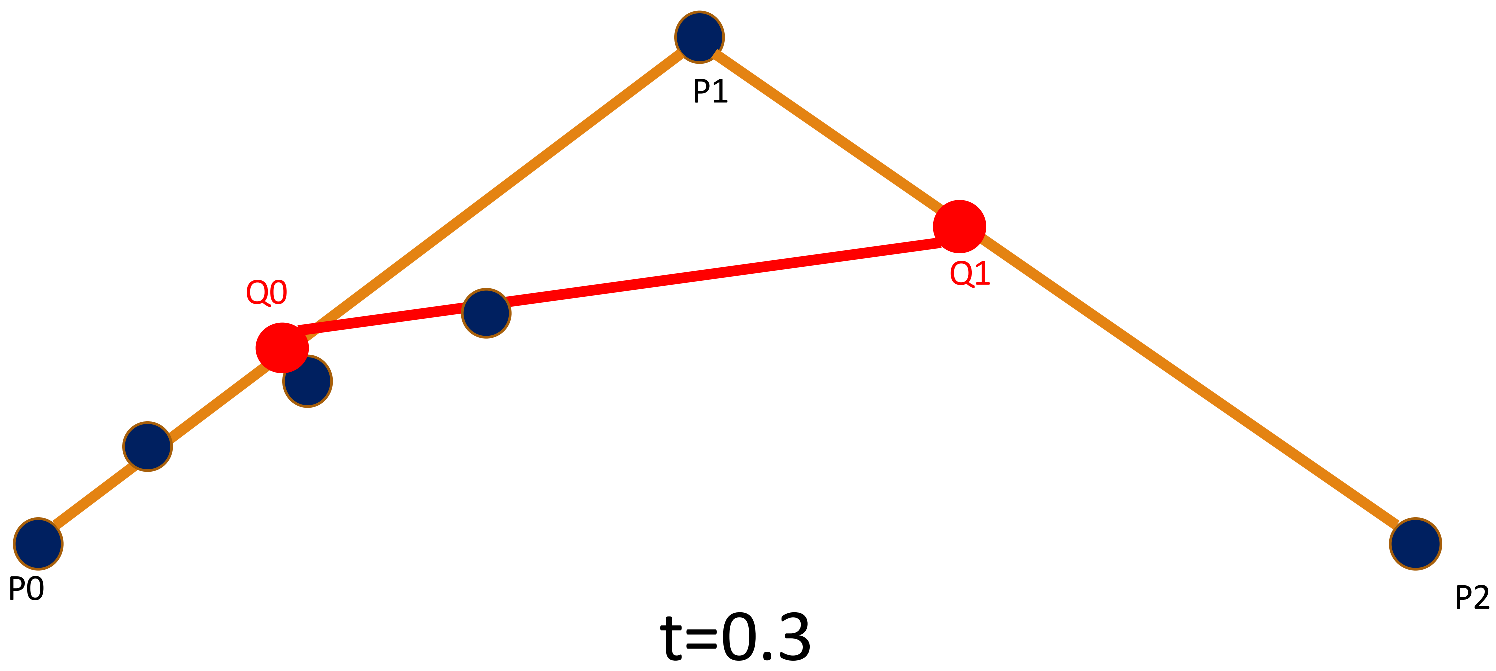


t=0.2

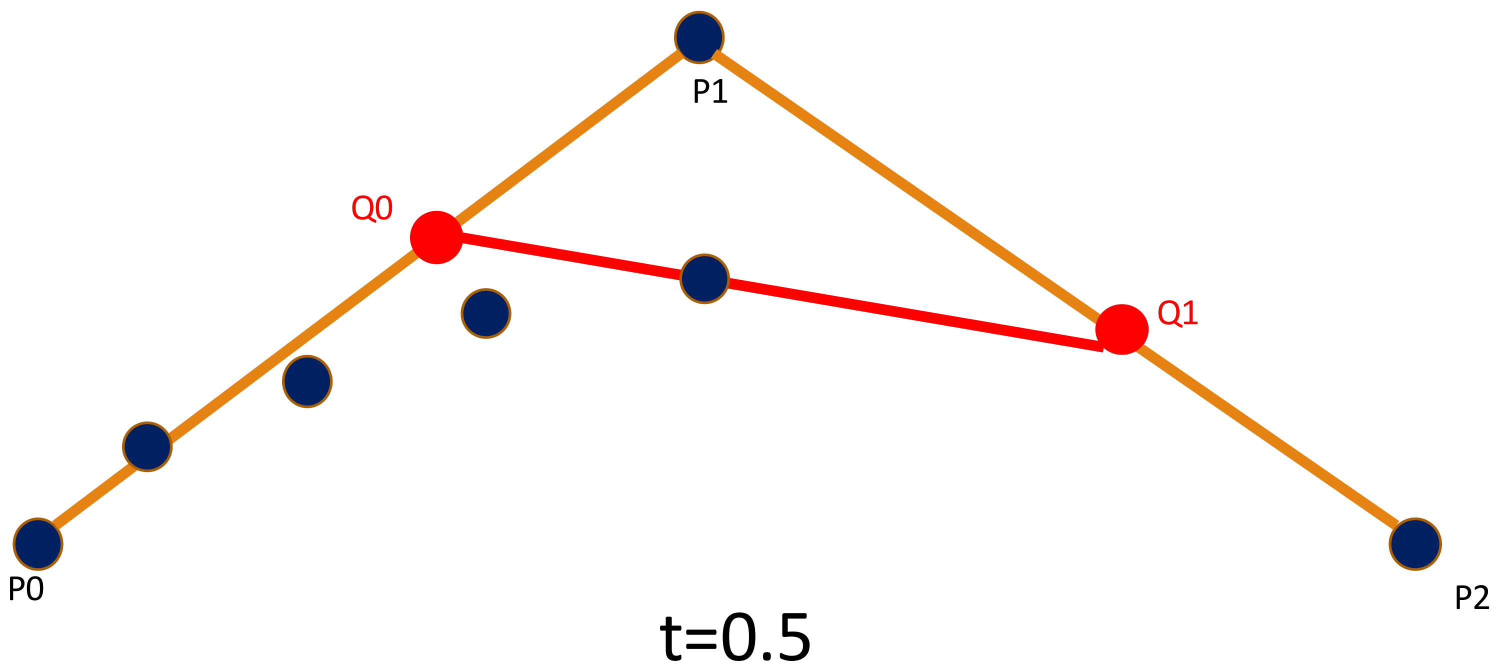
Bezier curves(Cuadratic)



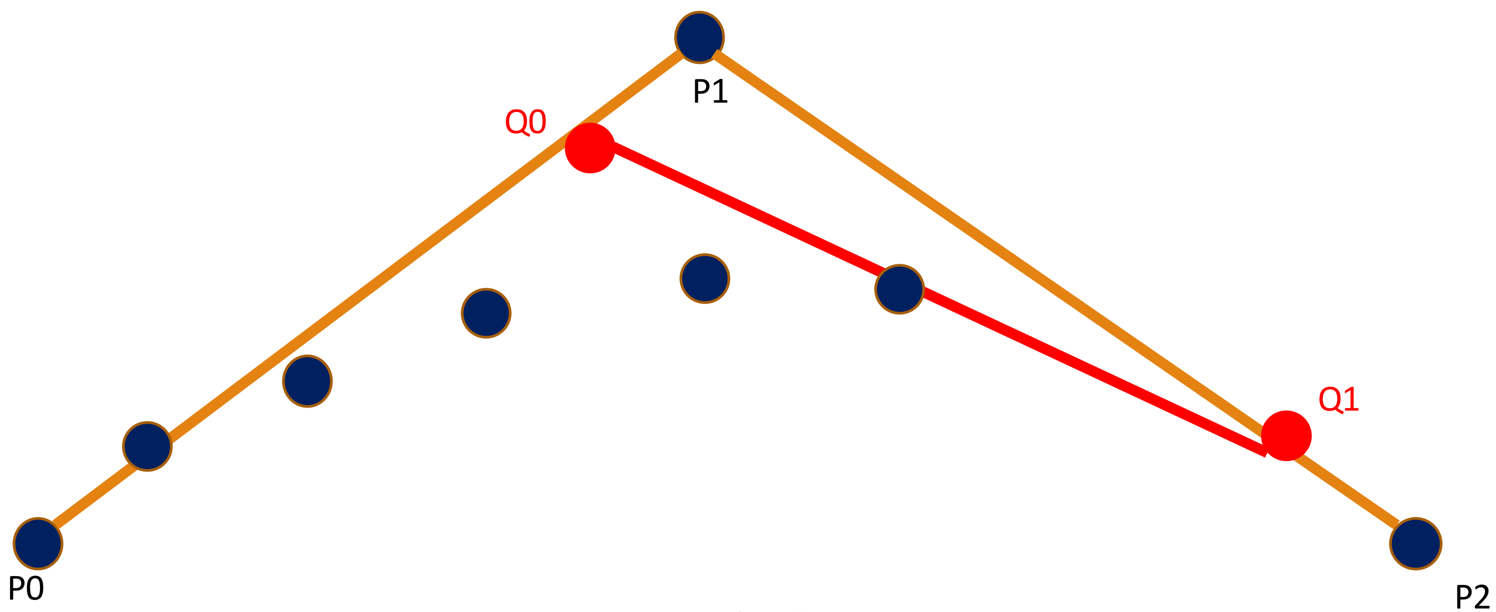
Bezier curves(Cuadratic)



Bezier curves(Cuadratic)

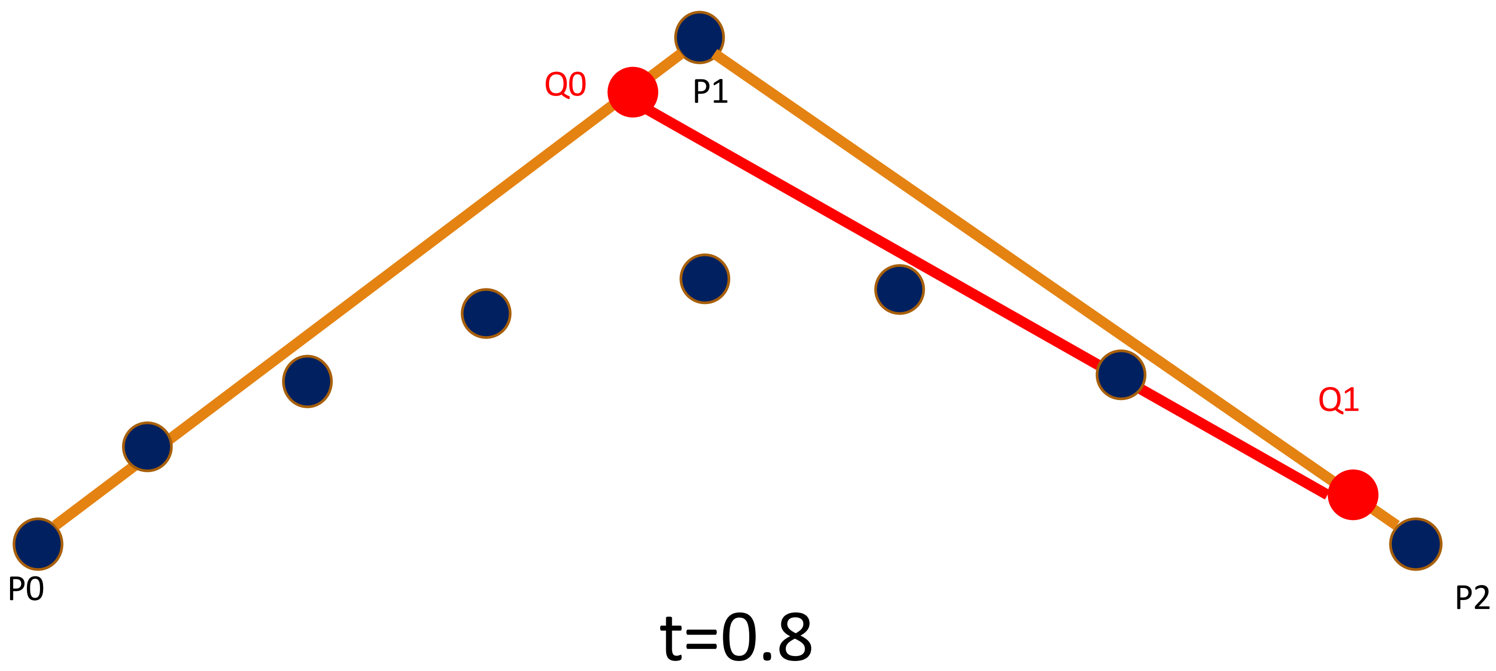


Bezier curves(Cuadratic)

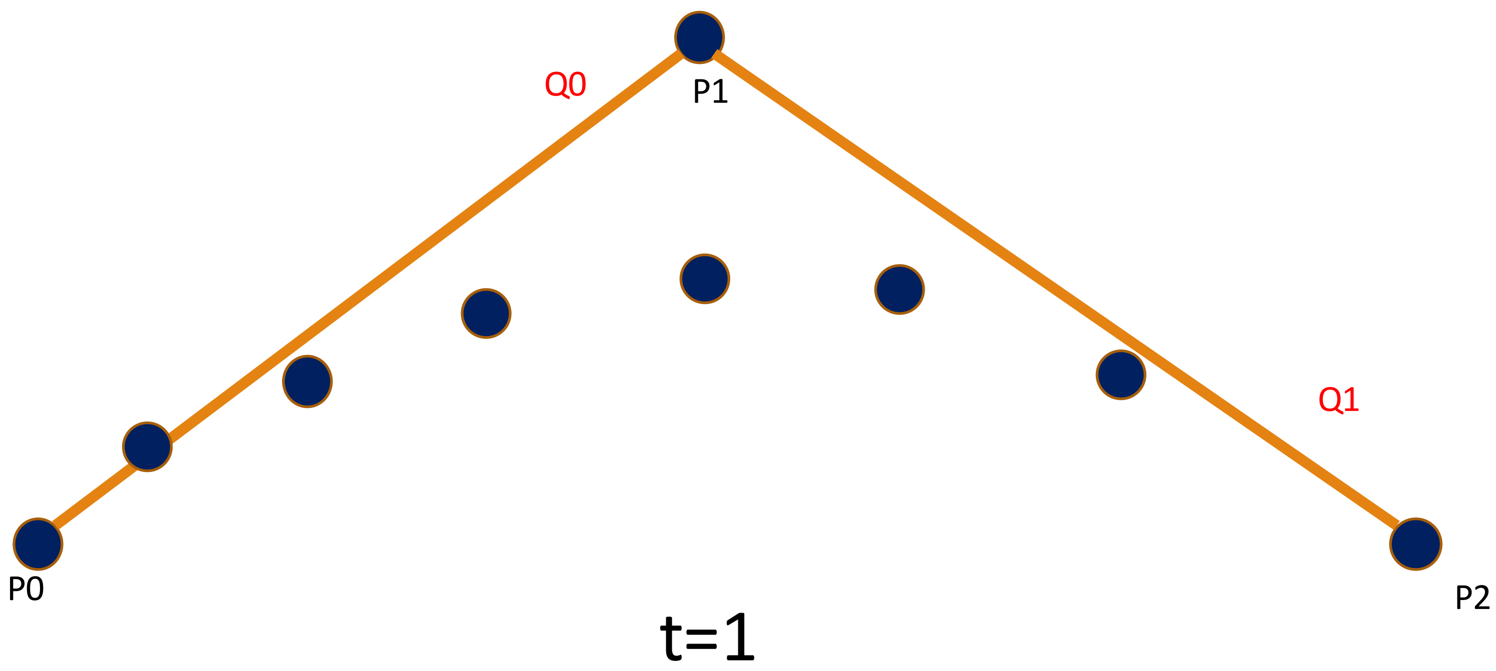


t=0.6

Bezier curves(Cuadratic)



Bezier curves(Cuadratic)



Bezier curves(Cuadratic)

$$Q_0 = (1-t)P_0 + tP_1$$

$$Q_1 = (1-t)P_1 + tP_2$$

$$Curve(t) = (1-t)Q_0 + tQ_1$$

$$\Downarrow$$

$$r(t) = (1-t)^2 P_0 + 2t(1-t)P_1 + t^2 P_2$$

Bezier Curves (Cubic)

$$r(t) = (1-t)^3 P_0 + 3(1-t)^2 t P_1 + 3(1-t) t^2 P2 + t^3 P_3$$

Example:

$$r(t) = \begin{array}{c} \begin{bmatrix} 0 & 8 & 12 & 16 \\ 0 & -4 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{matrix} \text{P0} & \quad \text{P1} & \quad \text{P2} & \quad \text{P3} \end{matrix} \end{array} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix} = \begin{bmatrix} 4t^3 - 12t^2 + 24t \\ -23t^3 + 33t^2 - 12t \\ 0 \end{bmatrix}$$

$$r(t) = \begin{bmatrix} 4t^3 - 12t^2 + 24t \\ -23t^3 + 33t^2 - 12t \end{bmatrix} \quad (0 \le t \le 1)$$