Adaptive Unscented Kalman Filter (AUKF) for Robust State Estimation in FPV Racing

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***Abstract*—In this proposal, we introduce the usage of the Adaptive Unscented Kalman Filter (AUKF) for autonomous drone racing (ADR), which pushes quadrotor trajectories to highly stiff and nonlinear flight profiles, often exceeding 5Gs in turns and 100 mph in straightaways. In these areas, low-cost MEMS (Micro-Electro-Mechanical System) Inertial Measurement Units (IMUs) exhibit heteroscedastic noise behavior. The sensor noise is not constant; it increases drastically with motor RPM (vibration) and G-loading (sensor saturation and non-linearity).**

***Index Terms*—Adaptive Unscented Kalman Filter (AUKF), Autonomous drone racing, Heteroscedastic noise, Quadrotor dynamics, Sensor fusion**

I. Introduction

Traditional Unscented Kalman Filters (UKF) rely on a static Measurement Noise Covariance matrix (R). This creates a fatal trade-off in racing:

- **Tuning for Hover (Low R):** The filter trusts the IMU heavily. During a high-G turn, vibration-induced noise is interpreted as valid motion, causing the state estimate to jitter and the control loop to oscillate or destabilize.
- **Tuning for Racing (High R):** The filter distrusts the IMU. In low-dynamic sections (precision gate entry), the filter ignores subtle sensor cues, leading to "drift" and gate collision.

ADR requires vehicles to navigate 3D gates at extreme velocities, such that the margin for error in state estimation is nearly zero. Precise flight control relies on state estimates, which are traditionally derived from the fusion of MEMS IMUs and computer vision

However, the standard assumptions that are necessary in the formulation of the linear Kalman Filter(stationary, Gaussian white noise) fails in autonomous drone racing. As motor RPM scales to provide the thrust necessary for 5G cornering, mechanical vibrations inject massive high-frequency noise into the accelerometer and gyroscope signals. This phenomenon, known as heteroscedasticity, implies that the measurement noise covariance R is a dynamic function of the flight regime. Relying on a statically tuned filter leads to either over-smoothing during low-speed flight or catastrophic filter divergence during high-speed maneuvers. This project aims to implement and simulate an Adaptive UKF that learns the noise environment in real-time.

II. Literature Review

A. Non-linear State Estimation in Robotics

The Unscented Kalman Filter (UKF) was introduced by Julier and Uhlmann [1] as a derivative-free alternative to the Extended Kalman Filter (EKF), specifically designed to overcome the instability caused by linearization. Further developed by Wan and Van Der Merwe [2], the UKF employs the Unscented Transform (UT) to propagate a set of deterministic sigma points through the non-linear system dynamics. This approach captures the posterior mean and covariance to the second order of the Taylor series expansion (third order if the distribution is Gaussian), providing a rigorous Bayesian framework for recursive estimation [3].

However, as detailed in the comprehensive survey by Crassidis et al. [4], the performance of these filters in attitude estimation is heavily dependent on the accuracy of the process and measurement noise covariance matrices (Q and R). Standard formulations assume these matrices are constant, an assumption that frequently fails in high-dynamic operating conditions.

B. Physical Impact of Vibrations

In the context of quadrotor flight, the assumption of stationary Gaussian noise is invalidated by mechanical stressors. Yang [5] demonstrated that airframe vibrations induce high-frequency signals that can be conflated with legitimate rigid-body motion, leading to fault conditions if not properly filtered. Furthermore, Allione et al. [6] showed that repetitive low-acceleration impacts and mechanical shocks, which are common in racing scenarios, degrade the performance of MEMS inertial measurement units (IMUs), introducing non-linear errors that standard filters cannot track.

C. Adaptive Filtering for Dynamic Environments

To address this, Adaptive UKFs (AUKF) tune parameters online. Zheng et al. [7] developed a corrective framework that fuses innovation-based noise estimates with prior values via a weighting factor. Complementing this, Zhang et al. [8] proposed a stability-focused method using a sensitivity parameter (η) to detect regime shifts and explicitly re-update the state covariance ($\mathbf{P}$). Recent applications extend these principles to extreme maneuvers: Lenzo et al. [9] dynamically scale process noise to estimate slip angles in drifting ground vehicles, while Esmaeili Yengejeh et al. [10] employ a dual-adaptation strategy for aerial systems to robustly reject measurement outliers during turbulent flight.

D. Proposed Extension: Physics-Informed Adaptation

Restricted by weight and power constraints, Autonomous Drone Racing (ADR) platforms avoid heavy LiDAR-based systems and opt for a sensor suite pairing high-frequency MEMS IMUs with low-latency Computer Vision (CV). While CV corrects drift by identifying visual features, the IMU remains the primary source for high-bandwidth control data. Yet, it is uniquely vulnerable to heteroscedastic noise from high-RPM motors and G-loading. Unlike existing adaptive methods [7]–[10] which are fundamentally reactive to statistical errors, our work introduces a predictive, physics-informed adaptation layer. By explicitly modeling measurement noise covariance ($\mathbf{R}$) as a function of control input (RPM) and inertial loading (G-force), we anticipate noise shifts before state corruption occurs. Cascaded with covariance re-updating [8], this feed-forward mechanism ensures the robust "inner-loop" stability required for reliable CV-based navigation during extreme maneuvers.

III. Methodology

A. The Heteroscedastic Noise Model

We consider a discrete-time nonlinear system representing the quadrotor kinematics, formulated using unit quaternions to avoid gimbal lock singularities inherent in Euler angle representations:

$$\begin{aligned} \mathbf{x}_k &= f(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) + \mathbf{w}_{k-1} \\ \mathbf{z}_k &= h(\mathbf{x}_k) + \mathbf{v}_k \end{aligned} \tag{1}$$

Where the state vector $\mathbf{x} \in \mathbb{R}^{10}$ comprises position, velocity, and the attitude quaternion:

$$\mathbf{x} = [\mathbf{p}^\top, \mathbf{v}^\top, \mathbf{q}^\top]^\top$$

Here, $\mathbf{q} = [q_w, q_x, q_y, q_z]^\top$ represents the rotation from the body frame to the inertial frame.

The critical challenge in high-dynamic flight is that the measurement noise $\mathbf{v}_k$ is *heteroscedastic* (having non-constant variance). We model this as a non-stationary Gaussian process:

$$\mathbf{v}_k \sim \mathcal{N}(0, \mathbf{R}_k), \quad \mathbf{R}_k = g(\text{RPM}_k, \mathbf{a}_k)$$

Our goal is to implement an adaptation law that estimates the instantaneous covariance $\mathbf{R}_k$ as a composite function of static sensor properties and the dynamic flight regime:

$$\mathbf{R}_k = \mathbf{R}_{\text{floor}} + \mathbf{R}_{\text{vibe}}(\Omega_k) + \mathbf{R}_{\text{load}}(\mathbf{a}_k) \tag{2}$$

Where the constituent matrices are defined by the following physics-informed relationships:

1. RPM-Induced Vibration: Given the rotational velocity of the four motors $\Omega_{i,k}$ (in RPM), the vibration noise injected into the IMU is modeled as proportional to the total kinetic energy of the rotors (proportional to frequency squared):

$$\mathbf{R}_{\text{vibe}}(\Omega_k) = \alpha \cdot \text{diag}\left(\sum_{i=1}^{4} \Omega_{i,k}^2\right) \cdot \mathbf{I}_{3\times3} \tag{3}$$

where α is a constant coefficient characterizing the airframe's structural resonance and dampening properties.

2. G-Loading Sensitivity: To account for MEMS sensor non-linearities during high-acceleration maneuvers[6], the covariance scales with the magnitude of the specific force (linear acceleration) $\|\mathbf{a}_k\|$:

$$\mathbf{R}_{\text{load}}(\mathbf{a}_k) = \beta \cdot \|\mathbf{a}_k\| \cdot \mathbf{I}_{3\times3} \tag{4}$$

where β is the empirical G-sensitivity coefficient of the accelerometer and gyroscope axes.

B. Validation

To validate the proposed AUKF, we will conduct a two-phase assessment: a controlled simulation in PyBullet and a real-world data benchmark using the TII Drone Racing Dataset selecting specific FPV runs by world class racer MinChan Kim.

Phase 1: High-Fidelity Simulation (PyBullet)

- **Environment:** A quadrotor model will autonomously navigate a 3D racing track (comprising split-S maneuvers and high-speed banks) using a Ground Truth-based PID controller to ensure consistent trajectory generation.
- **Sensor Simulation:** Inertial Measurement Unit (IMU): Sampled at 240Hz. We will implement a heteroscedastic noise model where the noise covariance scales linearly with the instantaneous G-load ($||\vec{a}||$) and motor RPM, mimicking MEMS sensor saturation.
- **Visual-Inertial Odometry (VIO):** Sampled at 30Hz. In order to prevent unbounded drift, we will simulate a vision-based position update by adding Gaussian white noise to the Ground Truth position ($\sigma = 0.05m$) and latency (30ms).
- **Estimators Under Test:**
  - Hover UKF: Standard UKF with static low process noise (Q) and measurement noise (R), optimized for stable hovering.
  - Race UKF: Standard UKF with static high Q and R, optimized for high-dynamic rejection.
  - Adaptive UKF: The adaptive filter using the motor RPM and G-Parameter noise scaling model.

Phase 2: Real-World Benchmarking

We will replay the TII Drone Racing Dataset (Blackbird/UZH-FPV) through all three filters. This dataset provides raw accelerometer/gyroscope logs from actual racing drones flown by human pilots. It serves as the definitive test for the filter's ability to handle the intended noise scaling alongside real aerodynamic disturbances (e.g., prop wash, ground effect) that are not accounted for in our simulation.

Evaluation Metrics

- **RMSE (Root Mean Square Error):** To quantify overall position accuracy.
- **NEES (Normalized Estimation Error Squared):** To evaluate filter consistency (i.e., verifying if the filter correctly estimates its own uncertainty).

Visualization & Analysis

We shall produce the following plots for both simulation and real-world cases to visualize performance:

- **Plot 1: The 3D Trajectory Comparison**
  - *Description:* A 3D line plot of the track.
  - *Expected Result:*
    - **Ground Truth:** Black Line (Perfect).
    - **Hover UKF:** Smooth but drifts wide on corners (overshoots).
    - **Race UKF:** Tracks corners well but exhibits noisy "jitter" on straight sections.
    - **GP-AUKF:** Tight on corners, smooth on straights.
- **Plot 2: Position Error vs. G-Force (Time-Series)**
  - *Axes:* X-Axis: Time (s); Y-Axis (Left): Euclidean Error (m); Y-Axis (Right, dotted): G-Force magnitude.
  - *Justification:* This correlates error spikes with dynamic maneuvers. We expect the "Hover UKF" error to spike synchronously with G-Force peaks (turns), whereas the GP-AUKF error should remain distinct from G-Force magnitude.
- **Plot 3: Adaptive Parameters (Q and R Evolution)**
  - *Axes:* X-Axis: Time; Y-Axis: Value of the Diagonal of R (Measurement Noise).
  - *Justification:* Demonstrates the adaptation mechanism in real-time. The value should be low during hover phases and spike high during high-G turns, visualizing the "G-Parameter" scaling.

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