Initial Value Theorem

1) Verify Initial and final value theorem for the function, $f(t) = 1 + e^{-t}(\sin t + \cos t)$.

Sol: Given:

$$f(t) = 1 + e^{-t}(\sin t + \cos t)$$

$$f(t) = 1 + [e^{-t}\sin t] + [e^{-t}\cos t]$$

$$L[e^{-t}\sin t] = \left[\frac{1}{s^2+1}\right]_{s \to s+1} = \frac{1}{(s+1)^2+1}$$

$$L[e^{-t}\cos t] = \left[\frac{s}{s^2+1^2}\right]_{s \to s+1} = \frac{s+1}{(s+1)^2+1}$$

$$\Rightarrow L[f(t)] = \frac{1}{s} + \frac{1}{(s+1)^2+1} + \frac{s+1}{(s+1)^2+1}$$

$$= \frac{1}{s} + \frac{s+1+1}{(s+1)^2+1}$$

$$L[f(t)] = \frac{1}{s} + \frac{s+2}{(s+1)^2+1}$$

$$\therefore F(s) = L[f(t)] = \frac{1}{s} + \frac{s+2}{(s+1)^2+1}$$

we know that, By I.V.T,

$$\boxed{\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)}$$

LHS

$$\lim_{t \to 0} [1 + e^{-t}\sin t + e^{-t}\cos t] \qquad \left[\because \begin{array}{l} e^{-0} = 1 \\ \sin 0 = 0 \\ \cos 0 = 1 \end{array}\right]$$

$$= 1 + 1$$

$$= 2.$$

RHS

$$\lim_{s \to \infty} sF(s) = \lim_{s \to \infty} s\left[\frac{1}{s} + \frac{s+2}{(s+1)^2+1}\right]$$

$$= \lim_{s \to \infty}\left[1 + \frac{s^2(1+2/s)}{s^2(1+2/s+2/s^2)}\right]$$

$$= 1 + 1 \qquad \left[\because \frac{1}{\infty} = 0\right]$$

$$= 2$$

So, LHS = RHS

Hence, Initial value theorem is verified //.

For Final Value Theorem

we know that, by F.V.T,

$$\boxed{\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)}$$

LHS $\lim_{t \to \infty} f(t) = \lim_{t \to \infty} [1 + e^{-t}\sin t + e^{-t}\cos t]$

$$= 1$$

RHS

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0} s\left[\frac{1}{s} + \frac{s+2}{(s+1)^2+1}\right]$$

$$= \lim_{s\to 0}\left[\frac{s}{s} + \frac{(s+2)s}{(s+1)^2+1}\right]$$

$$= \lim_{s\to 0}\left[1 + \frac{s^2+2s}{(s+1)^2+1}\right]$$

$$= 1+0$$

$$= 1$$

$\Rightarrow$ LHS = RHS

Hence, Final value theorem is verified//.

Periodic function.

2.) Find the Laplace Transformation of the triangular wave function,

$$f(t) = \begin{cases} t, & 0<t<a \\ 2a-t, & a<t<2a \end{cases}$$ with $f(t+2a) = f(t)$.

Sol: we know that,

$$\boxed{L[f(t)] = \frac{1}{1-e^{-Ps}}\int_0^P e^{-st} f(t)\,dt}$$

$\boxed{P = 2a}$.

$$= \frac{1}{1-e^{-2as}}\left[\int_0^a e^{-st}\cdot t\cdot dt + \int_0^{2a} e^{-st}(2a-t)dt\right]$$

$u_1 = t$, $dv = e^{-st}$, $u_2 = 2a-t$

$u_1' = 1$, $v_1 = \frac{e^{-st}}{-s}$, $u_2' = -1$

$u_1'' = 0$, $v_2 = \frac{e^{-st}}{s^2}$, $u_2'' = 0$

$$\therefore L[f(t)] = \frac{1}{1-e^{-2as}}\left\{\left[t\cdot\left(\frac{e^{-st}}{-s}\right) - \frac{e^{-st}(1)}{s^2}\right]_0^a + \left[(2a-t)\left(\frac{e^{-st}}{-s}\right) + \left(\frac{e^{-st}}{s^2}\right)\right]_0^{2a}\right\}$$

$$= \frac{1}{1-e^{-2as}}\left[\left(\frac{-a\cdot e^{-as}}{s} - \frac{e^{-as}}{s^2}\right) - \left(0-\frac{1}{s^2}\right)\right] + \left[\left(0+\frac{e^{-2as}}{s^2}\right) - \left(\frac{-ae^{-as}}{s} + \frac{e^{-as}}{s^2}\right)\right]$$

$$= \frac{1}{1-e^{-2as}}\left[\cancel{\frac{-ae^{-as}}{s}} - \frac{e^{-as}}{s^2} + \frac{1}{s^2} + \frac{e^{-2as}}{s^2} + \cancel{\frac{ae^{-as}}{s}} - \frac{e^{-as}}{s^2}\right]$$

$a^2 - 2ab + b^2$

$$= \frac{1}{1-e^{-2as}}\left[\frac{1-2e^{-as}+e^{-2as}}{s^2}\right]$$

(a^2-b^2)

$$\Rightarrow \frac{1}{s^2}\cdot\frac{1}{\cancel{(1-e^{-as})}(1+e^{-as})} \times \left[(1-e^{-as})^{\cancel{2}}\right]$$

$$= \frac{1}{s^2}\cdot\left(\frac{1-e^{-as}}{1+e^{-as}}\right)$$

$$= \frac{1}{s^2} \tanh\left(\frac{as}{2}\right) \quad \because \tanh\theta = \left(\frac{e^{\theta} - e^{-\theta}}{e^{\theta} + e^{-\theta}}\right)$$

3.) Find the Laplace Transformation of,

$$f(t) = \begin{cases} E, & 0 < t < a/2 \\ -E, & a/2 < t < a \end{cases}$$

Sol:

Given:

$$L[f(t)] = \frac{1}{1-e^{-ps}} \int_0^p e^{-st} \cdot f(t)\,dt$$

$$\boxed{p = a}$$

$$L[f(t)] = \frac{1}{1-e^{-as}} \left[\int_0^{a/2} e^{-st} \cdot E \cdot dt + \int_{a/2}^{a} e^{-st} \cdot -E \cdot dt \right]$$

$$= \frac{1}{1-e^{-as}} \left[E \int_0^{a/2} e^{-st} \cdot dt - E \int_{a/2}^{a} e^{-st} \cdot dt \right]$$

$$= \frac{E}{1-e^{-as}} \left[\left(\frac{e^{-st}}{-s}\right)_0^{a/2} - \left(\frac{e^{-st}}{-s}\right)_{a/2}^{a} \right]$$

$$= \frac{E}{1-e^{-as}} \left[\left(\frac{e^{-\frac{sa}{2}}}{-s} - \frac{1}{-s}\right) - \left(\frac{e^{-sa}}{-s} - \frac{e^{-\frac{sa}{2}}}{-s}\right) \right]$$

$$= \frac{E}{1-e^{-as}} \left[\frac{e^{-sa/2} - 1 - e^{-sa} + e^{-\frac{as}{2}}}{-s} \right]$$

$$= \frac{E}{1-e^{-as}} \left[\frac{1 + e^{-as} - 2e^{-\frac{as}{2}}}{s} \right]$$

$$= \frac{E}{1-e^{-as}} \times \frac{(1 - e^{-\frac{as}{2}})^2}{s}$$

$$= \frac{E}{\cancel{(1-e^{-as/2})}(1+e^{-as/2})} \times \frac{(1-e^{-\frac{sa}{2}})^{\cancel{2}}}{s}$$

$$= \frac{E}{s} \times \frac{(1 - e^{-as/2})}{(1 + e^{-as/2})}$$

$$\Rightarrow L[f(t)] = \frac{E}{s} \tanh\left(\frac{sa}{4}\right) \quad //$$

4) Find the Laplace Transformation of f

$$f(t) = \begin{cases} \sin\omega t, & 0 < t < \pi/\omega \\ 0, & \pi/\omega < t < \frac{2\pi}{\omega} \end{cases}$$

Sol:

Given: w.k.t,

$$L[f(t)] = \frac{1}{1-e^{-ps}} \int_0^p e^{-st} f(t)\,dt$$

$$\boxed{p = \frac{2\pi}{\omega}}$$

Note: $\int e^{ax} \sin bx = \frac{e^{ax}}{a^2+b^2}(a \sin bx - b\cos bx)$

$$\therefore L[f(t)] = \frac{1}{1-e^{-2\pi s/\omega}}\left[\int_0^{\pi/\omega} e^{-st}\sin\omega t \, dt + \int_{\pi/\omega}^{2\pi/\omega} e^{-st} \cdot 0 \cdot dt\right]$$

$$= \frac{1}{1-e^{\frac{-2\pi s}{\omega}}}\left[\frac{e^{-st}}{s^2+\omega^2}(-\sin\omega t \cdot s - \omega \cdot \cos\omega t)\right]_0^{\pi/\omega}$$

$$= \frac{1}{1-e^{\frac{-2\pi s}{\omega}}}\left[\left(\frac{e^{\frac{-s\pi}{\omega}}}{s^2+\omega^2}[-s(0)-\omega(-1)]\right)-\left(\frac{1}{s^2+\omega^2}(-s(0)-\omega(1))\right)\right]$$

$$= \frac{1}{1-e^{\frac{-2\pi s}{\omega}}}\left[\frac{e^{\frac{-s\pi}{\omega}}\cdot\omega}{s^2+\omega^2} + \frac{\omega}{s^2+\omega^2}\right]$$

$$= \left(\frac{1}{1-e^{\frac{-2\pi s}{\omega}}}\right) \times \frac{\omega}{s^2+\omega^2}\left[\left(1+e^{-s\pi/\omega}\right)\right]$$

$$= \frac{1}{(1-e^{-\pi s/\omega})\cancel{(1+e^{-\pi s/\omega})}} \times \frac{\omega}{s^2+\omega^2} \times \cancel{(1+e^{-s\pi/\omega})}$$

$$\Rightarrow L[f(t)] = \frac{\omega}{s^2+\omega^2} \times \left(\frac{1}{1-e^{-s\pi/\omega}}\right) \quad //$$

Convolution Theorem.

5.) Using convolution theorem, find

$$L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$$

Sol: Given. $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$

$$= L^{-1}\left[\frac{s}{s^2+a^2} \cdot \frac{s}{s^2+b^2}\right]$$

$$= L^{-1}\left[\frac{s}{s^2+a^2}\right] * L^{-1}\left[\frac{s}{s^2+b^2}\right]$$

$$= \cos at * \cos bt$$

By convolution,

$$\boxed{f(t) * g(t) = \int_0^t f(u)\, g(t-u)\, du}$$

$$\Rightarrow \cos at * \cos bt = \int_0^t \cos au - \cos(b(t-u)) \cdot du$$

$$\boxed{\cos A \cos B = \frac{\cos(A+B) + \cos(A-B)}{2}}$$

$$\Rightarrow \cos at * \cos bt = \int_0^t \cos(au) - \cos(bt-bu) \cdot du$$

Also, $\int \cos x = \sin x$, $\sin(-\theta) = -\sin\theta$

$$= \int_0^t \frac{\cos(au + bt - bu) + \cos(au - bt + bu)}{2} \cdot du$$

$$= \frac{1}{2} \int_0^t \left(\cos((a-b)u + bt) + \cos((a+b)u - bt)\right) \cdot du$$

$$= \frac{1}{2} \left[\frac{\sin((a-b)u + bt)}{a-b} + \frac{\sin((a+b)u - bt)}{a+b} \right]_0^t$$

$$= \frac{1}{2} \left[\left(\frac{\sin((a-b)t + bt)}{a-b} + \frac{\sin((a+b)t - bt)}{a+b} \right) - \left(\frac{\sin bt}{a-b} + \frac{\sin(-bt)}{a-b} \right) \right]$$

$$= \frac{1}{2} \left[\left(\frac{+\sin at}{a-b} + \frac{\sin at}{a+b} \right) - \left(\frac{\sin bt}{a-b} - \frac{\sin bt}{a+b} \right) \right]$$

$$= \frac{1}{2} \left[\sin at \left[\frac{1}{a-b} + \frac{1}{a+b} \right] - (\sin bt) \left[\frac{1}{a-b} - \frac{1}{a+b} \right] \right]$$

$$= \frac{1}{2} \left[\sin at \left(\frac{a + \cancel{b} + a - \cancel{b}}{(a-b)(a+b)} \right) - \sin bt \left(\frac{\cancel{a} + b - \cancel{a} + b}{(a-b)(a+b)} \right) \right]$$

$$= \frac{1}{2} \left[\sin at \cdot \frac{2a}{(a-b)(a+b)} - \sin bt \times \frac{2b}{(a-b)(a+b)} \right]$$

$$= \frac{1 \times \cancel{2}}{\cancel{2}\,(a-b)(a+b)} \times \left[a \sin at - b \sin bt \right]$$

$$= \frac{a \sin at - b \sin bt}{(a-b)(a+b)}$$

$$\Rightarrow \frac{a \sin at - b \sin bt}{a^2 - b^2}$$

$$\therefore L^{-1} \left[\frac{s^2}{(s^2+a^2)(s^2+b^2)} \right] = \frac{a \sin at - b \sin bt}{a^2 - b^2} \quad //$$

6.) Using convolution theorem, find $L^{-1} \left[\frac{1}{(s+a)(s+b)} \right]$

Sol: Given: $L^{-1} \left[\frac{1}{(s+a)(s+b)} \right] = L^{-1} \left[\frac{1}{s+a} \cdot \frac{1}{s+b} \right]$

$$= L^{-1} \left[\frac{1}{s+a} \right] * L^{-1} \left[\frac{1}{s+b} \right]$$

$$= e^{-at} * e^{-bt}.$$

by convolution theorem,

$$\boxed{f(t) * g(t) = \int_0^t f(u) \cdot g(t-u)\, du}$$

$$e^{-at} * e^{-bt} = \int_0^t e^{-au} \cdot e^{-b(t-u)}\, du$$

$$= \int_0^t e^{-au} \cdot e^{-bt} \cdot e^{bu} \cdot du$$

$$= e^{-bt}\left[\int_0^t e^{-u(a-b)}\right]$$

$$= e^{-bt}\left[\frac{e^{-u(a-b)}}{-(a-b)}\right]_0^t$$

$$= \frac{e^{-bt}}{-(a-b)} \cdot \left[e^{-t(a-b)} - 1\right]$$

$$= \frac{-e^{-bt}}{(a-b)}\left[e^{-ta} \cdot e^{bt} - 1\right]$$

$$= \frac{-1}{(a-b)}\left[e^{-ta} - e^{-bt}\right]$$

$$= \frac{e^{-bt} - e^{-at}}{a-b} \quad //$$

$$\Rightarrow L^{-1}\left[\frac{1}{(s+a)(s+b)}\right] = \frac{e^{-bt} - e^{-at}}{a-b} \quad //$$

7.) Using convolution theorem find, $L^{-1}\left[\frac{s+2}{(s^2+4s+13)^2}\right]$

Sol:
$$L^{-1}\left[\frac{s+2}{(s^2+4s+13)^2}\right] = L^{-1}\left[\frac{s+2}{(s^2+4s+13-4+4)^2}\right]$$

$$= L^{-1}\left[\frac{s+2}{((s+2)^2+3^2)^2}\right]$$

$$= L^{-1}\left[\frac{s+2}{(s+2)^2+3^2} \cdot \frac{1}{(s+2)^2+3^2}\right]$$

$$= L^{-1}\left[\frac{s+2}{(s+2)^2+3^2}\right] * L^{-1}\left[\frac{1}{(s+2)^2+3^2}\right]$$

$$= e^{-2t} \cdot L^{-1}\left[\frac{s}{s^2+3^2}\right] * e^{-2t} L^{-1}\left[\frac{1}{s^2+3^2}\right]$$

$$= e^{-2t}\cos 3t * e^{-2t} \cdot \frac{1}{3}\sin 3t.$$

by convolution theorem,

$$f(t) * g(t) = \int_0^t e^{-2u}\cos 3u \cdot e^{-2(t-u)} \cdot \frac{1}{3}\sin 3(t-u) \cdot du$$

$$= \int_0^t \cancel{e^{-2u}}\cos 3u \cdot e^{-2t} \cdot \cancel{e^{2u}} \cdot \frac{1}{3}\sin(3t-3u)\,du$$

$$= e^{-2t}\int_0^t \cos 3u \cdot \sin(3t-3u)\,du.$$

$$\because \cos A \cdot \sin B = \frac{\sin(A+B) - \sin(A-B)}{2}$$

$$\Rightarrow \frac{e^{-2t}}{3}\int_0^t \frac{\sin(\cancel{3u}+3t-\cancel{3u}) - \sin(3u-3t+3u)}{2}.\,du$$

$$= \frac{e^{-2t}}{6}\int_0^t \sin 3u - \sin(6u-3t)\,du$$

$$= \frac{e^{-2t}}{6}\left[\sin 3t\,[u]_0^t + \left[\frac{\cos(6u-3t)}{6}\right]_0^t\right]$$

$$= \frac{e^{-2t}}{6}\left[\sin 3t\,(t-0) + \left(\frac{\cancel{\cos 3t} - \cos(-3t)}{6}\right)\right]$$

$$= \frac{e^{-2t}}{6}\cdot t\sin 3t$$

$$\therefore\ L^{-1}\left[\frac{s+2}{(s^2+4s+13)^2}\right] = \frac{t\cdot e^{-2t}}{6}\sin 3t \quad //$$

DIFFERENTIAL EQUATION SOLVE. (Derivatives)

8.) Solve differential equation, $y''-3y'+2y = 4t+e^{3t}$ where $y(0)=1$ & $y'(0)=-1$.

Sol:

Given: $y''-3y'+2y = 4t+e^{3t} \Rightarrow (*)$

w.k.t,

$$L[y''(t)] = s^2L[f(t)] - sf(0) - f'(0) \quad - (1)$$

$$L[y'(t)] = sL[f(t)] - f(0) \rightarrow (2)$$

using L.T on both sides of $(*)$,

$$L[y''(t)] - 3L[y'(t)] + 2[y(t)] = 4L[t] + L[e^{3t}].$$

put (1) & (2),

$$\Rightarrow \left[s^2L[f(t)] - sf(0) - f'(0)\right] - 3\left[sL[f(t)] - f(0)\right] + 2\left[L[f(t)]\right] = \frac{4}{s^2} + \frac{1}{s-3}$$

$$= \left[s^2L[f(t)] - s(1) - (-1)\right] - 3\left[sL[f(t)] - 1\right] + 2\left[L[f(t)]\right] = \frac{4}{s^2} + \frac{1}{s-3}$$

$$= s^2L[f(t)] - s + 1 - 3sL[f(t)] + 3 + 2L[f(t)] = \frac{4(s-3)+s^2}{s^2(s-3)}$$

$$= L[f(t)]\left[s^2-3s+2\right] - s + 1 + 3 = \frac{4s-12+s^2}{s^2(s-3)}$$

$$L[f(t)](s^2-3s+2) - s + 4 = \frac{s^2+4s-12}{s^2(s-3)}$$

$$L[f(t)](s^2-3s+2) - s + 4 = \frac{s^2+4s-12}{s^2(s-3)}$$

$$L[f(t)](s^2-3s+2) = \frac{s^2+4s-12}{s^2(s-3)} + s - 4$$

$$L[f(t)](s^2-3s+2) = \frac{s^2+4s-12+(s-4)(s^3-3s^2)}{s^2(s-3)}$$

$$L[f(t)](s^2-3s+2) = \frac{s^2+4s-12+s^4-4s^3-3s^3+12s^2}{s^2(s-3)}$$

$$L[f(t)](s-1)(s-2) = \frac{s^4-7s^3+13s^2+4s-12}{s^2(s-3)}$$

$$L[f(t)] = \frac{s^4-7s^3+13s^2+4s-12}{s^2(s-3)(s-2)(s-1)} \rightarrow (A)$$

consider,

$$\frac{s^4-7s^3+13s^2+4s-12}{s^2(s-1)(s-2)(s-3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-3} + \frac{D}{s-2} + \frac{E}{s-1}.$$

$$s^4-7s^3+13s^2+4s-12 = A(s)(s-3)(s-2)(s-1) + B(s-3)(s-2)(s-1) + C(s^2)(s-2)(s-1) + D(s^2)(s-3)(s-1) + E(s^2)(s-3)(s-2)$$

Put $s=3$

$81-189+117+12-\cancel{12} = C(9)(1)(2)$

$9 = 2C \times \cancel{9}$

$1 = 2C$

$\Rightarrow \boxed{C = \frac{1}{2}}$

Put $s=2$

$16-56+52+8-12 = D(4)(-1)$

$-4D = 8$

$\boxed{D = -8}$

Put $s=1$

$1-7+13+4-12 = E(1)(-2)(-1)$

$-1 = 2E$

$\boxed{E = \frac{-1}{2}}$

Put $s=0$

$-12 = B(-3)(-2)(-1)$

$-12 = -6B$

$\boxed{B = 2}$

on coefficient of s^4,

$1 = A+C+D+E$

$1 = A + \frac{1}{2} - 2 - \frac{1}{2}$

$1 = A - 2$

$\boxed{A = 3}$

So, equation (A) can be written as,

$$\Rightarrow L[f(t)] = \frac{3}{s} + \frac{2}{s^2} + \frac{1/2}{s-3} + \frac{-2}{s-2} + \frac{-1/2}{s-1}.$$

$$f(t) = 3L^{-1}\left[\frac{1}{s}\right] + 2L^{-1}\left[\frac{1}{s^2}\right] + \frac{1}{2}L^{-1}\left[\frac{1}{s-3}\right] - 2L^{-1}\left[\frac{1}{s-2}\right] - \frac{1}{2}L^{-1}\left[\frac{1}{s-1}\right]$$

$$\Rightarrow f(t) = 3 + 2t + \frac{1}{2}e^{3t} - 2e^{2t} - \frac{1}{2}e^t$$

$$\therefore \boxed{f(t) = 3 + 2t + \frac{1}{2}e^{3t} - 2e^{2t} - \frac{1}{2}e^t}$$

9.) Solve: $(D^2+5D+6)y = 2$

Given: $y(0) = 0$; $y'(0) = 0$

Sol: Given $\rightarrow$ $(D^2+5D+6)y = 2$

$$y''(t) + 5y'(t) + 6y = 2.$$

using L.T on both sides,

$$L[y''(t)] + 5L[y'(t)] + 6L[y(t)] = 2L[1]$$

$$[s^2 L[y(t)] - s y(0) - y'(0)] + 5[sL[y(t)] - y(0)] + 6\,\cancel{[y(t)]}\,L[y(t)] = \frac{2}{s}$$

$$s^2 L[y(t)] - 0 - 0 + 5sL[y(t)] + 6L[y(t)] = \frac{2}{s}$$

$$L[y(t)](s^2+5s+6) = \frac{2}{s}$$

$$\Rightarrow L[y(t)] = \frac{2}{s(s^2+5s+6)}$$

$$L[y(t)] = \frac{2}{s(s+2)(s+3)}$$

$$\frac{2}{s(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$2 = A(s+2)(s+3) + B(s)(s+3) + C(s)(s+2)$$

Put $s=-2$

$2 = B(-2)(1)$

$2 = -2B$

$\boxed{B=-1}$

Put $s=0$

$2 = A(2)(3)$

$2 = 6A$

$A = \frac{2}{6} = \frac{1}{3}$

$\boxed{A = \frac{1}{3}}$

Put $s=-3$

$2 = C(-3)(-1)$

$2 = 3C$

$\boxed{C = \frac{2}{3}}$

$$\Rightarrow L[y(t)] = \frac{1/3}{s} + \frac{1}{s+2} + \frac{2/3}{s+3}$$

$$\therefore y(t) = \frac{1}{3}L^{-1}\left[\frac{1}{s}\right] + 1L^{-1}\left[\frac{1}{s+2}\right] + \frac{2}{3}L^{-1}\left[\frac{1}{s+3}\right]$$

$$= \frac{1}{3}(1) + 1(e^{-2t}) + \frac{2}{3}e^{-3t}$$

$$= \frac{1 + 3e^{-2t} + 2e^{-3t}}{3}$$

$$\Rightarrow \boxed{y(t) = \frac{1}{3}(1 + 3e^{-2t} + 2e^{-3t})}$$

10.) Solve: $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 2$

by $x(0) = 0$, $x'(0) = 5$ for $t=0$.

Sol:

Given $x'' - 3x' + 2x = 2$

W.k.t,

$$L[x''(t)] = s^2 L[x(t)] - s x(0) - x'(0) \rightarrow *$$

$$L[x'(t)] = sL[x(t)] - x(0) \rightarrow **$$

using L.T on both sides,

$$\rightarrow L[x''(t)] - 3L[x'(t)] + 2L[x(t)] = L[2]$$

$$\rightarrow [s^2 L[x(t)] - s x(0) - x'(0)] - 3[sL[x(t)] - x(0)] + 2L[x(t)] = \frac{2}{s}$$

$$\rightarrow s^2 L[x(t)] - 0 - 5 - 3sL[x(t)] + 0 + 2L[x(t)] = \frac{2}{s}$$

$$L[x(t)](s^2 - 3s + 2) = \frac{2}{s} + 5$$

$$L[x(t)] = \frac{2+5s}{s(s-1)(s-2)}$$

by partial fraction,

$$\frac{2+5s}{s(s-1)(s-2)} = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s-2}$$

$$2+5s = A(s-1)(s-2) + B(s)(s-2) + C(s)(s-1)$$

Put $s=0$	Put $s=1$	Put $s=2$
$2 = A(-1)(-2)$	$2+5 = B(1)(-1)$	$2+10 = C(2)(1)$
$2 = 2A$	$7 = -B$	$12 = 2C$
$\boxed{A=1}$	$\boxed{B=-7}$	$\boxed{C=6}$

$$L[x(t)] = \frac{1}{s} + \frac{-7}{s-1} + \frac{6}{s-2}$$

$$x(t) = L^{-1}\left[\frac{1}{s}\right] - 7L^{-1}\left[\frac{1}{s-1}\right] + 6L^{-1}\left[\frac{1}{s-2}\right]$$

$$= 1 - 7e^{t} + 6e^{2t}$$

$$\boxed{x(t) = 1 - 7e^{t} + 6e^{2t}}$$ //