Parallel Programming

Data-Parallel Primitives:

Split and Sort

Split and Sort

Primitive: Split
Input: $R_{in}[1, \ldots, n]$, $func(R_{in}[i]) \in [1,\ldots,F]$, i=1, …, n.
Output: $R_{out}[1, \ldots, n]$.
Function: $\{R_{out}[i],\ i=1,\ldots,\ n\} = \{R_{in}[i],\ i=1,\ \ldots,\ n\}$ and $func(R_{out}[i]) \le func(R_{out}[j]), \forall i,j \in [1,..,n], i \le j$.

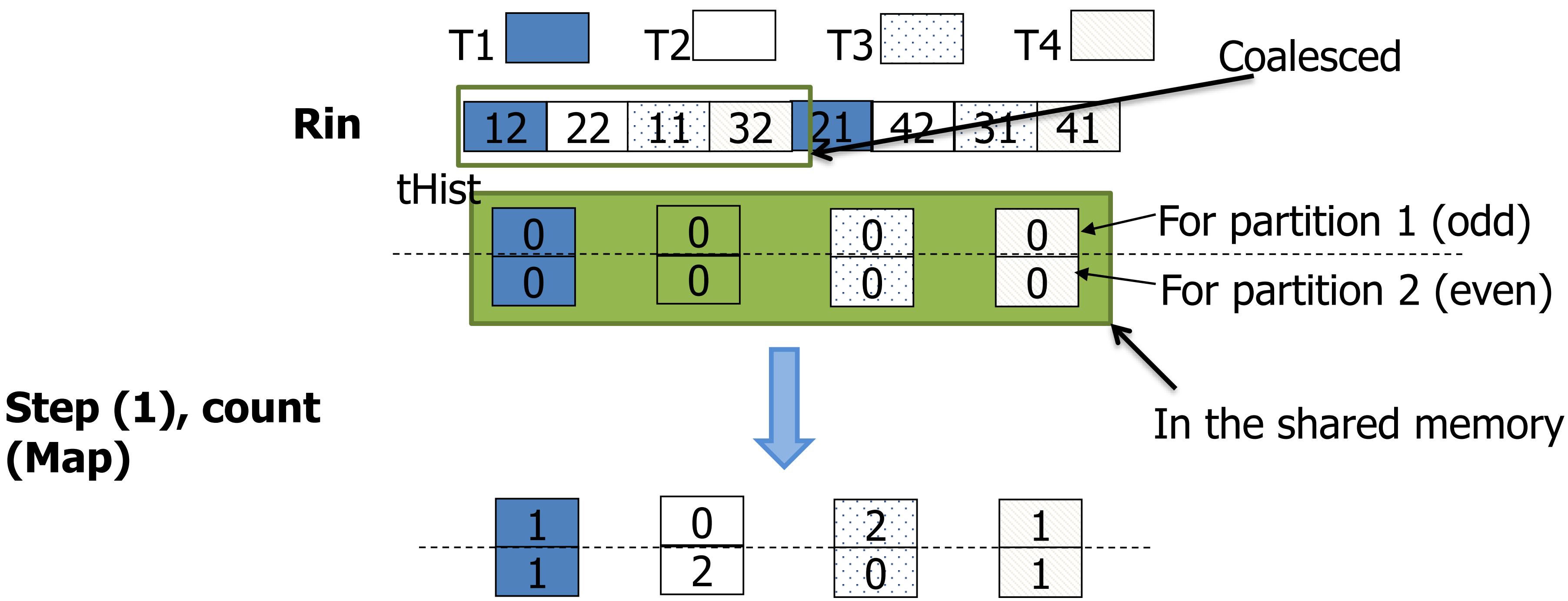
Primitive: Sort
Input: $R_{in}[1, \ldots, n]$.
Output: $R_{out}[1, \ldots, n]$.
Function: $\{R_{out}[i],\ i=1,\ldots,\ n\} = \{R_{in}[i],\ i=1,\ \ldots,\ n\}$ and $R_{out}[i] \le R_{out}[j], \forall i,j \in [1,..,n]$ and $i \le j$.

Algorithm for Split

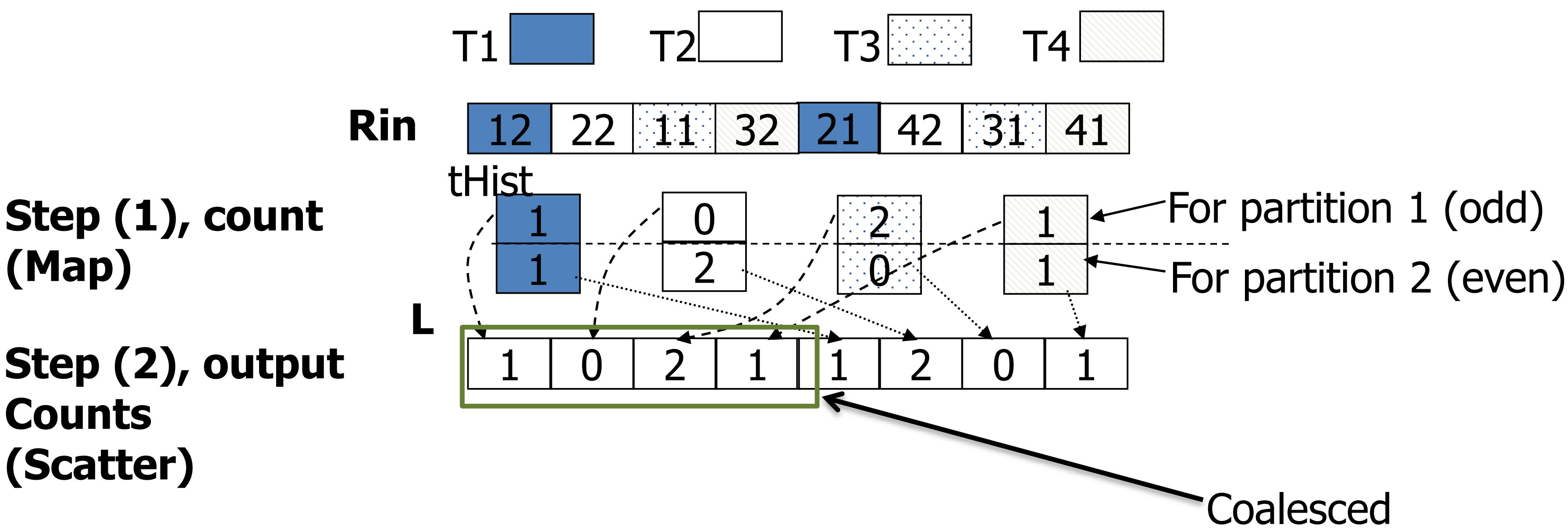
- A lock-free algorithm
  - Each thread is responsible for a portion of the input relation.
  - Each thread computes its local histogram (number of tuples in each output partition).
  - Given the local histograms, each thread computes its write locations.
  - Each thread writes the tuples to the output relation in parallel.

An Example of Split

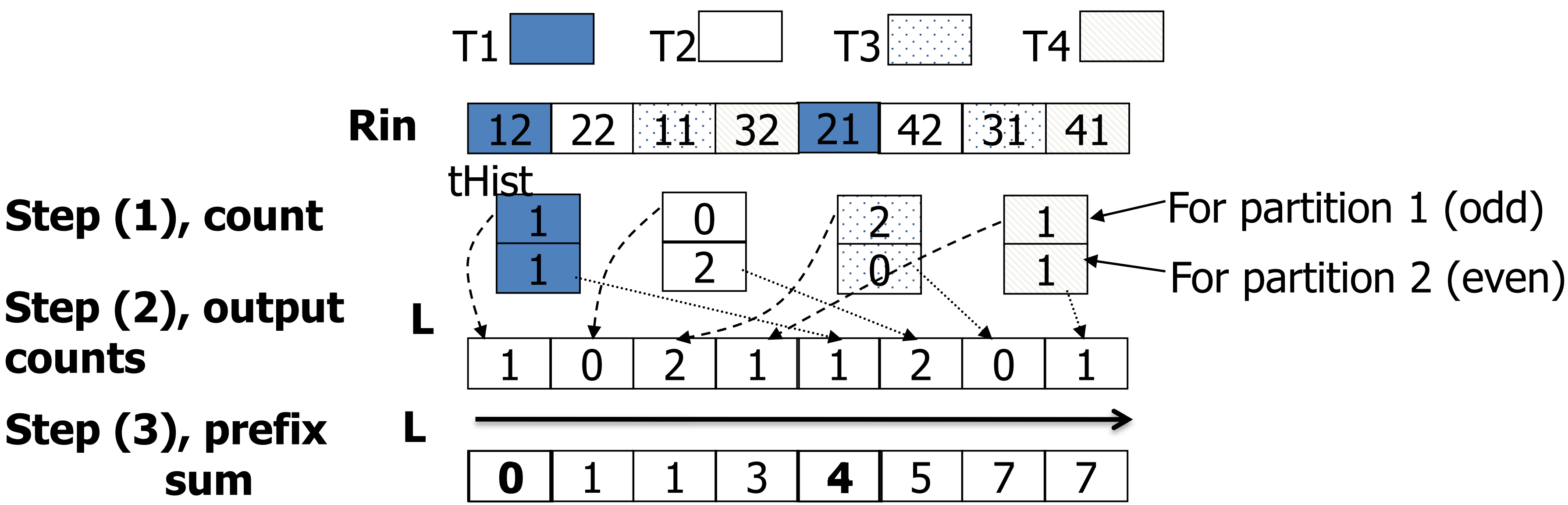
Split (Rin[1,…, 8], fcn, Rout[1,…,8]), fcn(x)=x mod 2



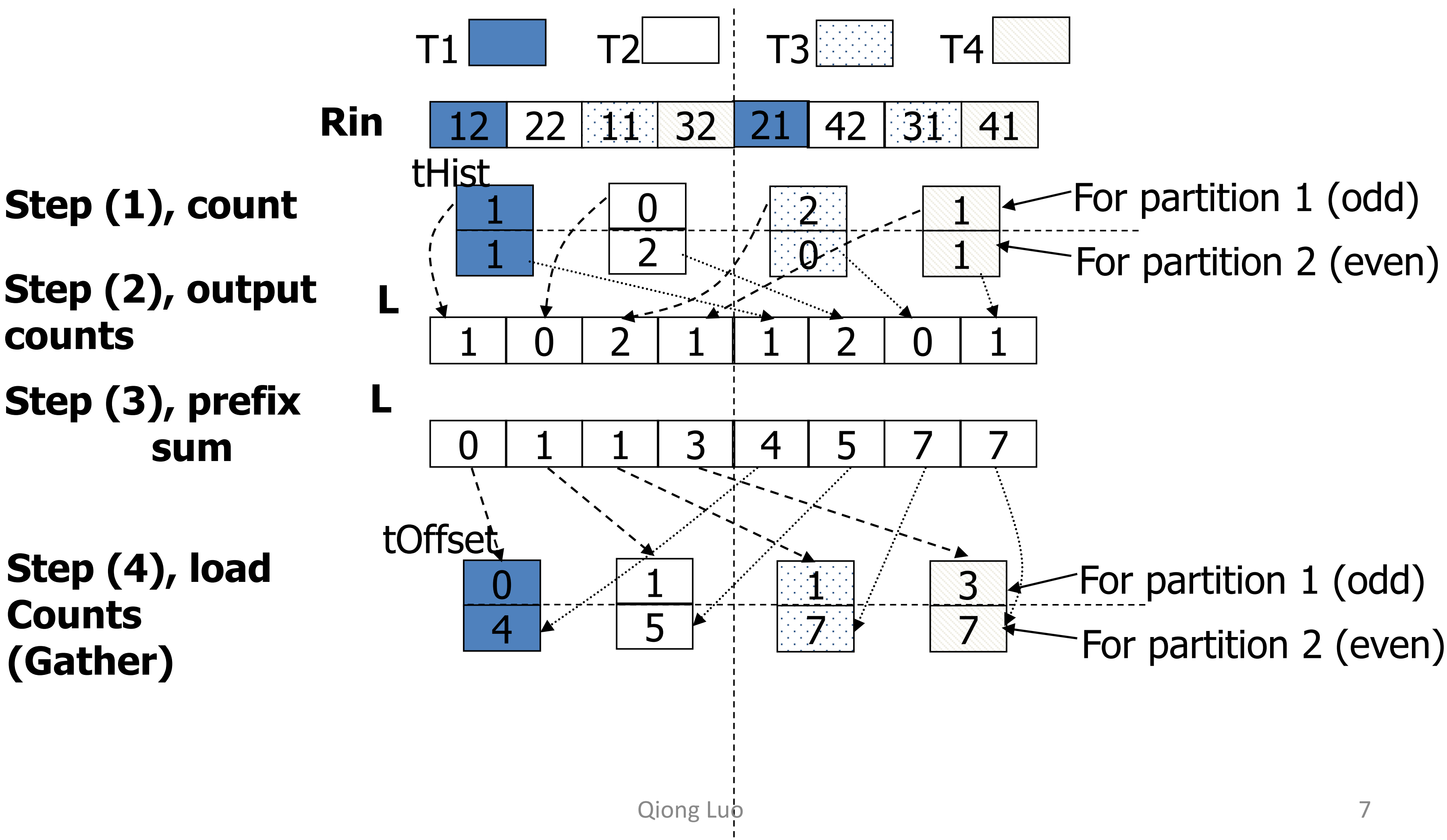
An Example of Split



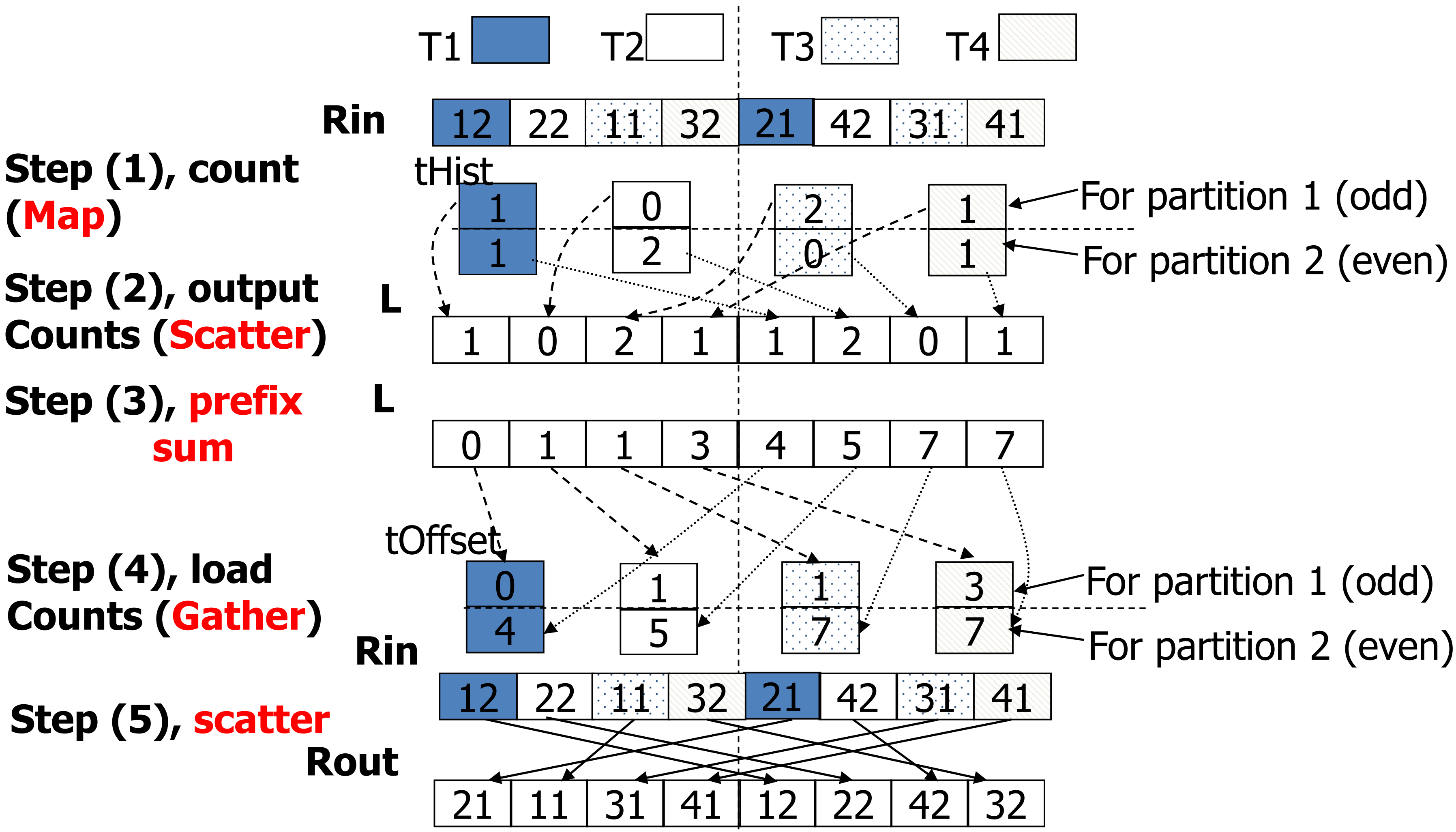
An Example of Split



An Example of Split



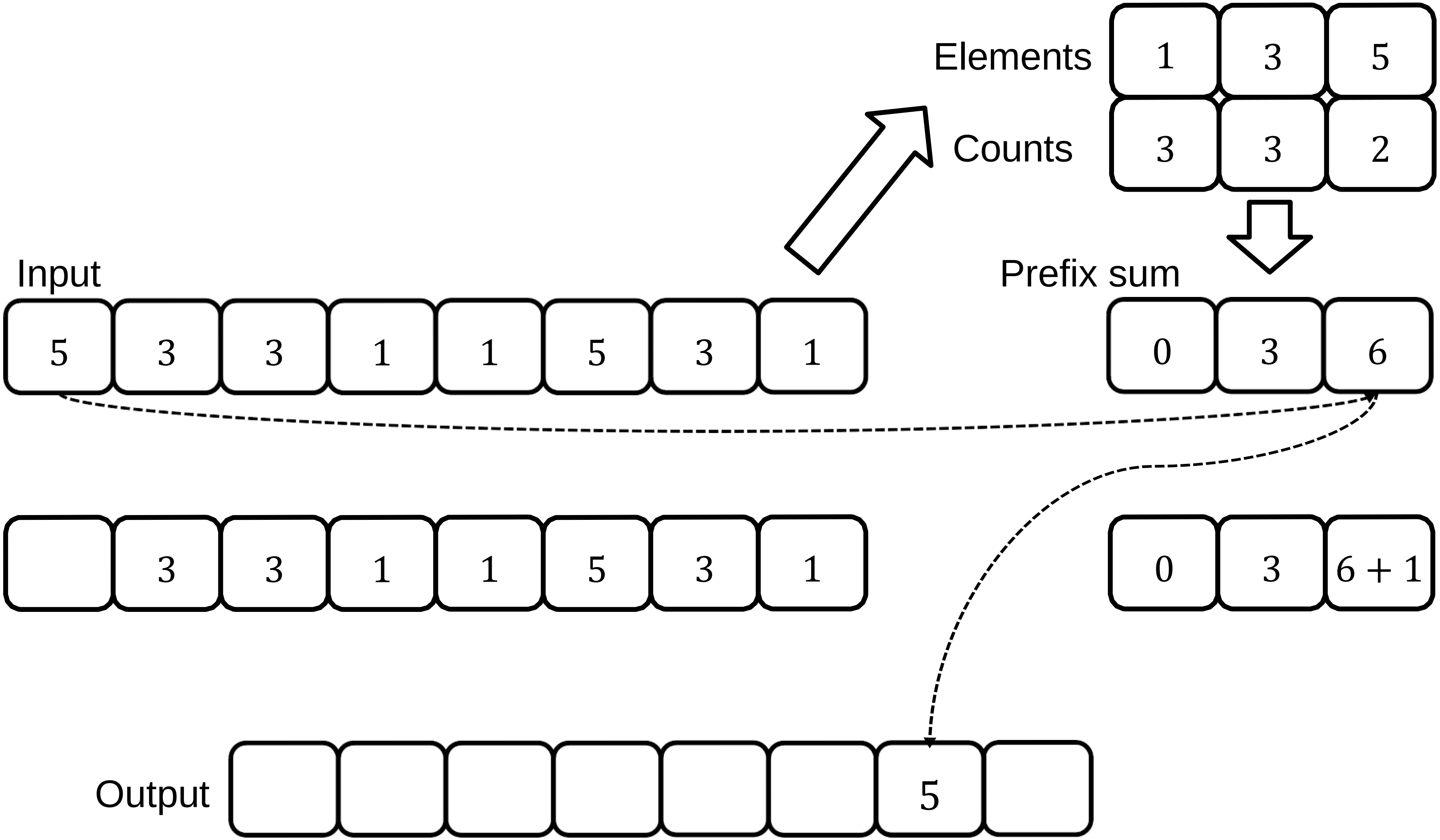
An Example of Split



Counting Sort Algorithm

- Sorting for equality-comparison elements, e.g., integers
- Assume
  - Constant number of possible values
  - Possible values known in advance
- Algorithm outline
  - Compute the histogram of each element
  - Prefix sum over the histogram
  - Move each element to its location
- Complexity (sequential) - O(n)

Counting Sort Example



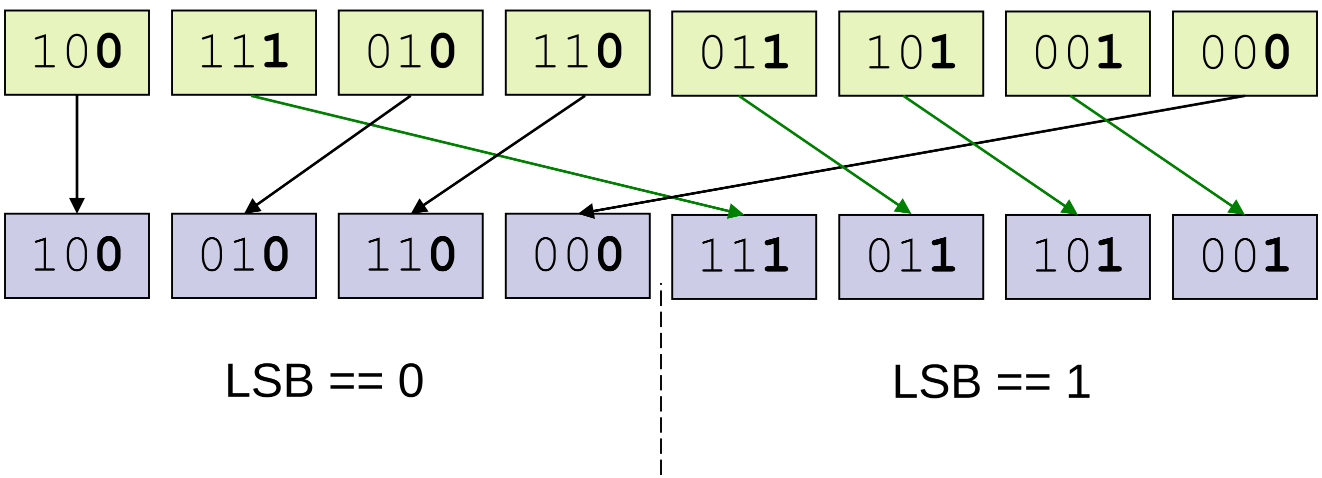
Radix Sort

- Iterated counting sort on individual digits (radix)
- Sort from the least significant bit (LSB) to the most significant bit (MSB)

Input	25	13	53	21	61	45	83	11
Sort 2nd digit	21	61	11	13	53	83	25	45
Sort 1st digit	11	13	21	25	45	53	61	83

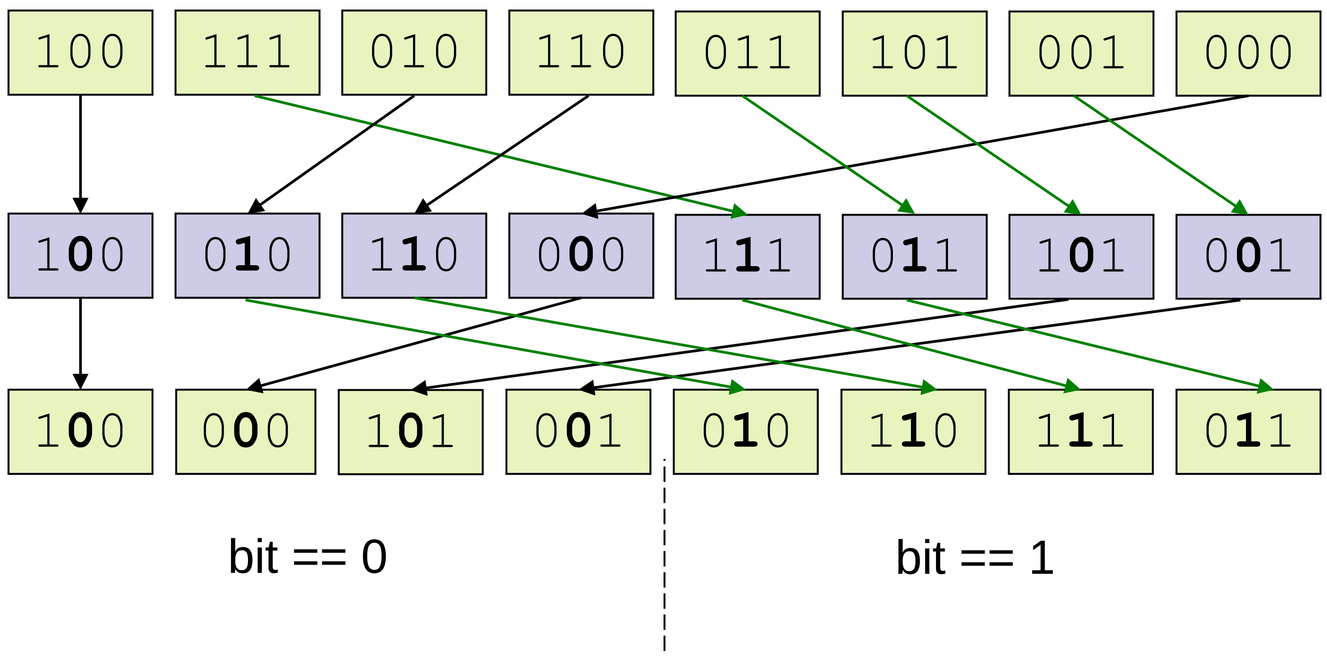
Radix Sort Example

- First pass: partition based on LSB



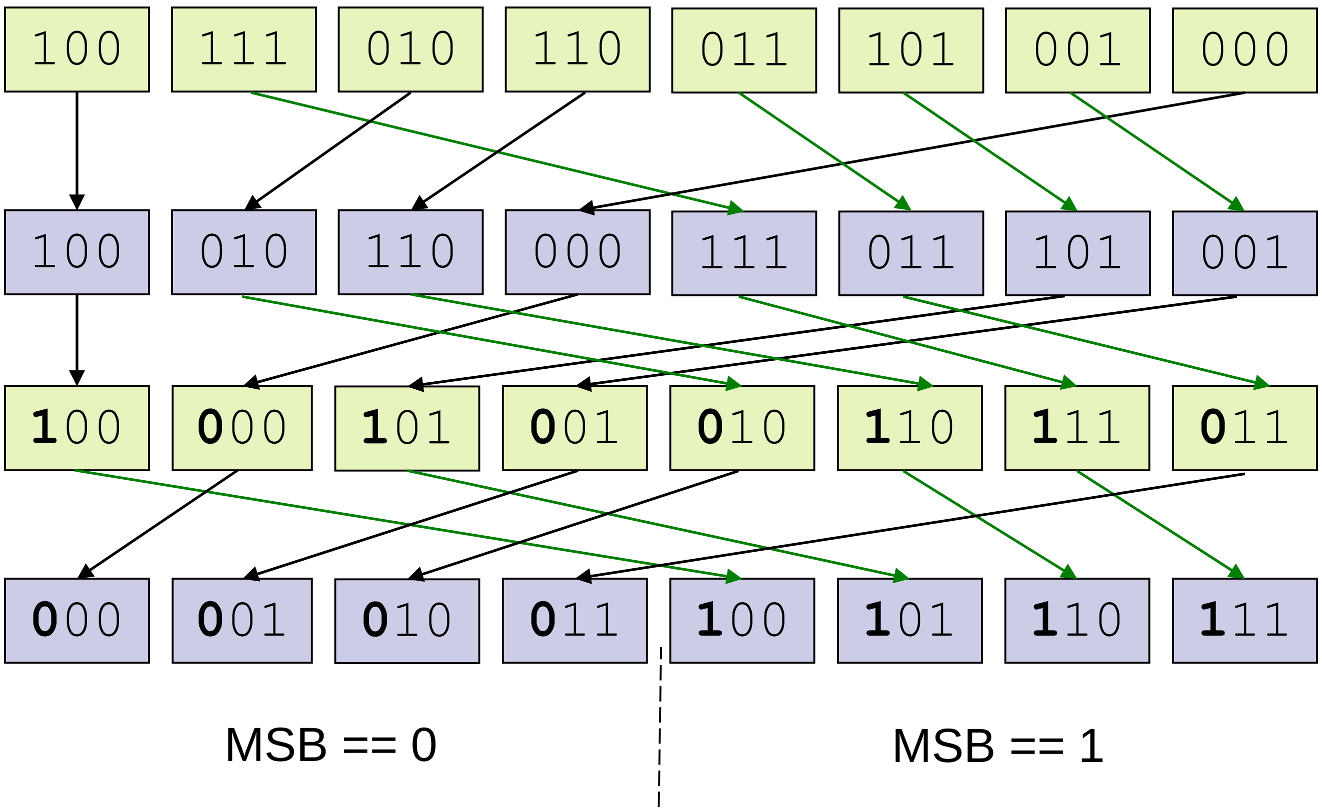
Radix Sort Example

- Second pass: partition based on second LSB



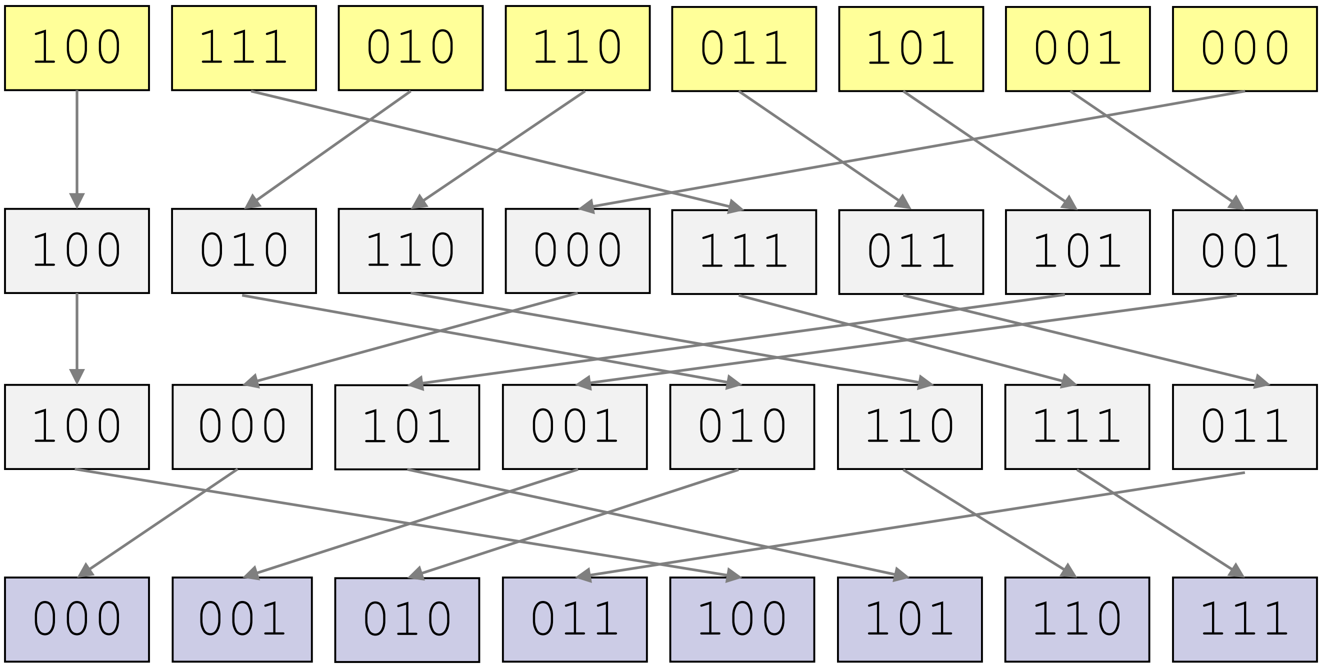
Radix Sort Example

- Final pass: partition based on MSB



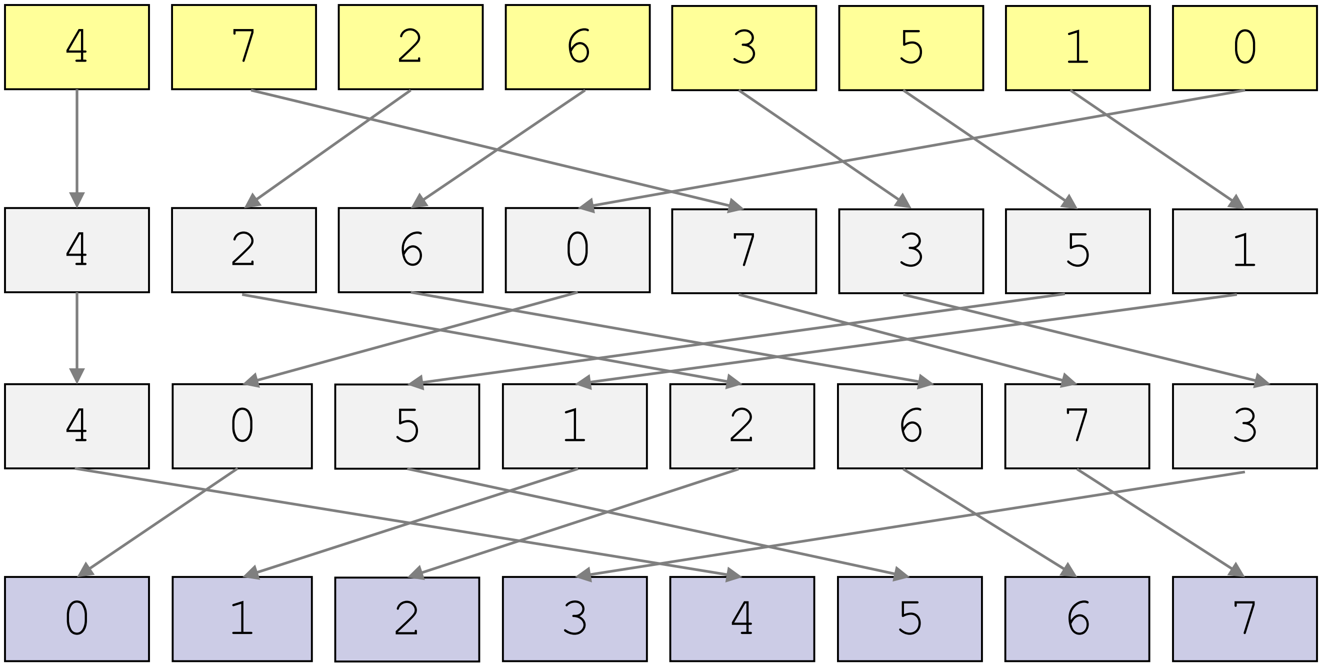
Radix Sort Example

- Completed:



Radix Sort Example

- Completed:



Parallel Radix Sort

1. Break input array into tiles
   – Each tile fits into shared memory for a thread block
2. Sort tiles in *parallel* with *radix sort*
3. Merge pairs of tiles using a *parallel bitonic merge* until all tiles are merged.

Our focus is on Step 2

Parallel Radix Sort

- Where is the parallelism?
  - Each tile is sorted in parallel
  - Where is the parallelism within a tile?
    - Each pass is done in sequence after the previous pass. No parallelism
    - Can we parallelize an individual pass? How?
  - Merge also has parallelism

Parallel Radix Sort in Each Pass

- Implement Split on the current bit under comparison
  - Histogram-based computation to count the number of 0/1s
  - Prefix scan to determine the output position of each element.
  - Efficient scatter of elements to target locations
  - Shared memory optimization
    - Histograms are stored in the shared memory.

Parallel Radix Sort

- Implement *split*. Given:
  - Array, *i*, at pass *n*:

100	111	010	110	011	101	001	000

  - Array, *b*, which is true/false for bit *n*:

0	1	0	0	1	1	1	0

- Output array with false keys before true keys:

100	010	110	000	111	011	101	001

Parallel Radix Sort

- Step 1: Compute *e* array

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array

Parallel Radix Sort

- Step 2: Exclusive Scan *e*

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array

Parallel Radix Sort

- Step 3: Compute *totalFalses*

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array

totalFalses = e[n – 1] + f[n – 1]
totalFalses = 1 + 3
totalFalses = 4

Parallel Radix Sort

- Step 4: Compute *t* array

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
								t array

t[i] = i – f[i] + totalFalses

totalFalses = 4

Parallel Radix Sort

- Step 4: Compute *t* array

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4								t array

t[0] = 0 – f[0] + totalFalses
t[0] = 0 – 0 + 4
t[0] = 4

totalFalses = 4

Parallel Radix Sort

- Step 4: Compute *t* array

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	$\boxed{1}$	1	2	3	3	3	3	f array
4	$\boxed{4}$							t array

t[1] = 1 – f[1] + totalFalses
t[1] = 1 – 1 + 4
t[1] = 4

totalFalses = 4

Parallel Radix Sort

- Step 4: Compute *t* array

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4	4	5						t array

t[2] = 2 – f[2] + totalFalses
t[2] = 2 – 1 + 4
t[2] = 5

totalFalses = 4

Parallel Radix Sort

- Step 4: Compute *t* array

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4	4	5	5	5	6	7	8	t array

t[i] = i – f[i] + totalFalses

totalFalses = 4

Parallel Radix Sort

- Step 5: Scatter based on address *d*

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4	4	5	5	5	6	7	8	t array
0								d[i] = b[i] ? t[i] : f[i]

Parallel Radix Sort

- Step 5: Scatter based on address *d*

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4	4	5	5	5	6	7	8	t array
0	4							d[i] = b[i] ? t[i] : f[i]

Parallel Radix Sort

- Step 5: Scatter based on address *d*

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4	4	5	5	5	6	7	8	t array
0	4	1						d[i] = b[i] ? t[i] : f[i]

Parallel Radix Sort

- Step 5: Scatter based on address *d*

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4	4	5	5	5	6	7	8	t array
0	4	1	2	5	6	7	3	d[i] = b[i] ? t[i] : f[i]

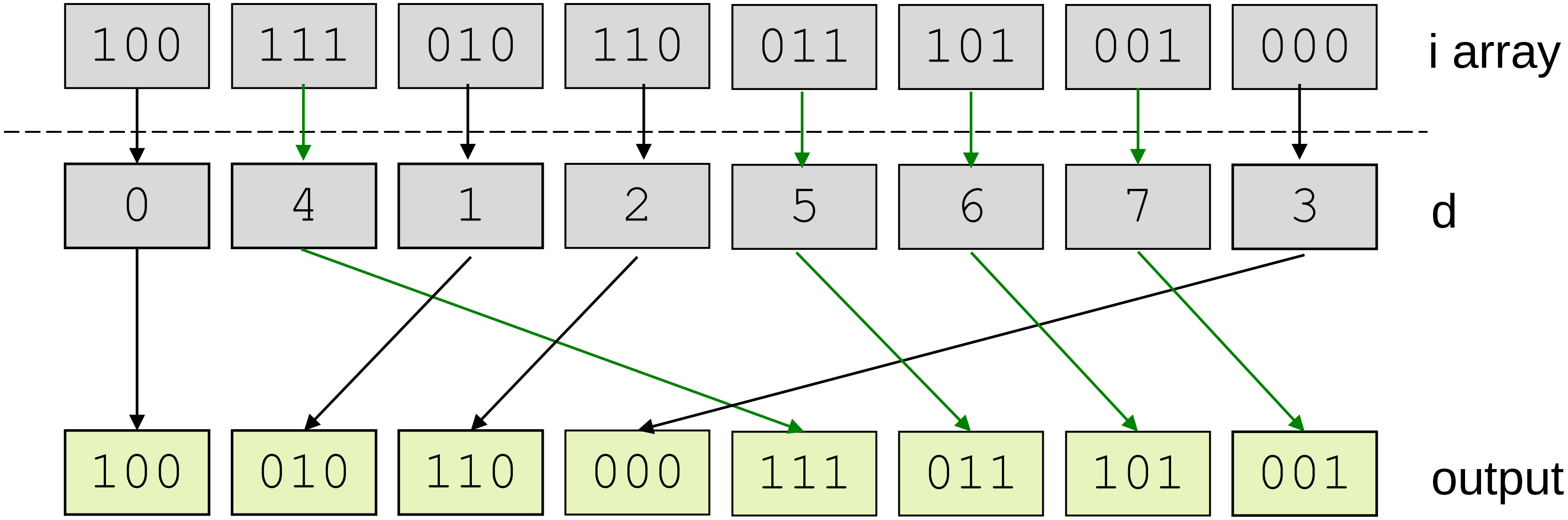
Parallel Radix Sort

- Step 5: Scatter based on address *d*

100	111	010	110	011	101	001	000	i array
0	4	1	2	5	6	7	3	d
								output

Parallel Radix Sort

- Step 5: Scatter based on address *d*



Radix Sort Analysis

- Integer sort
- Complexity $O(kn)$
  - n – number of elements
  - k– number of digits (constant)
- Parallel implementation (naïve)
  - For each radix (digit)
    - Compute radix histogram
    - Scan the histogram to compute offset for each element
    - Write the sorted elements
  - Work $O(kn)$
  - Time $O(k\log n)$

Summary

- Split and Sort are two common data-parallel primitives.
- They can be composed using simpler primitives.
- They are used widely in higher-level applications.
- Radix sort is currently the fastest sorting algorithm on the GPU.