#10)

$$\begin{cases} 4x-2y=2 \\ 5x-2y+z=7 \\ 3x+4y-z=3 \end{cases}$$

$$\left(\begin{array}{ccc|c} 4 & -2 & 0 & 2 \\ 5 & -2 & 1 & 7 \\ 3 & 4 & -1 & 3 \end{array}\right) \xrightarrow[R_1 \to \frac{1}{4}R_1]{} \left(\begin{array}{ccc|c} 1 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 5 & -2 & 1 & 7 \\ 3 & 4 & -1 & 3 \end{array}\right) \times(-5)$$

$-5 \quad \frac{5}{2} \quad 0 \quad -\frac{5}{2}$

$-3 \quad \frac{3}{2} \quad 0 \quad | \quad -\frac{1}{2}$

$$\downarrow$$

$$\left(\begin{array}{ccc|c} 1 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 1 & \frac{9}{2} \\ 0 & \frac{11}{2} & -1 & \frac{5}{2} \end{array}\right)$$

$$\downarrow$$

$$\left(\begin{array}{ccc|c} 1 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 2 & 9 \\ 0 & 11 & -2 & 5 \end{array}\right)$$

① $-11 \quad -22 \quad -99$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 2 & 9 \\ 0 & 0 & \cancel{+24}\ 1 & \cancel{-94}\ \frac{94}{24} \end{array}\right)$$

$$\begin{cases} 4x-2y=2 \\ 5x-2y+z=7 \\ 3x+4y-z=3 \end{cases}$$

$$\left(\begin{array}{ccc|c} 4 & -2 & 0 & 2 \\ 5 & -2 & 1 & 7 \\ 3 & 4 & -1 & 3 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 5 & -2 & 1 & 7 \\ 3 & 4 & -1 & 3 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 1 & \frac{9}{2} \\ 0 & \frac{11}{2} & -1 & \frac{3}{2} \end{array}\right)$$

$$\Downarrow$$

$$\left(\begin{array}{ccc|c} 1 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 2 & 9 \\ 0 & 0 & -24 & -96 \end{array}\right) \longleftarrow \left(\begin{array}{ccc|c} 1 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 2 & 9 \\ 0 & 11 & -2 & 3 \end{array}\right)$$

$0 \quad -11 \quad -22 \quad -99.$

$$\downarrow$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{array}\right) \Rightarrow \begin{array}{l} x=1 \\ y=1 \\ z=4 \end{array}$$

<연립일차방정식의 해와 행렬>

↳ 해의 개수 ?

연립일차방정식 → 해가 존재 → 유일하게 한 개 / 무한히 많음

연립일차방정식 → 해가 없음 → 0개

해의 개수

① 0 ⇒ 해가 없다? : 말이 안됨.

② 1 ⇒ 해가 유일하다? : 왼쪽이 단위(항등)행렬

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdots$

∞ ⇒ 해가 무수히 많다

연립일차 → 첨가행렬 ⟶ (기약)행사다리꼴

$\left[\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$ singular (특이행렬).

$\left[\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$ $0 \cdot x_1 + 0 \cdot x_2 + 0 \cdot x_3 + 0 \cdot x_4 + 0 \cdot x_5 = 0$

1. 연립일차방정식의 해의 개수?

- 해가 X　　$(0 \; 0 \; 0 \; 0 \mid \square)$
- 〃 1개　　항등
- 〃 ∞　　$0 \; 0 \; 0 \; 0 \mid 0$

2. 해들의 집합

ametric form.

$$\begin{bmatrix} 1 & -2 & 1 \\ -1 & 1 & 0 \\ 1 & -3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}.$$

using row reduction, find a solution of the vector eq

1. RREF

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 4 \\ -1 & 1 & 0 & -2 \\ 1 & -3 & 2 & 6 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & -2 & 1 & 4 \\ 0 & -1 & 1 & 2 \\ 0 & -1 & 1 & 2 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 4 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

2.

atrix is

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & -2 \\ 1 & 2 & 1 & 1 \\ 1 & -1 & 0 & 7 \end{array}\right]$$

-2

(해가 무수히 많을 때, 해를 나타내는 방법?!)

ex)
$$\begin{bmatrix} 1 & -2 & 1 \\ -1 & 1 & 0 \\ 1 & -3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 4 \\ -1 & 1 & 0 & -2 \\ 1 & -3 & 2 & 6 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 1 & 4 \\ 0 & -1 & 1 & 2 \\ 0 & -1 & 1 & 2 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} \boxed{1} & 0 & -1 & 0 \\ 0 & \boxed{+1} & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

선도 1 (leading one)

$x_1 \ x_2 \ x_3$.

pivot

x_3 : free-variable.

$x_1 - x_3 = 0$ $\quad$ $x_1 = x_3$.

$x_2 - x_3 = -2$

$x_2 = x_3 - 2$.

$$\begin{cases} x & -3y & +z & +2t & = 2 \\ & & z & +2t & = -1 \\ -x & +3y & -2z & -4t & = -1 \\ 2x & -6y & +4z & +8t & = 2 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & -3 & 1 & 2 & 2 \\ 0 & 0 & 1 & 2 & -1 \\ -1 & 3 & -2 & -4 & -1 \\ 2 & -6 & 4 & 8 & 2 \end{array}\right)$$

$$\begin{array}{cccc|c} -2 & 6 & -2 & -4 & -4 \end{array}$$

$$\longrightarrow \left(\begin{array}{cccc|c} 1 & -3 & 1 & 2 & 2 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & -1 & -2 & 1 \\ 2 & -6 & 4 & 8 & 2 \end{array}\right)$$

$$\downarrow$$

$$\left(\begin{array}{cccc|c} 1 & -3 & 1 & 2 & 2 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 4 & -2 \end{array}\right)$$

pivot

x_1 x_2 x_3 x_4

free free

$$\downarrow$$

$$\left(\begin{array}{cccc|c} 1 & -3 & 1 & 2 & 2 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & -3 & 0 & 0 & 3 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{cccc|c} \boxed{1} & -3 & 0 & 0 & 3 \\ 0 & 0 & \boxed{1} & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

x_2, x_4: free.

$x_1 = 3 + 3x_2$

$x_3 = -1 - 2x_4$

$$\begin{cases} x_2 + 4x_4 + x_5 = 5 + 2x_1 + x_3 \\ x_2 + 2x_3 + 3x_4 + 3x_5 = 2x_1 + 6 \\ 2x_1 - 5x_3 = 2x_4 + 5x_5 - 7 + x_2 \end{cases}$$

$$-2x_1 + x_2 - x_3 + 4x_4 + x_5 = 5$$

$$-2x_1 + x_2 + 2x_3 + 3x_4 + 3x_5 = 6$$

$$2x_1 - x_2 - 5x_3 - 2x_4 - 5x_5 = -7$$

$$\left[\begin{array}{ccccc|c} -2 & 1 & -1 & 4 & 1 & 5 \\ -2 & 1 & 2 & 3 & 3 & 6 \\ 2 & -1 & -5 & -2 & -5 & -7 \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} -2 & 1 & -1 & 4 & 1 & 5 \\ 0 & 0 & 3 & -1 & 2 & 1 \\ 0 & 0 & -3 & 1 & -2 & -1 \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} -2 & 1 & -1 & 4 & 1 & 5 \\ 0 & 0 & 3 & -1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} \boxed{1} & -\frac{1}{2} & \frac{1}{2} & -2 & -\frac{1}{2} & -\frac{5}{2} \\ 0 & 0 & \boxed{1} & -\frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} 1 & -\frac{1}{2} & 0 & -\frac{11}{6} & -\frac{5}{6} & -\frac{16}{6} \\ 0 & 0 & 1 & -\frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$0 \quad 0 \quad -\frac{1}{2} \quad \frac{1}{6} \quad -\frac{1}{3} \mid -\frac{1}{6}$

$0 \quad 0$

$x_1, x_3 \to$ pivot

$x_2, x_4, x_5 \Rightarrow$ free

$$\begin{cases} x_1 - \frac{1}{2}x_2 - \frac{11}{6}x_4 - \frac{5}{6}x_5 = -\frac{16}{6} \\ x_3 - \frac{1}{3}x_4 + \frac{2}{3}x_5 = \frac{1}{3} \end{cases}$$

$$x_1 = \frac{1}{2}x_2 + \frac{11}{6}x_4 + \frac{5}{6}x_5 - \frac{16}{6}$$

$$x_3 = \frac{1}{3}x_4 - \frac{2}{3}x_5 + \frac{1}{3}$$

$\frac{1}{3} \neq \frac{2h}{6}$ $h \neq 1$ $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

Question 4. [**5 pts**] For which values of h and k does the matrix-vector equation

$$\begin{bmatrix} 1 & 2h \\ 3 & 6 \end{bmatrix} \vec{x} = \begin{bmatrix} 2 \\ k \end{bmatrix}$$

$h, k = ?$

(a) have no solutions, $h = 1$, $k \neq 6$,
(b) have a unique solution,
(c) have infinitely many solutions?

-3 $-6h$ -6

$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$

(a) $\left[\begin{array}{cc|c} 1 & 2h & 2 \\ 3 & 6 & k \end{array}\right]$ $\left[\begin{array}{cc|c} 1 & 2h & 2 \\ 0 & 6-6h & k-6 \end{array}\right]$

$k \neq 6$.

$6-6h=0$,

$\left[\begin{array}{cc|c} 1 & 2h & 2 \\ 0 & -6h+6 & -6+k \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 2 & \frac{k}{3} \\ 0 & -6h+6 & -6+k \end{array}\right]$

(a) $h=1$, $k \neq 6$

$6-6h=0$. $k-6=0$.

(c) $h=1$, $k=6$

$\left[\begin{array}{cc|c} 1 & 2h & 2 \\ 0 & 6-6h & k-6 \end{array}\right]$

$$\boxed{\left[\begin{array}{cc|c} 1 & 2h & 2 \\ 0 & 6-6h & k-6 \end{array}\right]}$$

해가 존재

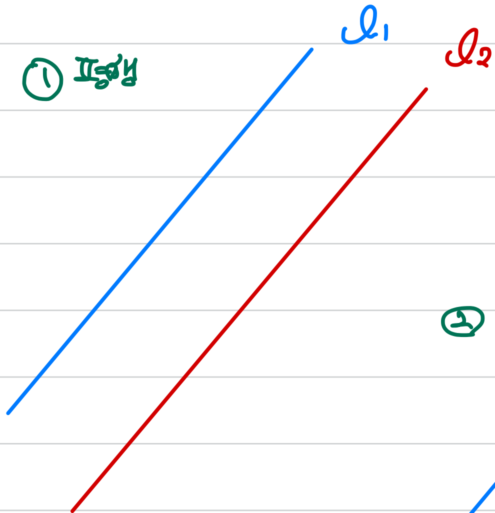
존재 X

$$\left(\begin{array}{cc|c} 1 & 4 & 2 \\ 0 & -6 & k-6 \end{array}\right)$$

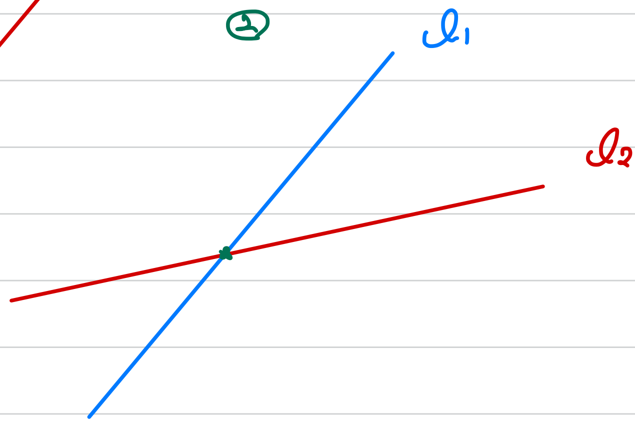
$$\left(\begin{array}{cc|c} 1 & 4 & 2 \\ 0 & 1 & \frac{6-k}{6} \end{array}\right)$$

평면 위의 두 직선의 위치관계

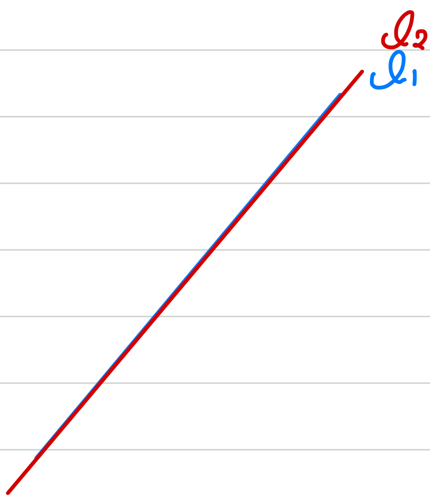
① 평행



②



③



서로 같음.

· 두 직선이 평행 = 기울기가 같고, y절편이 다름

$l_1 : y = a_1 x + b_1$ $l_2 : y = a_2 x + b_2$

$a_1 = a_2, \quad b_1 \neq b_2$

· 한 점에서 만난다.

$a_1 \neq a_2$

· 일치한다. $a_1 = a_2, \quad b_1 = b_2$.

$$\begin{pmatrix} \alpha & \beta \\ \gamma & \omega \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}$$

$$\begin{cases} \alpha x + \beta y = p \\ \gamma x + \omega y = q \end{cases}$$

$$\begin{cases} \alpha x + \beta y = p \\ \gamma x + \omega y = q \end{cases}$$

$$\beta y = -\alpha x + p \qquad y = -\frac{\alpha}{\beta}x + \frac{p}{\beta}$$

$$\omega y = -\gamma x + q$$

$$y = \frac{-\gamma}{\omega}x + \frac{q}{\omega}$$

① 평행 $-\dfrac{\alpha}{\beta} = -\dfrac{\gamma}{\omega}$ $\qquad \dfrac{p}{\beta} \neq \dfrac{q}{\omega}$

$$\frac{\alpha}{\gamma} = \frac{\beta}{\omega} \qquad \frac{p}{q} \neq \frac{\beta}{\omega}$$

평행: $\dfrac{\alpha}{\gamma} = \dfrac{\beta}{\omega} \neq \dfrac{p}{q}$

$$\begin{aligned} 2x + y &= -3 \\ -4x - 2x &= 4 \end{aligned}$$

② 한 점에서 만난다.

$$\frac{\alpha}{\gamma} \neq \frac{\beta}{\omega}$$

③ 일치한다.

$$\frac{\alpha}{\gamma} = \frac{\beta}{\omega} = \frac{p}{q}$$

(1)

$$\left(\begin{array}{ccc|c} 2 & -3 & 2 & a \\ 0 & -b & 3 & 3 \\ 1 & -1 & -1 & a+b \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & a+b \\ 0 & -b & 3 & 3 \\ 2 & -3 & 2 & a \end{array}\right), \quad -2 \quad 2 \quad 2 \quad -2(a+b)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & a+b \\ 0 & -b & 3 & 3 \\ 0 & -1 & 4 & -a-2b \end{array}\right),$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & a+b \\ 0 & 1 & -4 & a+2b \\ 0 & b & -3 & -3 \end{array}\right) \quad 0 \quad -b \quad +4b \;/\; -b(a+2b)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & a+b \\ 0 & 1 & -4 & a+2b \\ 0 & 0 & 4b-3 & -ab-2b^2-3 \end{array}\right)$$

$4b-3=0$, $-ab-2b^2-3=0$