# 2. Behoud van mechanische energie bij 2 deeltjes met inwendige interactie zonder uitwendig krachtenveld

2de WN:

$$\begin{cases} m_1 \dfrac{d^2\vec{r}_1}{dt^2} = \vec{G}_{12} \\ m_2 \dfrac{d^2\vec{r}_2}{dt^2} = \vec{G}_{21} \end{cases}$$

op 1 door 2

3de WN: $\vec{G}_{12} \parallel \vec{G}_{21}$ en

$$|\vec{G}_{12}| = |\vec{G}_{21}|$$

$$\Rightarrow \vec{v}_1 \cdot m_1 \frac{d^2\vec{r}_1}{dt^2} = \vec{G}_{12} \cdot \vec{v}_1 \quad [1]$$

" " [2]

$[1]+[2]$

$$m_1 \vec{v}_1 \frac{d\vec{v}_1}{dt} + m_2 \vec{v}_2 \frac{d\vec{v}_2}{dt} = \vec{v}_1 \vec{G}_{12} + \vec{v}_2 \vec{G}_{21}$$

$$\frac{d}{dt}\underbrace{\left(\frac{1}{2} m_1 \vec{v}_1^{\,2}\right)}_{T_1} + \frac{d}{dt}\underbrace{\left(\frac{1}{2} m_2 \vec{v}_2^{\,2}\right)}_{T_2} = \text{''}$$

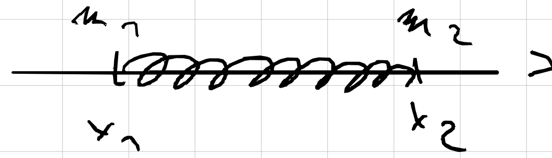
$$\int_{t_{in}}^{t_{fi}} \frac{d}{dt}\left(\frac{1}{2} m_1 \vec{v}_1^{\,2}\right) + \int_{t_{in}}^{t_{fi}} \frac{d}{dt}\left(\frac{1}{2} m_2 \vec{v}_2^{\,2}\right) = \int_{t_{in}}^{t_{fi}} \vec{G}_{12}\vec{v}_1 + \vec{G}_{21}\vec{v}_2 \, dt$$

$$\underbrace{(T_1+T_2)_{t_{fi}}}_{T_{tot,fi}} - \underbrace{(T_1+T_2)_{t_{in}}}_{T_{tot\,in}} = \int_{t_{in}}^{t_{fi}} \vec{G}_{12}\vec{v}_1 + \vec{G}_{21}\vec{v}_2 \, dt$$

$$\left(W = \int_{t_{in}}^{t_{fi}} \vec{F}\, d\vec{l} = \text{''} \; \vec{F}\cdot\vec{v}\, dt\right)$$

$$(T_1+T_2)_{t_{fi}} - (T_1+T_2)_{t_{in}} = W_{tot}\,(t_{in}, t_{fi})$$

$$\vec{G}_{12} = k(x_2 - x_1)\hat{i}$$




$$\vec{G}_{21} = k(x_1 - x_2)\hat{i}$$

$$\Rightarrow W_{tot}(t_{in}, t_{fi}) = \int_{t_{in}}^{t_{fi}} k(x_2 - x_1)\hat{i}\,\vec{v}_1 + k(x_1 - x_2)\hat{i}\,\vec{v}_2$$

Vorige week

$$\vec{F} = kr\hat{r}$$

$$\Leftrightarrow U = \underline{-\int \vec{F}\cdot d\vec{r}} \text{ of } dU = -\vec{F}$$

$$\Leftrightarrow \text{in ons geval} = -\frac{1}{2}kr^2 + C$$

$$\Rightarrow W_{tot}(t_{in}, t_{fi}) = \int_{t_{in}}^{t_{fi}} \left(k(x_2 - x_1)\frac{dx_1}{dt} + k(x_1 - x_2)\frac{dx_2}{dt}\right)dt$$

$$= -\int_{t_{in}}^{t_{fi}} \frac{d}{dt}\Big(\underbrace{\frac{1}{2}k(x_1 - x_2)^2}_{U^{\text{nw}}}\Big)dt$$

$$k(x_1 - x_2)\cdot(\dot{x}_1 - \dot{x}_2)$$

$$k(x_1 - x_2)\dot{x}_1 - \dot{x}_2 k(x_1 - x_2)$$

$$k(x_1 - x_2\ \dot{x}_1) + \dot{x}_2 k(x_2 - x_1)$$

$\Rightarrow$ OK! doe nog $k - 1$

$$= -\int_{U_{in}}^{U_{fi}} dU^{inew} = -U^{fi} + U^{in}$$

$\Leftrightarrow [3]$

$$[3]: \quad -\frac{1}{2}k(x_{1,fi} - x_{2,fi})^2 + \frac{1}{2}k(x_{1,in} - x_{2,in})^2$$

$$T_{tot,fi} - T_{tot,in} = W_{tot} = -U^{fi} + U^{in}$$

$$[4] \quad T_{tot,fi} + U^{fi} = T_{tot,in} + U^{in} \equiv E_{mech}$$

$$[3] \text{ en } [4] \Rightarrow \frac{1}{2}m_1\vec{v}_1^{\,2} + \frac{1}{2}m_2\vec{v}_2^{\,2} + \frac{1}{2}k(x_1 - x_2)^2 = E_{mech}$$

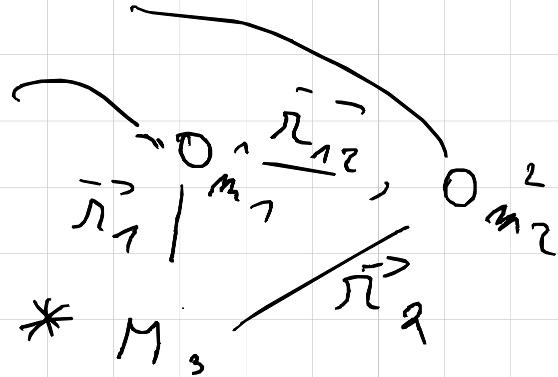
3 Verschillende deeltjes + uitwendig krachtenveld + inwendige (interactie) krachtenveld:

Systeem met 3 deeltjes:

$$E_{mech} = \frac{1}{2} m_1 \vec{v}_1^{\,2} + \frac{1}{2} m_2 \vec{v}_2^{\,2} + \frac{1}{2} m_3 \vec{v}_3^{\,2} + U_{12}^{inw} + U_{23}^{inw} + U_{13}^{inw}$$

$$+ U_1^{uitw} + U_2^{uitw} + U_3^{uitw} \ldots$$

Vb:




Het krachtenveld is $\vec{F}_G = -\frac{G M_1 M_2}{r^2}\hat{r}$

De bijhorende potentiële energie:

$$U = -\frac{G M_1 M_2}{r} + C$$

Dus $U_{12}^{inw} = -G\frac{m_1 m_2}{r_{12}}$, $U_1^{uit} + U_2^{uit} = -\frac{G m_1 M}{r_1} - \frac{G m_2 M}{r_2}$

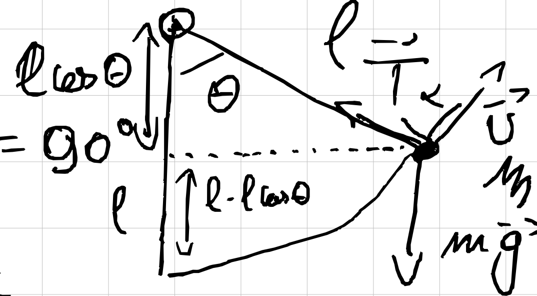
$$E_{kin} = \frac{1}{2} m_1 \vec{v}_1^2 + \frac{1}{2} m_2 \vec{v}_2^2 - G\frac{m_1 m_2}{r_{12}} - \frac{G m_1 M}{r_1} - \frac{G m_2 M}{r_2}$$

Slinger:

$$\vec{v} = \frac{dr}{dt}\hat{r} + r\frac{d\theta}{dt}\hat{\theta}$$

$\hat{r} = 0$

$$\vec{v} = r\frac{d\theta}{dt}\hat{\theta}$$




$W_{\vec{T}\vec{v}} = 0$ want $\alpha = 90°$

→ Span (draad) kracht

→ geen arbeid ⇒ Behoud mechanische energie

$$E^{mech} = \frac{1}{2} m \left(l\frac{d\theta}{dt}\right)^2 + mgl(1-\cos\theta)$$

$$\frac{d}{dt} \qquad 0 = \frac{2}{2}\cancel{m}l^{2}\cancel{\frac{d\theta}{dt}}\frac{d^2\theta}{dt^2} + \cancel{m}g\cancel{l}\sin\theta\cancel{\frac{d\theta}{dt}}$$

$$= l\frac{d^2\theta}{dt^2} + g\sin\theta$$

$$= \frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$$

Nieuw Hfdst : behoud E mech in (star) lichamen

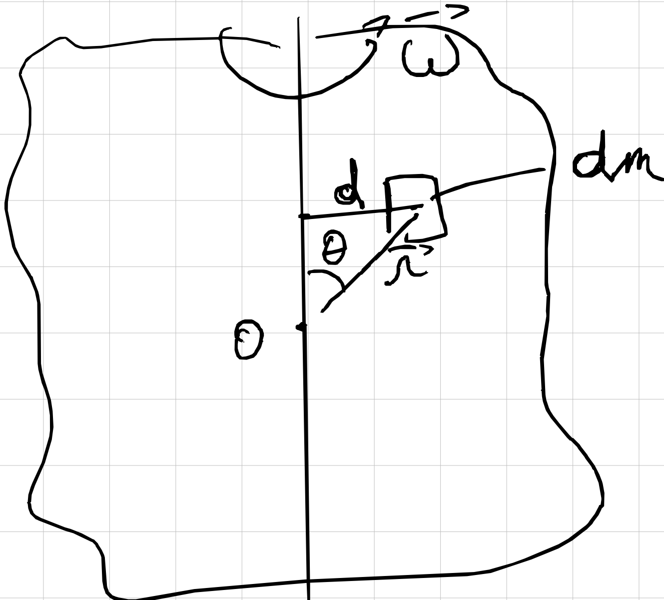
kan niet vervormen

Algemene beweging = translatie massacentrum + rotatie rond massacentrum

$\vec{\omega}$ ← snelheid als vector : grootte $\omega$, richting : rotatieas, zin : (rechterhandregel)

$\vec{\alpha}$ ← versnelling als vector : grootte $\alpha$,

Zuivere rotatie : - rotatie rond een vaste as




$$T = \frac{1}{2} m \vec{v}^2$$

$$K = \int \frac{1}{2} (\omega d)^2 dm$$

$d = r \sin\theta$

$$K = \int \frac{1}{2} (\omega r \sin\theta)^2 dm$$

$$= \int \frac{1}{2} |\underset{\vec{\omega} \to \omega\hat{\omega}}{\vec{\omega}} \times \vec{r}|^2 dm$$

$\equiv E_{kin\ rot}$

$$= \frac{1}{2} \underbrace{\int dm\, |\hat{\omega} \times \vec{r}|^2}_{\equiv I} \omega^2$$

$$= \frac{1}{2} I \omega^2$$

> traagheidsmoment (traagheid van rotatie)
"hoe moeilijk is de rot"