Data [illegible]

	romeo	juliet	happy	Dagger	live	Die	free	New-hampshire
d_1	1	1	0	0	0	0	0	0
d_2	0	1	1	1	0	0	0	0
d_3	1	0	0	1	0	1	0	0
d_4	0	0	0	0	1	1	1	1
d_5	0	0	0	0	0	0	0	1

Data: Documents

- d_1: Romeo and Juliet
- d_2: Juliet O happy dagger
- d_3: Romeo died by dagger
- d_4: Live free or die that is the New-Hampshire's motto
- d_5: Did you know New-Hampshire is in New-England

Document-Word matrix $A_{D\times V}, D = 5, V = 8$

Data 행렬

Data

행렬

분해 → Data 의 특징

Eigendecomposition (고유값 분해) ⇒ Square matrix

(# of row = # of column)

eigenvector

$Av = \lambda v$

eigenvalue

$v \neq 0$, $A \neq \lambda I$

$Av = \lambda I v$

$(A - \lambda I)v = 0$ zero-division

$\det(A - \lambda I) = 0$

특성다항식.

1. Similar matrix

For $A_{n\times n}$, $B_{n\times n}$, $\exists\, P_{n\times n}$ s.t. $A = P^{-1}BP$

$A = \begin{pmatrix} v_1 & v_2 & \cdots & v_n \end{pmatrix}$ $\quad B = \begin{pmatrix} w_1 & w_2 & \cdots & w_n \end{pmatrix}$

$\underbrace{\{v_1, v_2, \cdots, v_n\}}_{V}$ $\quad \{w_1, w_2, \cdots, w_n\}$ V

기저

같은 벡터 space의 기저

- ordered basis
- coordinate vector
- Matrix representation

QR decomposition

Square matrix A $\qquad A = QR$

Q: orthogonal matrix

R: upper triangular matrix

(orthogonal matrix) 직교행렬

$v_i, v_j \quad i \neq j$

$$\begin{pmatrix} v_1 & v_2 & \cdots, & v_n \end{pmatrix}$$

$\langle v_i, v_j \rangle = 0$

수직

orthonormal

orthogonal + normal $\qquad$ 크기 = 1

$$\begin{pmatrix} w_1 & w_2 & \cdots & w_n \end{pmatrix}$$

QR ~.

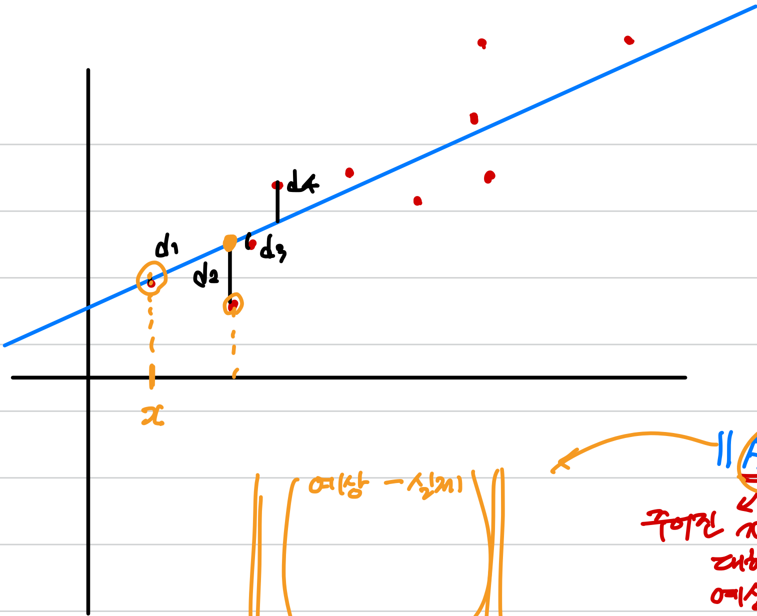
$\|Ax - b\|$ $\quad x = ?$

$\min \|\boxed{Ax} - b\|$ 최소제곱법

Statistics

$y = ax + b$

기준



$(d_1)^2 + (d_2)^2 + \cdots$

$\cdots + (d_n)^2$ 가장 작은값.

$\|Ax - b\|$

주어진 x에 대한 예상 data

실제 데이터

$\|$예상 − 실제$\|$

회귀분석

$|Ax-b|^2$ 최소화 $= (Ax-b)^t (Ax-b)$

$$Ax = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \quad b = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix}$$

$$Ax-b = \begin{pmatrix} v_1-u_1 \\ v_2-u_2 \\ \vdots \\ v_n-u_n \end{pmatrix}$$

$$\underbrace{\left\{ (v_1-u_1)^2 + (v_2-u_2)^2 + \cdots + (v_n-u_n)^2 \right\}}_{\|Ax-b\|^2}$$

$$\begin{pmatrix} v_1-u_1 \\ v_2-u_2 \\ \vdots \\ v_n-u_n \end{pmatrix}$$

$$(Ax-b)^t = (v_1-u_1, v_2-u_2, \ldots, v_n-u_n)$$

$$(Ax-b)^t (Ax-b)$$

$$= (v_1-u_1, v_2-u_2, \ldots, v_n-u_n) \begin{pmatrix} v_1-u_1 \\ v_2-u_2 \\ \vdots \\ v_n-u_n \end{pmatrix}$$

$$\| v_{n\times 1} \|^2 = v^t v$$

$\min \|Ax-b\| \iff \underbrace{\|Ax-b\|^2}_{(Ax-b)^t(Ax-b)}$ minimize

$(A-B)^T = A^T - B^T$

$(AB)^T = B^T A^T$

$$(Ax-b)^t(Ax-b)$$

$$= (x^t A^t - b^t)(Ax-b)$$

$$= x^t A^t A x - \underline{x^t A^t b} - b^t A x + b^t b$$

$x : n\times 1$ $\quad b : n\times 1$

$A : n\times n$

Note that $(x^t A^t b)^t = b^t A x$

$b^2 = \|b\|^2$

$$\underset{1\times n}{x^t}\ \underset{n\times n}{A^t}\ \underset{n\times 1}{b} = \text{숫자}$$

$$\|Ax-b\|^2 = \underline{x^t A^t A x - 2x^t A^t b + b^2}$$

$$\swarrow$$

$$0 = 2A^t A x - 2A^t b$$

$$2A^t b = 2A^t A x$$

$$A^t b = A^t A x$$

$$(A^t A)^{-1} A^t b = x.$$

$$A^{-1} = (QR)^{-1}$$
$$= R^{-1} Q^{-1}$$

$(A^t A)^{-1} A^t b = x.$ $\qquad A = QR$

$$A^{-1} \cancel{(A^t)^{-1}} \cancel{A^t} b = x$$

$$A^{-1} b = x$$

$x = A^{-1}b$ $\quad$ $Ax = b$

$x = A^{-1}b$

$A = QR$ $\quad$ $x = R^{-1}Q^{-1}b$

$A^{-1} = R^{-1}Q^{-1}$ $\quad$ $= R^{-1}Q^{T}b$

Orthogonal matrix $Q^{T} = Q^{-1}$

$i \neq j$ i^{th} column, j^{th} column ⇒ 수직

Orthogonal Matrix $\boxed{A^T = A^{-1}}$ $\|v_i\| = 1$.

Consider $A = \begin{pmatrix} v_1 & v_2 & \cdots & v_n \end{pmatrix}$

$A^T A$ $\quad v_i^t v_j = 0 \quad (i \neq j)$

$$\begin{pmatrix} v_1^t \\ v_2^t \\ v_3^t \\ \vdots \\ v_n^t \end{pmatrix} \begin{pmatrix} v_1 & v_2 & \cdots & v_n \end{pmatrix} = \begin{pmatrix} \overset{1}{\boxed{v_1^t v_1}} & \overset{0}{v_1^t v_2} & \cdots & \overset{0}{v_1^t v_n} \\ v_2^t v_1 & \overset{1}{v_2^t v_2} & \cdots & \\ 0 & & & \\ 0 & & & \end{pmatrix} = I$$

$\langle v_1, v_2 \rangle = v_1^t v_2$

$$\begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix}$$

$$\begin{pmatrix} w_1 & w_2 \end{pmatrix} \begin{pmatrix} \\ \\ \end{pmatrix}$$

Square matrix

Square

$A_{m \times n}$ matrix

submatrix

$A = USV^T$

orthogonal matrix

SVD

$A_{m\times n} = U\Sigma V^T$

V: $n\times n$ orthogonal matrix

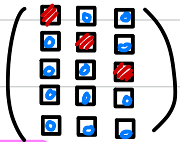
Σ: $m\times n$ diagonal matrix

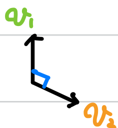
$$\begin{matrix} \sigma_1 & 0 & 0 & \cdots \\ 0 & \sigma_2 & \cdots & 0 \\ 0 & 0 & \sigma_3 \cdots & 0 \end{matrix}$$

U: $m\times m$ orthogonal matrix

$A^{-1} = A^T$ or $AA^T = I$

5×3 diagonal matrix





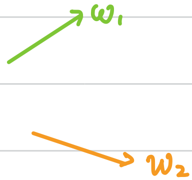
$v_1 \perp v_2$

linear transformation

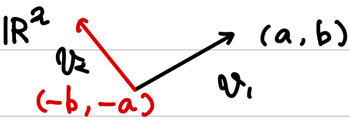
T

$Tv_1 = w_1$

$Tv_2 = w_2$



$w_1 \perp w_2$



$T(v_1) \overset{?}{\perp} T(v_2)$

이룰 수 있다?

$T(v_1) \perp T(v_2)$ 한번이상

$T(v_1)$ v_1

$$A = U\Sigma V^T \qquad V = (\vec{v}_1 \ \vec{v}_2)$$

$$\frac{Av_1}{\|Av_1\|} = u_1 \qquad = u_1 \qquad Av_2 = u_2 \qquad \frac{Av_2}{\|Av_2\|} = u_2 \qquad Av_1 \quad v_1$$

$$\Sigma_1 = \begin{pmatrix} \boxed{\sigma_1} & 0 \\ 0 & \boxed{\sigma_2} \end{pmatrix}$$

scale.

$$AV = U\Sigma$$

$$\Downarrow$$

$$\begin{pmatrix} Av_1 & Av_2 \end{pmatrix} = \begin{pmatrix} u_1 & u_2 \end{pmatrix} \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}.$$

$v_1 \quad v_2$

$$T(v) = Av$$

$$v_1 \perp v_2$$

A $$Av_1 \perp Av_2$$

$v_1 \quad v_2$

Av_1

Av_2

$$\begin{pmatrix} Av_1 & Av_2 \end{pmatrix} = \begin{pmatrix} \sigma_1 u_1 & \sigma_2 u_2 \end{pmatrix}$$

$$= \begin{pmatrix} u_1 & u_2 \end{pmatrix} \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}$$

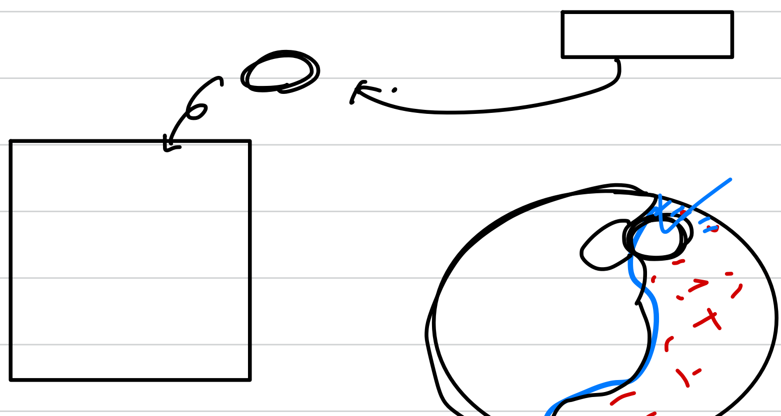
$$AV = U\Sigma$$

$$AVV^{-1} = U\Sigma V^{-1}$$

$$A = U\Sigma V^{-1} \qquad V^{-1} = V^t$$

$$= U\Sigma V^t$$

Ⓐ 정보량에 따라 여러개의 layer로 쪼개줌.
σ



square matrix

* Ⓐ 정보 A' 복원 부분적.

가지고 있는 정보 활용

linear transformation

$\boxed{A_{m\times n}}$ $v_1, v_2, \dots, v_n$ $v_1, v_2, \dots, v_n$

$Av_1 \quad Av_2 \quad \dots \quad Av_n$

선형변환

$v_1, v_2, \dots, v_n$ 이 주어짐

Ⓐ 찾기.

가중치

$A = U\Sigma V^t$

$(v_1 \; v_2 \; \cdots \; v_n)$

$(Av_1 \; Av_2 \; \cdots \; Av_n)$

$u_i = \dfrac{Av_i}{\|Av_i\|}$ $\quad \sigma_1 = \|Av_i\|$

$u_i\sigma_i = Av_i$