Unit – IV (Multiple Integrals)

1. $\int_a^b \int_0^x f(x,y)dxdy$ $x: a \ to \ b$ and $y: 0 \ to \ x$ (Here the first integral is w.r.t. y)
2. $\int_a^b \int_0^y f(x,y)dxdy$ $x: 0 \ to \ y$ and $y: a \ to \ b$ (Here the first integral is w.r.t. x)
3. Area $= \iint_R dxdy$ (or) $\iint_R dydx$

   To change the polar coordinate $x = r\cos\theta$, $y = r\sin\theta$ and $dxdy = rdrd\theta$.
4. Volume $= \iiint_V dxdydz$ (or) $\iiint_V dzdydx$

Unit – V (Differential Equations)

1. **ODE with constant coefficients:** Solution $y = \text{C.F} + \text{P.I}$

Complementary functions:

Sl.No.	Nature of Roots	C.F
1.	$m_1 = m_2$	$(Ax+B)e^{mx}$
2.	$m_1 = m_2 = m_3$	$\left(Ax^2+Bx+c\right)e^{mx}$
3.	$m_1 \neq m_2$	$Ae^{m_1x} + Be^{m_2x}$
4.	$m_1 \neq m_2 \neq m_3$	$Ae^{m_1x} + Be^{m_2x} + Ce^{m_3x}$
5.	$m_1 = m_2, \ m_3$	$(Ax+B)e^{mx} + Ce^{m_3x}$
6.	$m = \alpha \pm i\beta$	$e^{\alpha x}(A\cos\beta x + B\sin\beta x)$
7.	$m = \pm i\beta$	$A\cos\beta x + B\sin\beta x$

Particular Integral:

Type-I

If $f(x) = 0$ then $P.I = 0$

Type-II

If $f(x) = e^{ax}$ (or) $\sinh ax$ (or) $\cosh ax$

$$P.I = \frac{1}{\phi(D)} e^{ax}$$

Replace D by a. If $\phi(D) \neq 0$, then it is P.I. If $\phi(D) = 0$, then diff. denominator w.r.t D and multiply x in numerator. Again replace D by a. If you get denominator again zero then do the same procedure.

Type-III

Case: i If $f(x)=\sin ax\ (or)\ \cos ax$

$$P.I=\frac{1}{\phi(D)}\sin ax\ (\text{or})\ \cos ax$$

Here you have to replace only for D^2 not for D. D^2 is replaced by $-a^2$. If the denominator is equal to zero, then apply same procedure as in Type – I.

Case: ii If $f(x)=Sin^2 x\ (\text{or})\ \cos^2 x\ (\text{or})\ \sin^3 x\ (\text{or})\ \cos^3 x$

Use the following formulas $Sin^2 x=\frac{1-\cos 2x}{2}$, $\cos^2 x=\frac{1+\cos 2x}{2}$,

$\sin^3 x=\frac{3}{4}\sin x-\frac{1}{4}\sin 3x$, $\cos^3 x=\frac{3}{4}\cos x+\frac{1}{4}\cos 3x$ and separate $P.I_1$ & $P.I_2$

Case: iii If $f(x)=\sin A\cos B\ (or)\ \cos A\sin B\ (or)\ \cos A\cos B\ (or)\ \sin A\sin B$

Use the following formulas:

$$(i)\quad \sin A\cos B=\frac{1}{2}\left(sin(A+B)+\sin(A-B)\right)$$

$$(\text{ii})\quad \cos A\sin B=\frac{1}{2}\left(Sin(A+B)-\sin(A-B)\right)$$

$$(iii)\quad \cos A\cos B=\frac{1}{2}\left(\cos(A+B)+\cos(A-B)\right)$$

$$(iv)\quad \sin A\sin B=\frac{1}{2}\left(\cos(A-B)-\cos(A+B)\right)$$

Type-IV

If $f(x)=x^m$

$$P.I=\frac{1}{\phi(D)}x^m=\frac{1}{1+g(D)}x^m=\left(1+g(D)\right)^{-1}x^m$$

Here we can use Binomial formula as follows:

i) $(1+x)^{-1}=1-x+x^2-x^3+\ldots$

ii) $(1-x)^{-1}=1+x+x^2+x^3+\ldots$

iii) $(1+x)^{-2} = 1-2x+3x^2-4x^3+\dots$

iv) $(1-x)^{-2} = 1+2x+3x^2+4x^3+\dots$

Type-V

If $f(x)=e^{ax}V$ where $V=\sin ax, \cos ax, x^m$

$$P.I = \frac{1}{\phi(D)}e^{ax}V \;= e^{ax}\frac{1}{\phi(D+a)}V$$

Type-VI

If $f(x)=x^nV$ where $V=\sin ax, \cos ax$

$\sin ax = \text{I.P of } e^{iax}$

$\cos ax = \text{R.P of } e^{iax}$

Type-VII

If $f(x)=\sec ax \text{ (or) } \mathrm{cosec}\, ax \text{ (or) } \tan ax$

$$P.I = \frac{1}{D-a}f(x) = e^{ax}\int e^{-ax}f(x)dx$$

1. **ODE with variable co-efficient: (Euler's Method)**

The equation is of the form $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=f(x)$

Implies that $(x^2D^2+xD+1)y=f(x)$

To convert the variable coefficients into the constant coefficients

Put $z=\log x$ implies $x=e^z$

$xD=D'$

$x^2D^2=D'(D'-1)$ where $D=\frac{d}{dx}$ and $D'=\frac{d}{dz}$

$x^3D^3=D'(D'-1)(D'-2)$

The above equation implies that $\left(D'(D'-1)+D'+1\right)y=f(x)$ which is O.D.E with constant coefficients.

2. **Legendre's Linear differential equation:**

The equation if of the form $(ax+b)^2\frac{d^2y}{dx^2}+(ax+b)\frac{dy}{dx}+y=f(x)$

Put $z=\log(ax+b)$ implies $(ax+b)=e^z$

$(ax+b)D = aD'$

$(ax+b)^2 D^2 = a^2 D'(D'-1)$ where $D = \frac{d}{dx}$ and $D' = \frac{d}{dz}$

$(ax+b)^3 D^3 = a^3 D'(D'-1)(D'-2)$

3. Method of Variation of Parameters:

The equation is of the form $\boldsymbol{a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)}$

$C.F = Ay_1 + By_2$ and

$P.I = Py_1 + Qy_2$

where $P = -\int \frac{y_2 f(x)}{y_1 y_2' - y_1' y_2} dx$ and

$Q = \int \frac{y_1 f(x)}{y_1 y_2' - y_1' y_2} dx$

Textbook for Reference:

"ENGINEERING MATHEMATICS - I"

Publication: Sri Hariganesh Publications **Author: C. Ganesan**

Mobile: 9841168917, 8939331876

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----All the Best----