* 1. Evaluate $\int_0^a\int_y^a \frac{x\,dx\,dy}{x^2+y^2}$ by changing the order of integration. [1 time]

*** 2. Find the volume of a sphere $x^2+y^2+z^2=a^2$ using triple integration [2 times]

**** 3. Change the order of integration and hence evaluate $\int_0^a\int_{x^2/a}^{2a-x} xy\,dy\,dx$ [4 times]

** 4. Evaluate $\int_0^{\log a}\int_0^x\int_0^{x+y} e^{x+y+z}\,dx\,dy\,dz$ [2 times]

* 5. Using triple integration, find the volume of ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{x^2}{c^2}=1$ [1 time repeated]

* 6. Change the order of integration and hence evaluate $\int_0^1\int_y^{2-y} xy\,\cancel{dx}\,dy\,dx$

* 7. Evaluate $\int_2^4\int_0^x\int_0^{\sqrt{x+y}} z\,dz\,dy\,dx$

8. By changing into polar co-ordinates evaluate $\int_0^{2a}\int_0^{\sqrt{2a-x^2}}(x^2+y^2)\,dx\,dy$

* 9. Find the area of ~~cardo~~ cardiod $r=a(1+\cos\theta)$ using double integration

** 10. Find the volume of tetrahedron bounded by the planes $x=0, y=0, z=0$ and $\frac{x}{a}+\frac{y}{b}+\frac{c}{z}=1$ using integration. [2 times]

11. Show by double integration the area between the parabolas $y^2 = 4ax$, $x^2 = 4ay$ is $\frac{16a^3}{3}$

12. Using double integration, find the area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

13. find the value of $\int_0^1 \int_{x^2}^{x-y} xy(x+y)\,dy\,dx$

14 Evaluate $\int_0^1 \int_0^{\sqrt{1+y^2}} \frac{dy\,dx}{1+x^2+y^2}$

15. change the order of integration $\int_0^a \int_x^a (x^2+y^2)\,dy\,dx$ and hence evaluate it

16. Evaluate $\int_0^{\pi} \int_a^{a\sin\theta} r\,dr\,d\theta$

13. Find the value of $\int_0^1 \int_x^{\sqrt{x}} xy(x+y)\,dy\,dx$

***

17 change the order of integration and evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$. [3 times]

18. Find the area $r^2 = a^2\cos 2\theta$ using double integration.

***

19. Show that $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}$ [3 times]

20 Evaluate $\int_0^1 \int_0^x (x+y)\,dy\,dx$.

21. Find the volume bounded by the curves $y^2 = x$, $y = x^2$ and planes $z = 0$ and $x + y + z = 2$

22. Find the area that lies outside the circle $r = 2a\cos\theta$ and inside the circle $r = 4a\cos\theta$

*
23. Evaluate the changing the order of integration

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$$

24. Find the double integration smaller of area bounded by circle $x^2+y^2=9$, $x+y=3$

UNIT-2

**

1. Find the angle between normals to the surface $x^2 = yz$ at the points $(1,1,1)$ and $(2,4,1)$

2. A Fluid motion is given by $\bar{v} = (y+z)\vec{i} + (z+x)\vec{j} + (x+y)\vec{k}$ Is this motion irrotational? If so find scalar potential.

3. Find the Gauss divergence theorem for $\vec{F} = x^2\vec{i} + z\vec{y} + yz\vec{k}$ over a cube formed by $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$

***
4. Verify Green's theorem in the plane for $\iint [(3x^2 - 8y^2)dx + (4x - 6xy)]dy$ where c is boundary of the region defined by $x = y^2$, $y = x^2$

5.* verify divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$ taken over rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.
**

6. Find the angle between the surfaces $x^2-y^2-z^2=11$ and $xy+yz-zx=18$ at point $(6,4,3)$

7. * ~~Find~~ ~~to~~ show that $\vec{F}=(2xy+z^3)\vec{i}+x^2\vec{j}+3xz^2\vec{k}$ is conservative vector field hence find the Scalar potential.

8. ** Show that $r^n\bar{r}$ is an irrotational vector for any value of 'n' and is solenoidal only for $n=-3$

* 9. Find the workdone when a force $F=(x^2-y^2+x)\vec{i}-(2xy+y)\vec{j}$ displace a particle in the xy plane from $(0,0)$ to $(1,1)$ along the parabola $y^2=x$

10. Determine $f(r)$ so that $f(r)\vec{r}$ is both solenodial and irrotational.

11. ~~Find~~ Verify the gauss divergence theorem for $\vec{F}=x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$ taken over a cube bounded by the planes $x=0, x=1, y=0, y=1, z=0, z=1$

12. Find the unit normal to the surface $x^2y+2xz^2=8$ at the point $(1,0,2)$

13. Find the values of constants a,b,c so that $\vec{F}=(axy+bz^3)\vec{i}+(3x^2-cz)\vec{j}+(3xz^2-y)\vec{k}$ may be irrotational

14. Find the direction derivative of $\phi=2xy+z^3$ at the point $(1,-1,3)$ in the direction of $\vec{i}+2\vec{j}+2\vec{k}$

15. Verify Gauss divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$ taken over a rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$

16. Prove that the vector $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational and find scalar potential

17. Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{i} + (2xy + 3xz)\vec{j} + (3xy - 2xz + 2z)\vec{k}$ is bothe irrotational & solenoidal

18. Verify Gauss Divergence theorem for $F = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ taken over the cube bounded by the plane $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$

**
19. Verify stoke's theorem $\vec{F} = (y - z + 2)\vec{i} - (yz + 4)\vec{j} - (xz)\vec{k}$ over the surface of the cube $x = 0$, $y = 0$, $z = 0$, $x = 2$, $z = 2$, $y = 2$ above the x.y plane.

20. If $r = |\vec{r}|$ when $\vec{r}$ is position vector of the point (x, y, z) w.r.t origin, prove that $f(r)\vec{r}$ is an irrotational vector.

21. Prove that $\vec{F} = (y^2 \cos x + z^3)\vec{i} + (2y \sin x - 4)\vec{j} + 3xz^2\vec{k}$ is irrotational and find scalar potential

22. Find the direction derivative of $\phi = x^2 + y^2 + 4xyz$ at $(1, -2, 2)$ in the direction of $2\vec{i} - 2\vec{j} + \vec{k}$

*
23. Verify Gauss divergence for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ over the cuboid formed by $0 \le x \le a$, $0 \le y \le b$, $0 \le y \le c$.

24. If $\vec{F} = 3xy\,\vec{i} - y^2\vec{j}$, Evaluate $\int_C \vec{F}\cdot d\vec{r}$ where C is the arc of parabola $y=2x^2$ from $(0,0)$ to $(1,2)$

*
25. Prove that $\vec{A} = (2x+yz)\vec{i} + (4y-zx)\vec{j} - (6z-xy)\vec{k}$ is of soleniodal as well as irrotational. Also Find the scalar potential of $\vec{A}$,t

*
26. Verify Green's theorem in the plane for $\oint_C (xy+y^2)dx + x^2dy$ where C is the closed curve of region bounded by $y=x$, $y=x^2$ t≤2 ≤t≤4

27. If $\phi(x,y,z) = x^2y + y^2x + z^2$, fin $\nabla\phi$ at the point $(1,1,1)$

28 prove that $\nabla(r^n) = nr^{n-2}\cdot\vec{r}$, where r is positional vector of the point (x,y,z) wrt origin (t)
*

29. Verify Green's theorem for the intergal $\oint_C[(x^2+y)dx - xy^2dy]$ taken over the boundary of square whose vertices are $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$.)=f(t)

***
30. Show that the vector field $\vec{F} = (x^2-yz)\vec{i} + (y^2-zx)\vec{j} + (z^2-xy)\vec{k}$ is irrotational & find the scalar potential

31. Use stroke's theorem to evaluate $\oint_C \vec{F}\cdot d\vec{r}$ where $\vec{F} = (\sin x - y)\vec{i} - \cos x\,\vec{j}$ and C is boundary of triangle whose vertices $(0,0)$ $(\pi/2,0)$, $(\pi/2,1)$ e^-t

32. Find the angle b/w the normals of the curve $xy = z^2$ at the points $(-2,-2,2)$ and $(1,9,-3)$

33. Evaluate $\oint_C (x^2+y^2)\,dx - 2xy\,dy$ taken over the rectangle bounded by the lines $x = \pm a,\ y=0,\ y=b$

34. Evaluate divergence theorem $\iint_S (x+z)\,dydz + (x+y)\,dxdy$ over the sphere $(x^2+y^2+z^2=4)$

35. Find the constant a, b so that the surface $ax^2 - byz = (a+2)x$, $4x^2y+z^3=4$ cut orthogonally at $(1,-1,2)$

36. $\vec{F} = 2y\vec{i} - z\vec{j} + x\vec{k}$ evaluate $\oint_C \vec{F}\times d\vec{r}$ along the curve $x=\cos t,\ y=\sin t,\ z=2\cos t$ from $t=0$ to $\pi/2$

37. If $\bar{u}$ and $\bar{v}$ are irrotational vectors, then show that $\bar{u}\times\bar{v}$ is solenoidal vector

38. If $r=|\bar{r}|$ where $\bar{r}$ is positional vector of the point (x,y,z) wrt the origin, prove that $\boxed{\nabla f(r) = \frac{f'(r)}{r}\bar{r}}$

39. $\oint_C \vec{F}\times d\vec{r}$ where $\vec{F} = xy\vec{i} + z\vec{j} + x^2\vec{k}$ where $x=t^2,\ y=2t,\ z=t^3$ from 0 to 1

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UNIT - 3

1. Verify final value theorem for the function
$1 + e^{-t}(\sin t + \cos t)$

2. Solve using laplace transform method $y'' + 2y' - 3y = \sin t$ given $y(0) = y'(0) = 0$

3. (i) Using convolution theorem, Evaluate $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$

4. Find the Laplace transformation of $f(t) = \begin{cases} t & 0 \le t \le 2 \\ 4-t & 2 \le t \le 4 \end{cases}$ and satisfy $f(t+4) = f(t)$

5. Find $L^{-1}\left[\cot^{-1}\left[\frac{2}{s+1}\right]\right]$

6. If $L[f(t)] = \frac{1}{s(s+1)(s+2)}$, find $\lim_{t \to 0} f(t)$ & $\lim_{t \to \infty} f(t)$

7. Find $L\left[\frac{\sin 3t \sin t}{t}\right]$

8. Solve : $(D^2+6D+9)x = 6t^2e^{-2t}$, $x=0$, $Dx=0$ at $t=0$ using Laplace transform.

9. Find $L(f(t))$ if $f(t) = e^t$, $0 < t < 2\pi$, $f(t+2\pi) = f(t)$

10. Find $L\left[\frac{e^{-t} - e^{-3t}}{t}\right]$

11. Find $L^{-1}\left[\frac{e^{-s}}{(s+3)(s-2)}\right]$

12. Using convolution theorem find $L^{-1}\left[\frac{1}{(s+3)(s+1)}\right]$

13. Find $L[te^{-t}\sin t]$

14. Using Laplace transform, solve $y'' - 3y' + 2y = e^{-t}$ given $y(0) = 1$, $y'(0) = 0$

15. Evaluate $L\left[\int_0^t \frac{\cos 6t - \cos 4t}{t}\, dt\right]$

16) Solve ~~the~~ Using Laplace transform $y'' + 2y' - 3y = \sin t$
given $y(0) = 0,\ y'(0) = 0$.

17) Using convolution theorem, evaluate $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$

18) Find the Laplace transform of perodic function $f(t)$ with period 2 given by

$$f(t) = \begin{cases} 1 & 0 < t < 1 \\ -1 & 1 \le t < 2 \end{cases}$$

19) Using Laplace transform method solve $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^{-t}$
given $x(0) = 2,\ x'(0) = 1$

20) find $L^{-1}\left[\frac{(s-1)}{(s^2+2t+2)^2}\right]$

21) evaluate $L^{-1}\left[\log\left(\frac{s+1}{s-1}\right)\right]$

22) solve $(D^2 - 4D + 8)y = e^{2t}$, given $y(0) = 2,\ y'(0) = -2$
Using Laplace transform.

23) find $L\left[\frac{\sin 3t \cos t}{t}\right]$

24) Using convolution theorem, find $L^{-1}\left[\frac{1}{s^2(s+1)^2}\right]$

25) Evaluate $L\left[\frac{1-e^{-t}}{t}\right]$

26) Evaluate using Laplace transform $\int_0^\infty e^{-3t}\, t \sin t\, dt$

27) Using convolution theorem, find the Laplace transform of $\frac{1}{s(s+2)^3}$

28) Using Laplace transform method, solve
$(D^2+4)y = \cos 2t,\ y(0) = 3,\ y'(0) = 4$

(Q) Evaluate $\oint_c \frac{\sin^2 z}{(z-\pi/6)^3} dz$, $|z|=1$

(30) Expand $f(z)=\frac{z^2-1}{(z+2)(z+3)}$ as a Laurent's series if $|z|<2$, $2<|z|<3$.

(31) Find the residue of $F(z)=\frac{z}{(z-1)^2}$ at its poles.

(32) Find the Laplace^{-1} $L^{-1}\left[\frac{1}{(s^2+9)}\right]$

(33) $L\left[\frac{1-\cos t}{t}\right]$

(34) Find $L^{-1}\left[\frac{1}{(s+1)(s+2)}\right]$

(35) Solve $x''+3x'+2x=4$ given $x(0)=2$, $x'(0)=3$. using Laplace transform.

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UNIT-4

1. Find the constant a, b, c if $F(z) = x+ay+i(bx+cy)$ is analytic

2. ** Find the analytic function $F(z) = u+iv$ where $u = e^x(x\sin y + y\cos y)$

3. * Determine the region D of W-plane into which the triangular region D enclosed by lines $x=0, y=0, x+y=1$ is transformed under the transformation $W = 2Z$

4. Find the bilinear transformation which maps the points $z_1 = 1, z_2 = i, z_3 = -1$ into the points $w_1 = i, w_2 = 0, w_3 = -i$

5. Derive C-R Equations in polar ~~cor~~ form.

6. Determine the analytic function $f(z) = u+iv$ given that $3u + 2v = y^2 - x^2 + 16xy$.

7. *** Find the bilinear transformation which maps the point $z=0, z=1, z=\infty$ into the points $w=i, w=1, w=-i$

8. * State & prove ~~that~~ two important properties of an analytic function.

9. Show that $\sin z$ is an analytic function of z

10. *** If $F(z)$ is analytic function of z, show that $\nabla^2 |F(z)|^2 = 4|F'(z)|^2$

11. * Find the image of circle $|z-1| = 1$ under the mapping $w = 1/z$

12. *** Prove that an analytic with constant modulus is constant.

13. * Find the harmonic ~~go~~ conjugate of $u = \frac{1}{2}\log(x^2+y^2)$

14. * Find the bilinear transformation that maps the points $z = 1, i, -1$ in z-plane into the points $w = 2, i, -2$ in w plane.

15. Find the function [analytic] in terms of z if $u+v = (x-y)(x^2+4xy+y^2)$

16. ** Find the analytic function $f(z) = u+iv$ where $u-v = \dfrac{\sin 2x}{\cosh 2x - \cos 2x}$

17. * Find the bilinear transformation that maps the point $\infty, i, 0$ into $0, i, \infty$ respectively

18. Discuss the transformation $w = 1/z$

19. If $u+v = e^x[\cos y + \sin y]$, where $f(z) = u+iv$ is analytic, find the analytic function & hence find its derivative.

20. Find the bilinear transformation that maps the points $\infty, 0, i$ in z plane on the point $0, \infty, -i$ of the w-plane.

21. Show that the function $u = 2xy + 3y$ is harmonic and find their corresponding analytic function.

22. Find the harmonic conjugate $u = e^x \cos y$.

23. * Find the image of $|z-2i| = 2$ under the transformation $w = 1/z$

24. Bilinear $z = 1, i, -1$ onto $w = i, 0, -i$

25. * Find invariant points $w = \dfrac{2z+4i}{1+iz}$

26) Show that z^n is an analytic function.

27) Show that the function $u(x,y) = 3x^2y + 2x^2 - y^3 - 2y^2$ is hormonic.

28) Bilinear transformation $z = 1, i, -1$ on to $w = i, 0, -1$

29) ~~Verify intial value theorem~~ $\cancel{f(t) = 1 + e^{-t}[\sin t + \cos t}$

*

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UNIT-5

1. Find the family of curves $u=c_1$, $v=c_2$ cut orthogonally when $w=z^3$ ⟹ Unit-4
*
2. Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\cos\theta}$ by calculus of residue

***
3) Evaluate $\oint_C \frac{e^{2z}}{\cos \pi z} dz$ where C is circle $|z|=1$

***
4) Expand $F(z) = \frac{1}{(z-1)(z-2)}$ as Laurent series valid in the region (i) $|z|<1$, $1<|z|<2$, $|z|>2$

5) Expand e^{2z} about $z=2i$ in Taylor's series.

6) Find the image of triangular region in the z-plane bounded by the lines $x=0$, $y=0$, and $x+y=1$, under the transformation $\boxed{w=2z}$

7* Evaluate $\int_C \frac{z+4}{z^2+2z+5} dz$ where C is circle $|z+1+i|=2$ Using cauchy's integral formula

*
8. Evaluate $\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+1)(x^2+4)}$ using contour integration method.

**
9. Evaluate $\int_C \frac{z+1}{z(z+1)} dz$; $C: |z|=2$ Using cauchy's theorem.

~~10. Using cauchy's integral formula evaluate~~ $\cancel{\int_C \frac{z+4}{z^2+2z+5}}$

10. Evaluate $\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+9)(x^2+4)}$ Using ~~contour integration~~ method of residues
*

11. Expand $f(z) = \frac{z}{(z-1)(z-3)}$ as laurent series in the region $\boxed{0<|z-1|<2}$

12. Evaluate $\oint_C \tan z\, dz$ where c is circle $|z| = 2$

*13. Find the Laurent series of $\frac{1}{(z+1)(z+3)}$ into $0 < |z+1| < 2$

14. Evaluate $\oint_C \frac{ze^{2z}}{(z-1)^3} dz$ by using cauchy's integral formula where C is circle $|z+i| = 2$

15. Using cauchy's residue theorem, Evaluate $\oint_C \frac{z^2}{(z-1)^2(z+1)} dz$ where 'c' is $|z| = 2$.

16. $\int_0^{2\pi} \frac{d\theta}{1+\cos\theta}$ using contour method.

17 $f(z) = \frac{4z+3}{(z+1)z(z+3)}$ in laurent's series in the region given by

(i) $0 < |z+1| < 3$, $1 < |z| < 3$

18 Expand $\frac{z}{z^2+3z+2}$ using Taylor's series in the region $1 < |z| < 2$.

19. Evaluate using cauchy's integral formula $\int_C \frac{z\,dz}{(4z+1)(z-1)(z-2)}$ where c is the circle $|z| = 3$

20. Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$ using contour integration.

21 Expand $f(z) = \sin z$ in Talyor's series about $z = \pi/4$

22. Evaluate $\oint_C \frac{dz}{z^2(z+4)}$ where c is circle $|z| = 2$ using cauchy's integral formula.

23. $\int_C \frac{\cos \pi z^2}{(z-1)(z-2)} dz$ where c is circle $|z|=3$, using cauchy's integral formula.

24 * Using cauchy's residue theorem, Evaluate $\int_C \frac{\sin \pi z + \cos \pi z}{z+z^2} dz$ where c is circle $|z|=2$

* 25) Evaluate $\int_C \frac{(z+1)\,dz}{(z-1)(z-3)}$ where c is $|z|=2$ using residue.

* 26) * $\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta}$ using contour integration.

27) Evaluate $\oint_C \frac{e^{-z}}{e^{z+1}} dz$ where c is circle $|z|=2$ using cauchy's integral.

28) Expand $F(z) = \frac{1}{z(z-1)}$ as laurent's series in power valid in $|z|<1$ and $|z|>1$

29) Find the residue at pole of the function $F(z) = \frac{z}{z^2+1}$

30) Evaluate $\oint_C \frac{(\sin \pi z^2 + \cos \pi z^2)}{z+z^2} dz$, $|z|=2$ using cauchy's residue

31) Determine the poles of $F(z) = \frac{z^2}{(z-1)^2(z+2)}$ and then residues at one of the poles

32) ** Expand $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ in laurent's series $|z|<2$, $|z|>3$, $2<|z|<3$

33) $\int_0^{2\pi} \frac{1}{5+2\cos\theta} d\theta$ using contour integration.

$\underset{C}{\quad} \cos \pi z$

(34) $\oint_C \frac{(3z^2+z)}{z^2-1} dz$ where C is circle $|z-1| = 1$