1. The number of positive terms in the canonical form is called

   A. Signature of the quadratic form
   B. Index of the quadratic form
   C. Quadratic form
   D. Positive form

   ANSWER: B

2. The matrix of the quadratic form $3x_1^2 + 3x_2^2 - 5x_3^2 - 2x_1x_2 - 6x_2x_3 - 6x_3x_1$ is

   A. $\begin{pmatrix} 3 & -1 & -3 \\ -3 & -3 & 5 \\ -1 & 3 & -3 \end{pmatrix}$

   $B.$ $\begin{pmatrix} -3 & -3 & -5 \\ 3 & -1 & -3 \\ -1 & 3 & -3 \end{pmatrix}$

   $C.$ $\begin{pmatrix} 3 & -1 & -3 \\ -1 & 3 & -3 \\ -3 & -3 & -5 \end{pmatrix}$

   $D.$ $\begin{pmatrix} 3 & 3 & -5 \\ -1 & -1 & -3 \\ -3 & -3 & -3 \end{pmatrix}$

   ANSWER: C

3. The eigen values of A^2, if $A = \begin{pmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 2 \end{pmatrix}$ are

A. 1, 4, 9
B. 1, 9, 16
C. 2, 4, 6
D. 4, 9, 16

ANSWER: A

4. If two of the eigen values of a 3×3 matrix, whose determinant equals 4 are -1 and 2, then the third eigen value is

A. -8
B. -6
C. -4
D. -2

ANSWER: D

5. The nature of quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$ is

A. Positive Semidefinite
B. Indefinite
C. Positive definite
D. Negative definite

ANSWER: C

6. If 2 and 3 are the eigen values of $A = \begin{pmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{pmatrix}$, then the eigen values of A^{-1} are

A. $\frac{1}{2}, \frac{1}{3}, \frac{1}{3}$
B. $\frac{1}{2}, \frac{1}{2}, \frac{1}{4}$
C. $\frac{1}{2}, \frac{1}{2}, \frac{1}{3}$
D. $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$

ANSWER: C

7. The characteristic equation of the matrix $A = \begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix}$ is

A. $\lambda^2 + 5\lambda + 6 = 0$
B. $\lambda^2 - 5\lambda - 5 = 0$
C. $\lambda^2 - 5\lambda - 6 = 0$
D. $\lambda^2 - 6\lambda + 5 = 0$

ANSWER: C

8. The sum and the product of the eigen values of $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ are

A. 3 and 2
B. 2 and 1
C. 5 and 2
D. 5 and 3

ANSWER: D

9. Every square matrix satisfies its own

A. bilinear form
B. inverse of the equation
C. characteristic equation
D. quadratic equation

ANSWER: C

10. Let X_1 and X_2 be two column matrices, then X_1 and X_2 are orthogonal if

A. $X_1 + X_2 = 0$
B. $X_2 = 0$
C. $X_1 = 0$
D. $X_1^T X_2 = 0$

ANSWER: D

11. If $u = (x-y)^4 + (y-z)^4 + (z-x)^4$, then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} =$

A. $4(y-z)^3 - 4(z-x)^3$
B. $3(y-z)^4 - 3(z-x)^4$
C. $4(x-y)^3 - 4(x-z)^3$
D. $3(y-z)^4 - 3(z-x)^4$

ANSWER: A

12. If $u = x^3y^4$ where $x = t^3$ and $y = t^2$ then $\frac{du}{dt} =$

A. 17t

B. $17t^{16}$

C. $16t^{17}$

D. 16t

ANSWER: B

13. If $f(x, y) = 0$ is an implicit function then $\frac{dy}{dx} =$

A. $\frac{-\partial f/\partial x}{\partial f/\partial y}$

B. $\frac{-\partial f/\partial y}{\partial f/\partial x}$

C. $\frac{\partial f/\partial x}{\partial f/\partial y}$

D. $\frac{\partial f/\partial y}{\partial f/\partial x}$

ANSWER: A

14. If $f(x, y) = \tan^{-1}(\frac{y}{x})$, then $f_y(1, 1)$ is

A. $\frac{-1}{2}$

B. $\frac{1}{2}$

C. -1

D. 1

ANSWER: B

15. If $rt - s^2 < 0$ at (a, b) where $r = \frac{\partial^2 f}{\partial x^2}$, $s = \frac{\partial^2 f}{\partial x \partial y}$ and $t = \frac{\partial^2 f}{\partial y^2}$ then the point is a

A. maximum point
B. minimum point
C. saddle point
D. doubtful point

ANSWER: C

16. If $x = r\cos\theta$ and $y = r\sin\theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)} =$

A. r
B. r^2
C. 0
D. $2r$

ANSWER: A

17. The stationary points of $f(x, y) = x^3 + y^3 - 3axy$ are

A. (0, a) and (0, a)
B. (0, 0) and (a, a)
C. (a, 0) and (a, 0)
D. (0, a) and (a, 0)

ANSWER: B

18. If $r^2 = x^2 + y^2$ then $\frac{\partial r}{\partial x} =$

A. $\frac{y^2}{r}$
B. $\frac{y}{r}$
C. $\frac{x^2}{r}$

D. $\frac{x}{r}$

ANSWER: D

19. If $f(x, y) = x^2 y + \sin y + e^x$ then $f_{xy}(1, \pi) =$

A. $\pi + e$
B. $\pi - e$
C. 2
D. -2

ANSWER: C

20. If u, v and w are functionally dependent functions of three independent variables x, y and z then $\frac{\partial(u,v,w)}{\partial(x,y,z)} =$

A. 1
B. -1
C. neither 1 nor -1
D. 0

ANSWER: D

21. The particular integral of $(D^2 + 9)y = e^{-2x}$ is

A. $\frac{e^{-2x}}{13}$
B. $\frac{e^{-2x}}{14}$
C. $\frac{e^{2x}}{13}$
D. $\frac{e^{2x}}{14}$

ANSWER: A

22. The complete solution of $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$ is

A. $y = c_1e^{-3x} + c_2e^{-4x}$
B. $y = c_1e^{3x} + c_2e^{4x}$
C. $y = ce^{3x}$
D. $y = ce^{4x}$

ANSWER: B

23. An equation of the form $x^n\frac{d^ny}{dx^n} + a_1x^{n-1}\frac{d^{n-1}y}{dx^{n-1}} + \ldots + a_ny = F(x)$ can be transformed to a linear differential equation with constant coefficients by the transformation

A. $x = e^{2z}$
B. $ax + b = e^z$
C. $x = e^z$
D. $ax^2 + b = e^z$

ANSWER: C

24. The equation of the form $(ax + b)^n\frac{d^ny}{dx^n} + p_1(ax + b)^{n-1}\frac{d^{n-1}y}{dx^{n-1}} + \ldots + p_ny = F(x)$ is known as

A. Maclaurin's saeries
B. Taylor's series
C. Cauchy homogeneous linear equation
D. Legendre linear differential equation

ANSWER: D

25. If $3i$ and $-3i$ are the roots of the differential equation $(D^2+9)y=0$ then the complementary function is given by

A. $c_1e^{3x}+c_2e^{-3x}$
B. $c\sin 2x$
C. $c_1\cos 3x+c_2\sin 3x$
D. $c\cos 2x$

ANSWER: C

26. If $f_1=\cos x$ and $f_2=\sin x$, then the value of $f_1f_2^{'}-f_2f_1^{'}$ is

A. 1
B. -1
C. 2
D. -2

ANSWER: A

27. The solution of $(D^2+2D+1)y=8$ is

A. $y=(Ax+B)e^{-x}-8$
B. $y=Ae^{x}+8$
C. $y=Ae^{-x}+8$
D. $y=(Ax+B)e^{-x}+8$

ANSWER: D

28. The expansion of $(1 - \frac{D}{2})^{-1}$ is

A. $(1 + \frac{D}{2!} + \frac{D^2}{3!} + \frac{D^3}{4!} + \ldots)$
B. $(1 + \frac{D}{2} + \frac{D^2}{4} + \frac{D^3}{8} + \ldots)$
C. $(1 - \frac{D}{2} - \frac{D^2}{4} - \frac{D^3}{8} - \ldots)$
D. $(1 + \frac{D}{2} + \frac{D^2}{3} + \frac{D^3}{4} + \ldots)$

ANSWER: B

29. The auxillary equation for $(x^2D^2 + 4xD + 2)y = x\log x$ is

A. $m^2 + 2m + 3 = 0$
B. $m^2 + 3m - 2 = 0$
C. $m^2 + 3m + 2 = 0$
D. $m^2 - 3m + 2 = 0$

ANSWER: C

30. Using the transformation $z = \log x$, the differential equation $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12logx}{x^2}$ is transformed to a linear differential equation with constant coefficients as

A. $(D'^2 - 1)y = 12z$
B. $(D'^2 + D' + 1)y = 12z$
C. $(D'^2 + 1)y = 12z$
D. $D'^2y = 12z$

ANSWER: D

31. The curvature of a straight line is

A. 0

B. 1

C. -1

D. 2

ANSWER: A

32. The equation of the envelope of the family of curves $A\alpha^2 + B\alpha + C = 0$, where α being a parameter is

A. $\mathrm{B}^2 + 4AC = 0$

$B.$ $B^2 - AC = 0$

$C.$ $B^2 + AC = 0$

$D.$ $B^2 - 4AC = 0$

ANSWER: D

33. The radius of curvature in polar coordinates is

A. $\frac{(r^2 - r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2}$

B. $\frac{(r^2 + 2r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2}$

C. $\frac{(r^2 - 2r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2}$

D. $\frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2}$

ANSWER: D

34. The equation of circle of curvature at any point (x, y) with center of curvature $(\bar{x}, \bar{y})$ and radius of curvature ρ is

A. $(x + \bar{x})^2 + (y + \bar{y})^2 = \rho^2$
B. $(x - \bar{x})^2 + (y - \bar{y})^2 = \rho^2$
C. $(x - \bar{x})^2 + (y + \bar{y})^2 = \rho^2$
D. $(x + \bar{x})^2 + (y - \bar{y})^2 = \rho^2$

ANSWER: B

35. Evolute of a curve is the envelope of the ______ to that curve

A. tangent
B. locus
C. parallel
D. normals

ANSWER: D

36. The curvature of a circle is the reciprocal of its

A. radius
B. diameter
C. locus
D. tangent

ANSWER: A

37. The locus of centre of curvature is called

A. involute
B. evolute
C. envelope
D. space

ANSWER: B

38. The radius of curvature for $y = e^x$ at $x = 0$ is

A. $2\sqrt{2}$
B. 2
C. $\sqrt{2}$
D. $\frac{1}{\sqrt{2}}$

ANSWER: A

39. The value of $\Gamma\frac{1}{2}$ is

A. $-\pi$
B. 2π
C. π
D. $\sqrt{\pi}$

ANSWER: D

40. The relation between Beta function and Gamma function is

A. $\beta(m,n) = \frac{\Gamma m}{\Gamma m+n}$

B. $\beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$

C. $\beta(m,n) = \frac{\Gamma n}{\Gamma(m+n)}$

D. $\beta(m,n) = \Gamma m \Gamma n$

ANSWER: B

41. The sequence $a_n = 2^n$ is

A. convergent
B. divergent
C. oscillating
D. bounded

ANSWER: B

42. In the positive term series $\Sigma\, u_n$, if $\lim\limits_{n\to\infty} n(\frac{u_n}{u_{n+1}} - 1) = k$, then the series converges for

A. $k < 1$
B. $k > 1$
C. $k = 1$
D. $k \geq 1$

ANSWER: B

43. $\lim_{n\to\infty} \frac{2^n-2}{2^n+1} =$

A. ∞

B. 0

C. 1

D. 2

ANSWER: C

44. In a positive series $\Sigma\, u_n$, if $\lim\limits_{n\to\infty} (u_n)^{\frac{1}{n}} = \lambda$, then the series diverges for

A. $\lambda < 1$

B. $\lambda \geq 1$

C. $\lambda = 1$

D. $\lambda > 1$

ANSWER: D

45. An alternating series $u_1 - u_2 + u_3 - u_4 + \ldots$ converges if $\lim\limits_{n\to\infty} u_n = 0$ and

A. each term is numerically less than its preceding term

B. each term is numerically greater than its preceding term

C. conditionally Convergent

D. absolutely Convergent

ANSWER: A

46. The series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \ldots$ is

A. divergent

B. convergent

C. absolutely divergent

D. absolutely convergent

ANSWER: D

47. $\lim_{n\to\infty} \frac{n^2}{3^n} \times \frac{3^{n+1}}{(n+1)^2} =$

A. 3

B. 4

C. 5

D. 6

ANSWER: A

48. The series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots$ converges for

A. $p < 1$

B. $p = 1$

C. $p > 1$

D. $p \geq 1$

ANSWER: C

49. The series $1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + \ldots$ is

A. convergent

B. absolutely convergent

C. divergent

D. conditionally convergent

ANSWER: A

50. If $u_n = (\log n)^{-2n}$, then $\lim_{n\to\infty} (u_n)^{\frac{1}{n}} =$

A. ∞

B. 0

C. 1

D. 2

ANSWER: B