AMM Maths Problems

#1 Suppose there are two DEFI protocols P1: xy = c and P2: x+y = 2d.Given two curves meet on 2 points. Consider a situation where the difference in amount of token A on P1 and P2 is D1 and difference in amount of tokenB on P1 and P2 is D2, determine the number of tokenA and tokenB in P2 so that value of D1×D2 is maximum. And whats the max value of D1×D2 ? Is the derived amount of tokenA and tokenB the same? **[x** represents amount of tokenA and **y** represents amount of tokenB**; c** and **d** are constants**;** tokenA and tokenB are pegged to each other with **1:1]**.

#2 suppose there is a liquidity pool U1 with protocol P1: xy = 10 M^2 and P2: x+2y = c, A developer wants to create a liquidity pool U2 for tokenA and tokenB using protocol P2, how much total amount of tokenA and tokenB should he add to the pool so that amount of tokenA in pool U1 and U2 are equal and also amount of tokenB in pool U1 and U2 are equal, but only once. **[x** represents amount of tokenA and **y** represents amount of tokenB**; c** is constant; **1M = 1000000]**

#3 Consider the DEFI protocol P1: xy = c , where c is constant, suppose a user add liquidity by adding 'a' amount of tokenA and 'b' amount of tokenB now the curve changes to xy = c' , now in the new curve 'a' amount of token is swapped for tokenB, calculate the swapped amount of tokenB? Is the amount of tokenB obtained equal to 'b'? [The worth of **tokenA** and **tokenB** are equal]

#4 In the DEFI protocol P: xy = c, where c is constant; there is a flash swap by an user, the user A borrowed x_1 amount of token0 and he had to repay the borrowed amount in time t_1, where t_1 is the time duration between borrowing and repaying, another user during $t_1/2$ swapped 'p' amount of token0 for token1, suppose swapping works in such a way that input token is given first to protocol and then output the token back to the user, and the duration of the swapping transaction is t2, let us say $t_1 = t_2$. calculate the gain or loss to user B for the output token1.

#4 In DEFI protocol P: xy = c, where c is constant liquidity is added by amount x_1 and amount y_1 , let at point A on original curve i.e. xy = c; amount of tokenA and tokenB are same, calculate the minimum distance between point A and the new liquidated curve, in terms of, x_1 and y_1. [the worth of **tokenA** and **tokenB** are equal]

#6 In DEFI protocol P: xy = 8M*1M, where 1M=1000000; there is a point A with token0 = $\sqrt{8/\sqrt{3}}$ M and token1 = $\sqrt{8.\sqrt{3}}$ M. There is another point B on the curve, AB subtends 30 degrees to origin O. Determine the number of token0 and token1 at point B.**[x** represents amount of token0 and **y** represents amount of token1; the worth of token0 and token1 are equal**]**

#7 Suppose there are two hypothetical DEFI protocols P1 : xy = c1 and P2 : x + y = c2, where c1 and c2 are constants consider the situation S where amount of token0 in P1 and P2 are equal and amount of token1 in P1 and P2 are also equal, along with total liquidity of P1 is twice of P2, what's the relation between the curve of P1 and P2 at situation S ? **[x** denotes amount of token0 **y** denotes amount of token1**]**

#8 There are two hypothetical DEFI protocols P1 : x^2 + y^2 = c^2, and P2 : xy = 4c^2, where c is constant. Determine the position A on the curve of P1 such that spot price at P1 and P2 are equal, and distance of A from the curve of P2 is minimum. [The worth of two tokens are equal]

#9 There are two hypothetical DEFI protocols P1 : xy = 16c^2 and P2 : x + 4y = 10c, find the minimum difference between the amounts of token1 in protocol P1 and P2, where spot price on P1 is 1/4.
[x denotes amount of token0 **y** denotes amount of token1; c is constant and both token worth equally**].**

#10 suppose there are three hypothetical DEFI protocols P1 : xy = 9c^2, P2 : x +4y = c and P3 : 4x + y =c, some amount of token0 and token1 are removed from P1 to achieve the situation where amount of token0 in all protocols i.e P1, P2 and P3 are equal. Determine the ratio of new total liquidity to old total liquidity in P1.