Linear Algebra

⟨Review⟩

Matrix +, · (행렬끼리의 곱)

Systems of linear equations. → 행렬

1. Augmented Matrix

↓

2. Elementary row operation.

↓

3. (Reduced) Row Echelon form

(regular/non-singular)
exist

not exist
(singular)

Vector

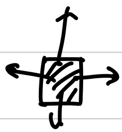
addition. inner product. cross product.

Vector space & Subspace.

Euclidean Vector space. ($\mathbb{R}^n$) *

isomorphic/ism.

(Basis. Span. linear (in)dependent, dimension
column space. solution space.)



<Span>

vector : $v_1, v_2, \cdots v_n$

$\beta = \{v_1, v_2, \cdots, v_n\}$

$v_1, \cdots, v_n$의 선형결합으로 나타낼 수 있는 벡터들의 집합.

$\mathrm{Span}(\beta) = \{v \mid a_1v_1 + a_2v_2 + \cdots + a_nv_n = v\}$

<linear combination>

선형결합

$v_1, v_2, \cdots, v_n$의 선형결합 ;

scalar (숫자) a_i에 대하여 $a_1v_1 + a_2v_2 + \cdots + a_nv_n$

<Theorem>

$v_1, v_2, \cdots, v_n \in V \Rightarrow \mathrm{span}\{v_1, \cdots, v_n\} \leq V$

$\mathrm{Span}\{\underset{\in \mathbb{R}^2}{(2,1)}, \underset{\mathbb{R}^2}{(1,-3)}\} \Rightarrow ? \leq \mathbb{R}^2$

(i) $\mathrm{span}\{(2,1), (1,-3)\} \overset{ok}{=} \mathbb{R}^2$

$a(2,1) + b(1,-3) = (x,y)$

$2a + b = x$
$a - 3b = y$

$a = \boxed{}$
$b = \boxed{}$

$$\begin{array}{l} 2a + b = x \\ -\ \underline{2a - 6b = 2y} \\ 7b = x - 2y \\ b = \frac{x-2y}{7} \end{array} \qquad \begin{array}{l} 6a + 3b = 3x \\ +\ \underline{a - 3b = y} \\ 7a = 3x + y \\ a = \frac{3x+y}{7} \end{array}$$

$2a+b=x \Rightarrow b = x-2a = x - \left(\frac{6x+2y}{7}\right) = \frac{x}{7} - \frac{2}{7}y.$

$a-3b=y$

$a-3(x-2a)=y$

$7a = 3x+y$

$a = \frac{3x+y}{7}$

$\mathrm{Span}\{(1,2,0),(3,-1,2),(7,0,4)\} \overset{?}{=} \mathbb{R}^3.$ X

$a(1,2,0)+b(3,-1,2)+c(7,0,4) = (x,y,z)$

$$\begin{cases} a+3b+7c = x \\ 2a-b = y \\ 2b+4c = z \end{cases}$$

$a+3b+7c=x$

$6a-3b = 3y$

$7a+7c = x+3y$

$$\begin{bmatrix} 1 & 3 & 7 \\ 2 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{pmatrix} 1 & 3 & 7 \\ 0 & -7 & -14 \\ 0 & 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ -2x+y \\ z \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 & 7 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ -2x+y+4z \\ z \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x+6x-3y-12z \\ -2x+y+4z \\ +4x-2y-8z+z \end{pmatrix}$$

Dimension ?

$\text{span}\{(1,2,0), (3,-1,2), (7,0,4)\} \subseteq \mathbb{R}^3$

Basis

$\boxed{V: \text{vector space}}$

$\beta = \{v_1, v_2, \cdots, v_n\}$, $v_i \in V$에 대하여

1. $\mathrm{span}(\beta) = V$
2. $v_1, \cdots, v_n$; linearly independent ?

· linearly independent (선형독립)

$a_1v_1 + a_2v_2 + \cdots + a_nv_n = 0$ 영벡터 일때,

$0 = a_1 = a_2 = \cdots = a_n$ 이면 선형독립

ex)

$(1, 2, 0), (3, -1, 2), (7, 0, 4)$ 가 선형독립? 종속?

$a(1,2,0) + b(3,-1,2) + c(7,0,4) = (0,0,0)$

$$\begin{aligned} a + 3b + 7c &= 0 \\ 2a - b &= 0 \rightarrow b = 2a \\ 2b + 4c &= 0 \rightarrow b = -2c \end{aligned}$$

$\frac{1}{2}b + 3b - \frac{7}{2}b = 0$

$b = 2$

$(a, b, c) = \left(\frac{b}{2}, b, -\frac{b}{2}\right) \Rightarrow$ 0이 된다 $\quad 1 \quad 2 \quad -1$

$$(-1)\cdot(-1,0,3) + (-2)\cdot(3,5,7) + 1\cdot(5,10,17) =$$

$(-1,0,3)$, $(3,5,7)$, $(5,10,17)$

$$-a+3b+5c=0$$
$$5b+10c=0 \rightarrow b=-2c$$
$$3a+7b+17c=0$$

$$\begin{cases} -a-6c+5c=0 \\ 3a-14c+17c=0 \end{cases}$$

$$+a+c=0$$
$$\cancel{3a}\ a=-c$$

$$(a,b,c)=(-c,-2c,c) \Rightarrow 0$$

선형종속.

$$av_1+bv_2+cv_3=0$$

$0=a=b=c$ 선형독립

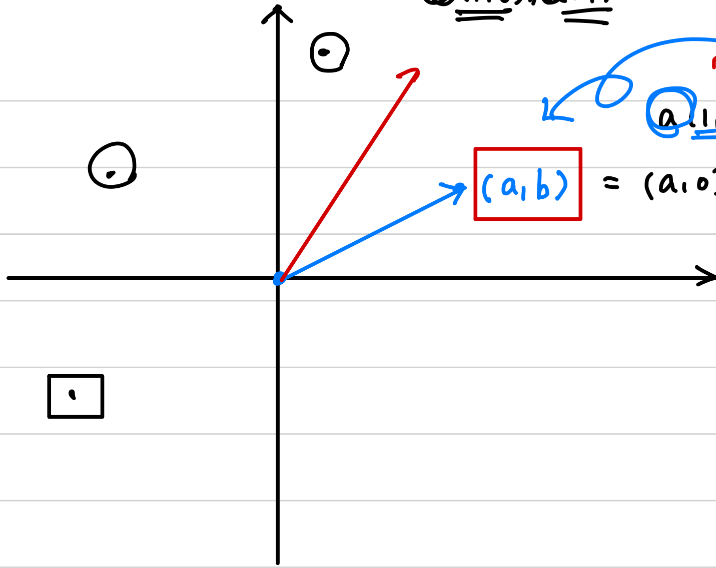
$\{(1,2,0), (3,-1,2), (7,0,4)\}$

$$\begin{aligned} a + 3b + 7c &= 0 \\ 2a - b &= 0 \\ 2b + 4c &= 0 \end{aligned} \Rightarrow \left(\begin{array}{ccc|c} 1 & 3 & 7 & 0 \\ 2 & -1 & 0 & 0 \\ 0 & 2 & 4 & 0 \end{array}\right)$$

○(1,0)+○(0,1)

basis.

$\mathbb{R}^2$

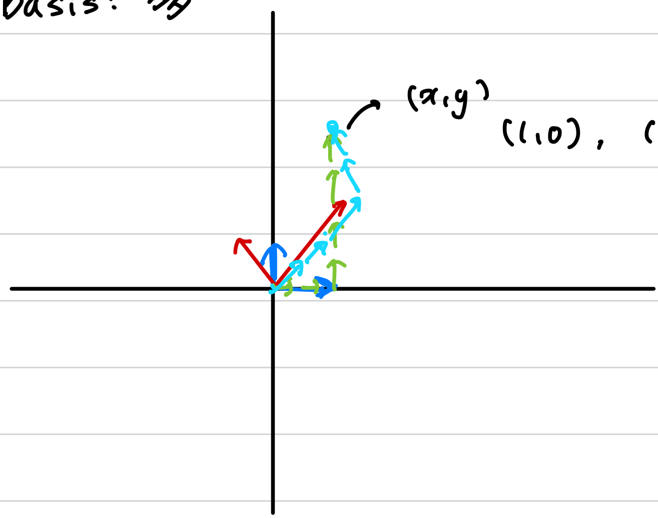


$a(1,0) + b(0,1)$

$\boxed{(a,b)} = (a,0) + (0,b)$

Basis: 기저

$(x,y) = x(1,0)$
$+ y(0,1)$.



$(1,0)$, $(0,1)$

Basis 공통점?

⇓

궁궁궁

Basis를 이루는 벡터의 개수가 일정,

‖

Dimension (차원)

☆ Dimension (차원) ☆

(Euclidean Vector space) : 유클리드 벡터공간 $= \mathbb{R}^n$

$\dim(\mathbb{R}^n) = n$

(, , ,) standard basis.

(1, .) (0, 1,)

↳ linearly independent 한 2개

다음 중 $\mathbb{R}^2$ Basis가 아닌 것은?

$a v_1 = -b v_2$

$a v_1 + b v_2 = 0$ $\quad v_1 = -\frac{b}{a} v_2$ x

$\{(1,0),(0,1)\}$ O

2개

$\{(1,1),(-1,-1)\}$ X linearly independent

$\{(1,2),(0,1)\}$ O

$\{(1,2),(2,1),(3,5)\}$ X X

$\{(-1,5),(6,3)\}$. O

1. Basis. linearly (in) dependent. Dimension

2. $\mathbb{R}^n$의 basis : linearly independent 한 n개의 vector들로 구성!

$\mathbb{R}^3$의 basis ?

$\{(1,2,-1), (-2,1,0), (1,1,1)\}$.

$a - 2b + c = 0$ $\qquad$ $2b = 0 \rightarrow b = 0$

$2a + b + c = 0$

$-a + c = 0 \longrightarrow a = -c$

$a = -c = 0$

$b = 0$

$c = 0$

set

Basis

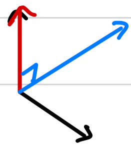
Ordered basis (순서 기저)

Basis ☆

$\Downarrow$

" 서로 수직인 기저 (수직기저)".

dot product = 0



수직기저

Ordered Basis

β_1 ; ordered basis of V $\Rightarrow$ $\beta = \{v_1, v_2, \cdots, v_n\}$

$v \in V$, $\quad v = a_1 v_1 + a_2 v_2 + \cdots + a_n v_n$

$$[v]_{\beta_1} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

$$(0, 11) = (2a - b, \ 11a)$$

$$a = 1, \ b = 2$$

$$[v]_{\beta_2} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$