THE VALUATION OF CRYPTOCURRENCY OPTIONS

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Abstract

The cryptocurrency transactions are recently thriving but a market for its contingent claims has not been built up yet. In this paper, we introduce two models, SVCJ and Meixner, to simulate the market dynamics and to price the European options based on the bitcoin (BTC) and the cryptocurrency index (CRIX).

SECTION 1

SVCJ Model and Simulation

Let $y_t = \ln S_t$. The stochastic volatility with correlated jumps (SVCJ) model dynamics are demonstrated as follows.

$$dy_t = \mu\, dt + \sqrt{v_t}\, dW_t^y + z_t^y\, dN_t, \tag{1.1}$$

$$dv_t = \kappa\,(\theta - v_t)\; dt + \sigma_v \sqrt{v_t}\, dW_t^v + z_t^v\, dN_t, \tag{1.2}$$

$$\mathrm{Cov}\,(dW_t^y, dW_t^v) = \rho\, dt, \tag{1.3}$$

$$P\,(dN_t = 1) = \lambda\, dt, \tag{1.4}$$

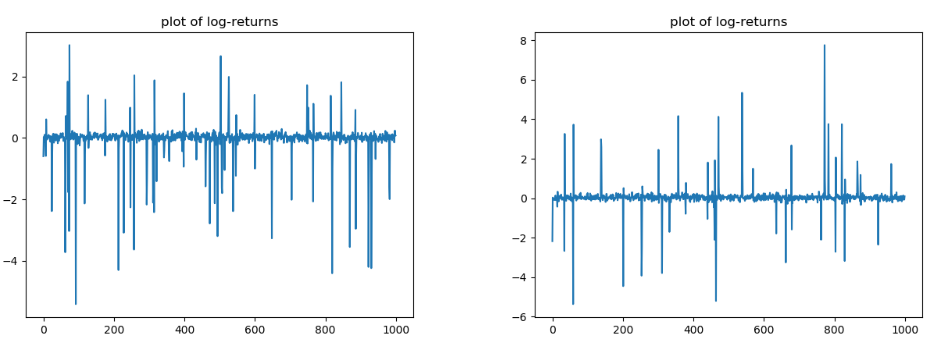
where κ and θ are the mean reversion rate and the mean reversion level respectively. Two Brownian motions W^y and W^v are correlated with the coefficient ρ. N_t is a pure jump process with a constant mean jump-arrival rate λ. The random jump sizes are z^y and z^v. We calibrate those parameters given the historical data from January 8th, 2014 to April 13th, 2019, and the results are shown as follows.

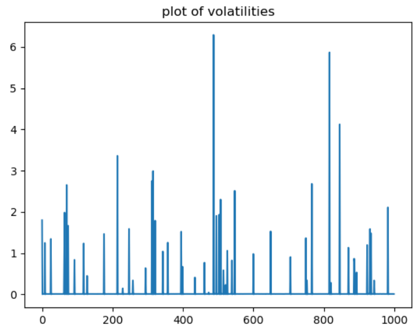
	BTC		CRIX	
	Mean	S.D.	Mean	S.D.
μ	0.0298	0.0064	0.0368	0.0075
μ_y	-0.5911	0.3310	-0.1941	0.3205
σ_y	1.9912	0.3782	2.4057	0.5068
λ	0.0336	0.0064	0.0454	0.0086
α	0.0095	0.0011	0.0099	0.0023
β	-0.0973	0.0087	-0.1220	0.0087
σ_v	0.0106	0.0021	0.0107	0.0027
ρ	0.1026	0.0430	0.3327	0.1269
ρ_j	-0.1818	0.3371	-0.5918	0.3697
μ_v	0.7514	0.1179	0.6433	0.1092

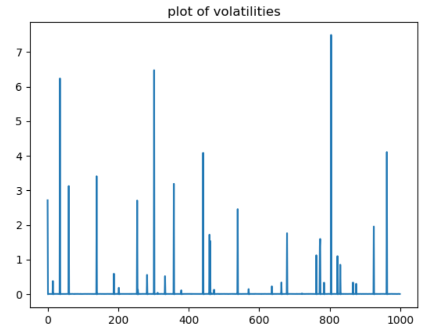
To simulate the process, we apply Euler's discretization as follows.

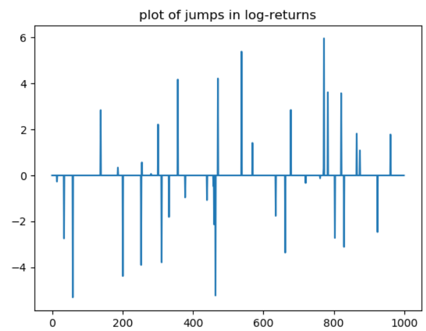
$$y_t = \mu + \sqrt{v_{t-1}}\epsilon_t^y + z_t^y J_t. \tag{1.5}$$

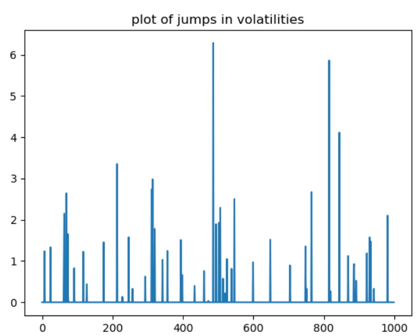
$$v_t = \alpha + \beta v_{t-1} + \sigma_v \sqrt{v_{t-1}}\epsilon_t^v + z_t^v J_t. \tag{1.6}$$

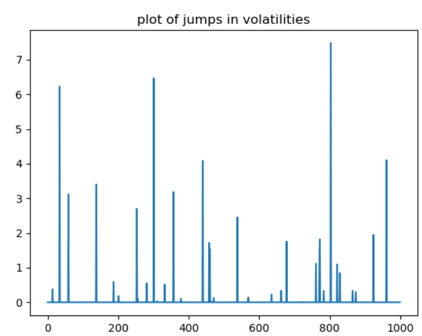












We plot the log-return and volatility dynamics of BTC on the left and CRIX on the right. From the figures above, it is obvious that there are much less jumps in CRIX than in BTC since CRIX is an index that contains more than thirty different cryptocurrencies and thus less volatile than BTC. Finally, we price the European call options by simulation. The simulation is done for 10000 iterations. We use the prices on April, 13th, 2019 as the current prices, and present the estimated prices in the following tables, assuming a BTC level of $S_T = 5000$ and a CRIX level of $S_T = 13000$. Although we can simulate the option prices by the SVCJ model, there are several drawbacks. The volatility is overestimated since we use the data from 2014 to 2019, the period in which cryptocurrencies are volatile. Compared with the options that depend on traditional underlying assets, we only have data from recent years and that might be not enough to generalize market dynamics. From the tables, the prices are not very accurate for the options close to expiry, and the in-the-money option prices seems to be less accurate than the out-of-the-money ones since they are relatively less liquid.

Strike	$T=1$	$T=7$	$T=30$	$T=60$	$T=90$	$T=180$	$T=360$	$T=720$
4000	1161.01	1161.69	1162.97	1164.18	1163.36	1170.52	1178.91	1193.32
4100	1061.01	1061.69	1062.97	1064.18	1063.36	1070.52	1078.91	1093.32
4200	961.01	961.69	962.96	964.18	963.35	970.52	978.91	993.32
4300	861.01	861.69	862.96	864.18	863.35	870.52	878.91	893.32
4400	761.01	761.69	762.96	764.18	763.35	770.52	778.91	793.39
4500	661.01	661.69	662.96	664.18	663.35	670.52	678.93	693.63
4600	561.01	561.69	562.96	564.18	563.35	570.52	579.01	594.28
4700	461.01	461.69	462.96	464.18	463.36	470.59	479.32	495.89
4800	361.01	361.69	362.96	364.20	363.38	370.85	380.43	399.97
4900	261.01	261.69	262.97	264.33	263.68	272.12	284.04	308.58
5000	161.01	161.71	163.35	165.62	166.06	177.27	194.06	225.48
5100	61.33	63.06	68.38	74.65	78.48	94.67	116.85	153.94
5200	1.24	2.50	7.63	14.85	20.48	36.61	59.81	96.88
5300	0.00	0.02	0.25	1.04	2.44	9.23	25.21	55.68
5400	0.00	0.00	0.00	0.07	0.23	1.47	8.61	29.00
5500	0.00	0.00	0.00	0.01	0.02	0.13	2.33	13.73
5600	0.00	0.00	0.00	0.00	0.00	0.00	0.55	5.65
5700	0.00	0.00	0.00	0.00	0.00	0.00	0.10	2.16
5800	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.69
5900	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18
6000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03

Option Prices of European Options on BTC

Strike	$T=1$	$T=7$	$T=30$	$T=60$	$T=90$	$T=180$	$T=360$	$T=720$
12100	1084.58	1089.31	1103.42	1128.89	1148.24	1215.23	1337.60	1599.03
12100	984.58	989.31	1003.43	1028.90	1048.37	1115.93	1239.75	1503.20
12200	884.58	889.31	903.46	928.94	948.62	1017.07	1142.89	1408.61
12300	784.58	789.31	803.50	829.12	849.17	918.95	1047.38	1315.40
12400	684.58	689.32	703.68	729.58	750.25	821.95	953.57	1223.92
12500	584.58	589.34	604.11	630.64	652.36	726.53	861.83	1134.48
12600	484.60	489.41	505.01	532.85	555.99	633.50	772.73	1047.23
12700	384.64	389.71	407.16	437.20	462.23	543.65	686.57	962.33
12800	284.78	290.81	311.99	345.05	372.57	458.34	604.14	880.30
12900	185.69	194.12	221.75	258.99	289.09	378.43	526.02	801.48
13000	93.22	104.80	140.44	182.24	214.30	305.32	452.74	725.91
13100	28.47	39.24	75.53	118.04	150.25	240.17	384.98	653.77
13200	4.30	10.33	34.94	69.57	99.08	183.99	323.38	585.22
13300	0.44	2.66	14.90	37.74	61.32	136.98	267.96	520.81
13400	0.05	0.71	6.07	19.48	36.00	98.84	218.83	460.66
13500	0.00	0.15	2.23	9.68	20.33	69.49	175.74	405.06
13600	0.00	0.02	0.80	4.54	11.07	47.61	139.03	354.00
13700	0.00	0.00	0.23	1.99	5.77	31.67	108.45	307.71
13800	0.00	0.00	0.07	0.80	2.82	20.44	83.24	265.89
13900	0.00	0.00	0.02	0.26	1.25	12.90	62.60	228.37
14000	0.00	0.00	0.00	0.07	0.57	7.80	46.24	194.82

Option Prices of European Options on CRIX

SECTION 2

Meixner Model and Estimation

The jump models often outperform the traditional stochastic models when they are used to describe the underlying asset price process with high volatility. Previously, we have used the SVCJ model to price the option prices. We now propose the Meixner model. Since it has a closed-formed PDF and a characteristic function, it can be constructed to price options faster. Its PDF is given by

$$M_1(x;a,b,d,m) = \frac{\left(2\cos\frac{b}{2}\right)^{2d}}{2a\pi\Gamma(2d)} \exp\left(\frac{b(x-m)}{a}\right) \left|\Gamma\left(d + \frac{i(x-m)}{a}\right)\right|^2 \tag{2.1}$$

where $a > 0$, $-\pi < b < \pi$, $d > 0$ and $m \in \mathbb{R}$. Since it belongs to the class of the infinitely divisible distributions, we have

$$M_k(x;a,b,d,m) = M_1(x;a,b,kd,km). \tag{2.2}$$

for $k > 0$. The characteristic function of the Meixner model is given by

$$\psi_k(u) = \mathbb{E}\left[e^{iuM_k}\right] = \left(\frac{\cos\left(\frac{b}{2}\right)}{\cosh\frac{au-ib}{2}}\right)^{2kd} e^{ikmu}. \tag{2.3}$$

We will apply two methods to estimate parameters in our model.

2.1 Moment Estimation

The theoretical moments of the Mexiner model are calculated as follows.

Mean	$m + ad\tan\frac{b}{2}$	Variance	$\frac{a^2 d}{\cos b+1}$	Skewness	$\frac{\sin b}{\sqrt{d(\cos b+1)}}$	Kurtosis	$3 - \frac{\cos b-2}{d}$

We first calculate four estimated moments given the sample data, and then solve the system of four equations with four variables.

$$a \approx -0.10314447262338783\,, \quad b \approx 2.000000000000000\,(3.141592653589793\,n - 0.11073778659061205)\,, \quad d \approx 0.2825790619887967\,, \quad m \approx -2.800220663783078\times10^{-44}\left(1.040863268392191\times10^{42}\tan(0.11073778659061205 - 3.141592653589793\,n) - 4.434631028511209\times 10^{40}\right), \quad n\in\mathbb{Z}$$

Computed by Wolfram|Alpha

$$a \approx -0.08723110162240453\,, \quad b \approx 2.000000000000000\,(3.141592653589793\,n - 0.2928082328320040)\,, \quad d \approx 0.3582248481829780\,, \quad m \approx 1.1984709815395192\times10^{-44}\left(1.2471693986496117\times10^{41} - 2.607351251457012\times10^{42}\tan(0.2928082328320040 - 3.141592653589793\,n)\right), \quad n\in\mathbb{Z}$$

Computed by Wolfram|Alpha

Given the solutions, we need to modify parameters to fit in the model. Let $n = 0$ and then $b \in (-\pi, \pi)$. We also flip the sign of a so that $a > 0$, arguing that the shape of the PDF will almost remain the same. The results are shown as follows.

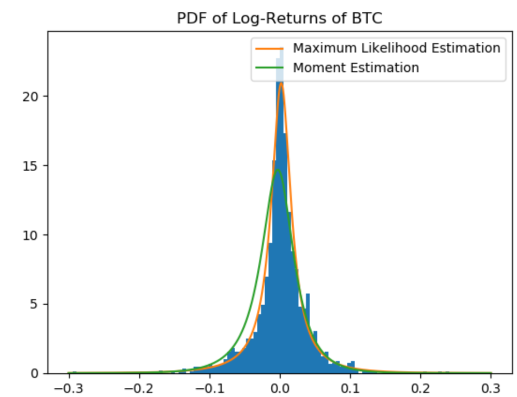
	$\hat{a}$	$\hat{b}$	$\hat{d}$	$\hat{m}$
BTC	0.1031	-0.2215	0.2826	-0.0020
CRIX	0.0872	-0.5856	0.3582	-0.0079

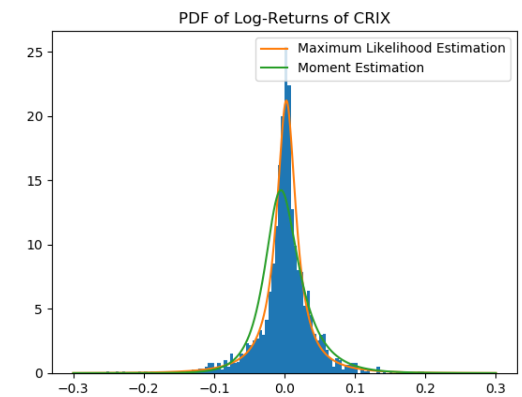
2.2 Maximum Likelihood Estimation

Since the previous estimation is not ideal, we now consider another approach. When the PDE is known, the maximum likelihood estimation is a common estimation method. Suppose we have n observations $x_1, x_2, \cdots, x_n$ and the PDF is $p(x_i; \theta)$ where θ is the set of parametrs to be estimated. Then we acquire $\hat{\theta}$ by maximizing $\sum_{i=1}^{n} \ln p(x_i; \theta)$. The results are shown as follows.

	$\hat{a}$	$\hat{b}$	$\hat{d}$	$\hat{m}$
BTC	0.1845	-0.1065	0.0926	0.0022
CRIX	0.1827	-0.1427	0.0921	0.0027

We plot the histogram and the PDFs to show how well we approximate the distributions.





It is obvious that the maximum likelihood estimation performs better. Now that we already acquire all parameters in the characteristic function, we can use the Fast Fourier Transform method to price the European call options.

Strike	$T = 1$	$T = 7$	$T = 30$	$T = 60$	$T = 90$	$T = 180$	$T = 360$	$T = 720$
3999.42	1162.06	1164.34	1173.10	1184.49	1195.85	1229.73	1296.60	1426.93
4100.67	1060.83	1063.16	1072.15	1083.84	1095.49	1130.25	1198.85	1332.51
4204.48	957.03	959.43	968.66	980.65	992.61	1028.28	1098.67	1235.77
4310.91	850.60	853.07	862.55	874.87	887.15	923.78	996.03	1136.69
4398.00	763.52	766.05	775.74	788.34	800.90	838.33	912.15	1055.74
4509.34	652.20	654.80	664.79	677.77	690.70	729.22	805.11	952.50
4600.43	561.12	563.79	574.04	587.36	600.63	640.11	717.77	868.32
4693.36	468.20	470.95	481.52	495.23	508.88	549.45	629.03	782.84
4812.18	349.40	352.29	363.36	377.70	391.95	434.14	516.42	674.46
4909.39	252.22	255.26	266.92	281.95	296.84	340.68	425.45	586.91
5008.57	153.09	156.39	168.97	185.08	200.90	246.93	334.50	499.18
5109.75	52.02	55.91	70.54	88.65	105.97	154.89	245.15	412.17
5212.97	0.20	1.33	6.35	15.51	27.84	72.45	161.10	327.40
5291.75	0.10	0.68	3.14	7.11	12.10	34.73	106.73	266.76
5398.65	0.05	0.37	1.66	3.68	6.12	16.60	56.86	192.89
5507.71	0.03	0.22	0.97	2.15	3.55	9.39	31.75	131.57
5590.95	0.02	0.15	0.68	1.49	2.45	6.43	21.52	96.17
5703.90	0.01	0.09	0.43	0.93	1.54	4.02	13.37	62.68
5790.10	0.01	0.07	0.30	0.67	1.10	2.88	9.55	45.64
5907.07	0.01	0.04	0.20	0.43	0.71	1.87	6.21	30.23
5996.34	0.01	0.03	0.14	0.32	0.52	1.36	4.54	22.37

Option Prices of European Options on BTC

Strike	$T = 1$	$T = 7$	$T = 30$	$T = 60$	$T = 90$	$T = 180$	$T = 360$	$T = 720$
11975.80	1063.64	1070.72	1097.88	1133.09	1168.12	1272.12	1475.75	1868.49
12096.16	943.31	950.52	978.20	1014.07	1049.71	1155.37	1361.76	1758.77
12217.73	821.76	829.14	857.43	894.04	930.37	1037.88	1247.19	1648.51
12278.97	760.54	768.01	796.64	833.67	870.39	978.90	1189.75	1593.22
12402.38	637.17	644.86	674.29	712.27	749.87	860.59	1074.65	1482.42
12527.02	512.57	520.53	550.98	590.14	628.80	742.04	959.52	1371.47
12589.82	449.81	457.94	489.00	528.86	568.11	682.80	902.06	1316.02
12716.35	323.35	331.93	364.58	406.18	446.89	564.76	787.65	1205.33
12780.09	259.67	268.54	302.25	344.96	386.54	506.19	730.87	1150.18
12908.53	131.41	141.23	178.13	223.79	267.50	390.92	618.75	1040.55
13038.26	5.20	16.90	59.89	109.16	154.75	280.59	509.69	932.25
13103.62	0.79	4.78	24.35	62.92	105.61	229.08	456.93	878.80
13235.31	0.37	2.50	11.68	27.50	48.61	141.21	356.82	773.82
13368.33	0.24	1.66	7.62	17.32	29.55	85.85	267.65	672.25
13435.34	0.20	1.39	6.37	14.36	24.28	69.15	228.84	623.14
13570.36	0.15	1.02	4.62	10.32	17.25	47.63	165.38	529.16
13638.39	0.13	0.88	3.99	8.87	14.78	40.39	140.76	484.71
13775.45	0.10	0.67	3.02	6.68	11.09	29.84	103.37	401.97
13844.50	0.09	0.58	2.64	5.84	9.68	25.91	89.31	364.08
13913.90	0.08	0.51	2.32	5.13	8.48	22.62	77.56	328.75
13983.64	0.07	0.45	2.04	4.51	7.46	19.82	67.67	296.06
14053.74	0.06	0.40	1.81	3.98	6.58	17.43	59.29	266.08

Option Prices of European Options on CRIX

SECTION 3

Conclusion

The Meixner process with maximum likelihood estimation can approximate log-return distribution with high accuracy. When it comes to pricing the European options, the Meixner model performs better than the SVCJ model, especially in the short term, due to its explicit PDF and characteristic function. However, the volatility in Meixner model is constant over the time periods that does not reflect the fluctuating cryptocurrency market dynamics, so we may introduce the stochastic volatility to the Meixner model in the future.

SECTION 4

Reference

[1] Hou, Ai Jun and Wang, Weining and Chen, Cathy Yi-Hsuan and Härdle, Wolfgang K. (2019). Pricing Cryptocurrency Options: The Case of Bitcoin and CRIX.
[2] Schoutens, W., (2002). The Meixner Process: Theory and Applications in Finance.
[3] Grigoletto, M. and Provasi, C. (2008). Simulation and Estimation of the Meixner Distribution. Communications in Statistics-Simulation and Computation, 38(1), 58-77.