Parallel Programming

Distributed Memory Programming With MPI (2)

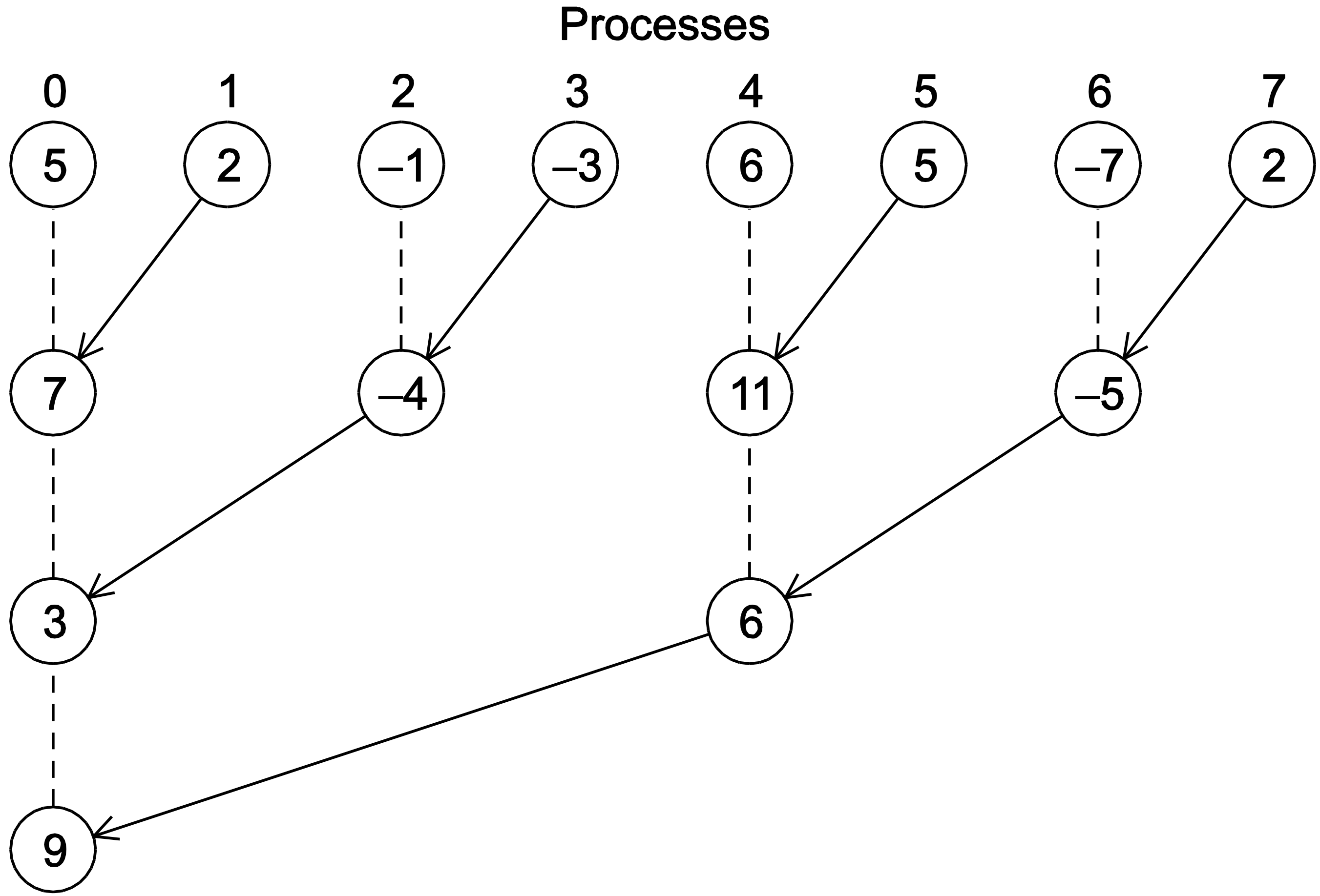
Slides adapted from the lecture notes by Peter Pacheco

Roadmap

- Writing your first MPI program.
- Using the common MPI functions.
- The Trapezoidal Rule in MPI.
- Collective communication.
- MPI derived datatypes.
- Performance evaluation of MPI programs.
- Parallel sorting.
- Safety in MPI programs.

COLLECTIVE COMMUNICATION

A tree-structured global sum

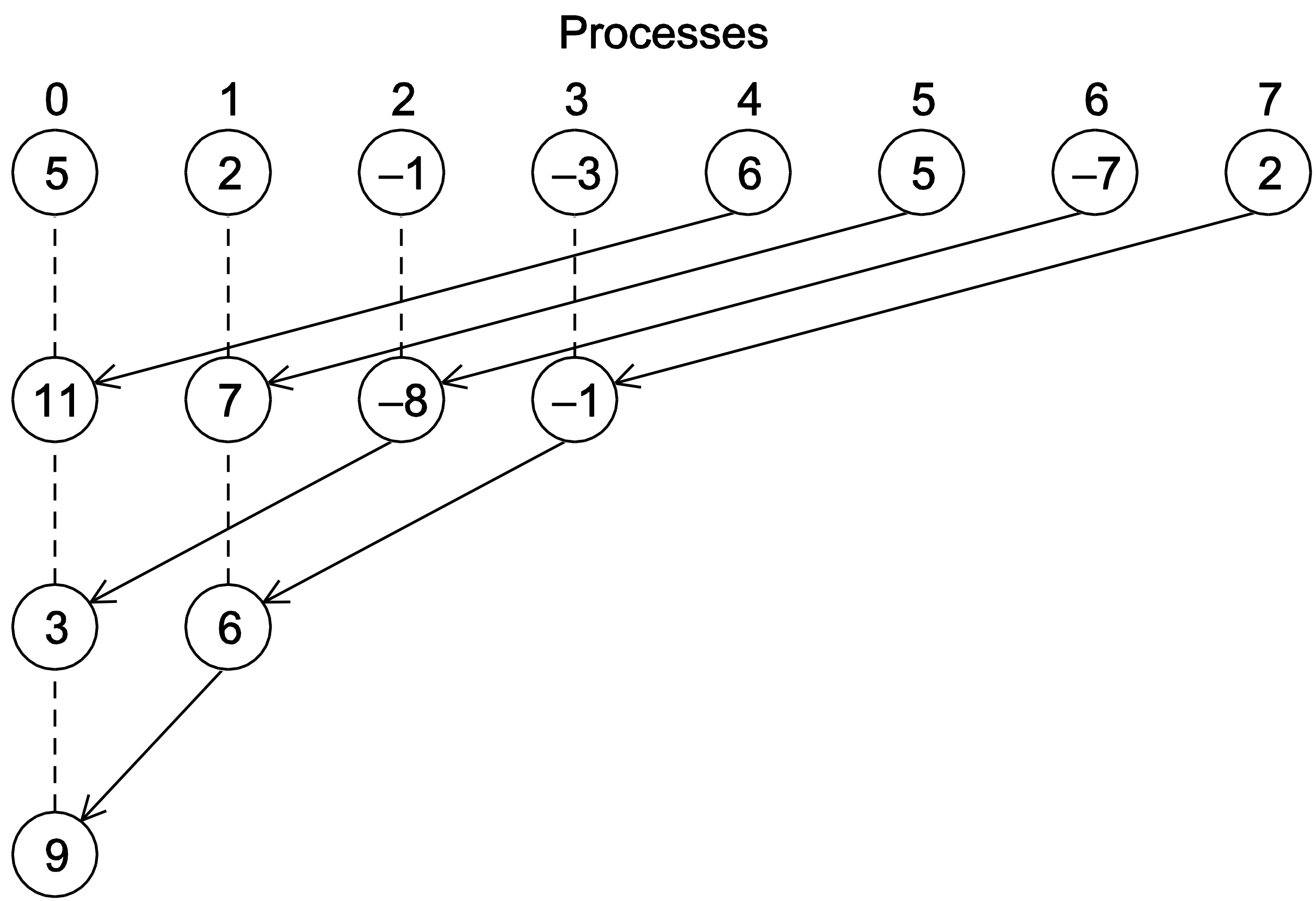


Tree-structured communication

1. In the first phase:
   (a) Process 1 sends to 0, 3 sends to 2, 5 sends to 4, and 7 sends to 6.
   (b) Processes 0, 2, 4, and 6 add in the received values.
   (c) Processes 2 and 6 send their new values to processes 0 and 4, respectively.
   (d) Processes 0 and 4 add the received values into their new values.

2. (a) Process 4 sends its newest value to process 0.
   (b) Process 0 adds the received value to its newest value.

An alternative tree-structured global sum



MPI_Reduce

```
int MPI_Reduce(
      void*          input_data_p   /* in  */,
      void*          output_data_p  /* out */,
      int            count          /* in  */,
      MPI_Datatype   datatype       /* in  */,
      MPI_Op         operator       /* in  */,
      int            dest_process   /* in  */,
      MPI_Comm       comm           /* in  */);
```

```
MPI_Reduce(&local_int, &total_int, 1, MPI_DOUBLE, MPI_SUM, 0,
      MPI_COMM_WORLD);
```

```
double local_x[N], sum[N];
. . .
MPI_Reduce(local_x, sum, N, MPI_DOUBLE, MPI_SUM, 0,
      MPI_COMM_WORLD);
```

Predefined reduction operators in MPI

Operation Value	Meaning
MPI_MAX	Maximum
MPI_MIN	Minimum
MPI_SUM	Sum
MPI_PROD	Product
MPI_LAND	Logical and
MPI_BAND	Bitwise and
MPI_LOR	Logical or
MPI_BOR	Bitwise or
MPI_LXOR	Logical exclusive or
MPI_BXOR	Bitwise exclusive or
MPI_MAXLOC	Maximum and location of maximum
MPI_MINLOC	Minimum and location of minimum

Collective vs. Point-to-Point Communications

- <u>All</u> the processes in the communicator must call the same collective function.

- For example, a program that attempts to match a call to MPI_Reduce on one process with a call to MPI_Recv on another process is erroneous, and, in all likelihood, the program will hang or crash.

Collective vs. Point-to-Point Communications

- The arguments passed by each process to an MPI collective communication must be "compatible."

- For example, if one process passes in 0 as the dest_process and another passes in 1, then the outcome of a call to MPI_Reduce is erroneous, and, once again, the program is likely to hang or crash.

Collective vs. Point-to-Point Communications

- The output_data_p argument is only used on dest_process.

- However, all of the processes still need to pass in an actual argument corresponding to output_data_p, even if it's just NULL.

Collective vs. Point-to-Point Communications

- Point-to-point communications are matched on the basis of tags and communicators.

- Collective communications don’t use tags.
- They’re matched solely on the basis of the communicator and the order in which they’re called.

Example (1)

Time	Process 0	Process 1	Process 2
0	`a = 1; c = 2`	`a = 1; c = 2`	`a = 1; c = 2`
1	`MPI_Reduce(&a, &b, ...)`	`MPI_Reduce(&c, &d, ...)`	`MPI_Reduce(&a, &b, ...)`
2	`MPI_Reduce(&c, &d, ...)`	`MPI_Reduce(&a, &b, ...)`	`MPI_Reduce(&c, &d, ...)`

Multiple calls to MPI_Reduce

Example (2)

- Suppose that each process calls MPI_Reduce with operator MPI_SUM, and destination process 0.

- At first glance, it might seem that after the two calls to MPI_Reduce, the value of b will be 3, and the value of d will be 6.

Example (3)

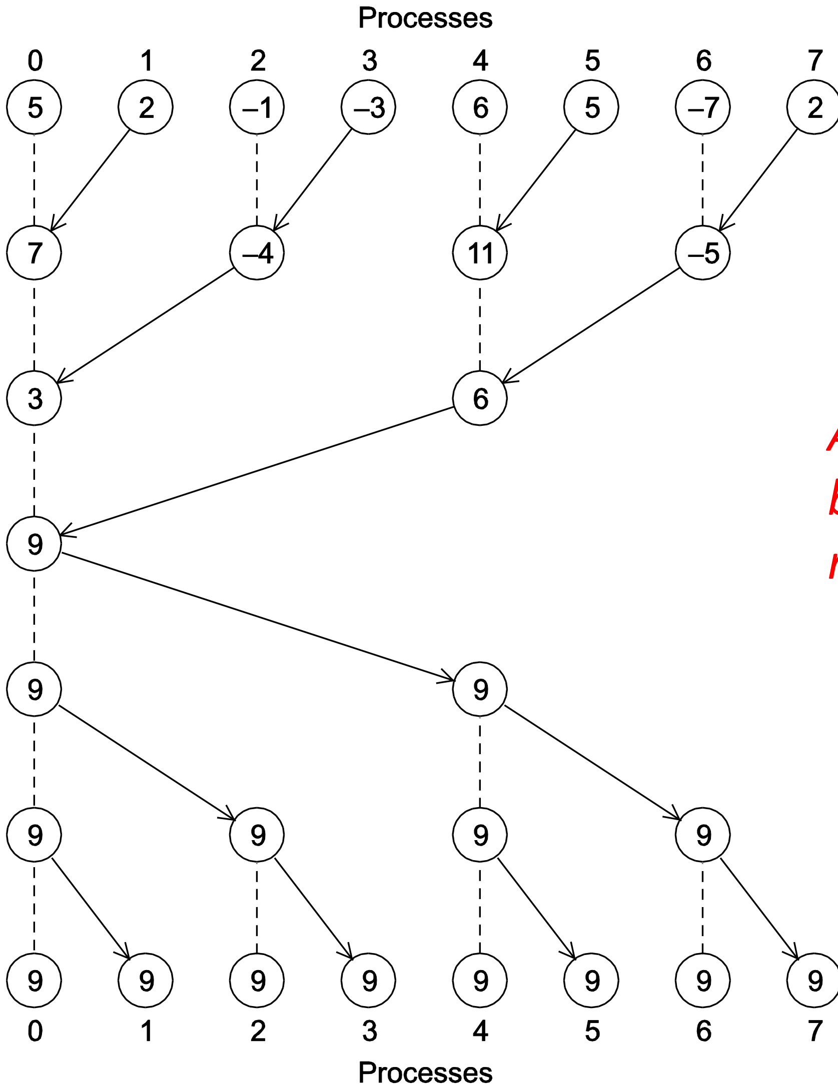
- However, the names of the memory locations are irrelevant to the matching of the calls to MPI_Reduce.

- The order of the calls will determine the matching so the value stored in b will be 1+2+1 = 4, and the value stored in d will be 2+1+2 = 5.

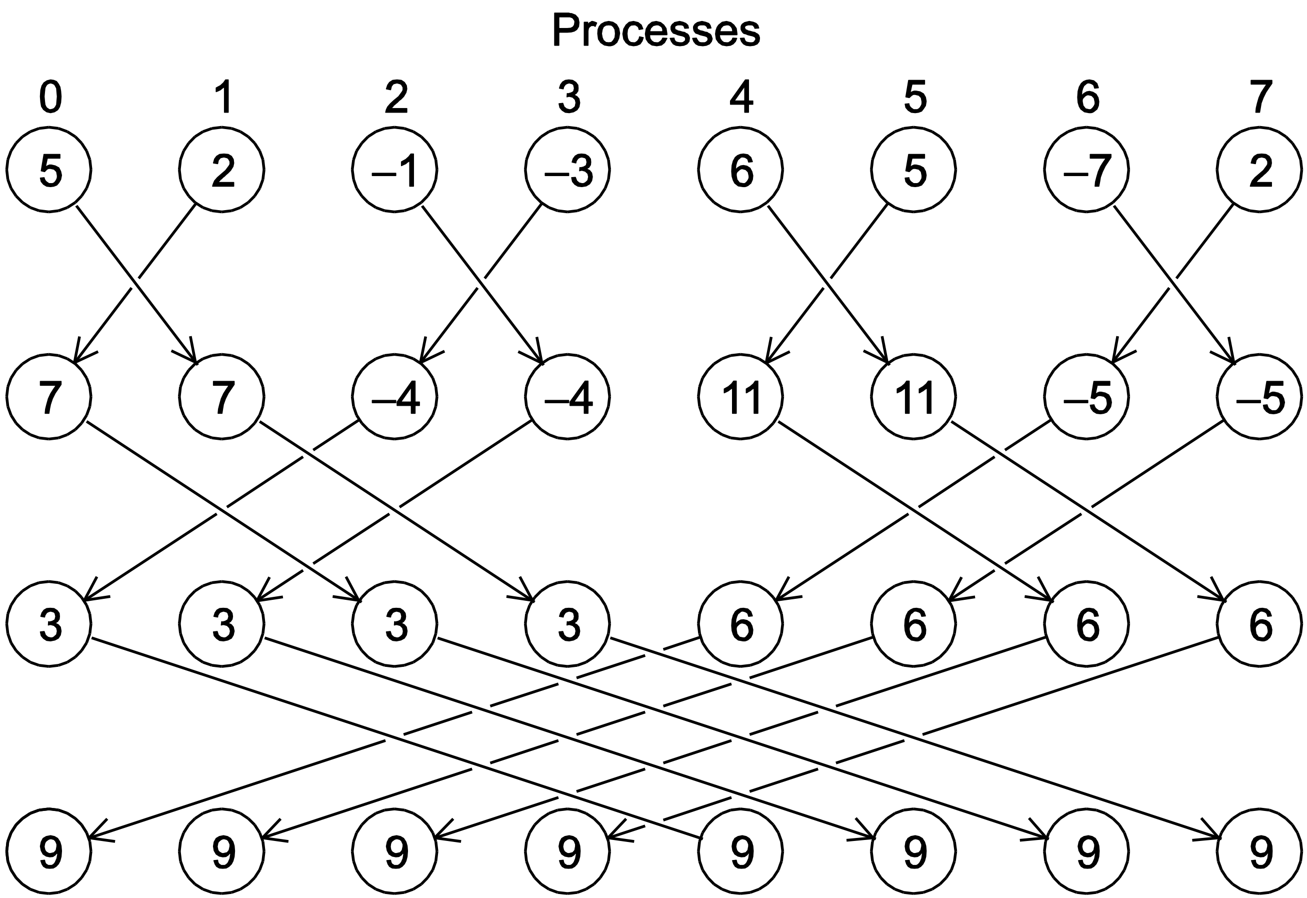
MPI_Allreduce

- Useful in a situation in which all of the processes need the result of a global sum in order to complete some larger computation.

```
int MPI_Allreduce(
        void*         input_data_p   /* in  */,
        void*         output_data_p  /* out */,
        int           count          /* in  */,
        MPI_Datatype  datatype       /* in  */,
        MPI_Op        operator       /* in  */,
        MPI_Comm      comm           /* in  */);
```



A global sum followed by distribution of the result.



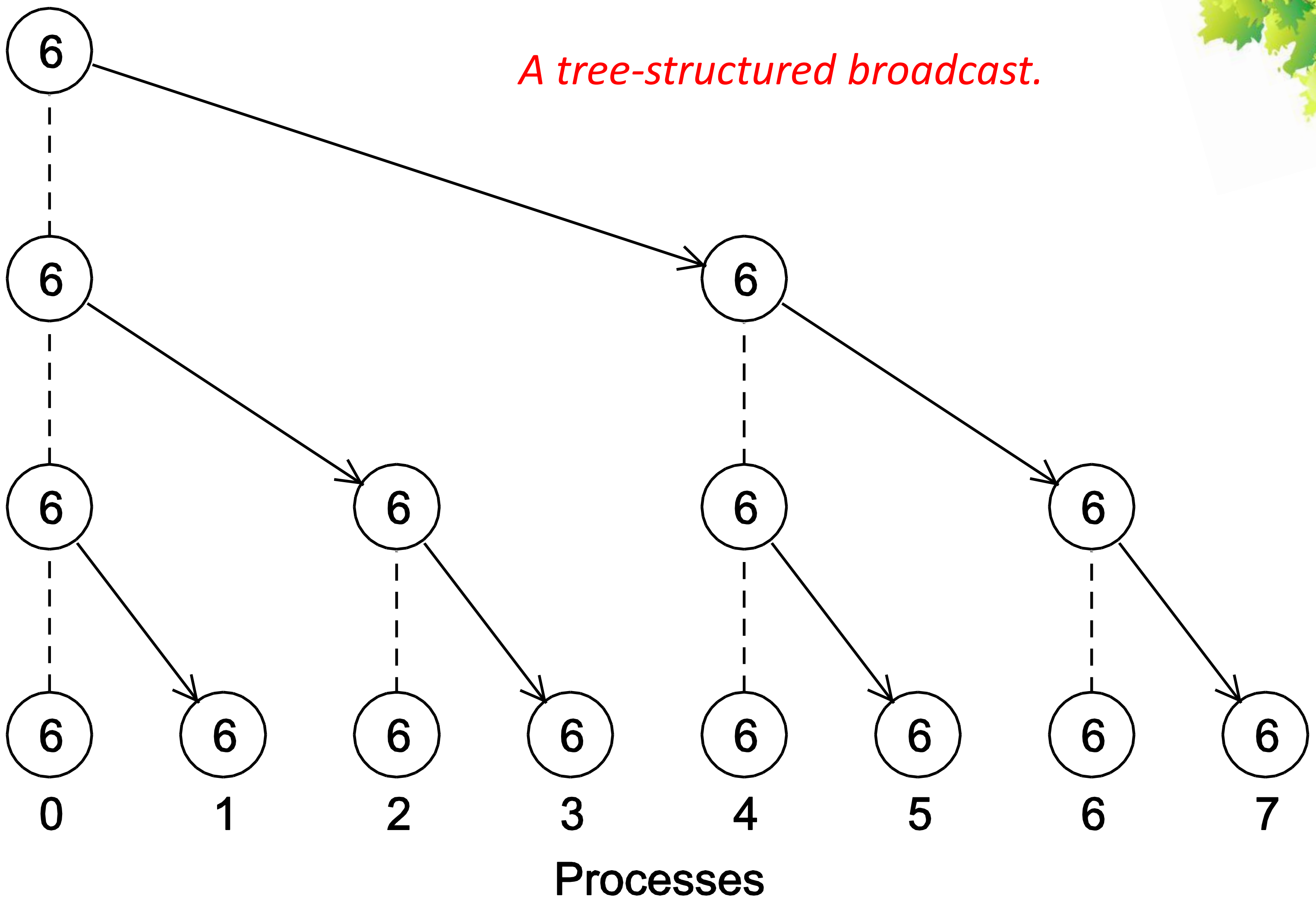
A butterfly-structured global sum.

Broadcast

- Data belonging to a single process is sent to all of the processes in the communicator.

```
int MPI_Bcast(
        void*         data_p       /* in/out  */,
        int           count        /* in      */,
        MPI_Datatype  datatype     /* in      */,
        int           source_proc  /* in      */,
        MPI_Comm      comm         /* in      */);
```

A tree-structured broadcast.



A version of Get_input that uses MPI_Bcast

```
void Get_input(
      int      my_rank  /* in  */,
      int      comm_sz  /* in  */,
      double*  a_p      /* out */,
      double*  b_p      /* out */,
      int*     n_p      /* out */) {

   if (my_rank == 0) {
      printf("Enter a, b, and n\n");
      scanf("%lf %lf %d", a_p, b_p, n_p);
   }
   MPI_Bcast(a_p, 1, MPI_DOUBLE, 0, MPI_COMM_WORLD);
   MPI_Bcast(b_p, 1, MPI_DOUBLE, 0, MPI_COMM_WORLD);
   MPI_Bcast(n_p, 1, MPI_INT, 0, MPI_COMM_WORLD);
}  /* Get_input */
```

Vector Addition Example

$$\begin{aligned}
\mathbf{x}+\mathbf{y} &= (x_0, x_1, \ldots, x_{n-1}) + (y_0, y_1, \ldots, y_{n-1}) \\
&= (x_0+y_0, x_1+y_1, \ldots, x_{n-1}+y_{n-1}) \\
&= (z_0, z_1, \ldots, z_{n-1}) \\
&= \mathbf{z}
\end{aligned}$$

Compute a vector sum.

Serial implementation of vector addition

```
void Vector_sum(double x[], double y[], double z[], int n) {
   int i;

   for (i = 0; i < n; i++)
      z[i] = x[i] + y[i];
}  /* Vector_sum */
```

Different partitions of a 12-component vector among 3 processes

Process	Components											
	Block				Cyclic				Block-cyclic Blocksize = 2			
0	0	1	2	3	0	3	6	9	0	1	6	7
1	4	5	6	7	1	4	7	10	2	3	8	9
2	8	9	10	11	2	5	8	11	4	5	10	11

Partitioning options

- Block partitioning
  - Assign blocks of consecutive components to each process.
- Cyclic partitioning
  - Assign components in a round robin fashion.
- Block-cyclic partitioning
  - Use a cyclic distribution of blocks of components.

Parallel implementation of vector addition

```
void Parallel_vector_sum(
      double  local_x[]  /* in  */,
      double  local_y[]  /* in  */,
      double  local_z[]  /* out */,
      int     local_n    /* in  */) {
   int local_i;

   for (local_i = 0; local_i < local_n; local_i++)
      local_z[local_i] = local_x[local_i] + local_y[local_i];
}  /* Parallel_vector_sum */
```

Scatter

- MPI_Scatter can be used in a function that reads in an entire vector on process 0 but only sends the needed components to each of the other processes.

```
int MPI_Scatter(
      void*          send_buf_p  /* in  */,
      int            send_count  /* in  */,
      MPI_Datatype   send_type   /* in  */,
      void*          recv_buf_p  /* out */,
      int            recv_count  /* in  */,
      MPI_Datatype   recv_type   /* in  */,
      int            src_proc    /* in  */,
      MPI_Comm       comm        /* in  */);
```

Reading and distributing a vector

```
void Read_vector(
      double    local_a[]   /* out */,
      int       local_n     /* in  */,
      int       n           /* in  */,
      char      vec_name[]  /* in  */,
      int       my_rank     /* in  */,
      MPI_Comm  comm        /* in  */) {

   double* a = NULL;
   int i;

   if (my_rank == 0) {
      a = malloc(n*sizeof(double));
      printf("Enter the vector %s\n", vec_name);
      for (i = 0; i < n; i++)
         scanf("%lf", &a[i]);
      MPI_Scatter(a, local_n, MPI_DOUBLE, local_a, local_n, MPI_DOUBLE,
            0, comm);
      free(a);
   } else {
      MPI_Scatter(a, local_n, MPI_DOUBLE, local_a, local_n, MPI_DOUBLE,
            0, comm);
   }
}  /* Read_vector */
```

Gather

- Collect all of the components of the vector onto process dest_proc.

```
int MPI_Gather(
      void*          send_buf_p   /* in  */,
      int            send_count   /* in  */,
      MPI_Datatype   send_type    /* in  */,
      void*          recv_buf_p   /* out */,
      int            recv_count   /* in  */,
      MPI_Datatype   recv_type    /* in  */,
      int            dest_proc    /* in  */,
      MPI_Comm       comm         /* in  */);
```

Print a distributed vector (1)

```
void Print_vector(
      double    local_b[]  /* in */,
      int       local_n    /* in */,
      int       n          /* in */,
      char      title[]    /* in */,
      int       my_rank    /* in */,
      MPI_Comm  comm       /* in */) {

   double* b = NULL;
   int i;
```

Print a distributed vector (2)

```
   if (my_rank == 0) {
      b = malloc(n*sizeof(double));
      MPI_Gather(local_b, local_n, MPI_DOUBLE, b, local_n, MPI_DOUBLE,
            0, comm);
      printf("%s\n", title);
      for (i = 0; i < n; i++)
         printf("%f ", b[i]);
      printf("\n");
      free(b);
   } else {
      MPI_Gather(local_b, local_n, MPI_DOUBLE, b, local_n, MPI_DOUBLE,
            0, comm);
   }
}  /* Print_vector */
```

Allgather

- Concatenates the contents of each process' send_buf_p and stores this in each process' recv_buf_p.
- As usual, recv_count is the amount of data being received from each process.

```
int MPI_Allgather(
        void*          send_buf_p   /* in  */,
        int            send_count   /* in  */,
        MPI_Datatype   send_type    /* in  */,
        void*          recv_buf_p   /* out */,
        int            recv_count   /* in  */,
        MPI_Datatype   recv_type    /* in  */,
        MPI_Comm       comm         /* in  */);
```

Matrix-vector multiplication

$A = (a_{ij})$ is an $m \times n$ matrix

$\mathbf{x}$ is a vector with n components

$\mathbf{y} = A\mathbf{x}$ is a vector with m components

$$y_i = a_{i0}x_0 + a_{i1}x_1 + a_{i2}x_2 + \cdots a_{i,n-1}x_{n-1}$$

i-th component of y

Dot product of the ith row of A with x.

Matrix-vector multiplication

$$
\begin{array}{|c|c|c|c|}
\hline
a_{00} & a_{01} & \cdots & a_{0,n-1} \\
\hline
a_{10} & a_{11} & \cdots & a_{1,n-1} \\
\hline
\vdots & \vdots & & \vdots \\
\hline
a_{i0} & a_{i1} & \cdots & a_{i,n-1} \\
\hline
\vdots & \vdots & & \vdots \\
\hline
a_{m-1,0} & a_{m-1,1} & \cdots & a_{m-1,n-1} \\
\hline
\end{array}
\begin{array}{|c|}
\hline
x_0 \\
\hline
x_1 \\
\hline
\vdots \\
\hline
x_{n-1} \\
\hline
\end{array}
=
\begin{array}{|c|}
\hline
y_0 \\
\hline
y_1 \\
\hline
\vdots \\
\hline
y_i = a_{i0}x_0 + a_{i1}x_1 + \cdots a_{i,n-1}x_{n-1} \\
\hline
\vdots \\
\hline
y_{m-1} \\
\hline
\end{array}
$$

Multiply a matrix by a vector

```
/* For each row of A */
for (i = 0; i < m; i++) {
   /* Form dot product of ith row with x */
   y[i] = 0.0;
   for (j = 0; j < n; j++)
      y[i] += A[i][j]*x[j];
}
```

Serial pseudo-code

C style arrays

$$\begin{pmatrix} 0 & 1 & 2 & 3 \\ 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 \end{pmatrix}$$

stored as

0 1 2 3 4 5 6 7 8 9 10 11

Serial matrix-vector multiplication

```
void Mat_vect_mult(
        double  A[]  /* in   */,
        double  x[]  /* in   */,
        double  y[]  /* out  */,
        int     m    /* in   */,
        int     n    /* in   */) {
    int i, j;

    for (i = 0; i < m; i++) {
        y[i] = 0.0;
        for (j = 0; j < n; j++)
            y[i] += A[i*n+j]*x[j];
    }
}  /* Mat_vect_mult */
```

An MPI matrix-vector multiplication function (1)

```
void Mat_vect_mult(
      double     local_A[]   /* in   */,
      double     local_x[]   /* in   */,
      double     local_y[]   /* out  */,
      int        local_m     /* in   */,
      int        n           /* in   */,
      int        local_n     /* in   */,
      MPI_Comm   comm        /* in   */) {
   double* x;
   int local_i, j;
   int local_ok = 1;
```

An MPI matrix-vector multiplication function (2)

```
    x = malloc(n*sizeof(double));
    MPI_Allgather(local_x, local_n, MPI_DOUBLE,
          x, local_n, MPI_DOUBLE, comm);

    for (local_i = 0; local_i < local_m; local_i++) {
       local_y[local_i] = 0.0;
       for (j = 0; j < n; j++)
          local_y[local_i] += local_A[local_i*n+j]*x[j];
    }
    free(x);
}  /* Mat_vect_mult */
```