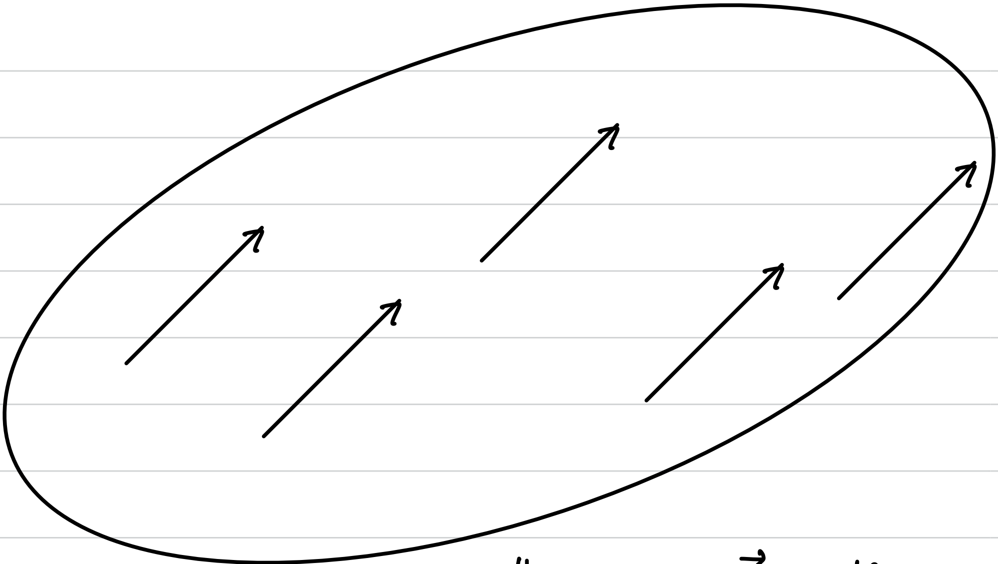
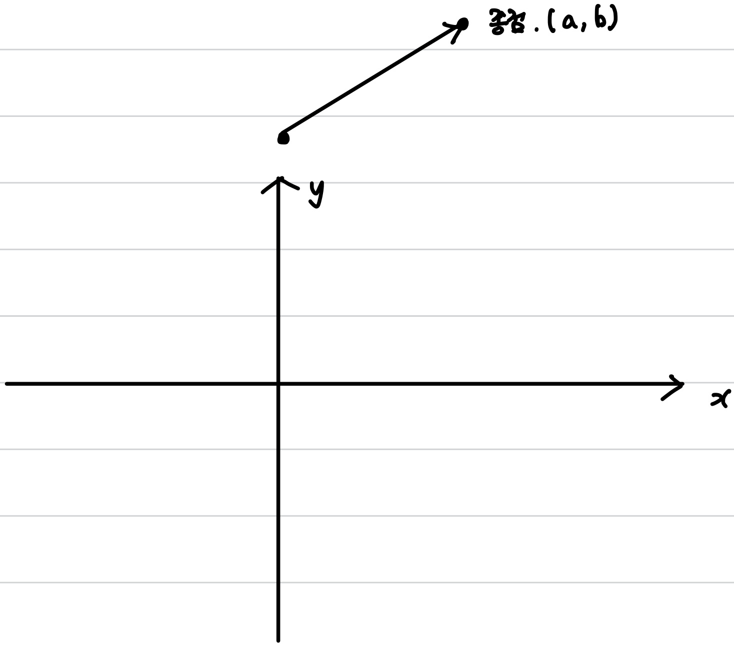
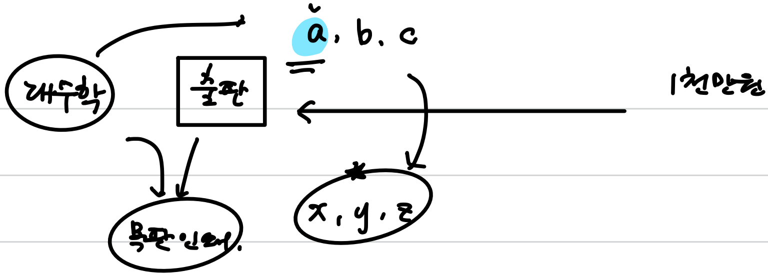


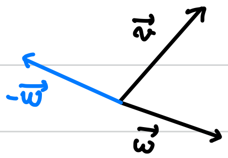
$\Downarrow$ $\vec{v} = (a, b)$





a, b, c
대수학
출판
1천만원
복판인쇄
x, y, z

+, −, ×, ÷



$$\vec{v}-\vec{w}=\vec{x}$$

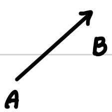
$$\Rightarrow \vec{v}+(\underbrace{-\vec{w}}_{(-1)\cdot\vec{w}})=\vec{x}$$

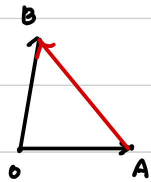
$A(2,4)$, $B=(-1,3)$

$$\boxed{\vec{AB}=\vec{OB}-\vec{OA}}$$

$$= (-1,3)-(2,4)$$

$$= (-3,-1).$$





$$\vec{OA}+\vec{AB}=\vec{OB}$$

$$\vec{AB}=\vec{OB}-\vec{OA}$$

연산, → 항등원, 역원

a b

$\boxed{\text{연산 } \circledast}$

□

$$a \circledast e = e \circledast a = a$$

역원

$$a \circledast b = e$$

$\circledast$에 대한 a의 역원.

수학 가장 아름다운 공식

$1 + e^{\pi i} = 0$

Euler

내적

Dot product.
(Inner product)

1. 내적과 노름(Norm) · 외적

2. 3차원에서의 직선 · 평면

3. 선형성

⟨Norm⟩

$x \in \mathbb{R}^n$

$x = (x_1, x_2, x_3, \cdots, x_n)$

$x = (1, 2, -1, -3, 5, -11)$

1. $\|x\|_1$: 1 - norm

$\|x\|_1$: $|x_1| + |x_2| + |x_3| + \cdots + |x_n|$

$\|x\|_1 = 24$

2. $\|x\|_2$

$\|x\|_2 = \sqrt{6+9+25+121}$

$= \sqrt{161}$

$\|x\|_2 = \sqrt{(x_1)^2 + (x_2)^2 + \cdots + (x_n)^2}$

3. $\|x\|_\infty$

$\|x\|_\infty = \max\{|x_1|, |x_2|, \cdots, |x_n|\}$

$\|x\|_\infty = 11$

내적의 성질

1. $x \cdot x \geq 0$, $x \cdot x = 0 \Leftrightarrow x = 0$

2 $x \cdot y = y \cdot x$

3. $(x+y) \cdot z = x \cdot z + y \cdot z$

4. $(kx) \cdot y = k \cdot (x \cdot y)$

$\mathbb{R}^n$의 두 벡터, $x = (x_1, x_2, \cdots, x_n)$ 가 이루는 각 θ

$y = (y_1, y_2, \cdots, y_n)$

$x \cdot y = \|x\| \|y\| \cos\theta.$

$x \cdot y = 0$?

영이 아닌 두 벡터 x, y에 대하여 $x \cdot y = 0$ 이라고 해보자

$$x \cdot y = \|x\| \|y\| \cos\theta$$

이때, x, y 둘다 영벡터가 아니므로 $\underset{\neq 0}{\|x\|} > 0$, $\underset{\neq 0}{\|y\|} > 0$ $\Rightarrow \cos\theta = 0$

$\downarrow$

$\theta = \frac{\pi}{2} = 90^\circ$

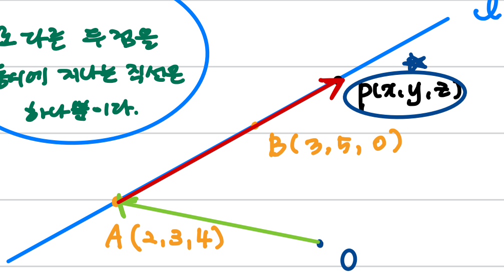
$\mathbb{R}^3$

직선과 평면의 방정식 ?

$\mathbb{R}^3$

$\mathbb{R}^3$에서의 직선과 평면의 방정식

1. 직선 … 서로 다른 두 점을 동시에 지나는 직선은 하나뿐이다.



$\overrightarrow{AP} \parallel \overrightarrow{AB}$

$\Downarrow$

$\overrightarrow{AP} = k\overrightarrow{AB}$

$\overrightarrow{OB} - \overrightarrow{OA} = (1,2,-4)$

$\overrightarrow{OP} = (x,y,z) = \overrightarrow{OA} + \overrightarrow{AP}$

$= (2,3,4) + k\cdot(1,2,-4)$ …▷ 매개변수 k를 이용

$(x,y,z) = (2,3,4) + k(1,2,-4)$

$(x,y,z) - (2,3,4) = k(1,2,-4)$

방향벡터.

$(x-2,\ y-3,\ z-4) = k(1,2,-4)$

$\boxed{x-2 = k}$, $\quad y-3 = 2k$, $\quad z-4 = -4k$

$k = \dfrac{y-3}{2} \qquad k = \dfrac{z-4}{-4}$

* 벡터의 평행

$\vec{x} \parallel \vec{y} \iff \vec{x} = k\cdot\vec{y}$

$$\boxed{k = x-2 = \frac{y-3}{2} = \frac{z-4}{-4}}$$

2. 평면

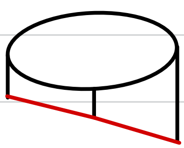
※ 평면의 결정조건

↳ 1. 한 직선위에 있지 않은 세 점을 지날때 ·

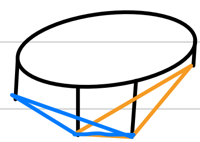
2. 한 직선과 직선밖의 한 점을 포함 ·

3. 평행한 두 직선을 포함 ·

4. 한점에서 만나는 두 직선을 포함. ·

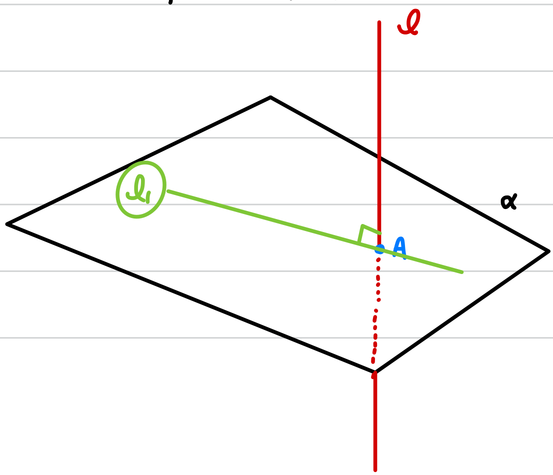


흔들?



흔들? O

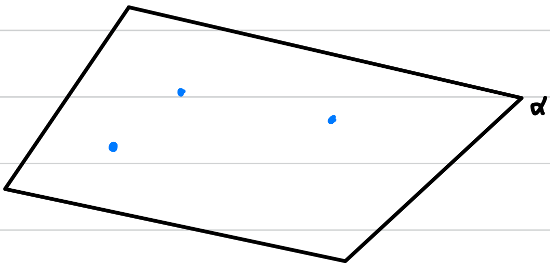
평면에 수직인 직선?

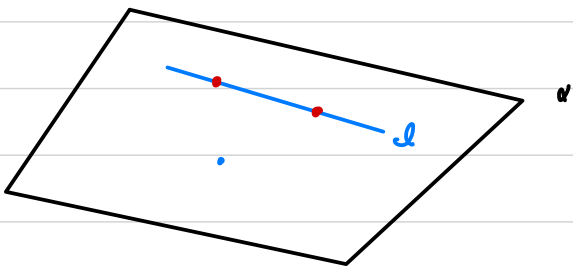


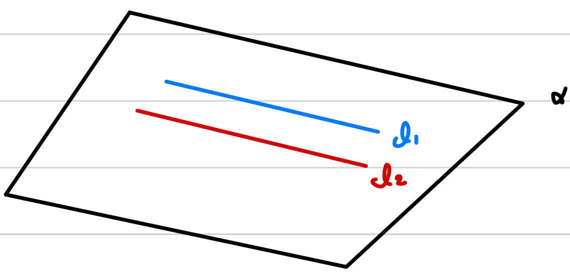
평면 α위의 점 A에서

α와 수직인 직선 = l

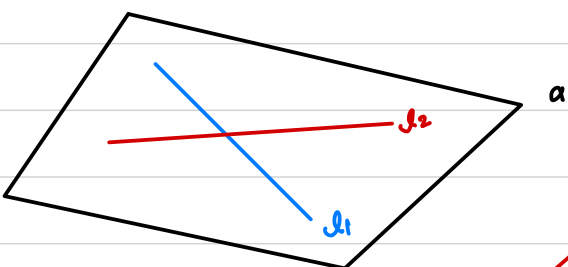
$l \perp \alpha$ $\Longleftrightarrow$ 점 A를 지나는 평면위의 직선 l_1에 대해 $l_1 \perp l$



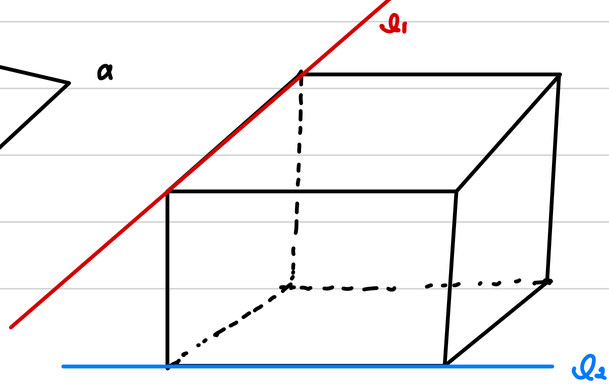




평면 ⇔ 평행하지 않다
= 두 직선이 만난다.

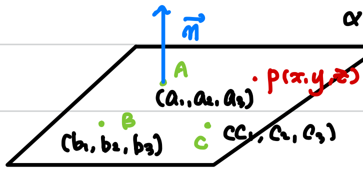


공간 속의 직선 ···> 꼬인위치



$\vec{m} = (a, b, c)$

$\vec{AB}$, $\vec{AC}$



$\vec{AB}$

$P(x, y, z)$?

* Step 1. $\vec{m}$: 점 A에서 평면 α에 수직인 벡터 구하기 ☆

(1) $\vec{m} = (a, b, c)$

(2) $\vec{AB}$, $\vec{AC}$ 구하기 → $\vec{AB} \times \vec{AC}$ (cross product ; 외적)

(3) $\vec{AB} \perp \vec{m}$, $\vec{AC} \perp \vec{m}$ ⇒ $\vec{AB} \cdot \vec{m} = 0$, $\vec{AC} \cdot \vec{m} = 0$ ⇒ m을 하나 찾기

* Step 2. $\vec{AP} \perp \vec{m}$; $\vec{AP} \cdot \vec{m} = 0$ ☆

(1) $\vec{AP} = \vec{OP} - \vec{OA} = (x - a_1, y - a_2, z - a_3)$

(2) $\vec{AP} \cdot \vec{m} = 0$

$\vec{x} \times \vec{y} = \vec{z}$

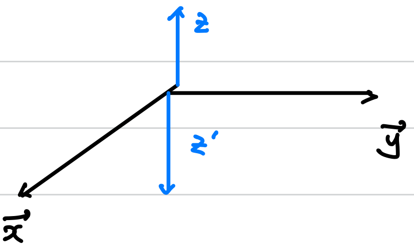
$\vec{x} \perp \vec{z}$

$\vec{y} \perp \vec{z}$

*** 벡터의 외적**

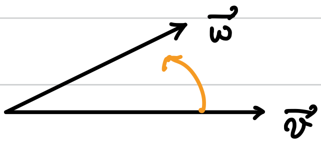
$\mathbb{R}^3$의 벡터 $\vec{x} = (x_1, x_2, x_3)$, $\vec{y} = (y_1, y_2, y_3)$ 에 대해서,

$$\vec{z} = \vec{x} \times \vec{y} = (x_2y_3 - x_3y_2,\ x_3y_1 - x_1y_3,\ x_1y_1 - x_2y_2)$$



$$\vec{x} \times \vec{y} \neq \vec{y} \times \vec{x}$$

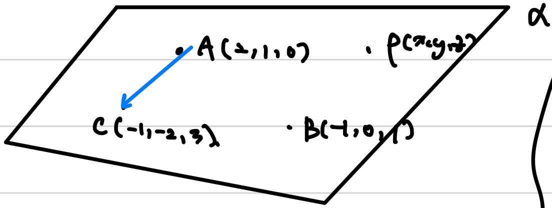
$$\vec{x} \times \vec{y} = -\vec{y} \times \vec{x}.$$



$\vec{v} \times \vec{w}$

$\vec{w} \times \vec{v}$

$$\overrightarrow{AP} = (x-2,\ y-1,\ z)\cdot \overset{\vec{n}}{(0,\ -6,\ -6)} = 0$$



$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$$
$$= (-3,\ -3,\ 3)$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$= (-1,0,1) - (2,1,0)$$
$$= (-3,\ -1,\ 1)$$

$$\overrightarrow{AC} \times \overrightarrow{AB}$$
$$= (0,\ -9+3,\ -6)$$
$$= (0,\ -6,\ -6)$$

$$0\cdot(x-2)$$
$$-6(y-1) - 6(z) = 0$$
$$y - 1 + z = 0$$