Reg. No.															

B.Tech. / M.Tech (Integrated) DEGREE EXAMINATION, JANUARY 2023

First Semester

21MAB101T – CALCULUS AND LINEAR ALGEBRA

(For the candidates admitted from the academic year 2022-2023)

Note:

(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40^{th} minute.

(ii) **Part – B** and **Part – C** should be answered in answer booklet.

Time: 3 Hours

Max. Marks: 75

PART – A (20 × 1 = 20Marks)

Answer **ALL** Questions

	Question	Marks	BL	CO	PO
1.	The number of positive terms in the canonical form is called (A) Signature (B) Index (C) Quadratic form (D) Positive definite	1	1	1	1
2.	Find the eigen values of A^2 if $A=\begin{pmatrix}1&2&4\\0&2&6\\0&0&5\end{pmatrix}$ (A) 1, 4, 25 (B) 2, 4, 20 (C) 4, 4, 25 (D) 1, 2, 25	1	1	1	1
3.	If A is an orthogonal matrix then $\|A\|$ is (A) 0 (B) ± 1 (C) 1 (D) -1	1	1	1	2
4.	Find the sum and product of the eigen values of the matrix $A=\begin{pmatrix}1&2&-1\\-2&0&0\\4&5&0\end{pmatrix}$ (A) $-1, -10$ (B) $1, -10$ (C) $-1, 10$ (D) $1, 10$	1	2	1	2
5.	If $z=2x^2+3y^2+5xy$, then find $\frac{\partial z}{\partial x}$ (A) $4x+5y$ (B) $4x-5y$ (C) $-4x+6y$ (D) $4x+6y+5xy$	1	2	2	2
6.	If f(x, y) is an implicit function, then $\frac{dy}{dx}$ is (A) $-\frac{(\partial f/\partial x)}{\partial f/\partial y}$ (B) $\frac{(\partial f/\partial x)}{\partial f/\partial y}$ (C) $\frac{(\partial f/\partial y)}{\partial f/\partial x}$ (D) $-\frac{(\partial f/\partial y)}{\partial f/\partial x}$	1	1	2	2

7. If $f(x,y)=x^2y+\sin y+e^x$, then find $f_x(1,\pi)$ 1 2 2 2

(A) 2π (B) $2\pi-e$
(C) $2\pi+e$ (D) 0

8. If u and v are functionally dependent, then their Jacobian value is 1 1 2 1

(A) Zero (B) Positive
(C) One (D) Negative

9. Solution of $(D^2+4)y=0$ 1 2 3 1

(A) $y=(Ax+B)e^{2x}$ (B) $y=A\cos\sqrt{2}x+B\sin\sqrt{2}x$
(C) $y=A\cos 2x+B\sin 2x$ (D) $y=Ae^{2x}+Be^{-2x}$

10. The value of $\frac{e^{ax}}{D-a}$ 1 1 3 1

(A) xe^{ax} (B) e^{ax}
(C) x^2e^{ax} (D) $\frac{x^2}{2}e^{ax}$

11. Solve $(D^2+5D+4)y=0$ 1 2 3 2

(A) $y=Ae^{x}+Be^{-4x}$ (B) $y=Ae^{x}+Be^{4x}$
(C) $y=Ae^{-x}+Be^{-4x}$ (D) $y=Ae^{-x}+Be^{4x}$

12. Find the Particular Integral of $(D^2-4)y=\cos 2x$ 1 2 3 2

(A) $\frac{x}{8}\cos 2x$ (B) $\frac{1}{8}\sin 2x$
(C) $\frac{1}{8}\cos 2x$ (D) $-\frac{1}{8}\cos 2x$

13. The curvature of the straight line is 1 1 4 1

(A) 1 (B) 2
(C) 0 (D) -1

14. The envelope of the family of curves of the form $A\alpha^2+B\alpha+C=0$, where α is the parameter is 1 1 4 1

(A) $B^2+AC=0$ (B) $B^2-4AC=0$
(C) $B^2+4AC=0$ (D) $B^2-AC=0$

15. The radius of curvature in polar co-ordinates is 1 1 4 1

(A) $\rho=[r^2+r'^2]^{3/2}/r^2-rr''+2r'^2$

(B) $\rho=[r^2-r'^2]^{3/2}/r^2-rr''+2r'^2$

(C) $\rho=[r^2-(r'')^2]^{3/2}/r^2-rr''+2r'^2$

(D) $\rho=[r^2+r'^2]^{3/2}/r^2+rr''+2r'^2$

16.	The equation of circle of curvature at any point (x, y) with centre of curvature $\overline{x}, \overline{y}$ and with radius of curvature ρ is (A) $(x-\overline{x})^2-(y+\overline{y})^2=\rho^2$ (B) $(x-\overline{x})^2+(y-\overline{y})^2=\rho^2$ (C) $(x+\overline{x})^2+(y-\overline{y})^2=\rho^2$ (D) $(x-\overline{x})^2+(y+\overline{y})^2=\rho^2$	1	1	4	1
17.	The series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent if (A) p = 1 (B) p > 1 (C) p = 0 (D) p < 1	1	1	5	1
18.	The convergence of $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^p}$ is tested by ________ (A) Ratio test (B) Root test (C) Leibnitz test (D) Raabe's test	1	2	5	2
19.	The value of $\underset{n\to\infty}{lt}\ (n)^{1/n}$ is equal to (A) 0 (B) 1 (C) 2 (D) $\frac{1}{2}$	1	2	5	2
20.	The series $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+......\infty$ is (A) Convergent (B) Absolutely convergent (C) Conditionally convergent (D) Divergent	1	2	5	2

PART – B (5 × 8 = 40 Marks)

Answer **ALL** Questions

		Marks	BL	CO	PO
21. a.	Find the Eigen values and Eigen vectors of the matrix $\begin{pmatrix}10 & -2 & -5\\ -2 & 2 & 3\\ -5 & 3 & 5\end{pmatrix}$.	8	3	1	1
	(OR)				
b.	Verify Cayley Hamilton theorem for the matrix $\begin{pmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{pmatrix}$.	8	4	1	2
22. a.	Expand $e^x \cos y$ at $\left(1,\frac{\pi}{4}\right)$ as a Taylor series upto second degree terms.	8	4	2	1
	(OR)				

	b. If $u = u(x,y)$ and $x = e^r \cos\theta$ and $y = e^r \sin\theta$ show that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = e^{-2r}\left[\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{\partial u}{\partial \theta}\right)^2\right]$.	8	3	2	1
23.	a. Solve $\left(D^2 + 2D + 1\right)y = e^{3x} + \sin 2x$.	8	3	3	1
	(OR)				
	b. Solve $\left(D^2 + 1\right)y = \tan x$ by the method of variation of parameters.	8	4	3	2
24.	a. Find the radius of curvature of the curve $r = a(1 + \cos\theta)$ at the point $\theta = \frac{\pi}{2}$.	8	3	4	2
	(OR)				
	b. Find the evolute of the parabola $y^2 = 4ax$.	8	4	4	2
25.	a. Test the convergence of the series $\sum_{n=1}^{\infty}\left(\sqrt{n^4+1} - \sqrt{n^4-1}\right)$.	8	4	5	2
	(OR)				
	b. Test the convergence of the series, $\frac{2}{3.4} + \frac{2.4}{3.5.6} + \frac{2.4.6}{3.5.7.8} + \frac{2.4.6.8}{3.5.7.9.10} + ...\infty$.	8	4	5	2

PART – C (1 × 15 = 15 Marks)
Answer **ANY ONE** Question

		Marks	BL	CO	PO
26.	Reduce the quadratic form $3x^2 - 2y^2 - z^2 - 4xy + 8xz + 12yz$ to canonical form by an orthogonal transformation. Discuss the nature of the quadratic form and also find rank, index, and signature.	15	4	1	1
27.	Find the dimensions of the rectangular box open at the top, of maximum capacity whose surface area is 432 sq.cm.	15	4	2	2

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