# * Vector Space * 중요*

유클리드 벡터공간

## · Euclidean Vector space. ($\mathbb{R}^n$)

### * Cartesian Product (데카르트 곱)

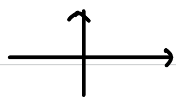
→ For two sets $A, B$, $A \times B$ is a Cartesian product of $A$ and $B$; $A \times B := \{(a,b) \mid a \in A \text{ and } b \in B\}$.

ex) $A = \{1, 2\}$, $B = \{a, b, c\}$

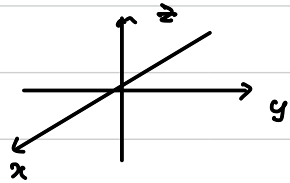
$A \times B = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\}$ //

$\mathbb{R}$ : 실수 전체

$\mathbb{R} \times \mathbb{R} = \{(x,y) \mid x \in \mathbb{R}, y \in \mathbb{R}\} = \mathbb{R}^2$



$\mathbb{R} \times \mathbb{R} \times \mathbb{R} = \mathbb{R}^3 = \{(x,y,z) \mid x \in \mathbb{R}, y \in \mathbb{R}, z \in \mathbb{R}\}$



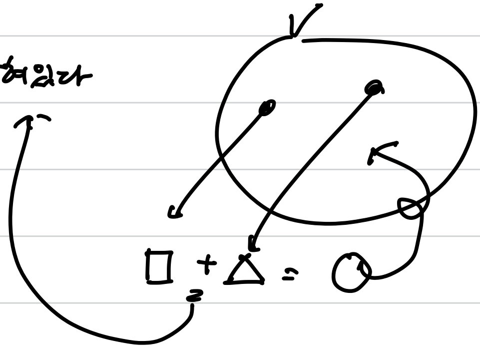
V ; 벡터들의 집합

2가지 연산

1. vector addition (벡터 덧셈)
2. scalar multiplication (스칼라배)

상수.

1. 연산에 대해 닫혀있음?




$\mathbb{R}^2$

? ↓

$(1,2) + (-1,0)$

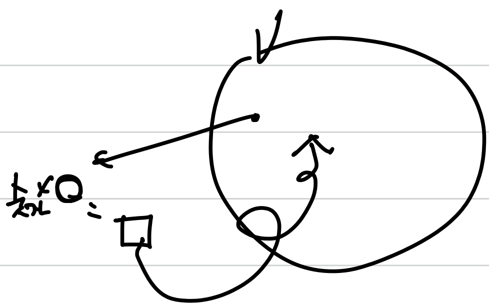
$=$

$(0,2)$

$3 \cdot (1,2)$

$=$

$(3,6)$



★ Ⓥ : 벡터 덧셈 / 스칼라곱 — 닫혀있음 ★

V의 원소 $\vec{u}, \vec{v}, \vec{w}$, , c, d ; 스칼라.

1) $\vec{u} + \vec{v} = \vec{v} + \vec{u}$ ; commutative (교환법칙),

2) $\{\vec{u} + \vec{v}\} + \vec{w} = \vec{u} + \{\vec{v} + \vec{w}\}$. ; associative (결합법칙)

3) zero vector $\in V$

덧셈에 대한 항등원

벡터.

4) $\vec{x} \in V$, $-\vec{x} \in V$ 역원.

5) $1 \cdot \vec{v} = \vec{v}$ 인 스칼라 존재.

6) $c(d\vec{v}) = (cd)\boxed{\vec{v}}$

7) $(c+d)\vec{v} = c\vec{v} + d\vec{v}$,

8) $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$.

5) c = 1

6) o

7) o

8) o

**18.** Let V $= \{(a_1, a_2) \colon a_1, a_2 \in R\}$. For $(a_1, a_2), (b_1, b_2) \in$ V and $c \in R$, define

$$(a_1, a_2) + (b_1, b_2) = (a_1 + 2b_1, a_2 + 3b_2) \quad \text{and} \quad c(a_1, a_2) = (ca_1, ca_2).$$

Is V a vector space over $R$ with these operations? Justify your answer.

1) $(a_1 + 2b_1, a_2 + 3b_2) \neq (b_1 + 2a_1, b_2 + 3a_2$

2) a

3) $(a_1 + 2b_1, a_2 + 3b_2) = (a_1, a_2) \to b_1 = 0, b_2 = 0$

4) $(a_1 + 2b_1, a_2 + 3b_2) = (0, 0) \to b_1 = -\frac{a_1}{2}, b_2 = -\frac{a_2}{3}$

# <General vector Space>

7) $(c+d)(a_1, a_2) = c(a_1, a_2) + d(a_1, a_2)$

$$((c+d)\cdot a_1, (c+d)\cdot a_2)$$

$$= (c\cdot a_1, c\cdot a_2) + (d\cdot a_1, d\cdot a_2)$$

$$\neq (c\cdot a_1 + 2\cdot d\cdot a_1, c\cdot a_2 + 3\cdot d\cdot a_2)$$

8) $c(a_1+2b_1, a_2+3b_2) = (c\cdot a_1, c\cdot a_2) + (c\cdot b_1, c\cdot b_2)$

$$= (c\cdot a_1 + 2\cdot c\cdot b_1, c\cdot a_2 + 3\cdot c\cdot b_2)$$

## <Example>

$+ (0,0)$

$(x, y)$

$(a_1, \overset{\shortparallel 0}{a_2}) + (b_1, b_2) = (0, 1)$

13. Let V denote the set of ordered pairs of real numbers. If $(a_1, a_2)$ and $(b_1, b_2)$ are elements of V and $c \in R$, define

$$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 b_2) \quad \text{and} \quad c(a_1, a_2) = (ca_1, a_2).$$

Is V a vector space over $R$ with these operations? Justify your answer.

$(a_1 + b_1, a_2 b_2)$

$= (0, 1)$

$b_1 = -a_1$

$b_2 = \frac{1}{a_2}$

$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2) \Leftarrow$ Define

$(2,3) + (3,1) = (5, 3)$

zero vector $= (z_1, z_2)$

$\boxed{(0,0)}$

$(a_1, a_2) + (z_1, z_2) = (a_1, a_2)$

↑ zero vector

$(0, 1)$

$(a_1 + z_1, a_2 z_2) = (a_1, a_2)$

$z_1 = 0$

$z_2 = 1$

$c(a_1, a_2) = (ca_1, a_2)$

$c\,1\,(a_1, a_2) = (a_1, a_2)$.

$1$

* 항등원과 역원 *

$-\vec{x}$

$a \circledast e = a$

항등원.

$a \circledast b = e$

3,

<Examples>

$\mathbb{R}^n$ : Euclidean Vector space.

- $M_{m\times n}$ : m행 n열 행렬 $\cong \mathbb{R}^{mn}$

- $P_n$ : (n차이하의 다항식). $\cong \mathbb{R}^{n+1}$

3차다항식 ← ( $x^3+2x+6$ , $-x^3+2x^2-x+1$ . )

$\cancel{2x^2+x+7}$

$\boxed{P_5}$ $(a_0, a_1, a_2, \cdots, a_5) \Rightarrow \mathbb{R}^6$ Isomorphism.

$a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5$

$2 + \quad x^2 - \frac{1}{2}x^3 \quad + x^5$

⇓

$(2, 0, 1, -\frac{1}{2}, 0, 1)$

Definition : 정의

Lemma : 보조정리

Theorem : 정리

Example : 예시.

Corollary : 추가정리

$1 \to 2$

$2 \to 3$

$3 \to 4$

귀류법 $a \to b$ True.

$\boxed{a} \to \boxed{\cancel{b}}$

$a \to b$

Suppose : 부정적

Assume : 긍정적

$\boxed{a} \to b$

# ⟨Subspace⟩ 부분공간

(Def)

Let V be a vector space.

Then, $S \subseteq V$ is a subspace of V when S is a vector space with same operations of V

1. Subset
2. Vector space 8개 ) ⇨ o.k.

$(0,0,0)$

(a) $W_1 = \{(a_1, a_2, a_3) \in \mathrm{R}^3 : a_1 = 3a_2 \text{ and } a_3 = -a_2\}$

$\mathbb{R}^3$ subspace

1. Subset ?

2. 1) zero vector ok

2) $(a_1, a_2, a_3) + (b_1, b_2, b_3) = (\boxed{a_1 + b_1}, a_2 + b_2, \boxed{a_3 + b_3})$

ok ① $a_1 + b_1 = 3(a_2 + b_2)$ ?

③ $a_3 + b_3 = -(a_2 + b_2)$ ?

$a_1 = 3a_2$
$b_1 = 3b_2$