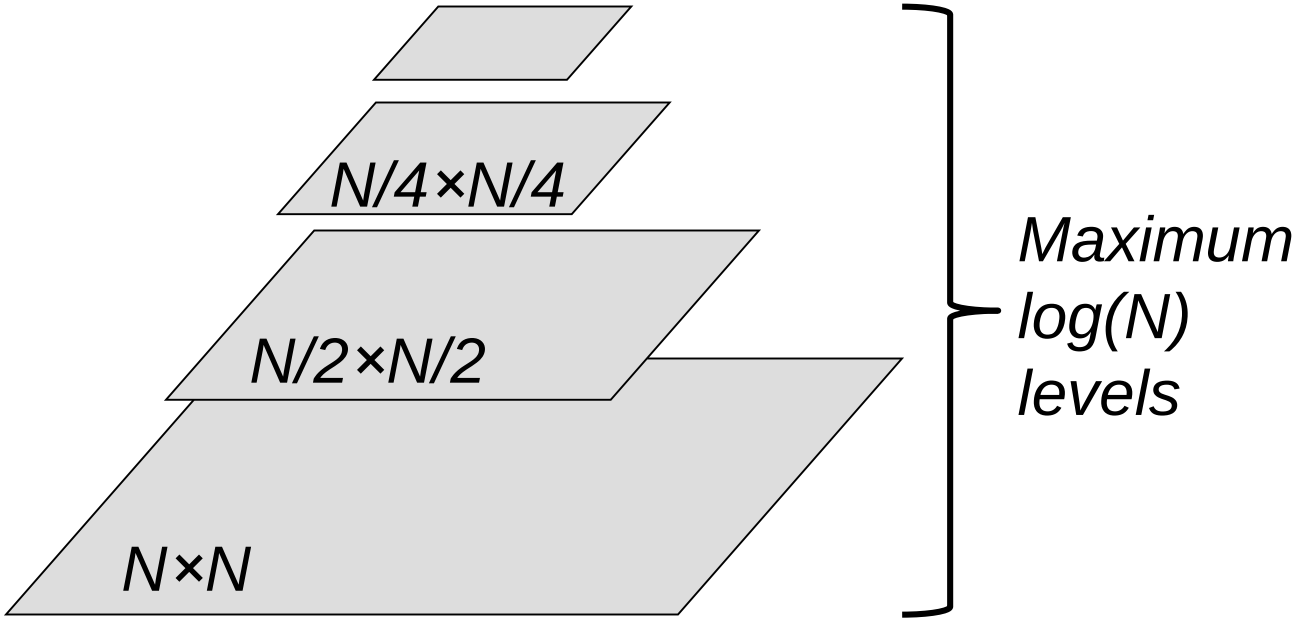
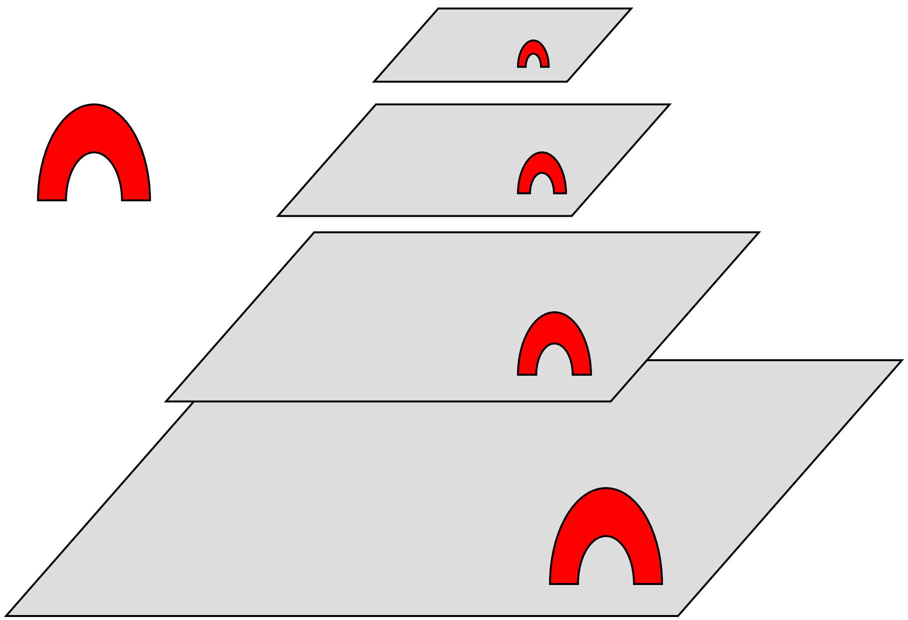
Image Pyramids



$$N^2 + \frac{1}{4}N^2 + \frac{1}{16}N^2 + \cdots = 1\frac{1}{3}N^2$$

Efficient Visual Search

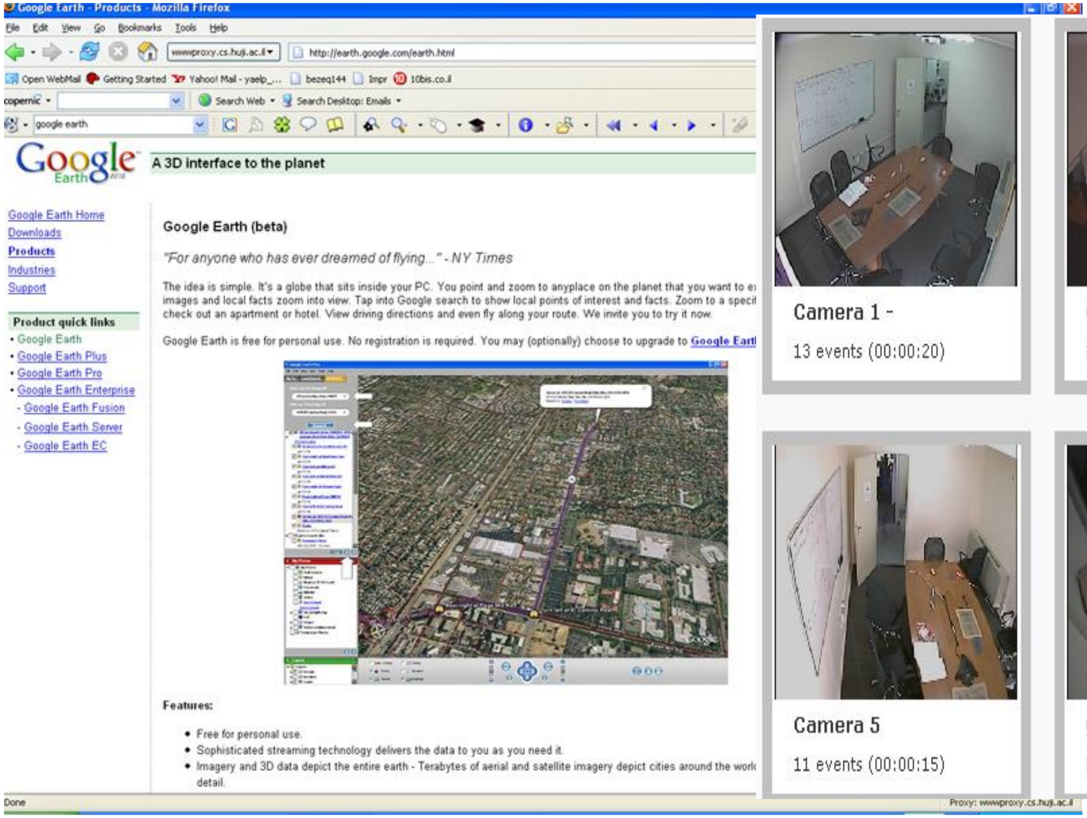


Search Area	Pattern Size		Total Oper.
± 16	4^2		2^{14}
± 32	8^2		2^{18}
± 64	16^2		2^{22}
± 128	32^2		2^{26}
$2^8 2^8$	$2^5 2^5$	$=$	2^{26}

- Pyramids: Start the search in a small image
- search area is small in larger levels (e.g. ±1) using the estimate from the smaller level

Applications for Pyramids

- Detection and Search (Esp. huge images)
- Browsing in Image Databases
- Motion Computation



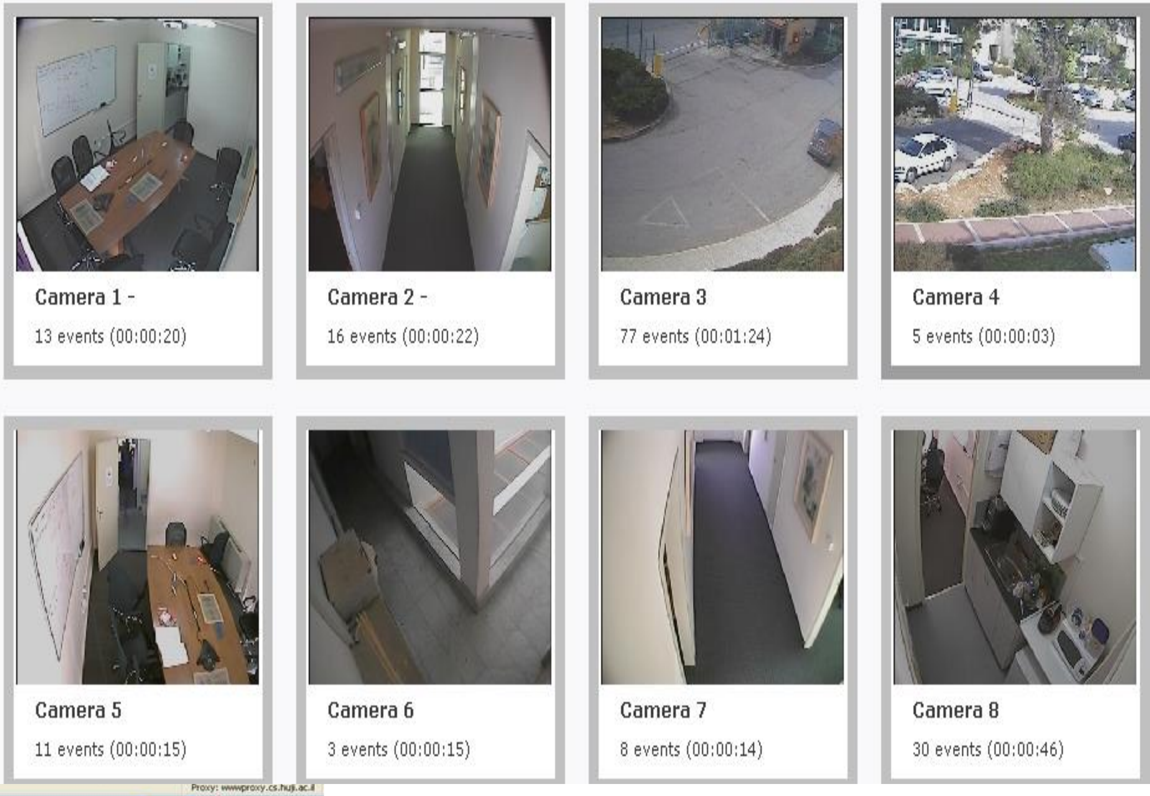


Image Resizing

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Reduce:

1. Blur
   – E.g. Convolve with a 3×3 filter $(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}) \times (\frac{1}{4}, \frac{1}{2}, \frac{1}{4})^T$, or a 5×5 filter $\frac{1}{16}(1, 4, 6, 4, 1) \times \ldots$ or larger
2. Sub-sample
   – Select only every 2^{nd} pixel in every 2^{nd} raw

Expand:

1. Zero Padding $(a_1, 0, a_2, 0, a_3, 0, \ldots)$
2. Blur
   - Note: Expand blur needs different normalizations!
   - Is zero padding followed by blur with $(\frac{1}{2}, 1, \frac{1}{2})$ OK?

Blur Kernels

Commonly Used - Binomial Coefficients

- Odd number of coefficients (have a center)
- Sum of coefficients normalized to 1
- Fast to compute (using shift and add)
- Asymptotically similar to a Gaussian

1 2 1	/ 4
1 4 6 4 1	/ 16
1 6 15 20 15 6 1	/ 64

$$(1\ 1) * \ldots^{2k} \ldots * (1\ 1) / 2^{2k}$$

Blur & Sub-sample
(Reduce)

2

1 2 1

1 2 3 4 5 6 5 4 3 2 1

1 2 1 * ¼

Blur & Sub-sample

2

1 2 1

1 2 3 4 5 6 5 4 3 2 1

1 2 1 * ¼

Blur & Sub-sample

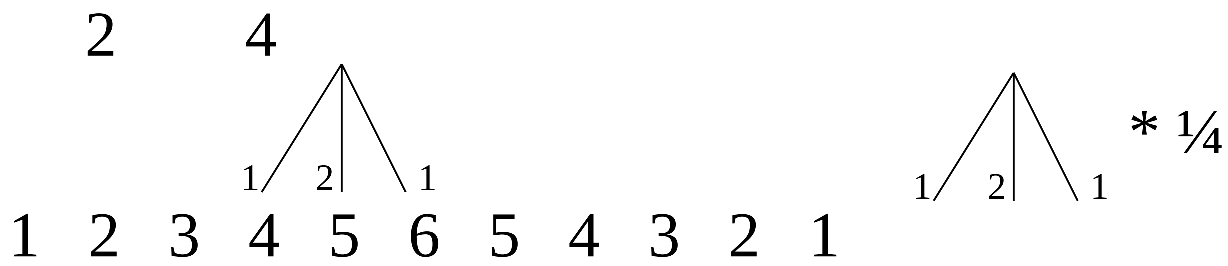
2 4

1 2 1

1 2 3 4 5 6 5 4 3 2 1

1 2 1 * ¼

Blur & Sub-sample



Blur & Sub-sample

2 4 5.5

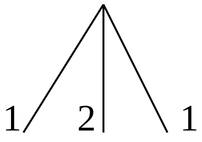
1 2 1

1 2 3 4 5 6 5 4 3 2 1

1 2 1 * ¼

Blur & Sub-sample

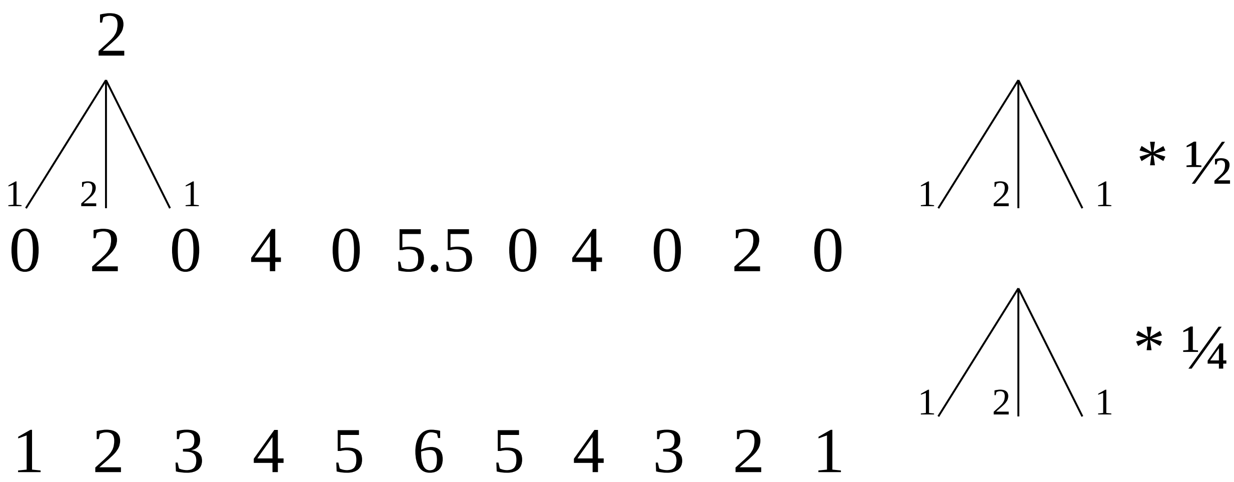
2 4 5.5 4 2



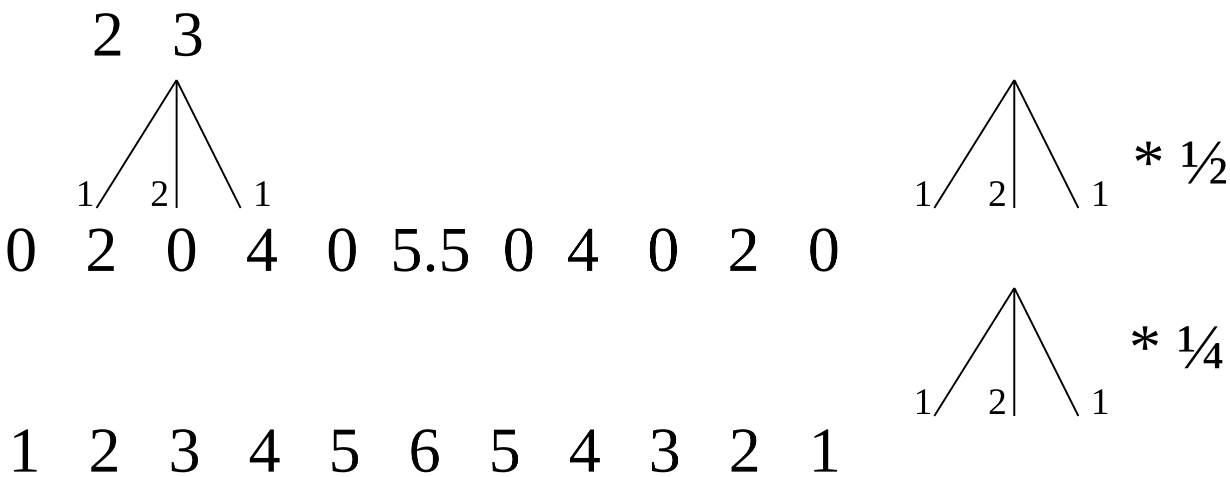
* ¼

1 2 3 4 5 6 5 4 3 2 1

Zero-Pad & Blur
(Expand)



Zero-Pad & Blur



Zero-Pad & Blur

2 3 4

1 2 1

1 2 1 * ½

0 2 0 4 0 5.5 0 4 0 2 0

1 2 1 * ¼

1 2 3 4 5 6 5 4 3 2 1

Zero-Pad & Blur

2 3 4 4¾

1 2 1

1 2 1 * ½

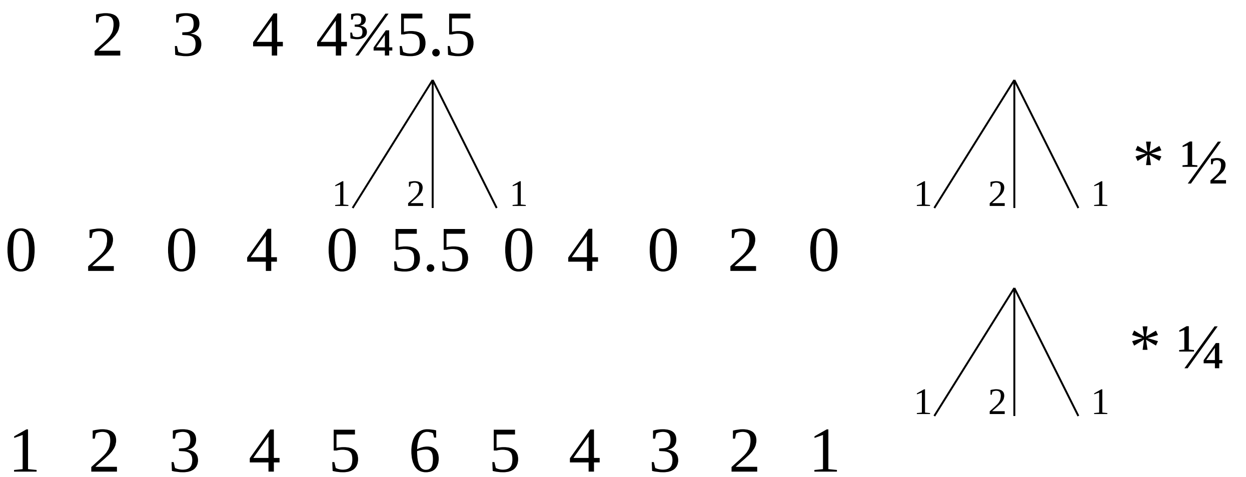
0 2 0 4 0 5.5 0 4 0 2 0

* ¼

1 2 1

1 2 3 4 5 6 5 4 3 2 1

Zero-Pad & Blur



Zero-Pad & Blur

2 3 4 4¾ 5.5 4¾

1 2 1

1 2 1 * ½

0 2 0 4 0 5.5 0 4 0 2 0

* ¼

1 2 1

1 2 3 4 5 6 5 4 3 2 1

Zero-Pad & Blur

2 3 4 4¾ 5.5 4¾ 4

1 2 1   1 2 1 * ½

0 2 0 4 0 5.5 0 4 0 2 0

1 2 1 * ¼

1 2 3 4 5 6 5 4 3 2 1

Zero-Pad & Blur

2 3 4 $4\frac{3}{4}$ 5.5 $4\frac{3}{4}$ 4 3 2

1 2 1 1 2 1 $* \frac{1}{2}$

0 2 0 4 0 5.5 0 4 0 2 0

1 2 1 $* \frac{1}{4}$

1 2 3 4 5 6 5 4 3 2 1

Handling Image Boundaries

Never Cyclic…

2 3 4 4¾5.5 4¾4 3 2

1 2 1   1 2 1 * ½

? 0 2 0 4 0 5.5 0 4 0 2 0

Mirror on last pixel. ?=2

Mirror after last pixel. ?=0

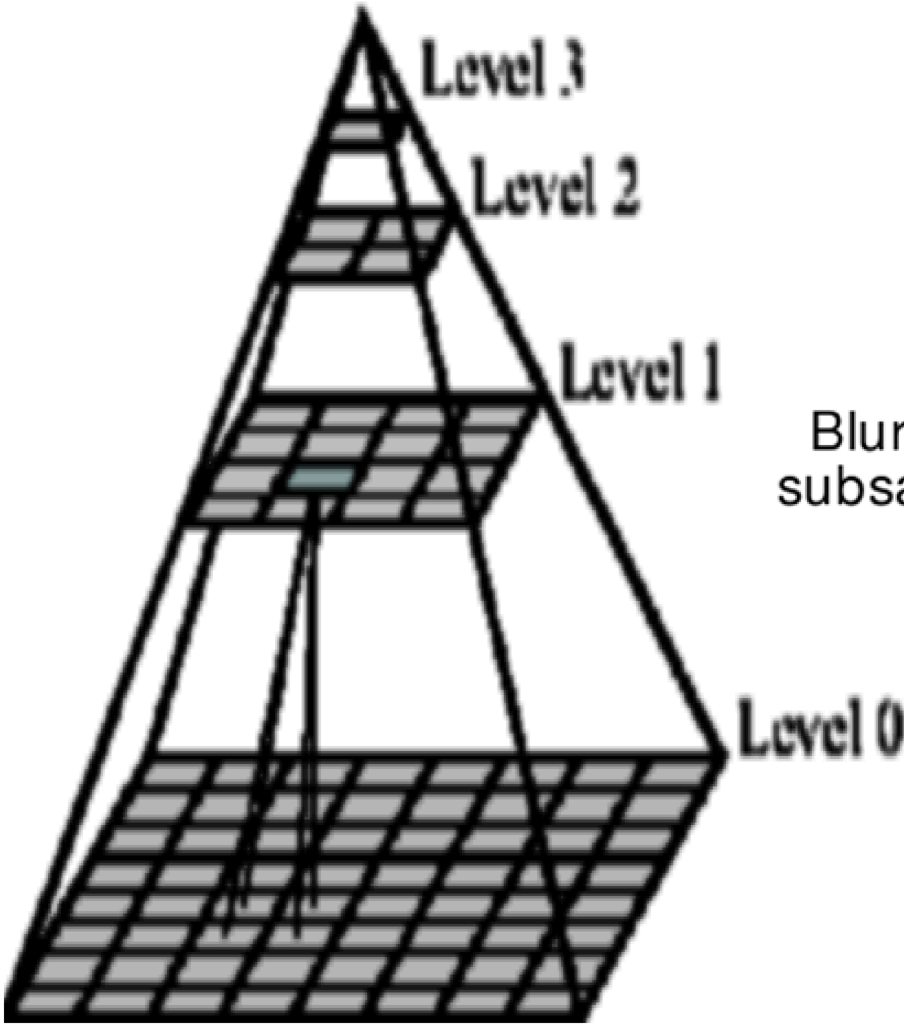
Duplicate last pixel. ?=0

Gaussian Pyramid

G_n	$Reduce\{G_{n-1}\}$
G_3	$Reduce\{G_2\}$
G_2	$Reduce\{G_1\}$
G_1	$Reduce\{G_0\}$
G_0	Original Image

- 2D Picture
  - Reduce Rows, Reduce Columns

5-Level Gaussian Pyramid
(Wikipedia)



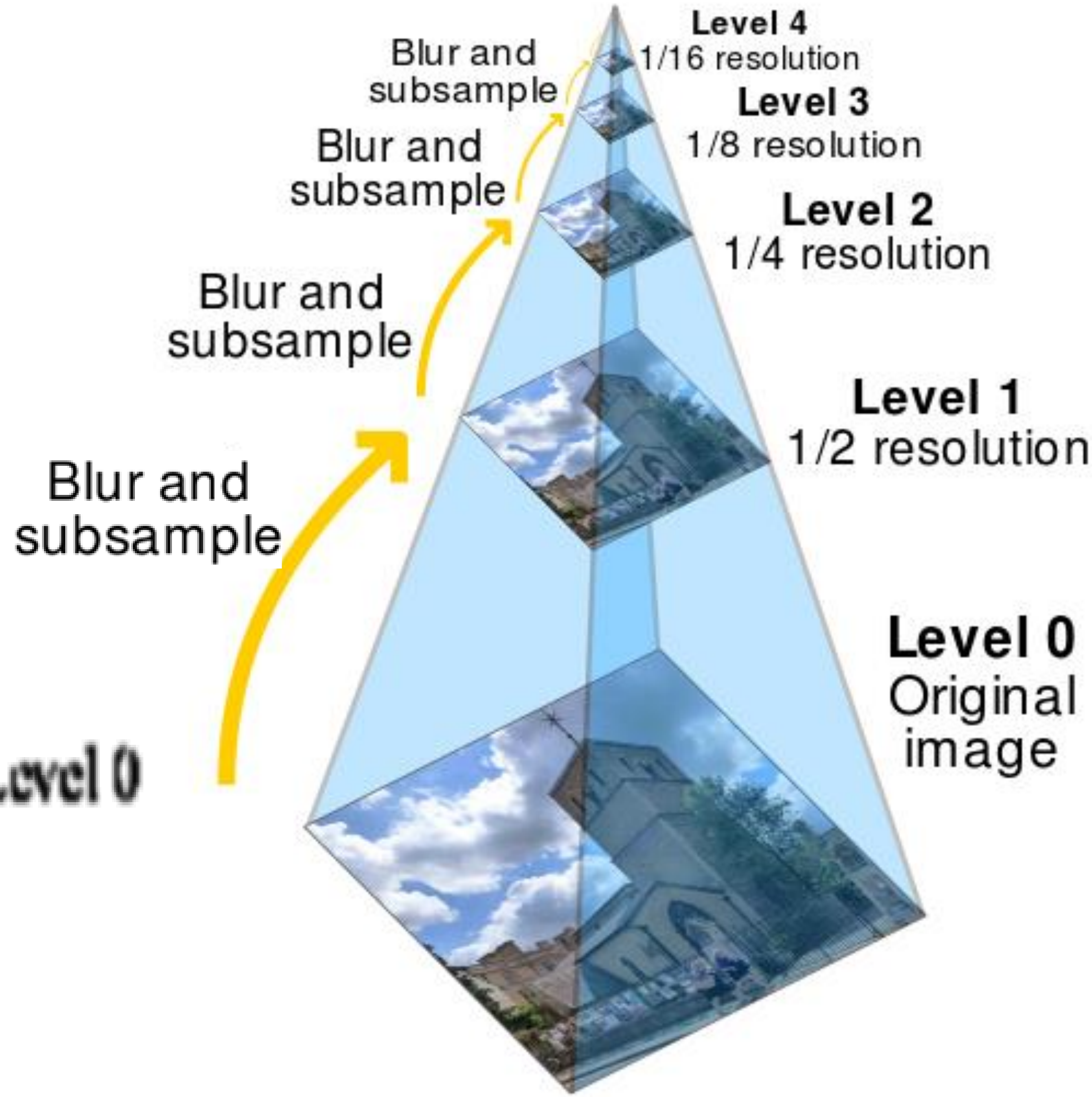
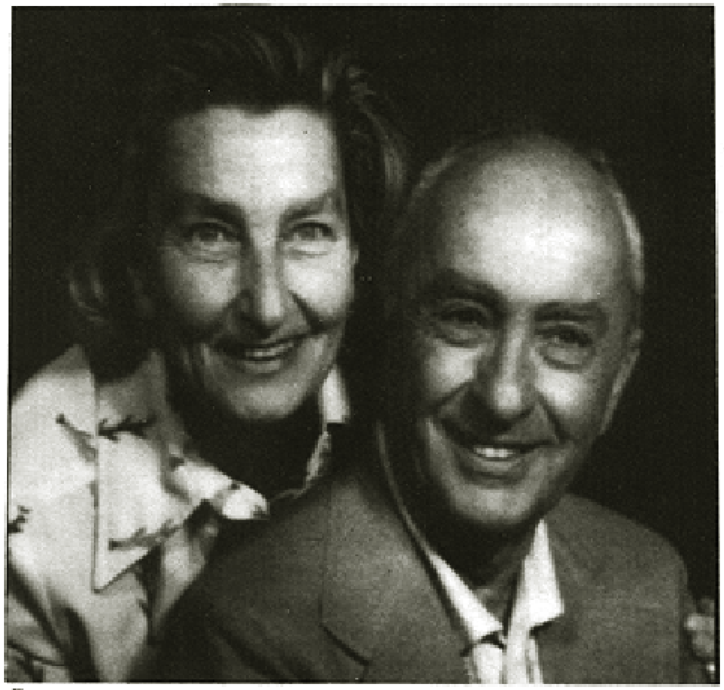
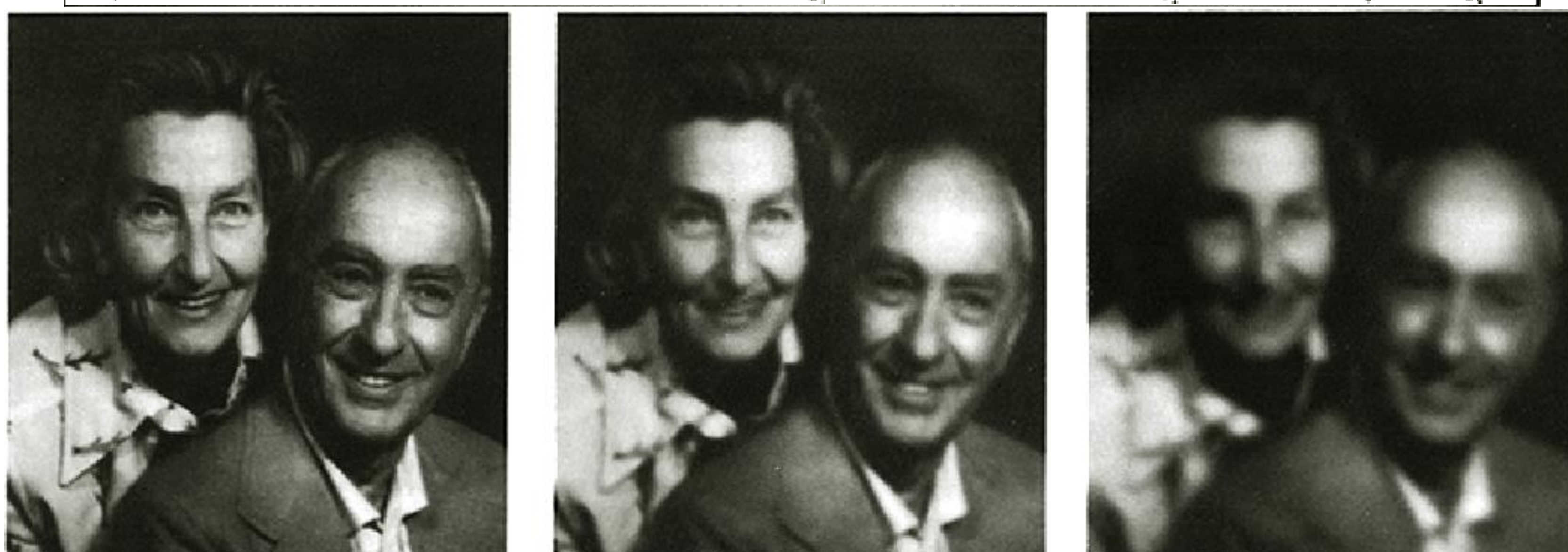


Fig. 2a. *The Gaussian pyramid. The original image, G_0 is repeatedly filtered and subsampled to generate the sequence of reduced resolution image G_1, G_2 etc. These comprise a set of lowpass-filtered copies of the original image in which the bandwidth decreases in one-octave steps.*



G_0

G_1 G_2 G_3 G_4



$G_{0,0}$ $G_{1,1}$ $G_{2,2}$

Laplacian Pyramid

Gaussian	Laplacian	
G_n	L_n	G_n
G_3	L_3	$G_3 - Expand\{Reduce\{G_3\}\}$
G_2	L_2	$G_2 - Expand\{Reduce\{G_2\}\}$
G_1	L_1	$G_1 - Expand\{G_2\}$
G_0	L_0	$G_0 - Expand\{G_1\}$

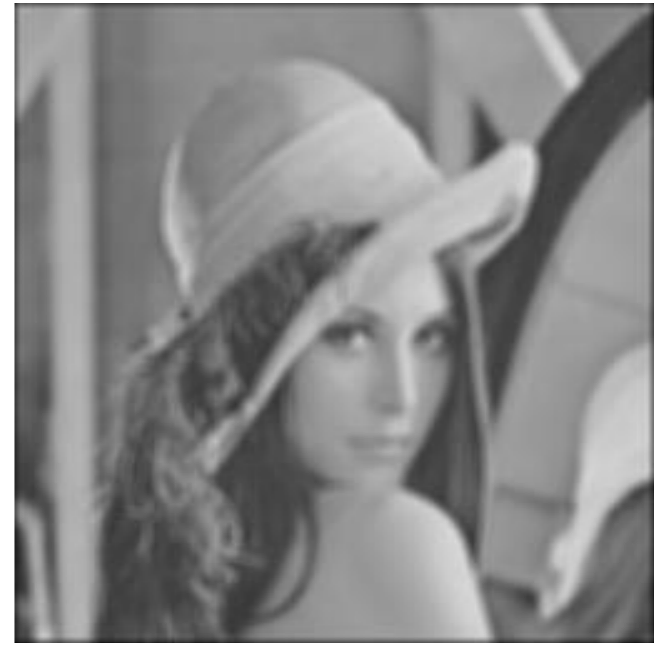
$$L_n + L_{n-1} = Expand\{L_n\} + L_{n-1} =$$

$$= Expand\{G_n\} + (G_{n-1} - Expand\{G_n\}) = G_{n-1}$$

$$\sum_{i=k}^{n}\left(L_i\right) = G_k \qquad \sum_{i=0}^{n}\left(L_i\right) = G_0$$

G_0

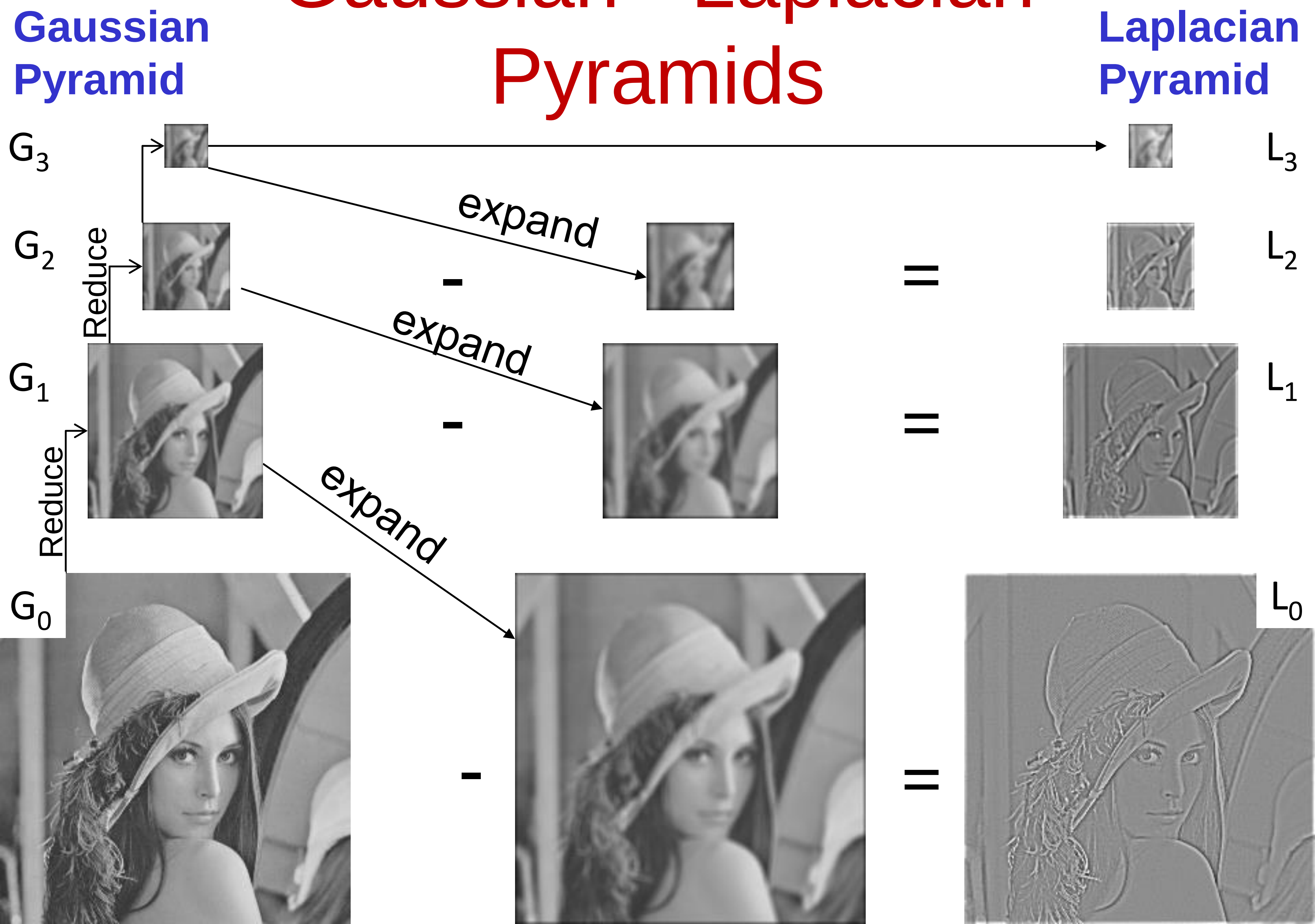
G_1



$Expand\{G_1\}$

$L_0 = G_0 - Expand\{G_1\}$

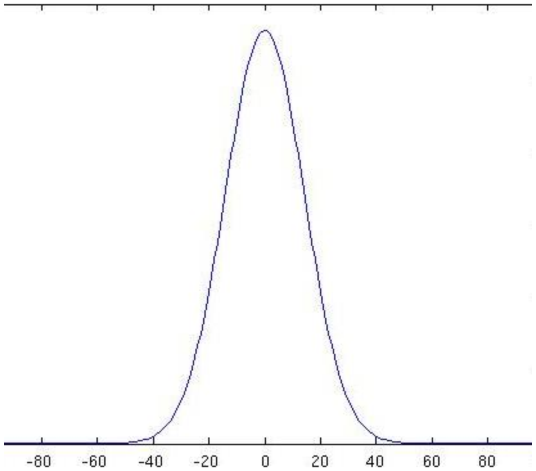
Gaussian - Laplacian Pyramids



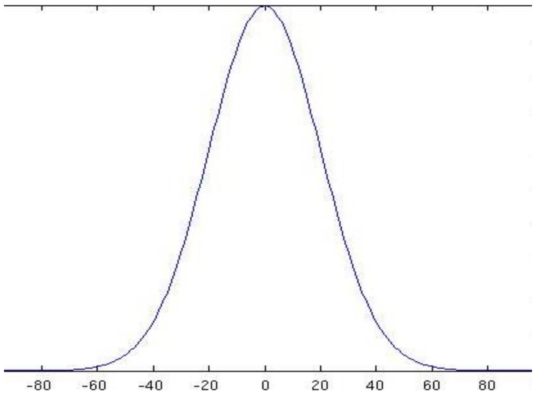
Laplacian Pyramid as a Band-Pass Filter

Gaussian Pyramid –

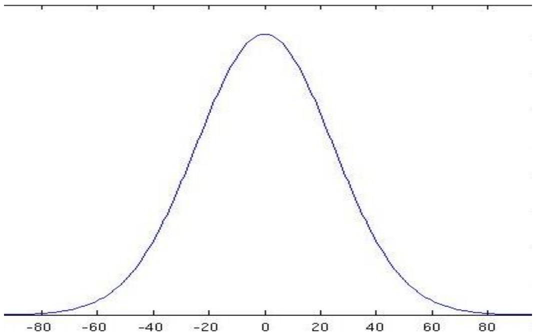
Convolution with a Gaussian filter



Level 1
Narrow



Level 2

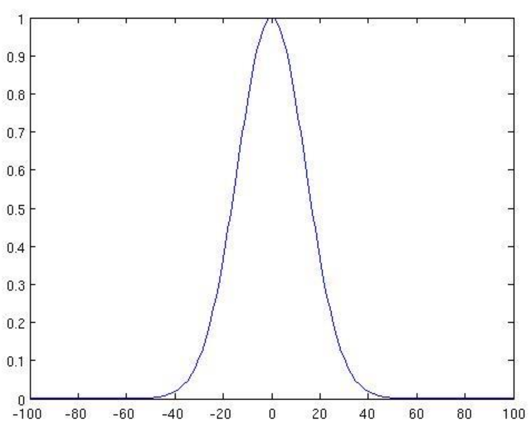


Level 3
Wide

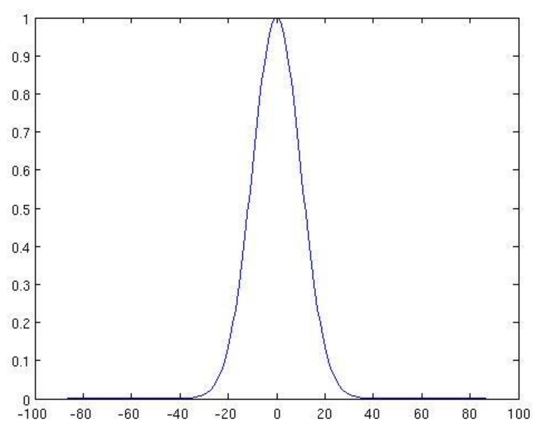
Laplacian Pyramid as a Band-Pass Filter

Gaussian Pyramid (in the Fourier domain) –

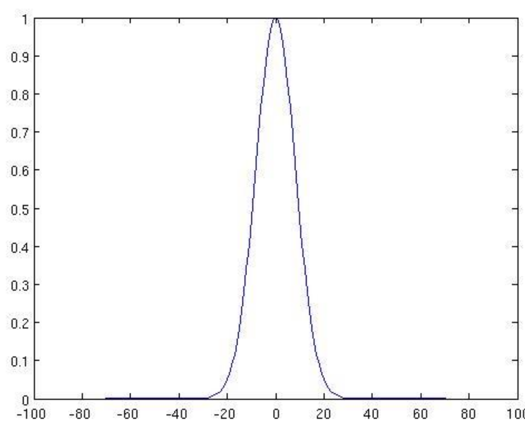
Multiplication with a Gaussian kernel



Level 1
Wide



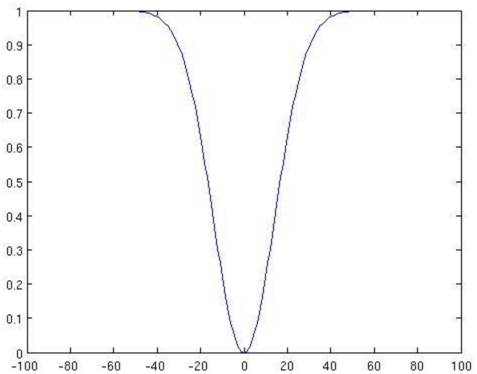
Level 2



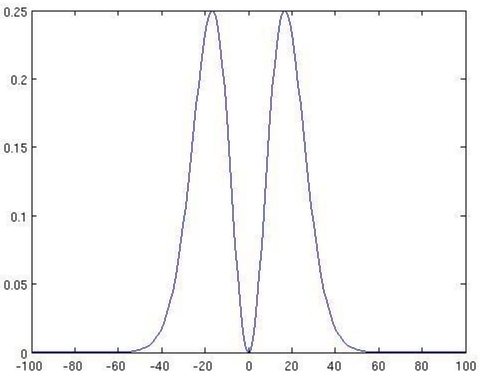
Level 3
Narrow

Laplacian Pyramid as a Band-Pass Filter

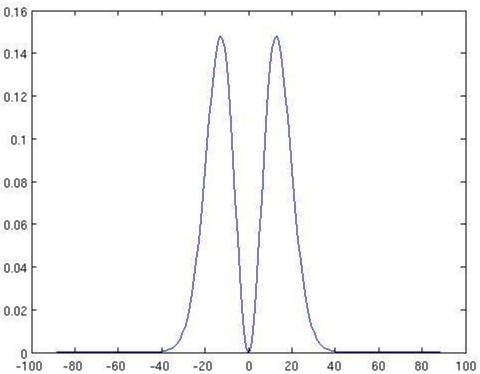
- In the Fourier domain, the Laplacian is the difference between two powers of Gaussian kernels:



Level 1



Level 2



Level 3

Pyramid Compression

- Build a Laplacian Pyramid
- Quantize pyramid values to 3-5 values
  - Optimal Quantization
- Compress using Entropy Compression
  - (Huffman, Lempel-Ziv)
- Reconstruct normally
- Next Generation: Wavelet Compression

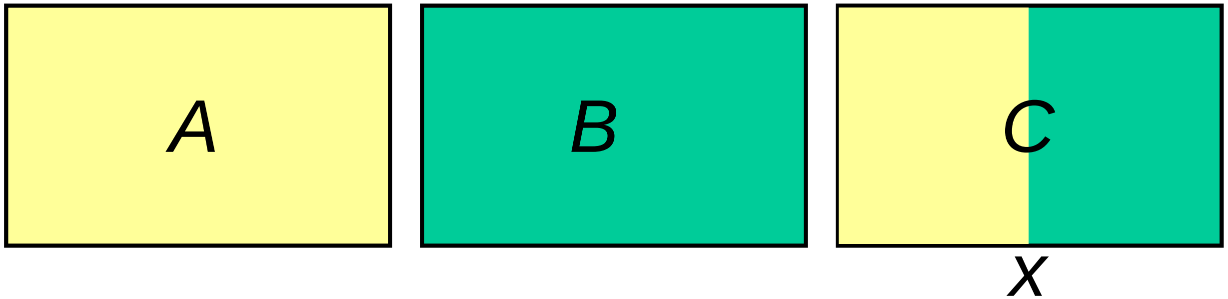
Pyramid Compression

8 bits/pixel | 1 bits/pixel | 0.5 bits/pixel

(a) (b) (c)

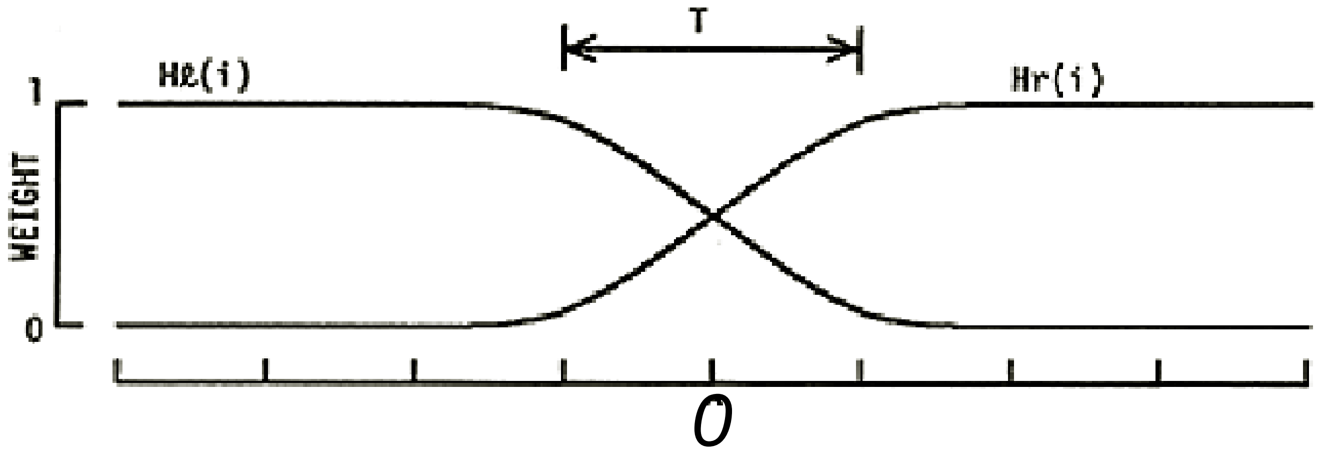
Fig. 5. *Pyramid data compression. The original image represented at 8 bits per-pixel is shown in (a). The node values of tbe Laplacian pyramid representation of this image were quantitized to obtain effective data rates of 1 b/p and 1/2 b/p. Reconstructed images (b) and (c) show relatively little degradation.*

Picture Merging with Spline



For every Row:

$$C(i) = H_l(i-x)A(i) + H_r(i-x)B(i)$$



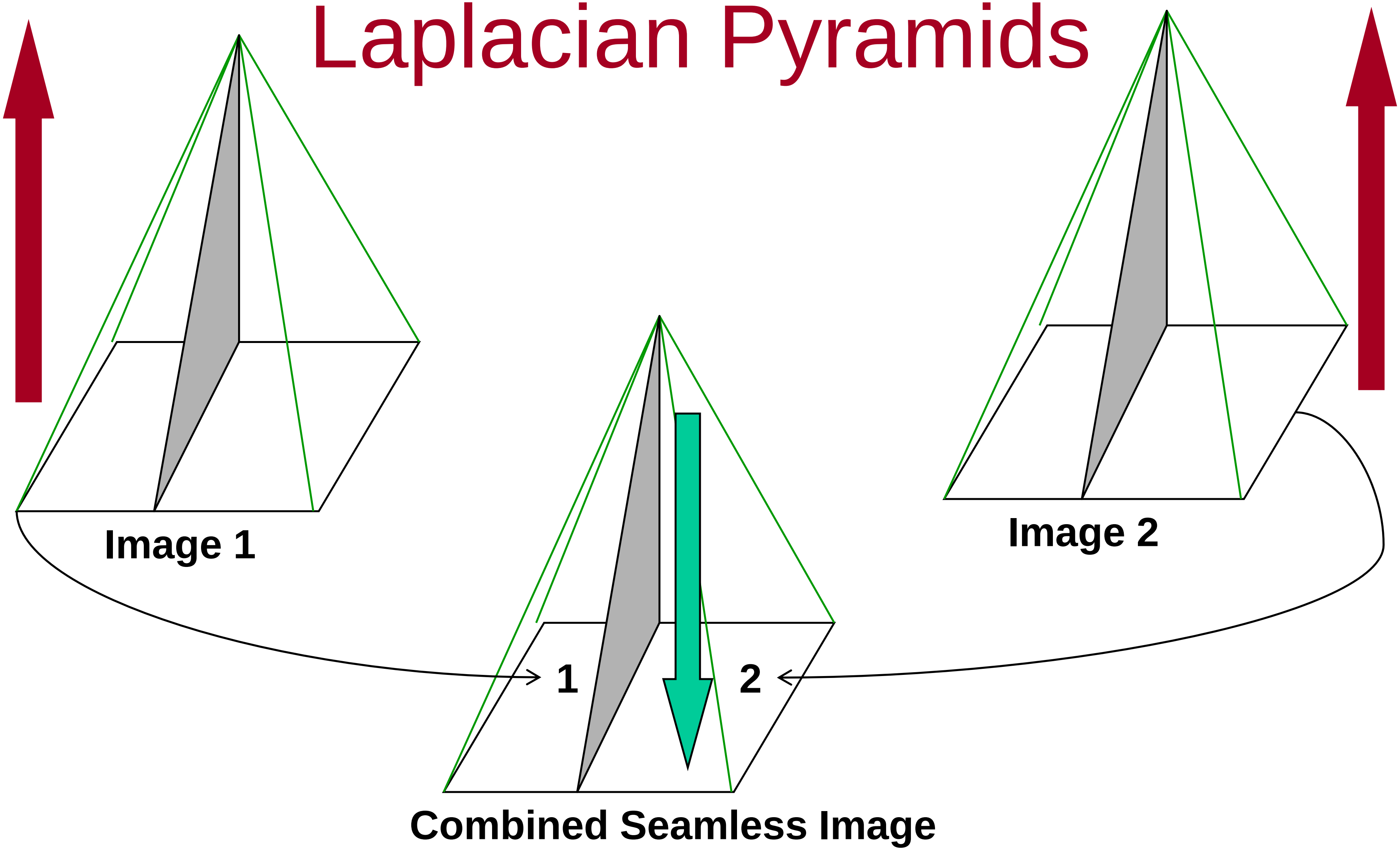
Multiresolution Spline

- Given two images *A* and *B* to be splined in middle
- Construct Laplacian Pyramid L_a and L_b
- Create a third Laplacian Pyramid L_c where for each level *k*

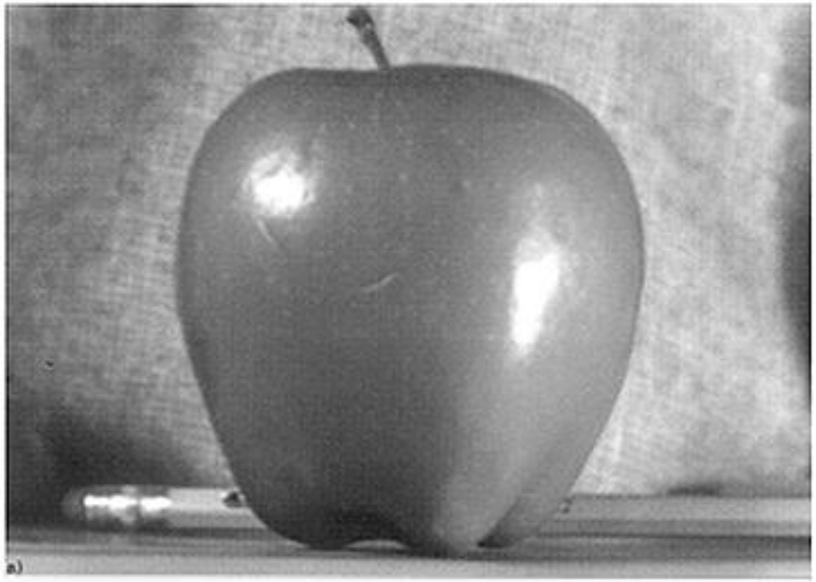
$$L_c(i,j) = \begin{cases} L_a(i,j) & \text{if } i < width/2 \\ (L_a(i,j) + L_b(i,j))/2 & \text{if } i = width/2 \\ L_b(i,j) & \text{if } i > width/2 \end{cases}$$

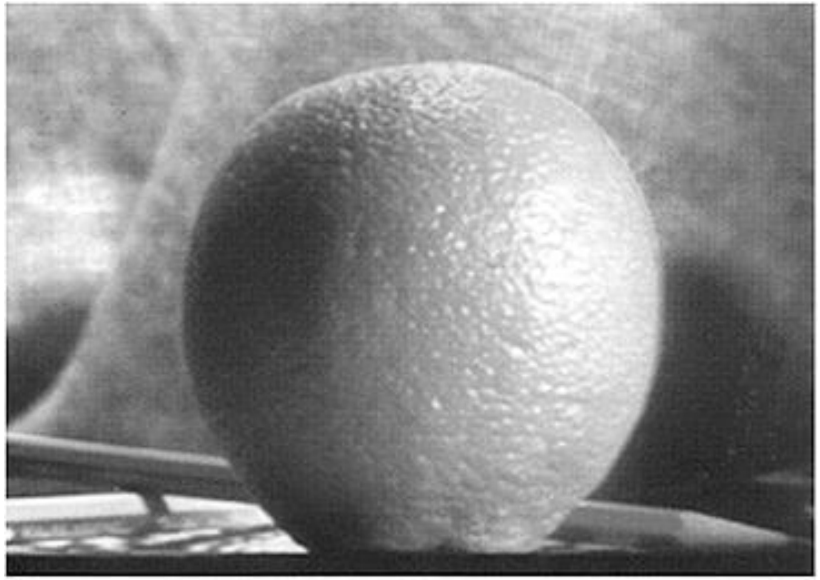
- Sum all levels in L_c to get the blended image

Image Merging with Laplacian Pyramids

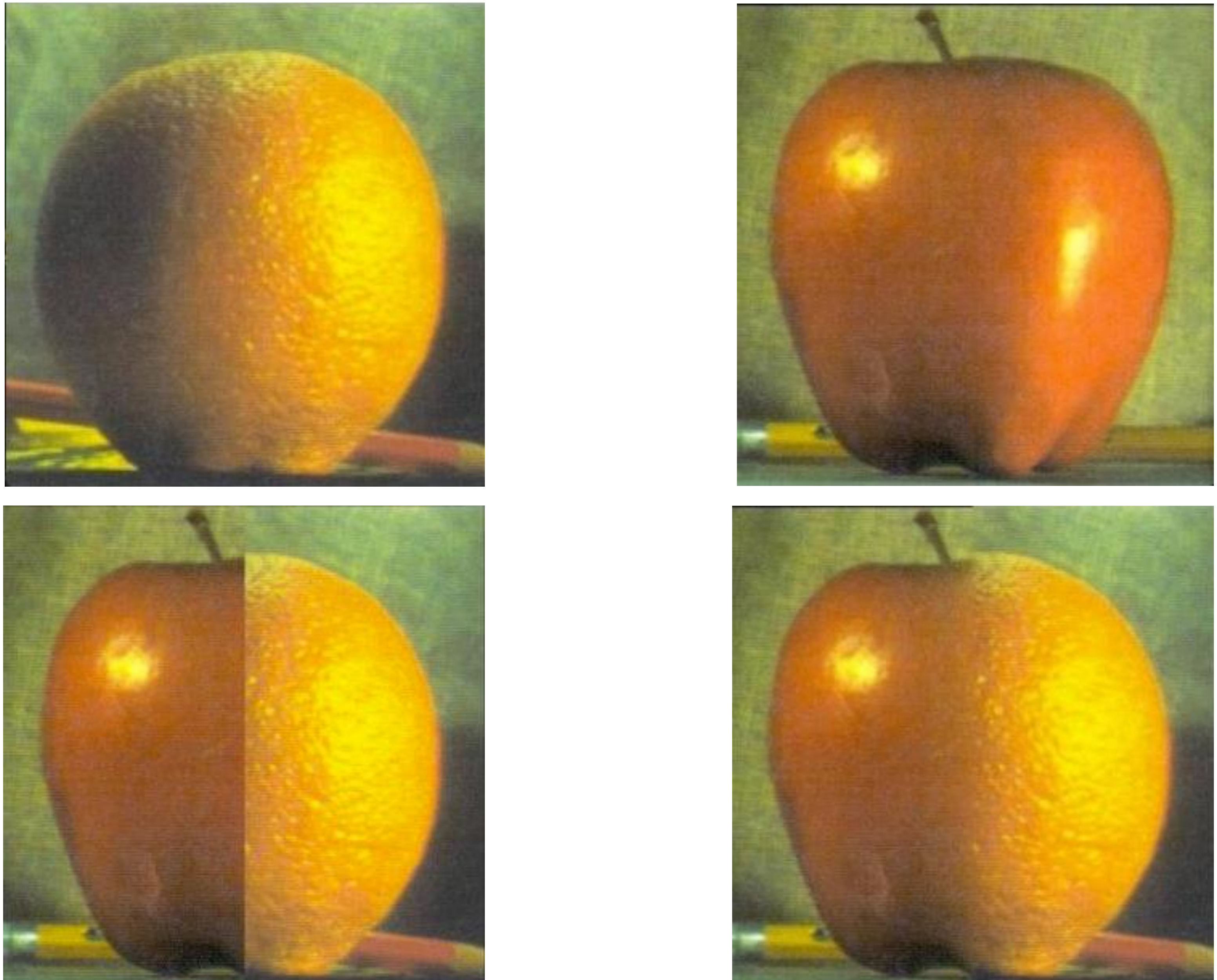


Pyramid Blending Example 1





Pyramid Blending Example 1



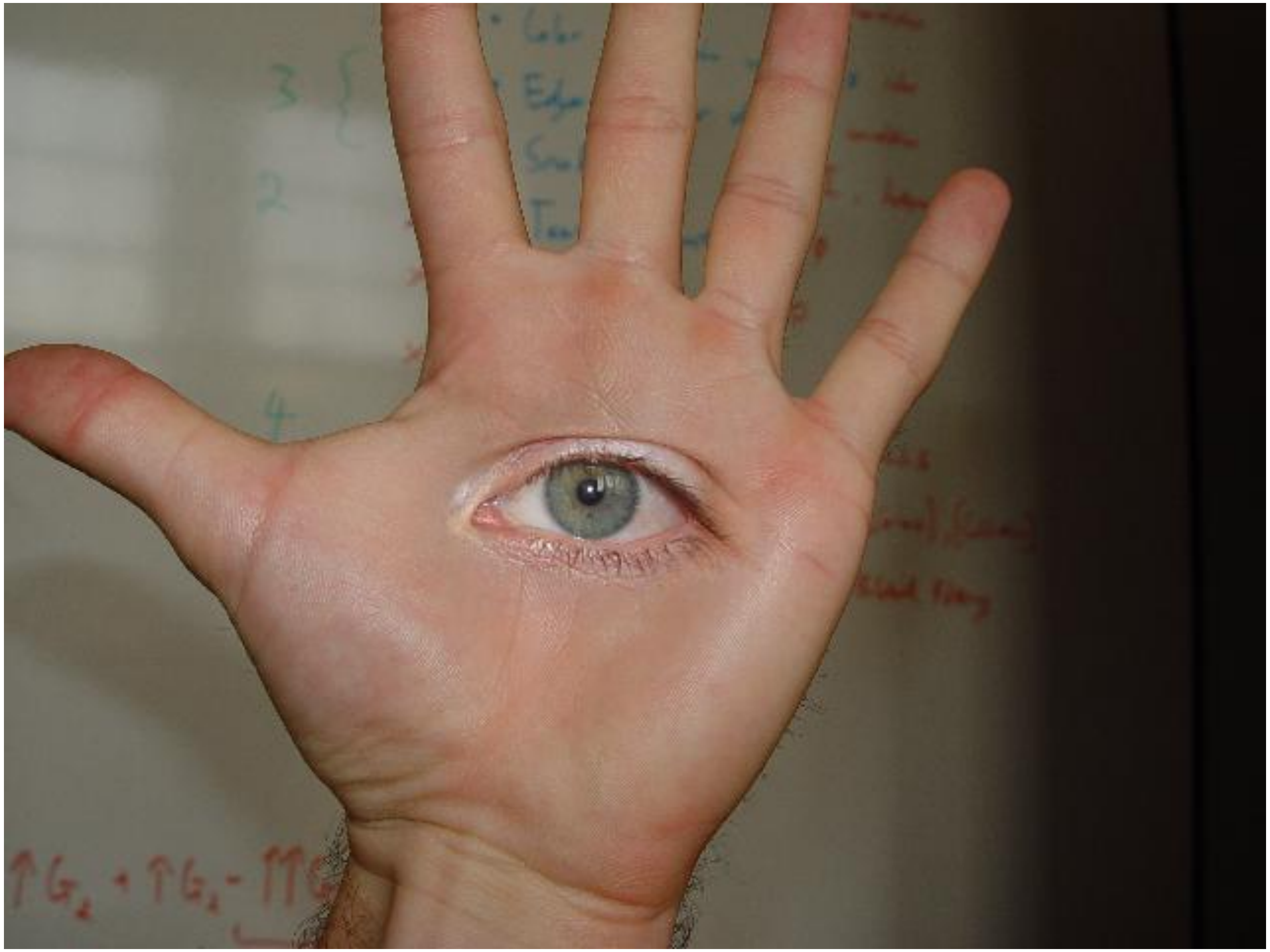
Pyramid Blending Arbitrary Shape

- Given two images *A* and *B*, and a binary mask *M*
- Construct Laplacian Pyramids L_a and L_b
- Construct a Gaussian Pyramid G_m
- Create a third Laplacian Pyramid L_c where for each level *k*

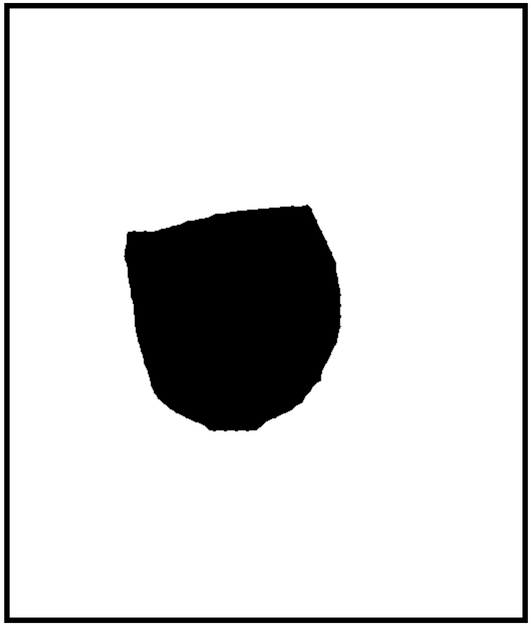
$$L_c(i,j) = G_m(i,j)L_a(i,j) + (1 - G_m(i,j))L_b(i,j)$$

- Sum all levels L_c in to get the blended image

Pyramid Blending Example 2



Pyramid Blending Example



Pyramid Blending Example

Pyramids (not Laplacian) & sea (talu)