MATHEMATICAL TOOLS IN COMPUTER SCIENCE - QUIZ 2

Name:
ID number:

Problem 1. Consider the matrix:

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

(1) What are A's eigenvalues?

(2) Show that if $v_1, v_2 \in \mathbb{R}^3$ are eigenvectors of A corresponding to different eigenvalues, then $\langle v_1, v_2 \rangle = 0$.

(3) Let $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ be a distribution (i.e. $x_1, x_2, x_3 \geq 0$ and $x_1 + x_2 + x_3 = 1$).

Prove that:

$$\lim_{n \to \infty} \left\| \frac{A^n x}{2^n} - \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix} \right\|_2 = 0$$