SRM Institute of Science and Technology
Ramapuram Campus
Department of Mathematics
18MAB101T - Calculus And Linear Algebra

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – II

FUNCTIONS OF SEVERAL VARIABLES

Part – B

1. **If $u = (x - y)(y - z)(z - x)$, then find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.**

   (A) 0 (B) 1 (C) 2 (D) 3

   Solution:

   Given $u = (x - y)(y - z)(z - x)$

$$\frac{\partial u}{\partial x} = (y - z)[(x - y)(-1) + (z - x)(1)]$$
$$= -(x - y)(y - z) + (y - z)(z - x)$$
$$\frac{\partial u}{\partial y} = (z - x)[(x - y)(1) + (y - z)(-1)]$$
$$= (x - y)(z - x) - (y - z)(z - x)$$
$$\frac{\partial u}{\partial z} = (x - y)[(y - z)(1) + (z - x)(-1)]$$
$$= (x - y)(y - z) - (x - y)(z - x)$$
$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0 \quad (\textbf{Option A})$$

2. **If $x = r\cos\theta, y = r\sin\theta$ find $\frac{\partial x}{\partial r}, \frac{\partial y}{\partial \theta}$.**

   (A) cos θ, sin θ (B) cos θ, *r* cos θ (C) *r* cos θ, sin θ (D) *r*, θ

   Solution:

$$\frac{\partial x}{\partial r} = \cos\theta$$
$$\frac{\partial y}{\partial \theta} = r\cos\theta \quad (\textbf{Option B})$$

3. **If** $f(x,y) = sin\left(\frac{x}{y}\right)$, $then\ find\ x\frac{\partial f}{\partial x} + y\ \frac{\partial f}{\partial y}$.

(A) 0 **(B) 1** **(C) 2** **(D) 3**

Solution:

$$\frac{\partial f}{\partial x} = cos\left(\frac{x}{y}\right)\frac{1}{y}, \quad \frac{\partial f}{\partial y} = cos\left(\frac{x}{y}\right)\left(-\frac{x}{y^2}\right)$$

$x\frac{\partial f}{\partial x} + y\ \frac{\partial f}{\partial y} = 0$ **(Option A)**

4. $Find\ \frac{dy}{dx}\ when\ x^3 + y^3 = 3axy.$

$(A) -\frac{x^2-ay}{y^2-ax}$ $(B)\frac{x^2-ay}{y^2-ax}$ $(C)\frac{y^2-ax}{x^2-ay}$ $(D) -\frac{y^2-ax}{x^2-ay}$

Solution:

Let $f(x,y) = x^3 + y^3 - 3axy$

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = -\frac{3x^2 - 3ay}{3y^2 - 3ax}$$

$$= -\frac{x^2 - ay}{y^2 - ax}$$ (**Option A**)

5. **If** $x = uv,\ y = \frac{u}{v}$, **find** $\frac{\partial(x,y)}{\partial(u,v)}$.

(A) $\frac{-2u}{v}$ **(B)** $\frac{2u}{v}$ **(C)** $\frac{-2v}{u}$ **(D)** $\frac{2v}{u}$

Solution:

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} v & u \\ \frac{1}{v} & \frac{-u}{v^2} \end{vmatrix} = -\frac{2u}{v}$$ (**Option A**)

6. If $f(x,y) = e^x \sin y$**, then find** $f_{yy}(0,0)$**.**

(A) 0 **(B) 1** **(C) 2** **(D) 3**

Solution:

$f_y = e^x \cos y$

$f_{yy}(x,y) = e^x(-\sin y)$

$f_{yy}(0,0) = 0$ **(Option A)**

7. If $x^y = y^x$, then find $\frac{dy}{dx}$.

(A) $\dfrac{y\,x^{y-1} - y^x \log y}{x\,y^{x-1} - x^y \log x}$ **(B)** $\dfrac{y\,x^{y-1} + y^x \log y}{x\,y^{x-1} - x^y \log x}$

(C) $\dfrac{y\,x^{y-1} + y^x \log y}{x\,y^{x-1} + x^y \log x}$ **(D)** $\dfrac{y\,x^{y-1} - y^x \log y}{x\,y^{x-1} + x^y \log x}$

Solution:

$f(x, y) = x^y - y^x = 0$

$$\frac{dy}{dx} = \frac{-f_x}{f_y} = \frac{y\,x^{y-1} - y^x \log y}{x\,y^{x-1} - x^y \log x}$$ **(Option A)**

8. If $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$, then find $f_x(x, y)$ at the point (1, 1).

(A) -1/2 **(B) 1** **(C) 1/2** **(D) 3**

Solution:

$f_x(x, y) = \dfrac{-y}{x^2 + y^2}$ $f_x(1,1) = -\dfrac{1}{2}$ **(Option A)**

9. If $x = r\cos\theta$, $y = r\sin\theta$, then find $\frac{\partial(x, y)}{\partial(r, \theta)}$.

(A) *r* **(B) 1/ *r*** **(C) 1/2** **(D) 1**

Solution:

$$\text{Now } \frac{\partial(x, y)}{\partial(r, \theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial v}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\left(\sin^2\theta + \cos^2\theta\right) = r(1) = r$$

(Option A)

10. If $u = 2xy$, $v = x^2 - y^2$, then find $\frac{\partial(u,v)}{\partial(x,y)}$.

(A) $-4y^2 - 4x^2$ **(B) $-4y^2 + 4x^2$**
(C) $4y^2 - 4x^2$ **(D) $4y^2 + 4x^2$**

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 2y & 2x \\ 2x & -2y \end{vmatrix} = -4y^2 - 4x^2 \quad \textbf{(Option A)}$$

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