NAME OF THE SUBJECT : Mathematics – I

SUBJECT CODE : MA6151

NAME OF THE MATERIAL : Formula Material

MATERIAL CODE : HG13AUM101

REGULATION : R2013

UPDATED ON : May-June 2015

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Name of the Student: Branch:

Unit – I (Matrices)

1. The Characteristic equation of matrix A is

   a) $\lambda^2 - S_1\lambda + S_2 = 0$ if A is 2 X 2 matrix

   Where $S_1 =$ **Sum of the main diagonal elements.**

   $S_2 = |A|$

   b) $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$ if A is 3 X 3 matrix

   Where $S_1 =$ **Sum of the main diagonal elements.**

   $S_2 =$ **Sum of the minors of the main diagonal elements.**

   $S_3 = |A|$

2. To find the eigenvectors solve $(A - \lambda I)X = 0$.

3. Property of eigenvalues:

   Let A be any matrix then

   a) Sum of the eigenvalues = Sum of the main diagonal.

   b) Product of the eigenvalues = $|A|$

   c) If the matrix A is triangular then diagonal elements are eigenvalues.

   d) If λ is an eigenvalue of a matrix A, the $\frac{1}{\lambda}$ is the eigenvalue of A^{-1}.

   e) If $\lambda_1, \lambda_2, \ldots \lambda_n$ are the eigenvalues of a matrix A, then $\lambda_1^m, \lambda_2^m, \ldots \lambda_n^m$ are eigenvalues of A^m.(m being a positive integer)

f) The eigenvalues of A & A^T are same.

4. **Cayley-Hamilton Theorem:**

   Every square matrix satisfies its own characteristic equation. (ie) $|A-\lambda I|=0$.

5. Matrix of the Quadratic form $=\begin{vmatrix} coeff\,(x_1^2) & \frac{1}{2}coeff\,(x_1x_2) & \frac{1}{2}coeff\,(x_1x_3) \\ \frac{1}{2}coeff\,(x_2x_1) & coeff\,(x_2^2) & \frac{1}{2}coeff\,(x_2x_3) \\ \frac{1}{2}coeff\,(x_3x_1) & \frac{1}{2}coeff\,(x_3x_2) & coeff\,(x_3^2) \end{vmatrix}$

6. Index = p = Number of positive eigenvalues

   Rank = r = Number of non-zero rows

   Signature = s = 2p-r

7. Diagonalisation of a matrix by orthogonal transformation (or) orthogonal reduction:

   Working Rules:

   Let A be any square matrix of order n.

Step:1 Find the characteristic equation.

Step:2 Solve the characteristic equation.

Step:3 Find the eigenvectors.

Step:4 Form a normalized model matrix N, such that the eigenvectors are orthogonal.

Step:5 Find $\mathbf{N}^{\mathbf{T}}$.

Step:6 Calculate $\mathbf{D}=\mathbf{N}^{\mathbf{T}}\mathbf{AN}$.

Note:

We can apply orthogonal transformation for symmetric matrix only.
If any two eigenvalues are equal then we must use a, b, c method for third eigenvector.

Unit – II (Sequences and Series)

1. **Convergent and Divergent sequence:**

If the sequence of real numbers $\{a_n\}_{n=1}^{\infty}$ has a limit L, then the sequence is said to be a convergent sequence. If it does not have it, then it is said to be divergent.

(i.e) $\lim\limits_{n\to\infty} a_n = L$

2. **Bounded Sequence:**

A Sequence $a_1, a_2, a_3 \ldots$ is bounded if there exist a number $M > 0$ such that $|a_n| < M$, $n \in \mathbb{N}$.

3. **Monotone Sequence:**

A sequence $\{a_n\}$ is non-decreasing if $a_n \le a_{n+1}$ for all n and non-increasing if $a_n \ge a_{n+1}$ for all n. A monotonic sequence is a sequence which is either non-decreasing or non-increasing.

Example:

- A non-decreasing sequence which is bounded above is convergent.
- A non-decreasing sequence is always bounded below.
- A non-increasing sequence which is bounded below is convergent.
- A non-increasing sequence is always bounded above.

4. **Comparison Test:**

If two series of non-negative terms $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ such that $a_n \le b_n$ for all n.

Then, if $\sum_{n=1}^{\infty} b_n$ is convergent then the given series $\sum_{n=1}^{\infty} a_n$ is convergent.

5. **Integral Test:**

Consider an integer *N* and a non-negative function *f* defined on the unbounded interval $[N, \infty)$, on which it is monotone decreasing. Then the infinite series $\sum_{n=N}^{\infty} f(n)$ converges to a real number if and only if the improper integral $\int_{N}^{\infty} f(x)dx$ is finite. In other words, if the integral infinite, then the series diverges.

6. **D'Alembert's ratio test Ratio Test:**

   In a series $\sum_{n=1}^{\infty} a_n$ of non-negative terms if $\sum_{n=1}^{\infty} \frac{a_{n+1}}{a_n} = L$ then the series $\sum_{n=1}^{\infty} a_n$ is converges if $L < 1$, diverges if $L > 1$ and test fails if $L = 1$.

7. **Alternating Series:**

   A series in which the terms are alternatively positive or negative that is $\sum_{n=1}^{\infty} (-1)^{n+1} a_n = a_1 - a_2 + a_3 - \ldots$ where a_n are positive, is called an alternating series.

8. **Leibnitz's Test:**

   Leibnitz's test is also known as the alternating series test. Given a series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ with $a_n > 0$, if a_n is monotonically decreasing as $n \to \infty$ and $\lim_{n\to\infty} a_n = 0$, then the series converges.

9. **Absolute and Conditional convergent:**

   An arbitrary series $\sum_{n=1}^{\infty} a_n$ is called **absolutely convergent** if $\sum_{n=1}^{\infty} |a_n|$ is convergent.

   If $\sum_{n=1}^{\infty} a_n$ is convergent and $\sum_{n=1}^{\infty} |a_n|$ is divergent we call the series **conditionally convergent**.

Unit – III (Applications of Differential Calculus)

1. Curvature of a circle = Reciprocal of it's radius
2. Radius of curvature with Cartesian form $\rho = \left| \frac{\left(1 + y_1^2\right)^{\frac{3}{2}}}{y_2} \right|$

3. Radius of curvature if $y_1 = \infty$, $\rho = \left| \dfrac{\left(1 + x_1^2\right)^{\frac{3}{2}}}{x_2} \right|$, where $x_1 = \dfrac{dx}{dy}$

4. Radius of curvature in implicit form $\rho = \left| \dfrac{\left(f_x^2 + f_y^2\right)^{\frac{3}{2}}}{f_{xx} f_y^2 - 2 f_{xy} f_x f_y + f_{yy} f_x^2} \right|$

5. Radius of curvature with parametric form $\rho = \left| \dfrac{\left(x'^2 + y'^2\right)^{\frac{3}{2}}}{x'y'' - x''y'} \right|$

6. Centre of curvature is $\left(\bar{x}, \bar{y}\right)$.

7. Circle of curvature is $\left(x - \bar{x}\right)^2 + \left(y - \bar{y}\right)^2 = \rho^2$.

   where $\bar{x} = x - \dfrac{y_1\left(1 + y_1^2\right)}{y_2}$, $\bar{y} = y + \dfrac{\left(1 + y_1^2\right)}{y_2}$

8. **Evolute:** The locus of centre of curvature of the given curve is called evolute of the curve. $\bar{x} = x - \dfrac{y_1\left(1 + y_1^2\right)}{y_2}$, $\bar{y} = y + \dfrac{\left(1 + y_1^2\right)}{y_2}$

9. **Envelope:** The envelope is a curve which meets each members of a family of curve.

   If the given equation can be rewrite as quadratic equation in parameter, (ie) $A\lambda^2 + B\lambda + C = 0$ where A, B, C are functions of x and y then the envelope is $B^2 - 4AC = 0$.

10. Evolute as the envelope of normals.

Equations	Normal equations
$y^2 = 4ax$	$y + xt = at^3 + 2at$
$x^2 = 4ay$	$x + yt = at^3 + 2at$
$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$	$\dfrac{ax}{\cos\theta} - \dfrac{by}{\sin\theta} = a^2 - b^2$

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$
$x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$	$x\cos\theta-y\sin\theta=a\cos 2\theta$
$xy=c^2$	$y-xt^2=\frac{c}{t}-ct^3$

Unit – IV (Differential Calculus of several variables)

1. **Euler's Theorem:**

If f is a homogeneous function of x and y in degree n, then

(i) $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=nf$ (first order)

(ii) $x^2\frac{\partial^2 f}{\partial x^2}++2xy\frac{\partial^2 f}{\partial x\partial y}+y^2\frac{\partial^2 f}{\partial y^2}=n(n-1)f$ (second order)

2. If $u=f(x,y,z)$, $x=g_1(t), y=g_2(t), z=g_3(t)$ then $\frac{du}{dt}=\frac{\partial u}{\partial x}\frac{dx}{dt}+\frac{\partial u}{\partial y}\frac{dy}{dt}+\frac{\partial u}{\partial z}\frac{dz}{dt}$

3. If $u=f(x,y), x=g_1(r,\theta), y=g_2(r,\theta)$ then

(i) $\frac{\partial u}{\partial r}=\frac{\partial u}{\partial x}\frac{\partial x}{\partial r}+\frac{\partial u}{\partial y}\frac{\partial y}{\partial r}$ (ii) $\frac{\partial u}{\partial \theta}=\frac{\partial u}{\partial x}\frac{\partial x}{\partial \theta}+\frac{\partial u}{\partial y}\frac{\partial y}{\partial \theta}$

4. **Maxima and Minima :**

Working Rules:

Step:1 Find f_x and f_y. Put $f_x=0$ and $f_y=0$. Find the value of x and y.

Step:2 Calculate $r=f_{xx}, s=f_{xy}, t=f_{yy}$. Now $\Delta=rt-s^2$

Step:3 i. If $\Delta>0$, then the function have either maximum or minimum.

1. If $r<0\Rightarrow$ Maximum
2. If $r>0\Rightarrow$ Minimum

ii. If $\Delta<0,$ then the function is neither Maximum nor Minimum, it is called Saddle Point.

iii. If $\Delta=0,$ then the test is inconclusive.

5. Maxima and Minima of a function using Lagrange's Multipliers:

   Let $f(x,y,z)$ be given function and $g(x,y,z)$ be the subject to the condition.

   Form $F(x,y,z)= f(x,y,z)+\lambda g(x,y,z)$, Putting $F_x = F_y = F_z = F_\lambda = 0$ and then find the value of x,y,z. Next we can discuss about the Max. and Min.

6. **Jacobian:**

   Jacobian of two dimensions: $J\left(\dfrac{u,v}{x,y}\right)=\dfrac{\partial(u,v)}{\partial(x,y)}=\begin{vmatrix}\dfrac{\partial u}{\partial x} & \dfrac{\partial u}{\partial y}\\ \dfrac{\partial v}{\partial x} & \dfrac{\partial v}{\partial y}\end{vmatrix}$

7. The functions u and v are called functionally dependent if $\dfrac{\partial(u,v)}{\partial(x,y)}=0$.

8. $\dfrac{\partial(u,v)}{\partial(x,y)}\times\dfrac{\partial(x,y)}{\partial(u,v)}=1$

9. **Taylor's Expansion:**

$$f(x,y)= f(a,b)+\frac{1}{1!}\left\{hf_x(a,b)+kf_y(a,b)\right\}+\frac{1}{2!}\left\{h^2 f_{xx}(a,b)+2hkf_{xy}(a,b)+k^2 f_{yy}(a,b)\right\}$$
$$+\frac{1}{3!}\left\{h^3 f_{xxx}(a,b)+3h^2kf_{xxy}(a,b)+3hk^2 f_{xyy}(a,b)+k^3 f_{yyy}(a,b)\right\}+\ldots$$

   where $h=x-a$ and $k=y-b$

Unit – V (Multiple Integrals)

1. $\int_a^b\int_0^x f(x,y)dxdy$ x : a to b and y : o to x (Here the first integral is w.r.t. y)
2. $\int_a^b\int_0^y f(x,y)dxdy$ x : 0 to y and y : a to b (Here the first integral is w.r.t. x)
3. Area $=\iint\limits_R dxdy$ (or) $\iint\limits_R dydx$

   To change the polar coordinate $\begin{aligned} x&=r\cos\theta\\ y&=r\sin\theta\\ dxdy&=rdrd\theta\end{aligned}$

4. Volume $=\iiint\limits_V dxdydz$ (or) $\iiint\limits_V dzdydx$

GENERAL:

1. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$ (or) $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x)$

2. $\int \frac{dx}{\sqrt{a^2+x^2}} = \log\left(x+\sqrt{a^2+x^2}\right)$ (or) $\int \frac{dx}{\sqrt{1+x^2}} = \log\left(x+\sqrt{1+x^2}\right)$

3. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$ (or) $\int \frac{dx}{1+x^2} = \tan^{-1}(x)$

4. $\int \sqrt{a^2-x^2}\ dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$

5. $\int_0^{\pi/2} \sin^n x\ dx = \int_0^{\pi/2} \cos^n x\ dx = \frac{n-1}{n}.\frac{n-3}{n-2}...\frac{2}{3}.1$ **if n is odd and $n \geq 3$**

6. $\int_0^{\pi/2} \sin^n x\ dx = \int_0^{\pi/2} \cos^n x\ dx = \frac{n-1}{n}.\frac{n-3}{n-2}...\frac{1}{2}.\frac{\pi}{2}$ **if n is even**

----*All the Best*----