Subspace

(Def)

For a Vector space V over F,

If $S \subseteq V$ and S: vector space. S: subspace of V

(Theorem)

For a vector space V and S, subset of V,

i) $\vec{0} \in S$ ➔ S is a non-empty set.

ii) $\forall \vec{x}, \vec{y} \in S$ then $\vec{x} + \vec{y} \in S$ ✓

iii) $\forall c \in F, \vec{x} \in S$ then $c \cdot \vec{x} \in S$.

Then, S is a subspace of V.

(d) $W_4 = \{(a_1, a_2, a_3) \in R^3 : a_1 - 4a_2 - a_3 = 0\}$

(e) $W_5 = \{(a_1, a_2, a_3) \in R^3 : a_1 + 2a_2 - 3a_3 = 1\}$

$\boxed{\textbf{(f)}\ W_6 = \{(a_1, a_2, a_3) \in R^3 : 5a_1^2 - 3a_2^2 + 6a_3^2 = 0\}}$ ✓

9. Let W_1, W_3, and W_4 be as in Exercise 8. Describe $W_1 \cap$

i) $5 \cdot 0 - 3 \cdot 0 + 6 \cdot 0 = 0$

ii) (x_1, x_2, x_3) $\quad 5x_1^2 - 3x_2^2 + 6x_3^2 = 0$

(y_1, y_2, y_3) $\quad 5y_1^2 - 3y_2^2 + 6y_3^2 = 0$

$$\underline{5(x_1+y_1)^2 - 3(x_2+y_2)^2 + 6(x_3+y_3)^2 \neq 0}$$

iii) $(x_1, x_2, x_3) \rightarrow 5x_1^2 - 3x_2^2 + 6x_3^2 = 0.$

$c(x_1, x_2, x_3) \Rightarrow$

(cx_1, cx_2, cx_3)

$5c^2x_1^2 - 3c^2x_2^2 + 6c^2x_3^2$

$= c^2(\overset{0}{5x_1^2 - 3x_2^2 + 6x_3^2})$

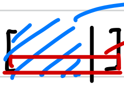
$= 0$

Linear systems

step 1. Augmented Matrix 형태로 표현

step 2. Elementary Row operation (기본행연산)

step 3. RREF 형태로 만들기

step 4. 

1. unique solution ⇒ I_n

2. infinitely many solution → zero row

3. Not exist (singular) ⇒ 

≠ zero.

zero

$$\left[\begin{array}{ccc|c} 1 & 2 & 6 & -1 \\ 2 & 1 & 1 & 8 \\ 3 & 1 & -1 & 18 \\ 1 & 3 & 10 & -5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 6 & -1 \\ 0 & -3 & -11 & 10 \\ 0 & -5 & -19 & 18 \\ 0 & 1 & 4 & -4 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 6 & -1 \\ 0 & 1 & 4 & -4 \\ 0 & -3 & -11 & 10 \\ 0 & -5 & -19 & 18 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 7 \\ 0 & 1 & 4 & -4 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = 3$

$x_2 = 4$

$x_3 = -2$

$$\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 5 \\ x_1 + 4x_2 - 3x_3 - 3x_4 &= 6 \\ 2x_1 + 3x_2 - x_3 + 4x_4 &= 8 \end{aligned}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 5 \\ 1 & 4 & -3 & -3 & 6 \\ 2 & 3 & -1 & 4 & 8 \end{array}\right).$$

$$\Downarrow$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 5 \\ 0 & 2 & -2 & -4 & 1 \\ 0 & -1 & 1 & 2 & -2 \end{array}\right)$$

$$0 \quad 0 \quad 0 \quad 0 \;\Big|\; \square \to \cancel{2}\ 3 .$$

$$\Downarrow$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 5 \\ 0 & 1 & -1 & -2 & \frac{1}{2} \\ 0 & 1 & -1 & -2 & 2 \end{array}\right)$$

$x_2 - x_3 - 2x_4 = \frac{1}{2}$

$'' \quad = 2 .$