## < Diagonalization>

Eigenvalue Eigenvector

## Diagonalization

(Def) (A is not a diagonal matrix).

For  $A_{mxm}$ ,  $V_{mx1}$ ,  $\lambda \in F$ ,

if  $A_{v} = \lambda v$ , then v: eigenvector,  $\lambda$ : eigenvalue.

a. How can we know a and ~?

<step 1>

Find  $\lambda$  s.t.  $\frac{\det(A-\lambda \mathbf{I})=0}{}$ 

4 Characteristic polynomial

「 みひ= みひ

AV=ඉ፲ሇ

 $A_{Y}-\lambda \pm 0 = 0$   $A_{Y}-\lambda \pm 0 = 0$   $A_{Y}-\lambda \pm 0 = 0$ 

(A - A I) N = 0 K

カーλエ キロ , か ‡ 0 Zero divisor (質智和). 」

(step2>

For A, An= Av => v vulon.

[Eigenspace]

For some A, then Ex = span [w | Aw = A, w]

Diagonalizable

# of linearly independent eigenvectors = n

Av= 910 , Av= 924

If  $\lambda_1 + \lambda_2$  then  $V_1$ ,  $V_2$  are chancerly independent.

There are n distinct eigenvalues = # of linearly independent eigenvector

Diagonalizable

2. For  $n_1$  with multiplicity =  $n_2$  if there are  $n_2$  linearly undependent eigenvectors when  $n_1 = n_1$ ,

Diagonalization.

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\frac{det}{det} \begin{pmatrix} (-\lambda & 1 & 1 \\ 0 & (-\lambda) & 0 \\ 0 & 1 & 4 & -\lambda \end{pmatrix} = \frac{(1-\lambda)}{(1-\lambda)(2-\lambda)}.$$

(i) 
$$\lambda = 1$$

$$(\beta - \lambda \mathbf{I}) \cdot \mathbf{v} = 0$$

$$\begin{pmatrix}
0 & 1 & 1 \\
0 & 0 & 0 \\
0 & 1 & 1
\end{pmatrix}
\begin{pmatrix}
U_1 \\
U_2 \\
U_3
\end{pmatrix} = \begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix}
\qquad
\begin{pmatrix}
U_2 + U_3 = 0 \\
U_2 = -U_3 \\
U_3
\end{pmatrix}$$

$$\begin{pmatrix}
U_1 \\
U_2 \\
U_3
\end{pmatrix} = \begin{pmatrix}
U_1 \\
U_2 \\
-U_M
\end{pmatrix}$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \\ -u_{21} \end{pmatrix}$$

$$\operatorname{span} \left\{ \binom{1}{2}, \binom{2}{1} \right\} \Rightarrow \dim \left( \right) = 2$$

$$\omega_2 = 0 \cdot \omega_2 = \omega_1$$

95 eigenvector \$3 apante space

$$dim (span(r)) = n$$

$$\beta = 1 \qquad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\lambda = 2 \qquad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$D = \begin{pmatrix} 0 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad D = \begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & \frac{1}{12} & 0 \\ 0 & 0 & \frac{1}{12} & 0 \end{pmatrix}$$

(e) 
$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}$$

1. x2 + 0.01 +1

+ (7/2 (1)

124

1+x-x+1 +1 = -3+x-x+1

(n-1)(n2+1). n= ±i

(カナリー(カナリ)=0

7=1.























$$A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{pmatrix}$$

$$A-\lambda I = \begin{pmatrix} 3-\lambda & 1 & 1 \\ 2 & 4-\lambda & 2 \\ -1 & -1 & 1-\lambda \end{pmatrix}$$

$$\lambda^2 - 4 \rightarrow 6$$

$$de+(A-\lambda 1) = (1)^{2} \cdot (3-\lambda) ((4-\lambda)(1-\lambda)+2)$$

$$+(-1)^{3} \cdot (2(1-\lambda)+2)$$

$$(A-\lambda I)U=0$$

$$A\begin{pmatrix} 3 & 1 \\ 2 & 4 \\ 1 & -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$2U_{1} + 3U_{2} + 2U_{3} = 0 \rightarrow 2U_{1} - 3U_{1} + 2U_{3} = 0$$

$$-U_{1} - U_{2} = 0 \rightarrow U_{2} = -U_{1}$$

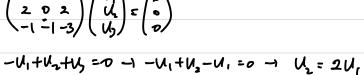
$$\left(\begin{array}{c} -\alpha_1 \\ \alpha_1 \end{array}\right) = \left(\begin{array}{c} -\alpha_1 \\ \alpha_1 \end{array}\right)$$

$$\begin{pmatrix} \Lambda^2 \\ \Lambda^{\prime} \end{pmatrix} = \begin{pmatrix} -\Lambda^1 \\ -\Lambda^1 \end{pmatrix} = \Lambda^1 \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 1 \\ 2 & 1 & 2 \\ -1 & -1 & -2 \end{pmatrix} \begin{pmatrix} u_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} * \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} u_1 \\ u_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} -u_3 \\ -u_3 \\ v_4 \end{pmatrix} = V_3 \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 2 & 0 & 2 \\ -1 & -1 & -3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$





24, +243=0 -> 43=-4,

 $\begin{pmatrix} U_1 \\ U_2 \\ 1 \end{pmatrix} = \begin{pmatrix} V_1 \\ 2U_1 \end{pmatrix} = V \cdot \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ 







$$A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{pmatrix} \qquad \begin{pmatrix} \lambda = 2 \\ (A - \lambda I) \cdot (I = D) \end{pmatrix}$$

$$\det \begin{pmatrix} 3 - \lambda & 1 & 1 \\ 2 & 4 - \lambda & 2 \\ -1 & -1 & (-\lambda) \end{pmatrix} \qquad \begin{pmatrix} (1 & 1) & 1 \\ 2 & 2 & 2 \\ -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} (I) & 1 \\ (I)$$

$$\det\begin{pmatrix} 3-\lambda & 1 & 1 \\ 2 & 2-\lambda & 2 \\ -1 & -1 & (-\lambda) \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2-\lambda & 2 \\ -1 & -1 & (-\lambda) \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(3-\lambda) \cdot \begin{pmatrix} \lambda^{2} - \lambda & \lambda + \lambda^{2} & -1 & (4-2\lambda) & +1 & (-2+4\lambda - \lambda) \\ (3-\lambda) & (\lambda-3)(\lambda-2) & -2(2-\lambda) & +(2-\lambda) \end{pmatrix}$$

$$(2-\lambda)^{\frac{5}{3}}(\eta_{-3})^{\frac{3}{2}}-2+1^{\frac{3}{4}}.$$

$$(2-\lambda)^{\frac{3}{2}}(\eta_{-3})^{\frac{3}{2}}-2+1^{\frac{3}{4}}.$$

$$(2-\lambda)^{\frac{3}{2}}(\eta_{-3})^{\frac{3}{2}}-2+1^{\frac{3}{4}}.$$

$$(2-\lambda)^{\frac{3}{2}}(\eta_{-2})^{\frac{3}{2}}=2+1^{\frac{3}{4}}.$$

$$(2-\lambda)^{\frac{3}{2}}(\eta_{-2})^{\frac{3}{2}}=2+1^{\frac{3}{4}}.$$

$$(2-\lambda)^{\frac{3}{4}}(\eta_{-2})^{\frac{3}{4}}=2+1^{\frac{3}{4}}.$$

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$$(2-\lambda)^{\frac{3}{4}}(\eta_{-2})^{\frac{3}{4}}=2+1^{\frac{3}{4}}.$$

$$h=2$$
 or  $4$ 

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{pmatrix}$$

$$(A - \lambda I) \cdot \mathcal{U} = 0$$

$$\frac{+U_2+U_3}{\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix}} = \begin{pmatrix} -U_2-U_3 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} -U_2-U_3 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} -U_2 \\ U_2 \\ U_3 \end{pmatrix} + \begin{pmatrix} -U_3 \\ 0 \\ U_3 \end{pmatrix}$$

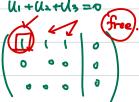
λ=2

U1 = - U2-U3

연결생사.

(A) (24) (27)

294 free variable.



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12= t

No = 5