

GAT Formulas

Results on Triangle:

Sum of the angles of a triangle is 180° .

The sum of any two sides of a triangle is greater than the third side.

Pythagoras Theorem:

In a right-angled triangle, $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$.

The line joining the mid-point of a side of a triangle to the opposite vertex is called the median.

The point where the three medians of a triangle meet, is called centroid. The centroid divides each of the medians in the ratio 2 : 1.

In an isosceles triangle, the altitude from the vertex bisects the base.

The median of a triangle divides it into two triangles of the same area.

The area of the triangle formed by joining the mid-points of the sides of a given triangle is one-fourth of the area of the given triangle.

Results on Quadrilaterals

The diagonals of a parallelogram bisect each other.

Each diagonal of a parallelogram divides it into triangles of the same area.

The diagonals of a rectangle are equal and bisect each other.

The diagonals of a square are equal and bisect each other at right angles.

The diagonals of a rhombus are unequal and bisect each other at right angles.

A parallelogram and a rectangle on the same base and between the same parallels are equal in area.

Of all the parallelogram of given sides, the parallelogram which is a rectangle has the greatest area.

IMPORTANT FORMULAE

1. Area of a rectangle = (Length x Breadth).

$$\therefore \text{Length} = \left(\frac{\text{Area}}{\text{Breadth}} \right) \text{ and Breadth} = \left(\frac{\text{Area}}{\text{Length}} \right).$$

2. Perimeter of a rectangle = $2(\text{Length} + \text{Breadth})$.

Area of a square = $(\text{side})^2 = \frac{1}{2}(\text{diagonal})^2$.

Area of 4 walls of a room = $2(\text{Length} + \text{Breadth}) \times \text{Height}$.

1. Area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$.

2. Area of a triangle = $s(s-a)(s-b)(s-c)$

where a, b, c are the sides of the triangle and $s = \frac{1}{2}(a + b + c)$.

3. Area of an equilateral triangle = $\frac{3}{4} \times (\text{side})^2$

4. Radius of incircle of an equilateral triangle of side $a = \frac{a}{2\sqrt{3}}$.

5. Radius of circumcircle of an equilateral triangle of side $a = \frac{a}{\sqrt{3}}$.

6. Radius of incircle of a triangle of area Δ and semi-perimeter $s = \frac{\Delta}{s}$

1. Area of parallelogram = (Base x Height).

2. Area of a rhombus = $\frac{1}{2} \times (\text{Product of diagonals})$.

3. Area of a trapezium = $\frac{1}{2} \times (\text{sum of parallel sides}) \times \text{distance between them}$.

1. Area of a circle = πR^2 , where R is the radius.
2. Circumference of a circle = $2\pi R$.
3. Length of an arc = $\frac{2\pi R\theta}{360}$, where θ is the central angle.
4. Area of a sector = $\frac{1}{2}(\text{arc} \times R) = \frac{\pi R^2\theta}{360}$.
1. Circumference of a semi-circle = πR .
2. Area of semi-circle = $\frac{\pi R^2}{2}$.

CUBOID

Let length = l , breadth = b and height = h units. Then

Volume = $(l \times b \times h)$ cubic units.

Surface area = $2(lb + bh + lh)$ sq. units.

Diagonal = $l^2 + b^2 + h^2$ units.

CUBE

Let each edge of a cube be of length a . Then,

Volume = a^3 cubic units.

Surface area = $6a^2$ sq. units.

Diagonal = $3a$ units.

CYLINDER

Let radius of base = r and Height (or length) = h . Then,

Volume = $(\pi r^2 h)$ cubic units.

Curved surface area = $(2\pi rh)$ sq. units.

Total surface area = $2\pi r(h + r)$ sq. units.

CONE

Let radius of base = r and Height = h . Then,

Slant height, $l = \sqrt{h^2 + r^2}$ units.

Volume = $\left(\frac{1}{3}\pi r^2 h\right)$ cubic units.

Curved surface area = (πrl) sq. units.

Total surface area = $(\pi rl + \pi r^2)$ sq. units.

SPHERE

Let the radius of the sphere be r . Then,

Volume = $\left(\frac{4}{3}\pi r^3\right)$ cubic units.

Surface area = $(4\pi r^2)$ sq. units.

HEMISPHERE

Let the radius of a hemisphere be r . Then,

Volume = $\left(\frac{2}{3}\pi r^3\right)$ cubic units.

Curved surface area = $(2\pi r^2)$ sq. units.

Total surface area = $(3\pi r^2)$ sq. units.

Note: 1 litre = 1000 cm³.

Work from Days:

If A can do a piece of work in n days, then A's 1 day's work = $\frac{1}{n}$.

Days from Work:

If A's 1 day's work = $\frac{1}{n}$, then A can finish the work in n days.

Ratio:

If A is thrice as good a workman as B, then:

Ratio of work done by A and B = 3 : 1.

Ratio of times taken by A and B to finish a work = 1 : 3.

IMPORTANT FACTS

Cost Price:

The price, at which an article is purchased, is called its cost price, abbreviated as C.P.

Selling Price:

The price, at which an article is sold, is called its selling prices, abbreviated as S.P.

Profit or Gain:

If S.P. is greater than C.P., the seller is said to have a profit or gain.

Loss:

If S.P. is less than C.P., the seller is said to have incurred a loss.

IMPORTANT FORMULAE

Gain = (S.P.) - (C.P.)

Loss = (C.P.) - (S.P.)

Loss or gain is always reckoned on C.P.

Gain Percentage: (Gain %)

$$\text{Gain \%} = \left(\frac{\text{Gain} \times 100}{\text{C.P.}} \right)$$

Loss Percentage: (Loss %)

$$\text{Loss \%} = \left(\frac{\text{Loss} \times 100}{\text{C.P.}} \right)$$

Selling Price: (S.P.)

$$\text{SP} = \left[\frac{(100 + \text{Gain \%})}{100} \times \text{C.P.} \right]$$

Selling Price: (S.P.)

$$\text{SP} = \left[\frac{(100 - \text{Loss \%})}{100} \times \text{C.P.} \right]$$

Cost Price: (C.P.)

$$\text{C.P.} = \left[\frac{100}{(100 + \text{Gain \%})} \times \text{S.P.} \right]$$

Cost Price: (C.P.)

$$\text{C.P.} = \left[\frac{100}{(100 - \text{Loss \%})} \times \text{S.P.} \right]$$

If an article is sold at a gain of say 35%, then S.P. = 135% of C.P.

If an article is sold at a loss of say, 35% then S.P. = 65% of C.P.

When a person sells two similar items, one at a gain of say $x\%$, and the other at a loss of $x\%$, then the seller always incurs a loss given by:

$$\text{Loss \%} = \left(\frac{\text{Common Loss and Gain \%}}{10} \right)^2 = \left(\frac{x}{10} \right)^2.$$

If a trader professes to sell his goods at cost price, but uses false weights, then

$$\text{Gain \%} = \left[\frac{\text{Error}}{(\text{True Value}) - (\text{Error})} \times 100 \right] \%. \quad \text{--- (1)}$$

Important Formulas on "Problems on Ages" :

1. If the current age is x , then n times the age is nx .

2. If the current age is x , then age n years later/hence $= x + n$.

3. If the current age is x , then age n years ago $= x - n$.

4. The ages in a ratio $a : b$ will be ax and bx .

5. If the current age is x , then $\frac{1}{N}$ of the age is $\frac{x}{n}$.

Average:

$$\text{Average} = \left(\frac{\text{Sum of observations}}{\text{Number of observations}} \right)$$

Average Speed:

Suppose a man covers a certain distance at x kmph and an equal distance at y kmph.

Then, the average speed during the whole journey is $\left(\frac{2xy}{x+y} \right)$ kmph.

Factorial Notation:

Let n be a positive integer. Then, factorial n , denoted $n!$ is defined as:

$$n! = n(n-1)(n-2) \dots 3.2.1.$$

Examples:

We define $0! = 1$.

$$4! = (4 \times 3 \times 2 \times 1) = 24.$$

$$5! = (5 \times 4 \times 3 \times 2 \times 1) = 120.$$

Permutations:

The different arrangements of a given number of things by taking some or all at a time, are called permutations.

Examples:

All permutations (or arrangements) made with the letters a, b, c by taking two at a time are (ab, ba, ac, ca, bc, cb) .

All permutations made with the letters a, b, c taking all at a time are:

$(abc, acb, bac, bca, cab, cba)$

Number of Permutations:

Number of all permutations of n things, taken r at a time, is given by:

$${}^n P_r = n(n-1)(n-2) \dots (n-r+1) = \frac{n!}{(n-r)!}$$

Examples:

$${}^6 P_2 = (6 \times 5) = 30.$$

$${}^7 P_3 = (7 \times 6 \times 5) = 210.$$

Cor. number of all permutations of n things, taken all at a time $= n!$.

An Important Result:

If there are n subjects of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of third kind and so on and p_r are alike of r^{th} kind,

such that $(p_1 + p_2 + \dots + p_r) = n$.

Then, number of permutations of these n objects is $= \frac{n!}{(p_1!)(p_2!) \dots (p_r!)}$

Combinations:

Each of the different groups or selections which can be formed by taking some or all of a number of objects is called a combination.

Examples:

Suppose we want to select two out of three boys A, B, C. Then, possible selections are AB, BC and CA.

Note: AB and BA represent the same selection.

All the combinations formed by a, b, c taking ab, bc, ca .

The only combination that can be formed of three letters a, b, c taken all at a time is abc .

Various groups of 2 out of four persons A, B, C, D are:

AB, AC, AD, BC, BD, CD.

Note that ab ba are two different permutations but they represent the same combination.

Number of Combinations:

The number of all combinations of n things, taken r at a time is:

$${}^nC_r = \frac{n!}{(r!)(n-r)!} = \frac{n(n-1)(n-2) \dots \text{to } r \text{ factors}}{r!}.$$

Note:

$${}^nC_n = 1 \text{ and } {}^nC_0 = 1.$$

$${}^nC_r = {}^nC_{(n-r)}$$

Examples:

$$\text{i. } {}^{11}C_4 = \frac{(11 \times 10 \times 9 \times 8)}{(4 \times 3 \times 2 \times 1)} = 330.$$

$$\text{ii. } {}^{16}C_{13} = {}^{16}C_{(16-13)} = {}^{16}C_3 = \frac{16 \times 15 \times 14}{3!} = \frac{16 \times 15 \times 14}{3 \times 2 \times 1} = 560.$$

Factors and Multiples:

If number a divided another number b exactly, we say that a is a factor of b .

In this case, b is called a multiple of a .

Highest Common Factor (H.C.F.) or Greatest Common Measure (G.C.M.) or Greatest Common Divisor (G.C.D.):

The H.C.F. of two or more than two numbers is the greatest number that divides each of them exactly.

There are two methods of finding the H.C.F. of a given set of numbers:

Factorization Method: Express the each one of the given numbers as the product of prime factors. The product of least powers of common prime factors gives H.C.F.

Division Method: Suppose we have to find the H.C.F. of two given numbers, divide the larger by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is required H.C.F.

Finding the H.C.F. of more than two numbers: Suppose we have to find the H.C.F. of three numbers, then, H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given number. Similarly, the H.C.F. of more than three numbers may be obtained.

Least Common Multiple (L.C.M.):

The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

There are two methods of finding the L.C.M. of a given set of numbers:

Factorization Method: Resolve each one of the given numbers into a product of prime factors. Then, L.C.M. is the product of highest powers of all the factors.

Division Method (short-cut): Arrange the given numbers in a row in any order. Divide by a number which divided exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers.

Product of two numbers = Product of their H.C.F. and L.C.M.

Co-primes: Two numbers are said to be co-primes if their H.C.F. is 1.

H.C.F. and L.C.M. of Fractions:

$$1. \text{ H.C.F.} = \frac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$$

$$2. \text{ L.C.M.} = \frac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$$

H.C.F. and L.C.M. of Decimal Fractions:

In a given numbers, make the same number of decimal places by annexing zeros in some numbers, if necessary. Considering these numbers without decimal point, find H.C.F. or L.C.M. as the case may be. Now, in the result, mark off as many decimal places as are there in each of the given numbers.

Comparison of Fractions:

Find the L.C.M. of the denominators of the given fractions. Convert each of the fractions into an equivalent fraction with L.C.M as the denominator, by multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.

Square Root:

If $x^2 = y$, we say that the square root of y is x and we write $y = x$.

Thus, $4 = 2$, $9 = 3$, $196 = 14$.

Cube Root:

The cube root of a given number x is the number whose cube is x .

We, denote the cube root of x by x .

Thus, $8 = 2 \times 2 \times 2 = 2$, $343 = 7 \times 7 \times 7 = 7$ etc.

Note:

1. $xy = x \times y$

$$2. \quad \frac{x}{y} = \frac{x}{y} = \frac{x}{y} \times \frac{y}{y} = \frac{xy}{y}.$$

Stock Capital:

The total amount of money needed to run the company is called the stock capital.

Shares or Stock:

The whole capital is divided into small units, called shares or stock.

For each investment, the company issues a 'share-certificate', showing the value of each share and the number of shares held by a person.

The person who subscribes in shares or stock is called a share holder or stock holder.

Dividend:

The annual profit distributed among share holders is called dividend.

Dividend is paid annually as per share or as a percentage.

Face Value:

The value of a share or stock printed on the share-certificate is called its Face Value or Nominal Value or Par Value.

Market Value:

The stock of different companies are sold and bought in the open market through brokers at stock-exchanges. A share or stock is said to be:

At premium or Above par, if its market value is more than its face value.

At par, if its market value is the same as its face value.

At discount or Below par, if its market value is less than its face value.

Thus, if a Rs. 100 stock is quoted at premium of 16, then market value of the stock = Rs. $(100 + 16)$ = Rs. 116.

Likewise, if a Rs. 100 stock is quoted at a discount of 7, then market value of the stock = Rs. $(100 - 7)$ = 93.

Brokerage:

The broker's charge is called brokerage.

(i) When stock is purchased, brokerage is added to the cost price.

(ii) When stock is sold, brokerage is subtracted from the selling price.

Remember:

The face value of a share always remains the same.

The market value of a share changes from time to time.

Dividend is always paid on the face value of a share.

Number of shares held by a person

$$= \frac{\text{Total Investment}}{\text{Investment in 1 share}} = \frac{\text{Total Income}}{\text{Income from 1 share}} = \frac{\text{Total Face Value}}{\text{Face of 1 share}}.$$

Thus, by a Rs. 100, 9% stock at 120, we mean that:

Face Value of stock = Rs. 100.

Market Value (M.V) of stock = Rs. 120.

Annual dividend on 1 share = 9% of face value = 9% of Rs. 100 = Rs. 9.

An investment of Rs. 120 gives an annual income of Rs. 9.

Rate of interest p.a = Annual income from an investment of Rs. 100

$$= \left(\frac{9}{120} \times 100 \right) \% = 7\frac{1}{2}\%.$$

Speed, Time and Distance:

$$\text{Speed} = \left(\frac{\text{Distance}}{\text{Time}} \right), \text{Time} = \left(\frac{\text{Distance}}{\text{Speed}} \right), \text{Distance} = (\text{Speed} \times \text{Time}).$$

km/hr to m/sec conversion:

$$x \text{ km/hr} = \left(x \times \frac{5}{18} \right) \text{ m/sec}.$$

m/sec to km/hr conversion:

$$x \text{ m/sec} = \left(x \times \frac{18}{5} \right) \text{ km/hr}.$$

If the ratio of the speeds of A and B is $a : b$, then the ratio of the

the times taken by them to cover the same distance is $\frac{1}{a} : \frac{1}{b}$ or $b : a$.

Suppose a man covers a certain distance at x km/hr and an equal distance at y km/hr. Then,

the average speed during the whole journey is $\left(\frac{2xy}{x+y} \right)$ km/hr.

Partnership:

When two or more than two persons run a business jointly, they are called partners and the deal is known as partnership.

Ratio of Divisions of Gains:

When investments of all the partners are for the same time, the gain or loss is distributed among the partners in the ratio of their investments.

Suppose A and B invest Rs. x and Rs. y respectively for a year in a business, then at the end of the year:

(A's share of profit) : (B's share of profit) = $x : y$.

When investments are for different time periods, then equivalent capitals are calculated for a unit of time by taking (capital \times number of units of time). Now gain or loss is divided in the ratio of these capitals.

Suppose A invests Rs. x for p months and B invests Rs. y for q months then,

(A's share of profit) : (B's share of profit) = $xp : yq$.

Working and Sleeping Partners:

A partner who manages the business is known as a working partner and the one who simply invests the money is a sleeping partner.

Odd Days:

We are supposed to find the day of the week on a given date.

For this, we use the concept of 'odd days'.

In a given period, the number of days more than the complete weeks are called odd days.

Leap Year:

(i). Every year divisible by 4 is a leap year, if it is not a century.

(ii). Every 4th century is a leap year and no other century is a leap year.

Note: A leap year has 366 days.

Examples:

Each of the years 1948, 2004, 1676 etc. is a leap year.

Each of the years 400, 800, 1200, 1600, 2000 etc. is a leap year.

None of the years 2001, 2002, 2003, 2005, 1800, 2100 is a leap year.

Ordinary Year:

The year which is not a leap year is called an ordinary year. An ordinary year has 365 days.

Counting of Odd Days:

1 ordinary year = 365 days = (52 weeks + 1 day.)

∴ 1 ordinary year has 1 odd day.

1 leap year = 366 days = (52 weeks + 2 days)

∴ 1 leap year has 2 odd days.

100 years = 76 ordinary years + 24 leap years

= (76 × 1 + 24 × 2) odd days = 124 odd days.

= (17 weeks + days) = 5 odd days.

∴ Number of odd days in 100 years = 5.

Number of odd days in 200 years = (5 × 2) = 3 odd days.

Number of odd days in 300 years = (5 × 3) = 1 odd day.

Number of odd days in 400 years = (5 × 4 + 1) = 0 odd day.

Similarly, each one of 800 years, 1200 years, 1600 years, 2000 years etc. has 0 odd days.

Day of the Week Related to Odd Days:

No. of days:	0	1	2	3	4	5	6
Day:	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.

Some Basic Formulae:

$$(a + b)(a - b) = (a^2 - b^2)$$

$$(a + b)^2 = (a^2 + b^2 + 2ab)$$

$$(a - b)^2 = (a^2 + b^2 - 2ab)$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$$

$$(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$$

$$(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$$

When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

Decimal Fractions:

Fractions in which denominators are powers of 10 are known as decimal fractions.

$$\text{Thus, } \frac{1}{10} = 1 \text{ tenth} = .1; \quad \frac{1}{100} = 1 \text{ hundredth} = .01;$$

$$\frac{99}{100} = 99 \text{ hundredths} = .99; \quad \frac{7}{1000} = 7 \text{ thousandths} = .007, \text{ etc.};$$

Conversion of a Decimal into Vulgar Fraction:

Put 1 in the denominator under the decimal point and annex with it as many zeros as is the number of digits after the decimal point. Now, remove the decimal point and reduce the fraction to its lowest terms.

$$\text{Thus, } 0.25 = \frac{25}{100} = \frac{1}{4}; \quad 2.008 = \frac{2008}{1000} = \frac{251}{125}.$$

Annexing Zeros and Removing Decimal Signs:

Annexing zeros to the extreme right of a decimal fraction does not change its value. Thus, $0.8 = 0.80 = 0.800$, etc.

If numerator and denominator of a fraction contain the same number of decimal places, then we remove the decimal sign.

$$\text{Thus, } \frac{1.84}{2.99} = \frac{184}{299} = \frac{8}{13}.$$

Operations on Decimal Fractions:

Addition and Subtraction of Decimal Fractions: The given numbers are so placed under each other that the decimal points lie in one column. The numbers so arranged can now be added or subtracted in the usual way.

Multiplication of a Decimal Fraction By a Power of 10: Shift the decimal point to the right by as many places as is the power of 10.

Thus, $5.9632 \times 100 = 596.32$; $0.073 \times 10000 = 730$.

Multiplication of Decimal Fractions: Multiply the given numbers considering them without decimal point. Now, in the product, the decimal point is marked off to obtain as many places of decimal as is the sum of the number of decimal places in the given numbers.

Suppose we have to find the product $(.2 \times 0.02 \times .002)$.

Now, $2 \times 2 \times 2 = 8$. Sum of decimal places = $(1 + 2 + 3) = 6$.

$\therefore .2 \times .02 \times .002 = .000008$

Dividing a Decimal Fraction By a Counting Number: Divide the given number without considering the decimal point, by the given counting number. Now, in the quotient, put the decimal point to give as many places of decimal as there are in the dividend.

Suppose we have to find the quotient $(0.0204 \div 17)$. Now, $204 \div 17 = 12$.

Dividend contains 4 places of decimal. So, $0.0204 \div 17 = 0.0012$

Dividing a Decimal Fraction By a Decimal Fraction: Multiply both the dividend and the divisor by a suitable power of 10 to make divisor a whole number.

Now, proceed as above.

$$\text{Thus, } \frac{0.00066}{0.11} = \frac{0.00066 \times 100}{0.11 \times 100} = \frac{0.066}{11} = .006$$

Comparison of Fractions:

Suppose some fractions are to be arranged in ascending or descending order of magnitude, then convert each one of the given fractions in the decimal form, and arrange them accordingly.

Let us to arrange the fractions $\frac{3}{5}$, $\frac{6}{7}$ and $\frac{7}{9}$ in descending order.

Now, $\frac{3}{5} = 0.6$, $\frac{6}{7} = 0.857$, $\frac{7}{9} = 0.777...$

Since, $0.857 > 0.777... > 0.6$. So, $\frac{6}{7} > \frac{7}{9} > \frac{3}{5}$.

Recurring Decimal:

If in a decimal fraction, a figure or a set of figures is repeated continuously, then such a number is called a recurring decimal.

In a recurring decimal, if a single figure is repeated, then it is expressed by putting a dot on it. If a set of figures is repeated, it is expressed by putting a bar on the set.

Thus, $\frac{1}{3} = 0.333... = 0.\dot{3}$; $\frac{22}{7} = 3.142857142857.... = 3.14285\bar{7}$.

Pure Recurring Decimal: A decimal fraction, in which all the figures after the decimal point are repeated, is called a pure recurring decimal.

Converting a Pure Recurring Decimal into Vulgar Fraction: Write the repeated figures only once in the numerator and take as many nines in the denominator as is the number of repeating figures.

Thus, $0.\dot{5} = \frac{5}{9}$; $0.5\bar{3} = \frac{53}{99}$; $0.06\bar{7} = \frac{67}{999}$, etc.

Mixed Recurring Decimal: A decimal fraction in which some figures do not repeat and some of them are repeated, is called a mixed recurring decimal.

Eg. $0.1733333... = 0.17\bar{3}$.

Converting a Mixed Recurring Decimal Into Vulgar Fraction: In the numerator, take the difference between the number formed by all the digits after decimal point (taking repeated digits only once) and that formed by the digits which are not repeated. In the denominator, take the number formed by as many nines as there are repeating digits followed by as many zeros as is the number of non-repeating digits.

$$\text{Thus, } 0.16 = \frac{16 - 1}{90} = \frac{15}{90} = \frac{1}{6} ; \quad 0.2273 = \frac{2273 - 22}{9900} = \frac{2251}{9900}.$$

Some Basic Formulae:

$$(a + b)(a - b) = (a^2 - b^2)$$

$$(a + b)^2 = (a^2 + b^2 + 2ab)$$

$$(a - b)^2 = (a^2 + b^2 - 2ab)$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$$

$$(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$$

$$(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$$

$$\text{When } a + b + c = 0, \text{ then } a^3 + b^3 + c^3 = 3abc.$$

Experiment:

An operation which can produce some well-defined outcomes is called an experiment.

Random Experiment:

An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance, is called a random experiment.

Examples:

Rolling an unbiased dice.

Tossing a fair coin.

Drawing a card from a pack of well-shuffled cards.

Picking up a ball of certain colour from a bag containing balls of different colours.

Details:

When we throw a coin, then either a Head (H) or a Tail (T) appears.

A dice is a solid cube, having 6 faces, marked 1, 2, 3, 4, 5, 6 respectively. When we throw a die, the outcome is the number that appears on its upper face.

A pack of cards has 52 cards.

It has 13 cards of each suit, name Spades, Clubs, Hearts and Diamonds.

Cards of spades and clubs are black cards.

Cards of hearts and diamonds are red cards.

There are 4 honours of each unit.

There are Kings, Queens and Jacks. These are all called face cards.

Sample Space:

When we perform an experiment, then the set S of all possible outcomes is called the sample space.

Examples:

In tossing a coin, $S = \{H, T\}$

If two coins are tossed, the $S = \{HH, HT, TH, TT\}$.

In rolling a dice, we have, $S = \{1, 2, 3, 4, 5, 6\}$.

Event:

Any subset of a sample space is called an event.

Probability of Occurrence of an Event:

Let S be the sample and let E be an event.

Then, $E \subseteq S$.

$$\therefore P(E) = \frac{n(E)}{n(S)}.$$

Results on Probability:

$$P(S) = 1$$

$$0 \leq P(E) \leq 1$$

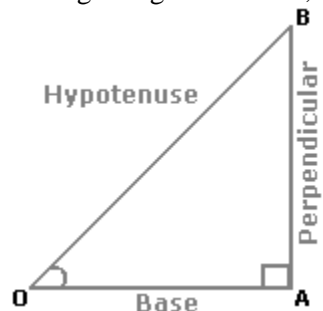
$$P(\Phi) = 0$$

For any events A and B we have : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A denotes (not-A), then $P(A) = 1 - P(A)$.

Trigonometry:

In a right angled Δ OAB, where $\angle BOA = \theta$,



$$\text{i. } \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{OB};$$

$$\text{ii. } \cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{OA}{OB};$$

$$\text{iii. } \tan \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AB}{OA};$$

$$\text{iv. } \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{OB}{AB};$$

$$\text{v. } \sec \theta = \frac{1}{\cos \theta} = \frac{OB}{OA};$$

$$\text{vi. } \cot \theta = \frac{1}{\tan \theta} = \frac{OA}{AB};$$

Trigonometrical Identities:

$$\sin^2 \theta + \cos^2 \theta = 1.$$

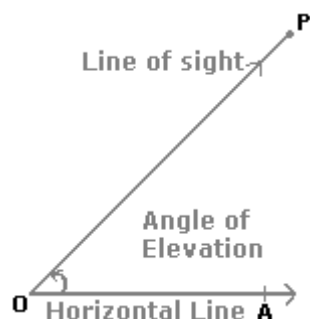
$$1 + \tan^2 \theta = \sec^2 \theta.$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta.$$

Values of T-ratios:

θ	0°	$(\pi/6)$ 30°	$(\pi/4)$ 45°	$(\pi/3)$ 60°	$(\pi/2)$ 90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	1
$\cos \theta$	1	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{3}$	1	3	not defined

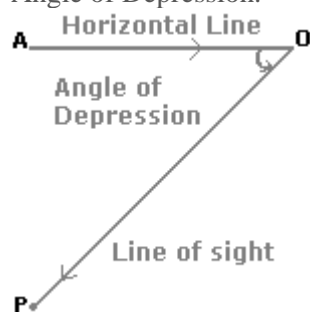
Angle of Elevation:



Suppose a man from a point O looks up at an object P, placed above the level of his eye. Then, the angle which the line of sight makes with the horizontal through O, is called the angle of elevation of P as seen from O.

∴ Angle of elevation of P from O = $\angle AOP$.

Angle of Depression:



Suppose a man from a point O looks down at an object P, placed below the level of his eye, then the angle which the line of sight makes with the horizontal through O, is called the angle of depression of P as seen from O.

Concept of Percentage:

By a certain percent, we mean that many hundredths.

Thus, x percent means x hundredths, written as $x\%$.

To express $x\%$ as a fraction: We have, $x\% = \frac{x}{100}$.

$$\text{Thus, } 20\% = \frac{20}{100} = \frac{1}{5}.$$

To express $\frac{a}{b}$ as a percent: We have, $\frac{a}{b} = \left(\frac{a}{b} \times 100 \right)\%$.

$$\text{Thus, } \frac{1}{4} = \left(\frac{1}{4} \times 100 \right)\% = 25\%.$$

Percentage Increase/Decrease:

If the price of a commodity increases by $R\%$, then the reduction in consumption so as not to increase the expenditure is:

$$\left[\frac{R}{(100 + R)} \times 100 \right]\%$$

If the price of a commodity decreases by $R\%$, then the increase in consumption so as not to decrease the expenditure is:

$$\left[\frac{R}{(100 - R)} \times 100 \right]\%$$

Results on Population:

Let the population of a town be P now and suppose it increases at the rate of $R\%$ per annum, then:

$$1. \text{ Population after } n \text{ years} = P \left(1 + \frac{R}{100} \right)^n$$

$$2. \text{ Population } n \text{ years ago} = \frac{P}{\left(1 + \frac{R}{100} \right)^n}$$

Results on Depreciation:

Let the present value of a machine be P. Suppose it depreciates at the rate of R% per annum. Then:

$$1. \text{ Value of the machine after } n \text{ years} = P \left(1 - \frac{R}{100} \right)^n$$

$$2. \text{ Value of the machine } n \text{ years ago} = \frac{P}{\left(1 - \frac{R}{100} \right)^n}$$

$$3. \text{ If A is R\% more than B, then B is less than A by } \left[\frac{R}{(100 + R)} \times 100 \right] \%.$$

$$4. \text{ If A is R\% less than B, then B is more than A by } \left[\frac{R}{(100 - R)} \times 100 \right] \%.$$

Minute Spaces:

The face or dial of watch is a circle whose circumference is divided into 60 equal parts, called minute spaces.

Hour Hand and Minute Hand:

A clock has two hands, the smaller one is called the hour hand or short hand while the larger one is called minute hand or long hand.

In 60 minutes, the minute hand gains 55 minutes on the hour on the hour hand.

In every hour, both the hands coincide once.

The hands are in the same straight line when they are coincident or opposite to each other.

When the two hands are at right angles, they are 15 minute spaces apart.

When the hands are in opposite directions, they are 30 minute spaces apart.

Angle traced by hour hand in 12 hrs = 360°

Angle traced by minute hand in 60 min. = 360° .

If a watch or a clock indicates 8.15, when the correct time is 8, it is said to be 15 minutes too fast.

On the other hand, if it indicates 7.45, when the correct time is 8, it is said to be 15 minutes too slow.

'BODMAS' Rule:

This rule depicts the correct sequence in which the operations are to be executed, so as to find out the value of given expression.

Here B - Bracket,

O - of,

D - Division,

M - Multiplication,

A - Addition and

S - Subtraction

Thus, in simplifying an expression, first of all the brackets must be removed, strictly in the order (), {}, and ||.

After removing the brackets, we must use the following operations strictly in the order:

(i) of (ii) Division (iii) Multiplication (iv) Addition (v) Subtraction.

Modulus of a Real Number:

Modulus of a real number a is defined as

$$|a| = \begin{cases} a, & \text{if } a > 0 \\ -a, & \text{if } a < 0 \end{cases}$$

Thus, $|5| = 5$ and $|-5| = -(-5) = 5$.

Virnaculum (or Bar):

When an expression contains Virnaculum, before applying the 'BODMAS' rule, we simplify the expression under the Virnaculum.

Ratio:

The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as $a : b$.
In the ratio $a : b$, we call a as the first term or antecedent and b , the second term or consequent.

Eg. The ratio $5 : 9$ represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Eg. $4 : 5 = 8 : 10 = 12 : 15$. Also, $4 : 6 = 2 : 3$.

Proportion:

The equality of two ratios is called proportion.

If $a : b = c : d$, we write $a : b :: c : d$ and we say that a, b, c, d are in proportion.

Here a and d are called extremes, while b and c are called mean terms.

Product of means = Product of extremes.

Thus, $a : b :: c : d \Leftrightarrow (b \times c) = (a \times d)$.

Fourth Proportional:

If $a : b = c : d$, then d is called the fourth proportional to a, b, c .

Third Proportional:

$a : b = c : d$, then c is called the third proportion to a and b .

Mean Proportional:

Mean proportional between a and b is ab .

Comparison of Ratios:

We say that $(a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$.

Compounded Ratio:

The compounded ratio of the ratios: $(a : b), (c : d), (e : f)$ is $(ace : bdf)$.

Duplicate Ratios:

Duplicate ratio of $(a : b)$ is $(a^2 : b^2)$.

Sub-duplicate ratio of $(a : b)$ is $(a : b)$.

Triplicate ratio of $(a : b)$ is $(a^3 : b^3)$.

Sub-triplicate ratio of $(a : b)$ is $(a^{1/3} : b^{1/3})$.

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. [componendo and dividendo]

Variations:

We say that x is directly proportional to y , if $x = ky$ for some constant k and we write, $x \propto y$.

We say that x is inversely proportional to y , if $xy = k$ for some constant k and

we write, $x \propto \frac{1}{y}$.

Races: A contest of speed in running, riding, driving, sailing or rowing is called a race.

Race Course: The ground or path on which contests are made is called a race course.

Starting Point: The point from which a race begins is known as a starting point.

Winning Point or Goal: The point set to bound a race is called a winning point or a goal.

Winner: The person who first reaches the winning point is called a winner.

Dead Heat Race: If all the persons contesting a race reach the goal exactly at the same time, the race is said to be dead heat race.

Start: Suppose A and B are two contestants in a race. If before the start of the race, A is at the starting point and B is ahead of A by 12 metres, then we say that 'A gives B, a start of 12 metres'.

To cover a race of 100 metres in this case, A will have to cover 100 metres while B will have to cover only $(100 - 12) = 88$ metres.

In a 100 race, 'A can give B 12 m' or 'A can give B a start of 12 m' or 'A beats B by 12 m' means that while A runs 100 m, B runs $(100 - 12) = 88$ m.

Games: 'A game of 100, means that the person among the contestants who scores 100 points first is the winner'.

If A scores 100 points while B scores only 80 points, then we say that 'A can give B 20 points'.

IMPORTANT CONCEPTS

Suppose a man has to pay Rs. 156 after 4 years and the rate of interest is 14% per annum. Clearly, Rs. 100 at 14% will amount to Rs. 156 in 4 years. So, the payment of Rs. now will clear off the debt of Rs. 156 due 4 years hence. We say that:

Sum due = Rs. 156 due 4 years hence;

Present Worth (P.W.) = Rs. 100;

True Discount (T.D.) = Rs. $(156 - 100) = \text{Rs. } 56 = (\text{Sum due}) - (\text{P.W.})$

We define: T.D. = Interest on P.W.; Amount = $(\text{P.W.}) + (\text{T.D.})$

Interest is reckoned on P.W. and true discount is reckoned on the amount.

IMPORTANT FORMULAE

Let rate = R% per annum and Time = T years. Then,

$$1. \text{ P.W.} = \frac{100 \times \text{Amount}}{100 + (R \times T)} = \frac{100 \times \text{T.D.}}{R \times T}$$

$$2. \text{ T.D.} = \frac{(\text{P.W.}) \times R \times T}{100} = \frac{\text{Amount} \times R \times T}{100 + (R \times T)}$$

$$3. \text{ Sum} = \frac{(\text{S.I.}) \times (\text{T.D.})}{(\text{S.I.}) - (\text{T.D.})}$$

$$4. (\text{S.I.}) - (\text{T.D.}) = \text{S.I. on T.D.}$$

$$5. \text{ When the sum is put at compound interest, then } \text{P.W.} = \frac{\text{Amount}}{\left(1 + \frac{R}{100}\right)^T}$$