$$
\begin{aligned}
2x_1 + x_2 + 3x_3 + x_4 &= 3 \\
x_1 + x_2 + x_3 - x_4 &= 6 \\
x_1 - x_2 + 3x_3 + 5x_4 &= -12 \\
4x_1 + x_2 + 7x_3 + 5x_4 &= -3
\end{aligned}
$$

$$
\left(\begin{array}{cccc|c}
2 & 1 & 3 & 1 & 3 \\
1 & 1 & 1 & -1 & 6 \\
1 & -1 & 3 & 5 & -12 \\
4 & 1 & 7 & 5 & -3
\end{array}\right)
$$

$$
\left(\begin{array}{cccc|c}
2 & 1 & 3 & 1 & 3 \\
1 & 1 & 1 & -1 & 6 \\
0 & -2 & 2 & 6 & -18 \\
0 & -1 & 1 & 3 & -9
\end{array}\right)
$$

$$
\Rightarrow \left(\begin{array}{cccc|c}
1 & 1 & 1 & -1 & 6 \\
2 & 1 & 3 & 1 & 3 \\
0 & 1 & -1 & -3 & 9 \\
0 & 0 & 0 & 0 & 0
\end{array}\right)
$$

$$
\Rightarrow \left(\begin{array}{cccc|c}
1 & 1 & 1 & -1 & 6 \\
0 & -1 & 1 & 3 & -9 \\
0 & 1 & -1 & -3 & 9 \\
0 & 0 & 0 & 0 & 0
\end{array}\right) \Rightarrow
$$

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 6 \\ 0 & -1 & 1 & 3 & -9 \\ 0 & 1 & -1 & -3 & 9 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 6 \\ 0 & 1 & -1 & -3 & 9 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 2 & 2 & -3 \\ 0 & 1 & -1 & -3 & 9 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} \boxed{1} & 0 & 2 & 2 & -3 \\ 0 & \boxed{1} & -1 & -3 & 9 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1$ $\qquad$ $x_1 = -3 - 2x_3 - 2x_4$

$x_2$ $\qquad$ $x_2 = 9 + x_3 + 3x_4$

$[x_3, x_4]$ : free variable

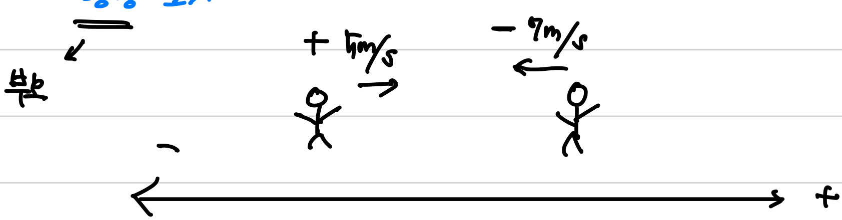
「 Real Number 」의 집합 $\mathbb{R}$ $\nearrow$ $\mathbb{R}$

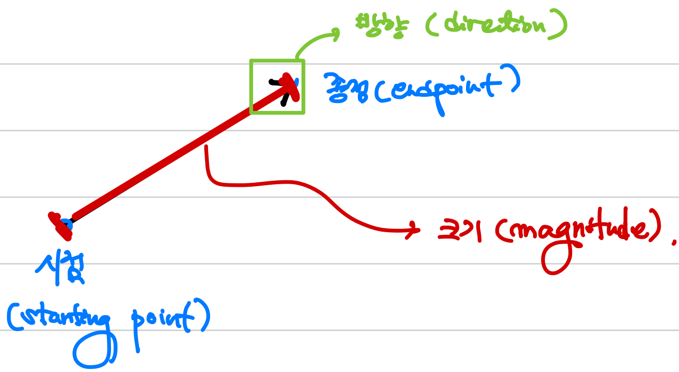
# [Vector]

(Def) 크기와 방향을 갖는 물리량 = Vector (벡터)

ex) 속도 / 속력 → scalar    ex) 힘

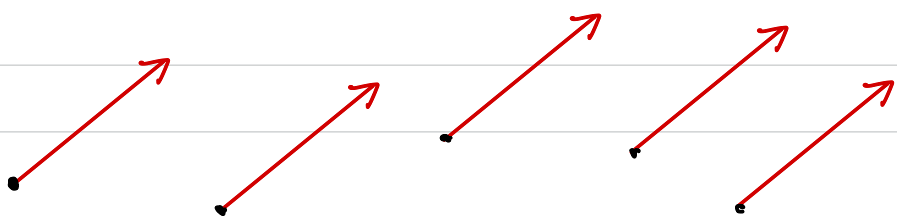
크기만 존재



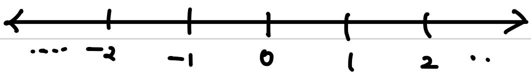

$\vec{v}$ , C

※서로 같은 두 벡터 : 크기 & 방향이 서로 같음.

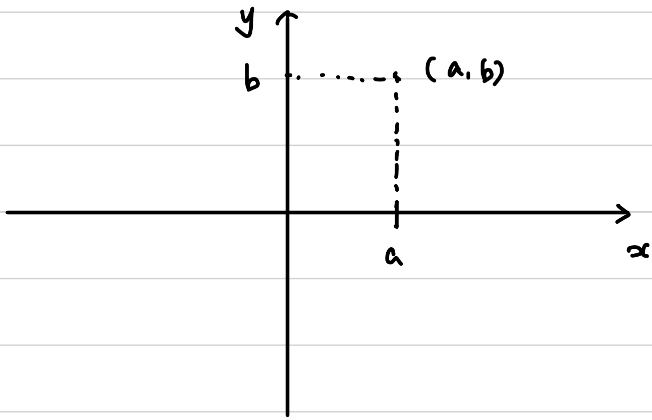


# <유클리드 벡터 공간>

$\mathbb{R}, \mathbb{R}^2, \mathbb{R}^3$ 좌표공간 / 평면



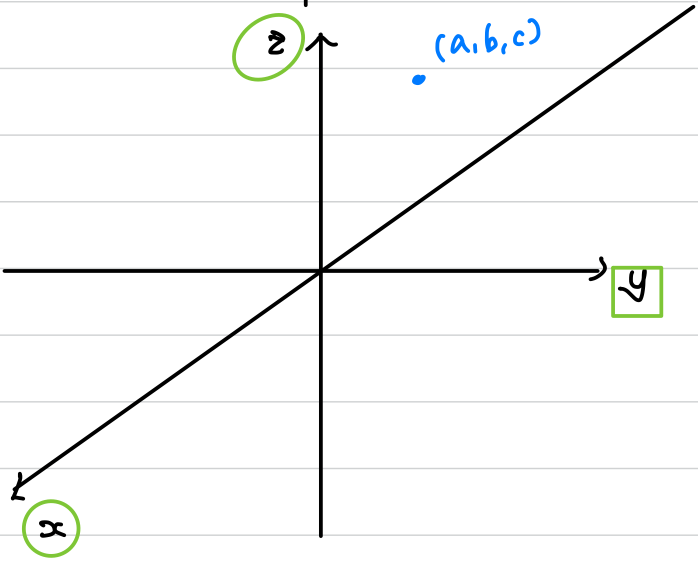

$\mathbb{R}$
실수(Real Number).




좌표평면

$\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$

(a,b) tuple




좌표공간

$\mathbb{R} \times \mathbb{R} \times \mathbb{R}$

$= \mathbb{R}^3$

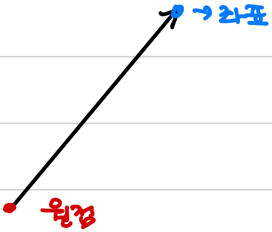
Positive Orientation.

# 벡터 ⇒ 좌표

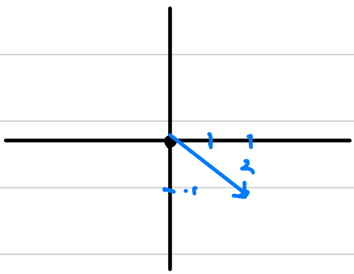
1차원

$\mathbb{R}$, $\mathbb{R}^2$, $\mathbb{R}^3$ → 3차원

2차원



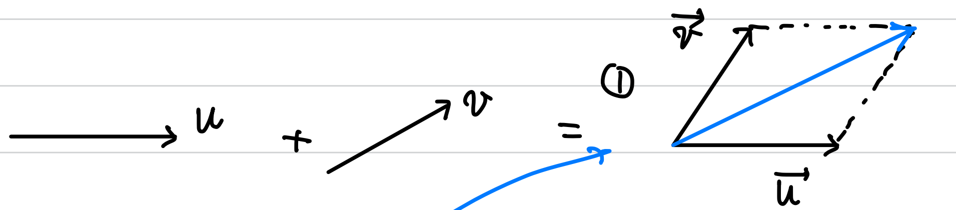

(2, −1)



# 벡터의 연산

## 1. Vector Addition

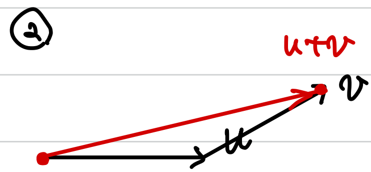
$\vec{u} + \vec{v} = ?$




$\mathbb{R}^2$

$(a,b) + (c,d)$

$= (a+c, b+d)$



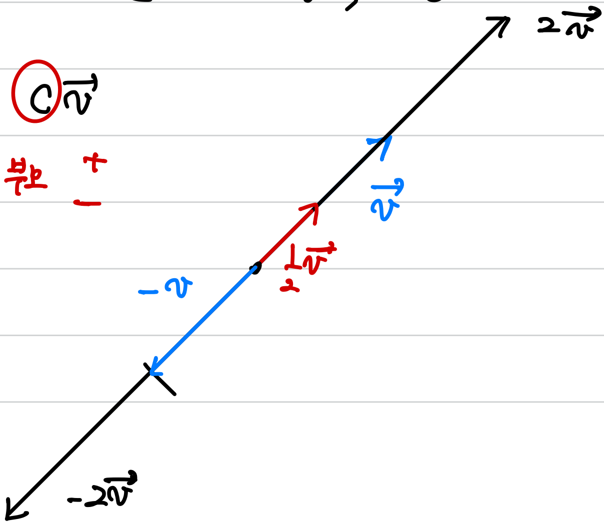

축소 or 유지

## 2. 스칼라곱

$\vec{v}$, c

$c\vec{v}$

1. 부호 + −




$0 \le |c| \le 1$

2. $|c|$

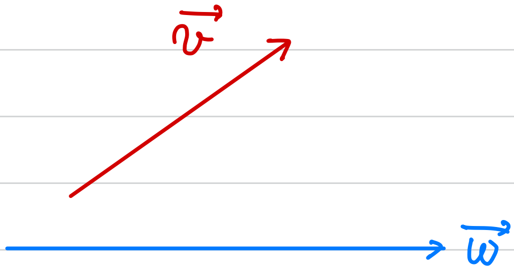
$c = 1$ $c = -1$

$c = 2$

$c = \frac{1}{2}$

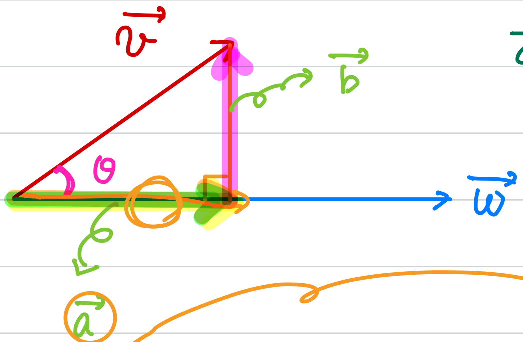
$1 < |c|$

# Inner Product (dot product)



$$\vec{v} \cdot \vec{w} = \|\vec{v}\| \, \|\vec{w}\| \cos\theta$$

1.



$\vec{a} + \vec{b} = \vec{v}$

$\vec{b} = \vec{v} - \vec{a}$

$$\cos\theta = \frac{\|\vec{a}\|}{\|\vec{v}\|}$$

$$\underbrace{\|\vec{v}\| \, \|\vec{w}\| \cos\theta}_{\|\vec{w}\| \, \|\vec{a}\|} = \vec{v} \cdot \vec{w}$$

$\vec{a} = ?$

벡터 $\vec{v}$에서, $\vec{w}$로의 수직인 선을 그었을 때,

## Norm of Vectors.

$\|\vec{v}\|$: $\vec{v}$의 크기.

좌표를 통한 내적?

$\mathbb{R}^2$

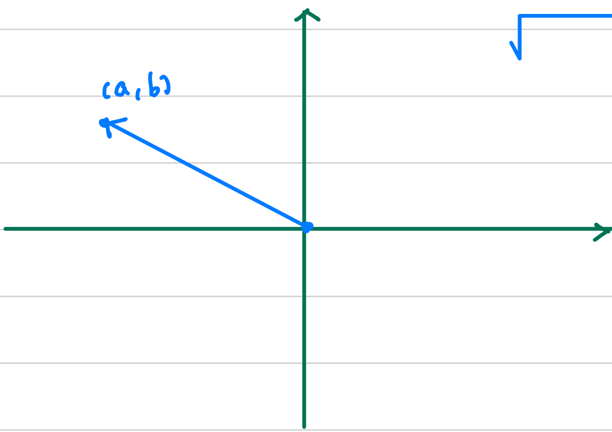
$$(a,b)\cdot(c,d) = ac+bd$$

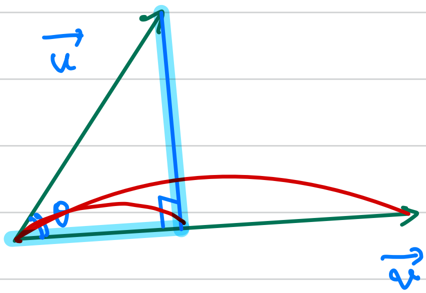
$\mathbb{R}^3$

$$(a,b,c)\cdot(d,e,f) = ad+be+cf$$

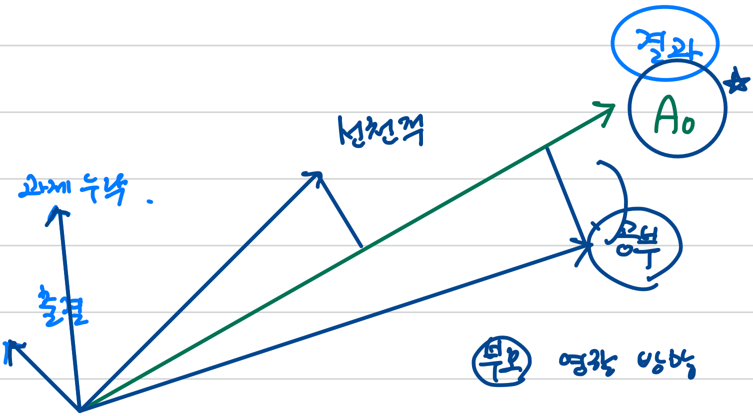
Norm : 벡터의 길이

$$\|\vec{v}\| = \sqrt{\vec{v}\cdot\vec{v}}$$






$\vec{u} \cdot \vec{v}$




내적

결과-원인

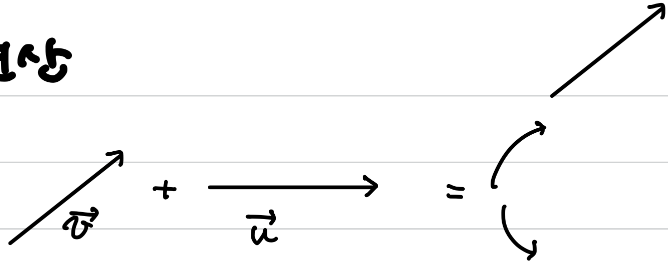
공부 ⇒ $A_0$

부호 영향 방향

얼마나 영향?

# 벡터의 연산

1. $\vec{v} + \vec{u}$ :



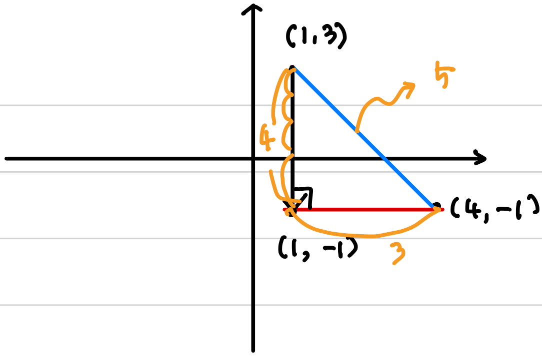

2. $c \cdot \vec{v}$

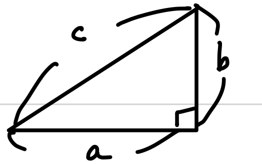
3. $\vec{u} \cdot \vec{v} = \langle \vec{u}, \vec{v} \rangle$

4. $\vec{v} \times \vec{u}$ (only in $\mathbb{R}^3$)

좌표를 통한 벡터.

직선, 평면




$c^2=a^2+b^2$

(a,b) 와 (c,d) 사이의 거리

$= \sqrt{(a-c)^2+(b-d)^2}$

$\sqrt{\vec{v}\cdot\vec{v}}$

$\shortparallel$

$\sqrt{a^2+b^2}$

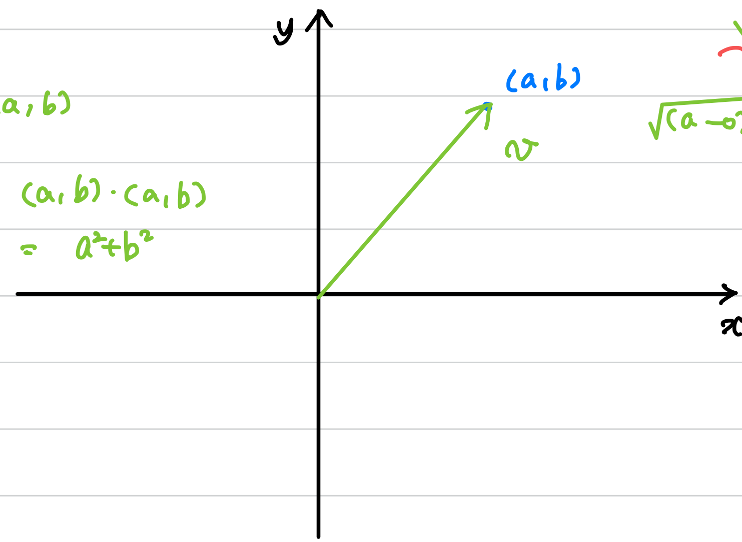
$\shortparallel$

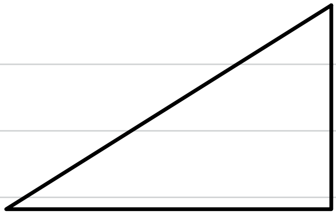
$\sqrt{(a-0)^2+(b-0)^2}$

$\vec{v}=(a,b)$

$\vec{v}\cdot\vec{v}=(a,b)\cdot(a,b)$

$= a^2+b^2$





$$\|\vec{v}\| = \sqrt{\vec{v}\cdot\vec{v}}$$

$$\vec{v}\cdot\vec{u} = (a,b)\cdot(c,d) = ac+bd$$

$$\vec{v}\cdot\vec{u} = (a,b,c)\cdot(d,e,f) = ad+be+cf$$

$$\vec{u}\cdot\vec{v} = \underbrace{\|\vec{u}\|\,\|\vec{v}\|}\cos\theta$$

$$\vec{u} \quad \vec{v}$$

$$\underbrace{(1,3)}, \quad \underbrace{(-1,\ 5)} \qquad \theta \qquad \cos\theta = ?$$

$$\sqrt{10} \times \sqrt{26} \times \cos\theta = -1+15 = 14$$

$$2\sqrt{5}\sqrt{13}\cos\theta = 14$$

$$\cos\theta = \frac{7}{\sqrt{65}}$$

$$\vec{u} = (1,2,3), \quad \vec{v} = (1,-1,0) \qquad \theta$$

$$\cos\theta = ?$$

$$\sqrt{1+4+9} \times \sqrt{2} \times \cos\theta = 1-2$$

$$\sqrt{14} \times \sqrt{2} \times \cos\theta = -1 \qquad \cos\theta = -\frac{1}{\sqrt{28}}$$

$$= -\frac{1}{2\sqrt{7}}$$

$i = (1,0,0)$ , $j = (0,1,0)$ , $k = (0,0,1)$

$3i + j = 3(1,0,0) + (0,1,0)$

$= (3,0,0) + (0,1,0) = (3,1,0)$

$u = (3,-1,4)$

$v = -i - 3j + k$

$w = (-1,1,2)$

(a) $u \cdot v \rightarrow -3 + 3 + 4 = 4$

$= (-1,-3,1)$

(b) $(3u) \cdot (-2v)$

$\downarrow$

$(9,-3,12) \cdot (2,6,-2)$

$18 - 18 - 24 = -24$

X (h) $(u) \cdot (\underline{v \cdot w})$ $\quad v \cdot w =$ [illegible]

$(3,-1,4) \cdot (1 - 3 + 2)$

$(3,-1,4) \cdot 0$

$= (0,0,0)$

$\vec{v}+\vec{w}$
$\shortparallel$
$\vec{u}$

$c \cdot \overset{\mathscr{G}}{\vec{v}} = \vec{w}$

$\overset{\curvearrowleft 6}{\vec{v}} \cdot \overset{3}{\vec{w}} = \|\vec{v}\| \|\vec{w}\| \cos\theta$

$\vec{u} \perp \vec{v} = \vec{u} \cdot \vec{v} = 0$

## 벡터의 평행과 수직

$\vec{u} \parallel \vec{v} \Rightarrow \vec{u} = c \cdot \vec{v}$