SRM Institute of Science and Technology
Ramapuram Campus
Department of Mathematics

18MAB101T - Calculus And Linear Algebra

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – IV

DIFFERENTIAL CALCULUS

Part – B

1. Envelope of the curve $y = mx + \frac{a}{m}$ (where m is the parameter) is

(A) $x^2 + a\,y = 0$ (B) $x + 4\,a\,y = 0$

(C) $y^2 - 4\,a\,x = 0$ (D) $y^2 + 4ax = 0$

Solution: Given: $y = mx + \frac{a}{m}$

$$y = \frac{m^2 x + a}{m}$$

$$m^2 x - y\,m + a = 0$$

The above equation is a quadratic equation in 'm'.

The discriminant is $b^2 - 4ac = 0$.

The envelope of the curve is $y^2 - 4\,a\,x = 0$. **(Option C)**

2. The radius of curvature of the curve $y = e^x$ at $x = 0$ is

(A) $2\sqrt{2}$ (B) $\sqrt{2}$

(C) 2 (D) 4

Solution:

$y_1 = e^x$ at $x = 0$ is 1

$y_2 = e^x$ at $x = 0$ is 1

$$\rho = \frac{\left(1 + y_1^{\ 2}\right)^{\frac{3}{2}}}{y_2}$$

$\rho = 2\sqrt{2}$ **(Option A)**

3. The radius of curvature of the curve *y* = 4 sin *x* at *x* = $\frac{\pi}{2}$ is

(A) $\frac{1}{2}$ (B) $\frac{-1}{2}$

(C) $\frac{1}{4}$ (D) $\frac{3}{4}$

Solution:

$y_1 = 4\cos x \ at\ x = \frac{\pi}{2} is\ 0$

$y_2 = -4\sin x\, at\ x = \frac{\pi}{2} is - 4$

$$\rho = \frac{\left(1 + y_1^{\ 2}\right)^{\frac{3}{2}}}{y_2}$$

$|\rho| = \frac{1}{4}$ **(Option C)**

4. The envelope of family of lines y = $mx + am^2$ (where *m* is the parameter) is

(A) $x^2 + 2ay = 0$ (B) $x^2 + 4\,a\,y = 0$

(C) $y^2 + 2ax = 0$ (D) $x^2 + 4\,a\,x = 0$

Solution:

The given equation is quadratic in 'm'.

The discriminant is $b^2 - 4ac = 0$.

Envelope of the family of lines is $x^2 + 4ay = 0$. **(Option B)**

5. The envelope of the family of lines $\frac{x}{t} + y\,t = 2c$, t being the parameter is

(A) $x^2 + y^2 = c^2$ (B) $x\,y = c^2$

(C) $x^2\,y^2 = c^2$ (D) $x^2 - y^2 = c^2$

Solution:

Simplifying the equation $\frac{x}{t} + y\,t = 2c$, we get $yt^2 - 2ct + x = 0$

The discriminant is $b^2 - 4ac = 0$.

Envelope of the family of lines is $x\,y = c^2$. **(Option B)**

6. The radius of curvature of the curve $r = e^{\theta}$ at any point on it is

(a) $2\sqrt{2}$ (b) $\sqrt{2}\,r$

(c) 2 (d) 4

Solution:

$r' = e^{\theta}$

$r'' = e^{\theta}$

$$\rho = \frac{\left(r^2 + r'^2\right)^{\frac{3}{2}}}{r^2 - rr'' + 2r'^2}$$

$\rho = \sqrt{2}\,r$

7. The radius of curvature at the point (3, 4) on the curve $x^2 + y^2 = 25$ is

(A) 5 (B) 4 (C) 0 (D) 2

Solution:

We know that the radius of curvature of the circle is equal to its radius.

$\rho = 5$ **(Option A)**

8. B (5/2, 1/2) = _______.

(A) 1 (B) 4

(C) 3π/8 (D) π

Solution:

$$B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

$$B(5/2,1/2) = \frac{\Gamma(5/2)\Gamma(1/2)}{\Gamma(3)} = \frac{3\pi}{8}$$ **(Option C)**

9. $\Gamma(-5/2)$ **=____________.**

(A) 1 (B) 4

(C) 1/2 (D) $\frac{-8\sqrt{\pi}}{15}$

Solution:

$$\Gamma\left(-n+\frac{1}{2}\right) = \frac{(-2)^n}{1.3.5\cdots(2n-1)}\sqrt{\pi}$$

$$\Gamma\left(-\frac{5}{2}\right) = \Gamma\left(\frac{-6+1}{2}\right)$$

$$= \Gamma\left(-3+\frac{1}{2}\right) = \frac{-8}{15}\sqrt{\pi}$$

(Option D)

10. Evaluate $\int_0^\infty e^{-x} x^4 \, dx$.

(A) 1 (B) 24

(C) 1/2 (D) $\frac{-8\sqrt{\pi}}{3}$

Solution

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} \, dx$$

$$\int_0^\infty e^{-x} x^4 \, dx = \int_0^\infty e^{-x} x^{5-1} \, dx = \Gamma(5) = 4! = 24$$

(Option B)

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