

# 2-D Transient-State Heat Sink Thermal Flow Simulation

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## Introduction

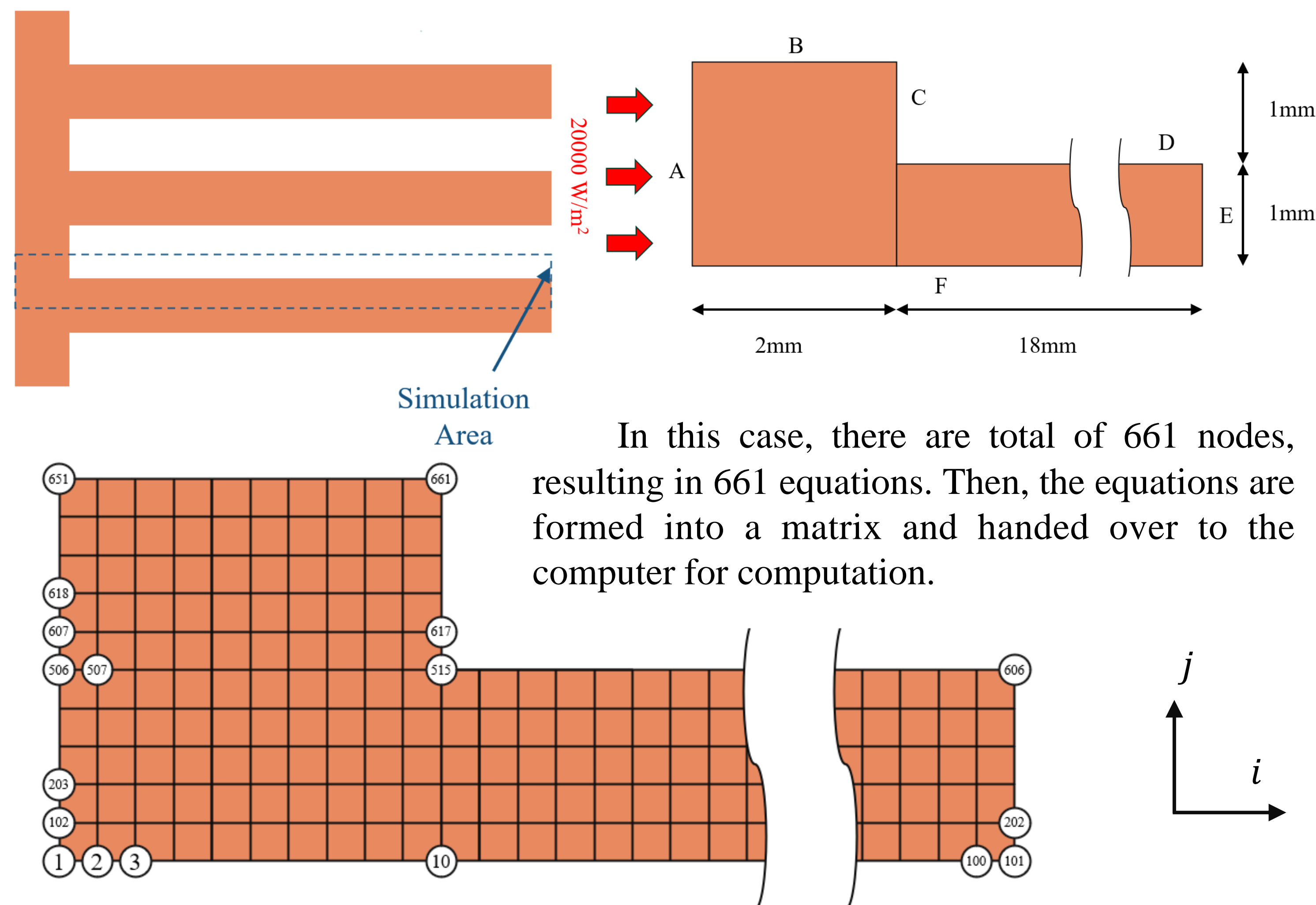
This project aims to simulate the temperature changes over time and the relationship with the heat convection coefficient  $h$  in the cross-section of a heat sink in a transient system. The objective is to understand basic heat transfer theory and explore the use of numerical methods to turn the model to be simulated into a large matrix for computation. Additionally, computer simulations will be used to display a complete temperature distribution map. Finally, by using the computation results from different heat convection coefficient choices, the relationship between the heat convection coefficient, the steady state temperature, and the time will be investigated.

Determine the specifications for a cross-section of a pure aluminum heat sink and its thermal conductivity properties, then apply constant heat flux on it. Next, choose air as the cooling fluid, and determine its heat transfer properties using a fitting formula provided by a senior. After establishing the grid size, using finite difference formulas for each point and convert them into a matrix. Finally, create charts to analyze the relationship between different average heat convection coefficient and the steady state temperature and its convergence.

## Mesh and Specification

A cross-section of a heat sink is composed of numerous slots and fins, so its temperature distribution should be symmetrically distributed. Therefore, analyzing an L-shaped cross-section composed of half a slot and half a fin will be sufficient to determine the complete temperature distribution of the heat sink cross-section.

After determining the grid size, draw the grid and mark the intersection points. At each point, apply the Finite Difference Equation. If a point is on the boundary, confirm its boundary conditions and incorporate them into the equation.



• Initial Condition :  
At  $T = 0$ , all points on the heat sink are at  $20^\circ\text{C}$ .

• Boundary Condition [1] :  
A :  
 $-k \frac{\partial T}{\partial x} = 20000 \left( \frac{\text{W}}{\text{m}^2} \right)$   
B ,F :  
 $\frac{\partial T}{\partial y} = 0$   
C ,E :  
 $-k \frac{\partial T}{\partial x} = h(T - T_\infty)$   
D :  
 $-k \frac{\partial T}{\partial y} = h(T - T_\infty)$

• Other properties :  
 $q'' = 20000 \left( \frac{\text{W}}{\text{m}^2} \right)$   
 $\Delta x = \Delta y = 0.2 \cdot 10^{-3} (\text{m})$   
 $\Delta t = 0.04 \cdot 10^{-3} (\text{s})$   
 $T_\infty = 20^\circ\text{C}$

Pure aluminum [1] :  
 $k = 237 \left( \frac{\text{W}}{\text{mK}} \right)$   
air [1] :  
 $\alpha = 0.097 \cdot 10^{-3} (\text{m}^2/\text{s})$

• Governing Equation [1]:  
 $\frac{1}{\alpha} \frac{\partial T}{\partial t} = \nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}$

• The relationship between air flow speed and the average heat transfer coefficient from the fitting formula. [2]

v (m/s)	0.5	1	1.5	2	2.5
$h_{\text{avg}}$	30.7378	36.5277	41.5978	46.0269	49.976

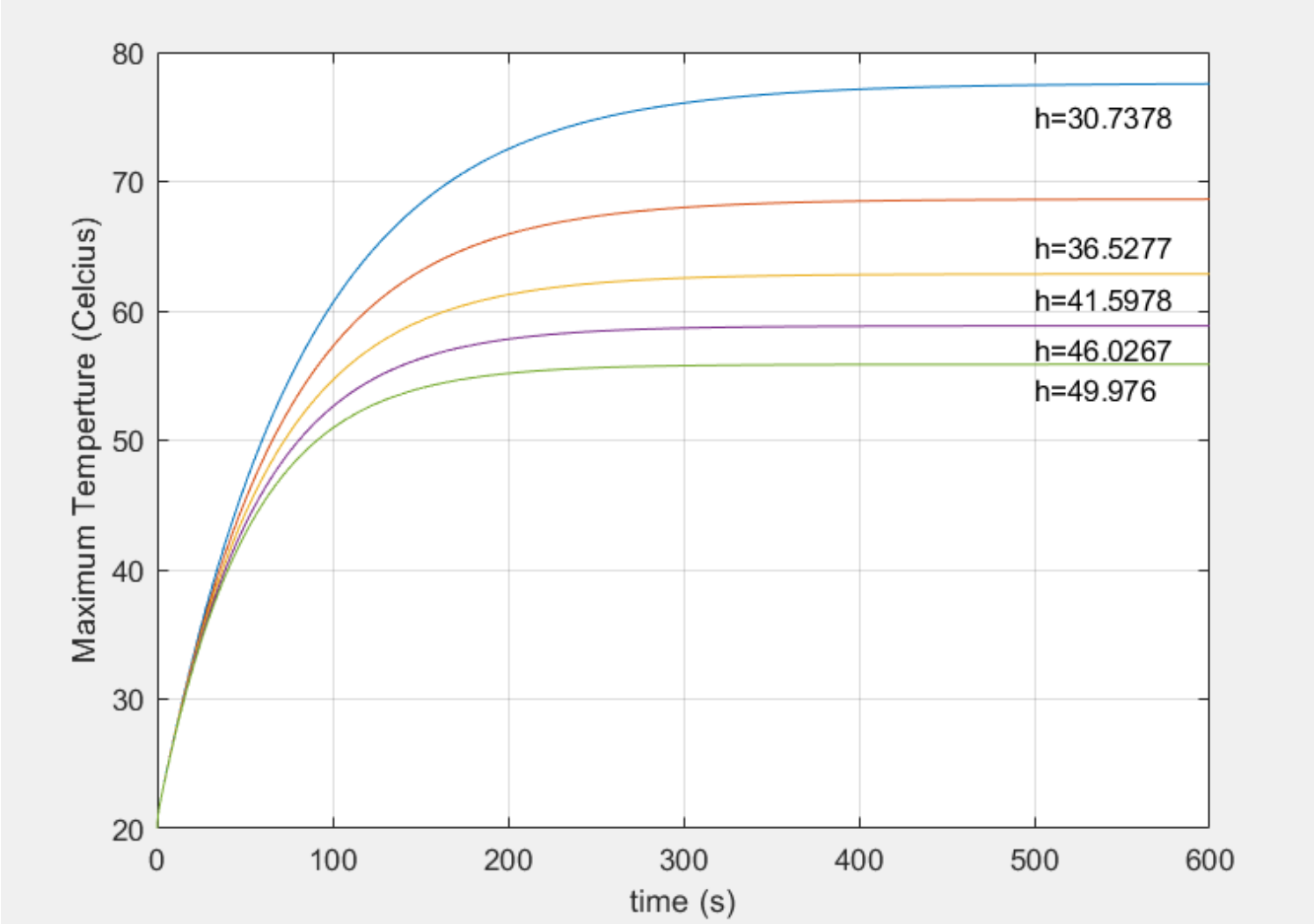
## Finite Difference Equation

- Using Finite Difference Equation, replace the partial derivatives in the above equation with approximations calculated from adjacent points.

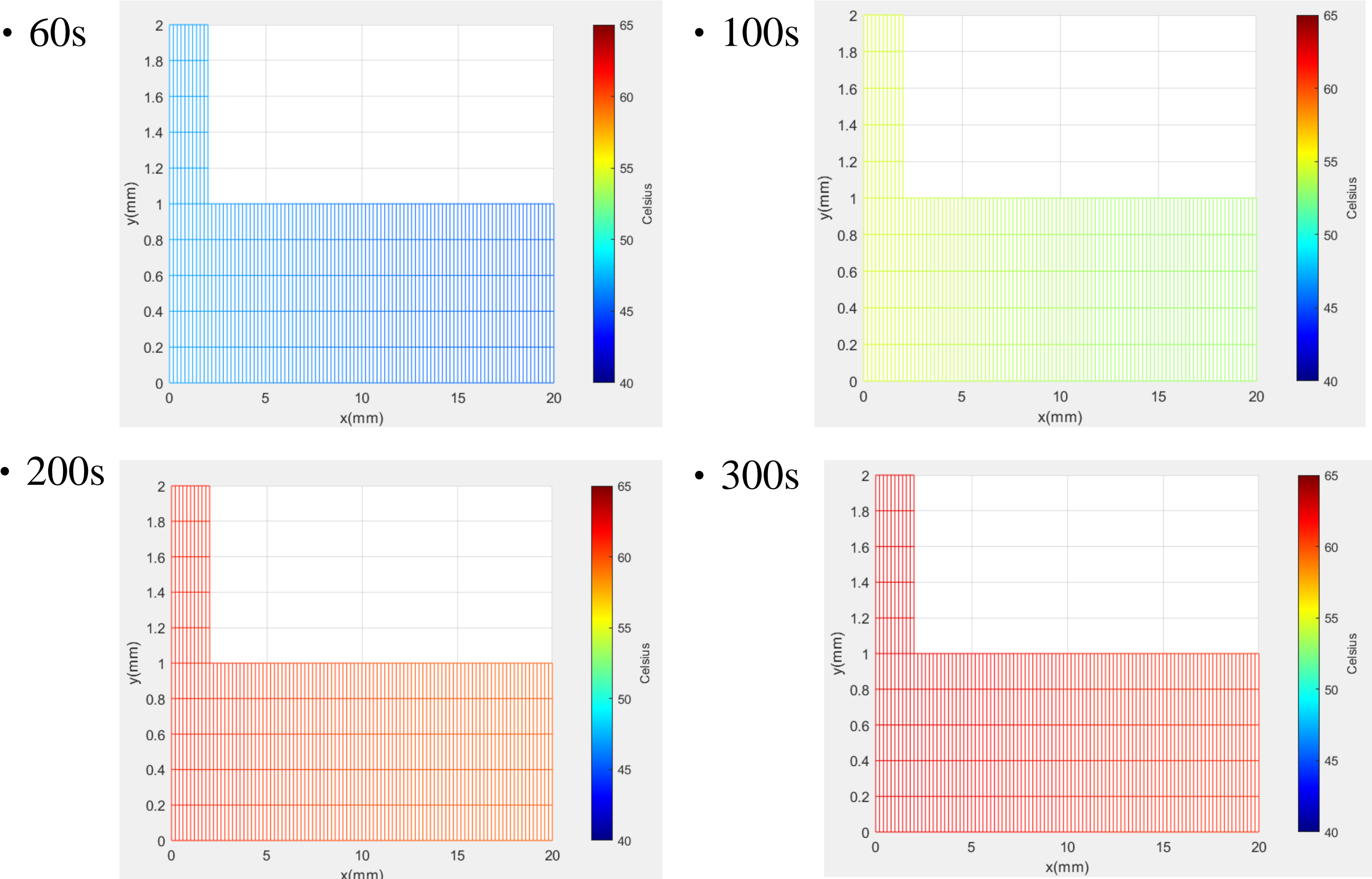
$$\frac{\partial T}{\partial t} = \frac{T^n - T^{n-1}}{\Delta t}, \quad \frac{\partial T}{\partial x} = \frac{T_{i,j} - T_{i-1,j}}{\Delta x}, \quad \frac{\partial T}{\partial y} = \frac{T_{i,j} - T_{i,j-1}}{\Delta y}$$
$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2}, \quad \frac{\partial^2 T}{\partial y^2} = \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2}$$

## Result

- highest temperature on cross-section after iterating 15,000,000 times (10 minutes) with different  $h$  values.



- Temperture distribution with  $h = 41.5978$  in different time.



## Conclusion

The results indicate that as the heat transfer coefficient increases, the convergence speed of the temperature also accelerates, leading to lower steady-state temperatures. Moreover, the rate of decrease in steady-state temperature slows down over time. However, since the variation in  $h$  itself is not constant, the relationship between the two is not easily discernible. Yet, observing the relationship between fluid flow velocity and heat transfer coefficient, it becomes apparent that while fluid flow velocity increases linearly, the steady-state temperature does not exhibit a linear decrease; instead, it gradually slows down.

Currently, the project progress is limited to simulating a single size heat sink. Next, we aim to make the computer model applicable to heat sinks of different specifications and varying mesh densities, enabling broader applications without the need to redraw the grid each time specifications are slightly altered. Additionally, we plan to integrate our previous research on flow velocity and heat convection coefficient to conduct more practical evaluations. This includes investigating how flow velocity affects temperature convergence results and determining the optimal flow velocity to maximize cooling benefits.

## Reference

[1] INCROPERA, FRANK P. *Incropera's Principle of Heat and Mass Transfer*. Global Edition, Wiley, 2017.

[2] Yovanovich, M. Michael. *Asymptotes and Asymptotic Analysis for Development of Compact Models for Microelectronics Cooling*.