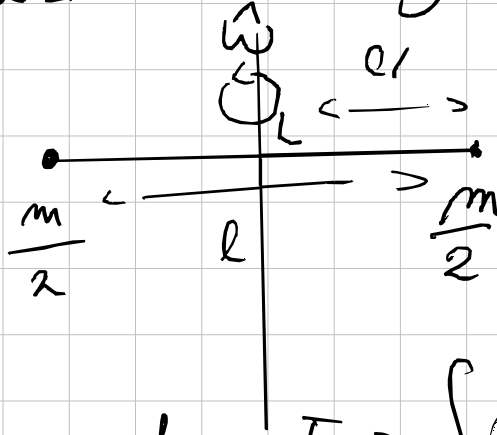


Trägheitsmoment I :

$$I = \int |\vec{\omega} \times \vec{r}|^2 dm$$

①

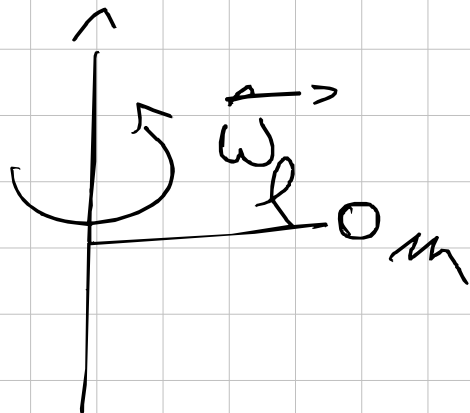


$\Rightarrow \sin \theta = 1$, also $I = \int r^2 dm$

$$K = \frac{1}{2} I \omega^2 = 2 \left(\frac{m}{2} \left(\frac{l}{2} \right)^2 \right) = \frac{m l^2}{4}$$

$$= \frac{1}{8} m l^2 \omega^2$$

②

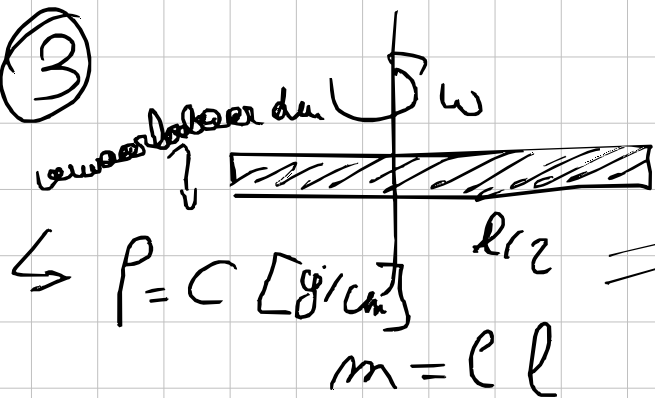


$$I = \int r^2 dm$$

$$= l^2 m$$

$$K = \frac{1}{2} l^2 m \omega^2$$

③



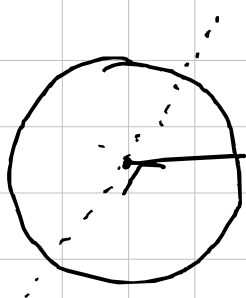
$$I = \int r^2 dm$$

$$dm = \rho dx$$

$$I = 2\rho \int_0^{l/2} x^2 dx$$

$$= \frac{2}{3} \rho \frac{l^3}{8} = \frac{\rho l^3}{12} = \frac{m l^2}{12}$$

③



$$r \sin \theta = r$$

$$I = \int r^2 dm$$

$$dm = \rho dA$$

$$dm = \rho 2\pi r dr$$

$$A = \pi R^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$dA = 2\pi r dr$$

$$I = \int_0^R r^2 \rho 2\pi r dr$$

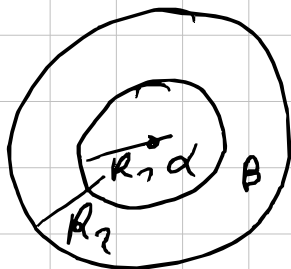
$$I = 2\pi \rho \int_0^R r^3 dr$$

$$I = 2\pi \rho \left[\frac{r^4}{4} \right]_0^R$$

$$I = \pi \rho \frac{R^4}{2} = \frac{m R^2}{2}$$

$$\begin{aligned} m &= \int_0^R \rho dl \\ &= 2\pi \rho \int_0^R r dr \\ &= \pi \rho R^2 \end{aligned}$$

4



$\vec{\omega} \perp \text{achse}$

$\rho [g/cm^2]$

$$I_B = \int r^2 dm$$

$\alpha + \beta = \text{tot}$

$$M_B = \pi \rho (R_2^2 - R_1^2)$$

$\leadsto I_{\text{tot}} = I_B + I_\alpha$

$$I_B = \frac{\pi}{2} \rho R_2^4 - \frac{\pi}{2} \rho R_1^4$$

$$= \frac{\pi}{2} \rho (R_2^2 - R_1^2) (R_2^2 + R_1^2)$$

$$= \frac{M_B}{2} (R_1^2 + R_2^2)$$

