

Name: \_\_\_\_\_  
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$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

- (1) What are  $A$ 's eigenvalues?
- (2) Show that if  $v_1, v_2 \in \mathbb{R}^3$  are eigenvectors of  $A$  corresponding to different eigenvalues, then  $\langle v_1, v_2 \rangle = 0$ .
- (3) Let  $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$  be a distribution (i.e.  $x_1, x_2, x_3 \geq 0$  and  $x_1 + x_2 + x_3 = 1$ ).

$$\lim_{n \rightarrow \infty} \left\| \frac{A^n x}{2^n} - \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix} \right\|_2 = 0$$
This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.