SRM Institute of Science and Technology
Ramapuram Campus
Department of Mathematics
18MAB101T – Calculus And Linear Algebra

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – I

MATRICES

Part – B

1. Find the characteristic equation of the matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$.

(A) $\lambda^2 - 3\lambda + 2 = 0$ **(B)** $\lambda^2 + 3\lambda + 2 = 0$

(C) $\lambda^2 - 3\lambda - 2 = 0$ **(D)** $\lambda^2 + 3\lambda - 2 = 0$

Solution: Let A = $\begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$. Its characteristic equation is $\lambda^2 - S_1\lambda + S_2 = 0$ where $S_1 = sum\ of\ the\ main\ diagonal\ elements = 1 + 2 = 3$,

$S_2 = Determinant\ of\ A = |A|$ = 1(2) – 2(0) = 2

Therefore, the characteristic equation is $\lambda^2 - 3\lambda + 2 = 0$ **(Option A)**

2. Find the characteristic equation of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

(A) $\lambda^3 - 18\lambda^2 + 45\lambda = 0$ **(B)** $\lambda^3 - 28\lambda^2 + 45\lambda = 0$

(C) $\lambda^3 - 18\lambda^2 + 35\lambda = 0$ **(D)** $\lambda^3 - 18\lambda^2 - 45\lambda = 0$

Solution: Its characteristic equation is $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$, where $S_1 = sum\ of\ the\ main\ diagonal\ elements$ = 8 + 7 + 3 = 18, $S_2 = Sum\ of\ the\ minors\ of\ the\ main\ diagonal\ elements = \begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix} = 5 + 20 + 20 = 45$, $S_3 = Determinant\ of\ A = |A|$ = 8(5)+6(-10)+2(10) = 40 – 60 + 20 = 0

Therefore, the characteristic equation is $\lambda^3 - 18\lambda^2 + 45\lambda = 0$ **(Option A)**

3. Find the eigen values of the matrix $A = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$.

(A) $2, -2$ **(B) $1, -1$**

(C) $3, -3$ **(D) $2, 2$**

Solution: Let A = $\begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$ which is a non-symmetric matrix.

The characteristic equation of A is $\lambda^2 - S_1\lambda + S_2 = 0$ where $S_1 =$ $sum\ of\ the\ main\ diagonal\ elements = 1 - 1 = 0$,

$S_2 = Determinant\ of\ A = |A|$ = 1(-1) – 1(3) = - 4

Therefore, the characteristic equation is $\lambda^2 - 4 = 0$ i.e., $\lambda^2 = 4$ or $\lambda = \pm 2$

Therefore, the eigen values are 2, -2. **(Option A)**

4. Find the sum and product of the eigen values of the matrix $A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$.

(A) $-3, 4$ **(B) $-3, -4$**

(C) $3, 4$ **(D) $-3, -4$**

Solution: Sum of the eigen values = Sum of the main diagonal elements = -3

Product of the eigen values = $|A|$ = -1 (1 – 1) -1(-1 – 1) + 1(1- (-1)) = 2 + 2 = 4

(Option A)

5. Find the sum and product of eigen values of the matrix A^T where $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

(A) $18, 0$ **(B) $18, 2$**

(C) $28, 0$ **(D) $18, -2$**

Solution: Since matrix A is symmetric , A and A^T have same eigen values.

Sum of Eigen value of A^T = trace(A) = 8+7+3=18

Product of Eigen value of $A^T = |A|$ = 8(5) + 6(−10)+2(10) = 40 −60 + 20 = 0 **(Option A)**

6. If 1, 1, 5 are the eigen values of A = $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$, find the eigen values of 5A.

(A) $5, 5, 2$ **(B) $5, 5, 25$** **(C) $2, 3, 2$** **(D) $7, 8, 7$**

Solution: By the property "If $\lambda_1, \lambda_2, \lambda_3$ are the eigen values of A, then $k\lambda_1, k\lambda_2, k\lambda_3$ are the eigen values of kA, the eigen values of 5A are 5(1), 5(1), 5(5) ie., 5,5,25**.** **(Option B)**

7. Find the eigen values of the matrix $2A^{-1}$ where A= $\begin{pmatrix} 3 & 8 & 9 \\ 0 & 1 & 3 \\ 0 & 0 & -2 \end{pmatrix}$.

(A) $\frac{2}{3}, 2, -1$ **(B) $\frac{1}{3}, 2, -4$** **(C) $\frac{2}{3}, 2, 1$** **(D) $\frac{2}{3}, 1, -2$**

Solution: Since given matrix is triangular matrix, the Eigen values are its diagonal elements.

$$\therefore \quad \lambda_1 = 3,\ \lambda_2 = 1,\ \lambda_3 = -2$$

Eigen values of $2A^{-1}$ are $\frac{2}{3}$, 2 , -1**(Option A)**

8. If the sum of two eigen values and trace of a 3 × 3 matrix A are equal, find the value of $|A|$.

(A) 5 **(B) 25** **(C) 2** **(D) 0**

Solution: Sum of the eigen values = $\lambda_1 + \lambda_2 + \lambda_3$ = sum of the diagonal elements

Given $\lambda_1 + \lambda_2$ = trace of A.

i.e., $\lambda_1 + \lambda_2 = \lambda_1 + \lambda_2 + \lambda_3$

Therefore $\lambda_3 = 0$. Then $|A|$ = Product of Eigen values = $\lambda_1 \lambda_2 \lambda_3 = 0$ **(Option D)**

9. Write the matrix corresponding to the quadratic form $x^2 + 2yz$.

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}$ **(C)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ **(D)** $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$

Solution: Given $\boldsymbol{X^T AX = x^2 + 2yz}$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \textbf{ (Option C)}$$

10. If 2, 3, –1 are the eigen values of 3 × 3 matrix, find rank, index and signature of the quadratic form.

(A) $\mathbf{5, 5, 2}$ **(B)** $\mathbf{5, 5, 25}$ **(C)** $\mathbf{3, 2, 1}$ **(D)** $\mathbf{1, 2, 3}$

Solution: Rank (r) = number of non zero terms in canonical form = 3

Index (p) = Number of positive terms in canonical form = 2

Signature (s) = Difference between number of positive terms and negative terms

= 2p – r

= 4 – 3

= 1 **(Option C)**

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