SRM Institute of Science and Technology
Ramapuram Campus
Department of Mathematics

18MAB101T - Calculus And Linear Algebra

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – V

SEQUENCE AND SERIES

Part – B

1. The sequence $\left\{\frac{1}{n}\right\}$ **converges to _________.**

(A) 0 **(B) 1** **(C) ½** **(D) ∞**

Solution:

$$a_n = \frac{1}{n}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} = 0$$

$\{a_n\}$ converges to 0. **(Option A)**

2. The sequence $\left\{\frac{n+1}{2n+3}\right\}$ **converges to _________.**

(A) 0 **(B) 1** **(C) ½** **(D) ∞**

Solution:

$$a_n = \frac{n+1}{2n+3}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n\left(1+\frac{1}{n}\right)}{n\left(2+\frac{3}{n}\right)} = \frac{1}{2}$$

$\{a_n\}$ converges to ½ . **(Option C)**

3. Test the convergence of the series $\sum \frac{1}{\sqrt{n+1}}$ **.**

(A) converges **(B) diverges**

(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{1}{\sqrt{n+1}} = \frac{1}{\sqrt{n\left(1+\frac{1}{n}\right)}}$$

Let $v_n = \frac{1}{\sqrt{n}}$

Now $\frac{u_n}{v_n} = \frac{1}{\sqrt{1+\frac{1}{n}}}$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = 1$$

$\sum v_n = \sum \frac{1}{\sqrt{n}} = \sum \frac{1}{n^{1/2}}$ is divergent.

Hence by comparison test, $\Sigma\, u_n$ is divergent. **(Option B)**

4. Test the convergence of the series $1+\frac{1}{3}+\frac{1}{5}+\cdots$ **.**

(A) converges **(B) diverges**

(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{1}{2n-1} = \frac{1}{n\left(2-\frac{1}{n}\right)}$$

Let $v_n = \frac{1}{n}$

Now $\frac{u_n}{v_n} = \frac{1}{2-\frac{1}{n}}$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \frac{1}{2}$$

$\sum v_n = \sum \frac{1}{n}$ is divergent.

Hence by comparison test, $\Sigma\, u_n$ is divergent. **(Option B)**

5. Test the convergence of the series $\sum \frac{x^n}{n\,!}$ where $x > 0$.

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{x^n}{n\,!}, \quad u_{n+1} = \frac{x^{n+1}}{(n+1)\,!}$$

Now $\frac{u_{n+1}}{u_n} = \frac{x}{n+1} = \frac{x}{n\left(1+\frac{1}{n}\right)}$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{x}{n\left(1+\frac{1}{n}\right)} = 0 < 1$$

Hence by Ratio test, $\Sigma\, u_n$ is convergent. **(Option A)**

6. Test the convergence of the series $\sum \frac{n\,!}{n^n}$.

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{n\,!}{n^n}, \quad u_{n+1} = \frac{(n+1)\,!}{(n+1)^{n+1}}$$

Now $\frac{u_{n+1}}{u_n} = \frac{1}{\left(1+\frac{1}{n}\right)^n}$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \frac{1}{e} < 1$$

Hence by Ratio test, $\Sigma\, u_n$ is convergent. **(Option A)**

7. The series $1+\frac{x}{1!}+\frac{x^2}{2!}+\cdots$ is ________________.

(A) absolutely convergent **(B) diverges to + ∞**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{x^{n-1}}{(n-1)\,!}, \quad u_{n+1} = \frac{x^n}{n\,!}$$

Now $\dfrac{u_{n+1}}{u_n} = \dfrac{x}{n}$

$\lim\limits_{n\to\infty}\left|\dfrac{u_{n+1}}{u_n}\right| = 0 < 1$

Hence the series is absolutely convergent. **(Option A)**

8. Test the convergence of the series $\sum \dfrac{n^3}{3^n}$**.**

(A) converges **(B) diverges**

(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$u_n = \dfrac{n^3}{3^n}, \quad (u_n)^{1/n} = \dfrac{(n^{1/n})^3}{3}$

$\lim\limits_{n\to\infty}(u_n)^{1/n} = \dfrac{1}{3} < 1$

Hence by Root test, $\Sigma\, u_n$ is convergent. **(Option A)**

9. Test the convergence of the series $\sum \dfrac{3^n\, n\,!}{n^n}$**.**

(A) converges **(B) diverges**

(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$u_n = \dfrac{3^n\, n\,!}{n^n}, \quad u_{n+1} = \dfrac{3^{n+1}\,(n+1)\,!}{(n+1)^{n+1}}$

Now $\dfrac{u_{n+1}}{u_n} = \dfrac{3}{\left(1+\dfrac{1}{n}\right)^n}$

$\lim\limits_{n\to\infty}\dfrac{u_{n+1}}{u_n} = \dfrac{3}{e} > 1$

Hence by Ratio test, $\Sigma\, u_n$ is divergent. **(Option B)**

10. Test the convergence of the series $\sum \dfrac{1}{n^2}$**.**

(A) converges **(B) diverges**

(C) oscillates finitely **(D) oscillates infinitely**

Solution:

By Harmonic Series test or p-test, $\sum \dfrac{1}{n^2}$ converges. **Option (A)**

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