PROFESSIONAL DEVELOPMENT FOR QUALITY EDUCATION

MATHEMATICS

Grade IX-X

Teachers' Guide

Directorate of Staff Development, Wahdat Colony, Lahore.

Preface

The Government of Punjab has a strong desire to improve the quality of teaching and learning in the classroom. Various initiatives have been undertaken for provision of quality education to students in the Province. Provision of quality education at secondary level is an important step towards building an education system meant to contribute meaningfully towards development of our society. To achieve the desired goal, activity oriented training for secondary school teachers based on modern teaching methodologies has been considered imperative and crucial.

Directorate of Staff Development (DSD) has been training in-service and pre-service public school teachers and developing educational material since its inception. Considering the quality work produced over the years, the task of development of the Teachers' Guides for secondary school teachers in the subjects of English, Physics, Chemistry, Biology and Mathematics was assigned to the Directorate of Staff Development by the Provincial Government.

DSD worked in collaboration with over three hundred professionals i.e. Teachers, Book Writers and Teacher Trainers from both public and private educational institutions in the subject of English, Physics, Chemistry, Biology and Mathematics who worked in groups to develop these comprehensive Teachers' Guides. These Teachers' Guides with textbooks are aimed to achieve Students' Learning Outcomes (SLOs) through the teaching materials and methodologies which suit varying teaching and learning contexts of Punjab. These Teachers' Guides will help secondary school teachers to deliver and further plan their content lessons, seek basic information on given concepts and topics, and assess students' understanding of the taught concepts.

The DSD team acknowledges the cooperation extended by various public & private, national and international organizations in the preparation of Teachers' Guides. DSD is especially grateful to German International Cooperation Agency (GIZ) for extending its full cooperation and support in conduction of workshops, development of material, quality management, layout and designing of these Guides. DSD recognizes the contribution made by all developers and reviewers belonging to following organizations including Institute of Education and Research (IER) Punjab University, Government Science College, International School of Choueifat, Crescent Model Higher Secondary School, Punjab Textbook Board, Lahore Grammar School, Himayat-e-Islam Degree College, SAHE, PEAS, NEEC, HELP Foundation, Ali Institute of Education, Beaconhouse School System, ALBBS, The Educators, Divisional Public School, The City School, AFAQ, Portal, LACAS, Children's Library Complex (CLC) and GICW Lahore, Govt. Higher Secondary Schools and Govt. Colleges for Elementary Teachers in Punjab.

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UNIT 1

Lesson Plan 1

TOPIC

Introduction to the Matrices

Matrices and Determinents

Grade IX

Students' Learning Outcomes

Define

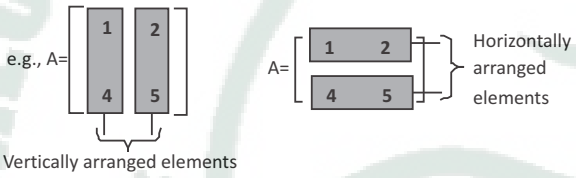
- A matrix with real entries and relate its rectangular layout (formation) with real life,
- Rows and columns of a matrix,
- The order of matrix,
- Equality of two matrices.

Information for Teachers

- A matrix is an ordered set of numbers listed in rectangular form.
- A general matrix can be written in a bracketed rectangular array of mxn elements, arranged in m rows and n columns as shown in fig.1

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ . & . & . & & . \\ . & . & . & & . \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$ Fig 1

- Usually capital letters such as A,B,C,X,Y, etc. are used to represent the matrices and small letters such as a,b,c,l,m,n, a_{12}, a_{13},....., to indicate the entries or elements of the matrices.
- Each matrix consists of horizontally and vertically arranged elements. Horizontally arranged elements are said to form **rows** whereas vertically arranged elements are said to form **column.**



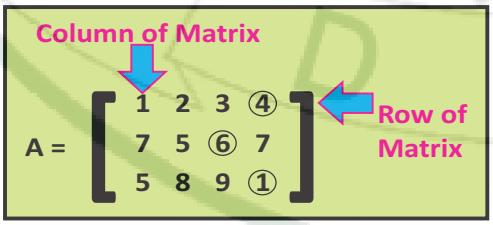
- **Order of a matrix:**

  If a matrix A has m number of rows and n number of columns, then the order of the matrix A is m x n. We read m x n as m cross n or m by n. It may be noted that m x n is not a product of m and n.

- **Equal matrices:**

  Two matrices A and B are said to be equal if and only if they are of the same order and their corresponding elements are equal. In this case we say that A=B.

Example. Let A denote the matrix.

- 

  Matrix A has three rows and four columns. It could be said that A is 3 x 4 Matrix.

- Every entry in the rectangular array is called element of matrix and can be located with the position.

  Example $a_{2,3}$ is element of matrix in second row and third column which is 6. Similarly $a_{3,4}$= 1 and $a_{1,4}$=4.

- Matrices are used to organize information having two variables.

  Example: Marks of five groups of students in different subjects, Number of different grades books of different subjects in library etc.

Duration/Number of Period

80 mins/2period

Material/Resources required

Different grade level Books of different subject, worksheet, Calendar

Introduction

Activity

- Arrange students in groups of five students.
- Give worksheet (four by four tables) to students in groups (sample worksheet is provided below)
- Told them to ask and write time their group fellows spend for different daily activities
- Ask students to fill tables through the small scale survey.
- Use developed tables to paste on board.
- Discuss benefits of using table for information organization.

Sample Worksheet

	School	Games	Home work	Watch TV
Ali	6 hours	2 hours	3 hours	1 hours
Sana				
Zia				
Anum				

Development

Activity 1:

(Groups Activity)

- Arrange students in groups.
- Give few books of mathematics, science and English of grade 8th, 9th and 10th to students in groups.
- Ask them to think different ways to record number of books using different ways.
- Encourage students to explore different data organization layouts.
- Ask groups to present their explored technique for data organization.
- If they used rectangular array then discuss, otherwise told them rectangular array to organize data.
- Told students that the number of categories horizontally and vertically are called rows and columns of the rectangular array and the rectangular array is called matrix.
- Discuss the size of matrix with the order which is 3x3.

Number of books in Library

	Maths	Science	English
8th	12	5	10
9th	11	5	9
10th	15	11	9

Activity 2:

(Individual Activity)

- Ask students to record marks of their last four weeks, test e.g, marks of mathematics, science, English and Urdu.
- Ask them to use rectangular array (matrix) to organize information.
- Ask them to write how many rows and column are required for the given situation.
- Also ask them how many entries will be required to fill information.
- Told them all entries are called elements of matrix.
- Ask students to determine order of the matrix used to organize information.

Activity 3:

(Group or individual activity)

- Give a calendar to students.
- Ask them to count and write how many Sunday, Saturday and Fridays in the month of June, November and March.
- Ask them to use rectangular array.
- Also ask them to count element, rows, column and order of matrix.
- Discuss the use of matrices to organize information in daily life context along with rows, column and order of matrices.

Activity 4:

(Group or individual activity)

- Discuss the equality of Matrices by taking different examples.

Conclusion/Sum up

- In the situations when information having two variables is involved we can use rectangular array called matrix.
- Each element of matrix associated with the categories in the row and column.
- Each element is also identified with row and column.
- Order of matrix is used to determine size of matrix which is number of rows by number of columns.

Assessment

Write more matrices and ask students to present matrices along with rows, columns, elements and order of given matrix.

- Write $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ on board and ask what could be the meaning of a matrix A. Ask about number of elements, rows and columns of A. Also ask about of order of A.
- Share population of Punjab, Sindh, Baluchistan in year 2000, 2005 and 2010 and ask students to organize information using matrix, ask them how many rows and columns will be used, also ask order of matrix.

Follow-up

- Ask students to select any situation from their surroundings where they could use rectangular array to organize information. Also ask them to determine rows, column, elements and order of the matrix used to organize information.
- Which of the following matrixes are equal and which of them are not?

$$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix},\ B = \begin{bmatrix} 3 & 4 \end{bmatrix},\ C = \begin{bmatrix} 5 \\ 6 \end{bmatrix},\ D = \begin{bmatrix} 2+3 \end{bmatrix}$$

$$E = \begin{bmatrix} 5+1 \\ 2+2 \end{bmatrix},\ F = \begin{bmatrix} 2+1 & 3+2 \end{bmatrix},\ G = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix},$$

$$H = \begin{bmatrix} 1+2 & 2+1 \\ 2+1 & 2+3 \end{bmatrix},\ I = \begin{bmatrix} a & b \\ c & d \end{bmatrix},\ J = \begin{bmatrix} a+0 & b \\ c & d+0 \end{bmatrix}$$

- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

Lesson Plan 2

TOPIC

Addition and subtraction of Matrices

Grade IX

Students' Learning Outcomes

- Add and subtract matrices.

Information for Teachers

- Matrices are rectangular array used to organize information.
- Each element is associated with the two variables in accordance to horizontal and vertical categories.
- Two matrices of same order and same horizontal and vertical categories can be added (demonstrated below).

A =

	Maths	English	Science	Urdu
Group 1	21	31	11	20
Group 2	12	14	19	17
Group 3	24	20	14	30

B =

	Maths	English	Science	Urdu
Group 1	15	14	10	21
Group 2	24	14	11	14
Group 3	25	20	24	16

A+B=

	Maths	English	Science	Urdu
Group 1	21+15=36	45	21	41
Group 2	36	28	30	31
Group 3	49	40	38	46

- Similarly two matrices of same order and arrangement can be subtracted (demonstrated below).

A =

	Maths	English	Science	Urdu
Group 1	21	31	11	20
Group 2	12	14	19	17
Group 3	24	20	14	30

B =

	Maths	English	Science	Urdu
Group 1	15	14	10	21
Group 2	24	14	11	14
Group 3	25	20	24	16

A−B=

	Maths	English	Science	Urdu
Group 1	21-15=6	17	1	-1
Group 2	-12	0	8	3
Group 3	-1	0	-10	14

Duration/Number of Periods

80 mins / 2 period

Material/Resources required

Worksheets, three different grade books of different subjects

Introduction

Activity

- Arrange students into group of five students.
- Give worksheet (four by four tables) in groups of five students (sample worksheet is provided).
- Ask students to find time they spend for

different daily activities in two consecutive weeks from remaining four class fellows and fill tables through the small scale survey.

- Ask them to tell how much total time they spent for different daily activity in two weeks.
- Ask them to explain their working.
- Discuss how corresponding elements of two matrices added.
- Discuss addition of two matrices. Highlight order of the matrices should be same for addition.

Sample worksheet

Week 1=

Names	School	Games	Home work	Watch TV
Ali	6 hours			
Sana				
Sara				
Zia				

Week 2=

Names	School	Games	Home work	Watch TV
Ali	6 hours			
Sana				
Sara				
Zia				

Week 1 + Week 2 =

Names	School	Games	Home work	Watch TV
Ali	6+6=12 hours			
Sana				
Sara				
Zia				

Development

Activity 1

(Group Activity)

- Arrange students in groups.
- Give few books of maths, science and English of grade 8th, 9th and 10th to students in groups.
- Ask groups to organize number of books using 3x3 rectangular array.
- Arrange groups together and ask them to make rows and columns of their categories (sequence remain same).
- Add their matrices by adding corresponding elements of both matrices.
- After addition engage students in discussion about addition of matrices.
- Ask them that can we add two matrices of different order.

Activity 2

(Group or individual activity)

- Give following information to students and ask them to organize information with matrices.

Population in 2000			
	Kids	Women	Men
City A	2200	2152	2000
City B	1500	1780	1804
City C	980	789	753

Population in 2010			
	Kids	Women	Men
City A	4252	4752	4698
City B	3562	3625	3789
City C	2500	3687	3210

- Ask students to add both matrices after representation.
- Ask students to present addition of matrices in front of the class.
- Discuss what will happen if we interchange rows with columns or columns with rows in both matrices.
- Also ask students to describe increase of population in 2000 and 2010. For this ask students to subtract population of three city kids, women and men of 2010 from 2000 accordingly.
- Discuss addition and subtraction of matrices.

Activity 3

(Group activity)

- Arrange students into group of five.
- Ask them to prepare a drama in which they have to take a situation in which they have to raise value of matrices from daily life situation.
- Ask them to demonstrate addition and subtraction of matrices through drama.
- After the preparation they have to present drama in front of their fellows.
- Discuss application of addition and subtraction of matrices.

Conclusion/Sum up

- Two matrices can be added also subtracted.
- To add or subtract two matrices, order need to be same.
- To add or subtract corresponding variables should be in same sequence and will be operated accordingly.

Assessment

- Give two matrices of 2x2, 2x3, 3x2 and 3x3

e.g $A=\begin{bmatrix}2 & 3\\ 1 & 4\end{bmatrix}$ and $B=\begin{bmatrix}3 & 6\\ 5 & 7\end{bmatrix}$ or

$C=\begin{bmatrix}1 & 2 & 3\\ 5 & 4 & 1\\ 1 & 3 & 7\end{bmatrix}$ and $D=\begin{bmatrix}3 & 2 & 5\\ 1 & 3 & 7\\ 2 & 4 & 5\end{bmatrix}$ and ask them to find sum and difference of two matrices.

- Provide information about population of male and female in year 2000, 2005 and 2010 of Punjab. Also provide same kind of information for any other province. Ask students to organize information using two matrices. Add and subtract both matrices.

Follow-up

1. Assign matrices of 2x2,3x3 and 4x4 order e.g

$A=\begin{bmatrix}1 & 3\\ 1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}3 & 7\\ 1 & 5\end{bmatrix}$

to students to find their sum and difference.

2. Ask students to search and note application of matrices addition and subtraction from daily life situation.

- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

Lesson Plan 3

TOPIC

Solution of Simultaneous Linear Equations

Grade IX

$$3x + 2y = 7 \qquad 9x + 4y = 9$$

$$-bx + 6y = 6 \qquad -bx + 5y = 8$$

$$\begin{bmatrix} 3 & 2 \\ -b & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix} \qquad \begin{bmatrix} 9 & 4 \\ -b & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \end{bmatrix}$$

$$A \quad x \quad b \qquad A \quad x \quad b$$

Students' Learning Outcomes

- Solve a system of two linear equations and related real life problems in two unknowns using Cramer's rule.

Information for Teacher

- The theory of matrices and determinants is highly developed branch of Algebra.
- Simultaneous equation $C_1 = a_1X + b_1Y$ and $C_2 = a_2X + b_2Y$ can be represented and solved with the help of matrices and determinants.
- A determinant $\begin{vmatrix} 2 & 3 \\ 4 & 7 \end{vmatrix}$ can be represented as $(2x7 - 3x4) = (14-12)=2$.
- Process of Cramer's Rule
  Given the system of linear equations
  $a_1x + b_1y = c_1$
  $a_2x + b_2y = c_2$

The above equation can be written in matrix form as: Ax=B

where $A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$ $B = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \end{bmatrix}$

This system has the unique solution

$$x = \frac{D_x}{D} \quad \text{and} \quad y = \frac{D_y}{D}$$

where

$$|A| \text{ or } D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \text{ or } D_2 \quad D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} \text{ or } D_2 \quad D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} \quad D \neq 0$$

When solving a system of equations using Cramer's Rule, remember the following:

1. Three different determinants are used to find x and y. The determinants in the denominators are identical.

2. The elements of D, the determinant in the denominator, are the coefficients of the variables in the system; coefficients of x in the first column and coefficients of y in the second column.

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

3. D_x, the determinant in the numerator of x, is obtained by replacing the x-coefficients, a_1 and a_2, in D with the constants from the right sides of the equations, c_1 and c_2.

As $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ then $D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$

4. D_y, the determinant in the numerator for y, is obtained by replacing the y-coefficients, , b_1 and b_2, in D with the constants from the right side of the equation, c_1 and c_2.

As $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ then $D_y = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$

Duration/Number of Periods

40 mins / 1 Period

Material/Resources Required

Board, loose sheet, word problems sheet

Introduction

Activity

(Individual or group activity)

- Give simultaneous equations to students.
- Ask them to solve the equations with the help of different methods.

- Encourage them to represent simultaneous equation with the help of matrices.
- Ask them that they can also use concept of matrices and determinants to find the values of x and y.

Development

Activity 1

(Group Activity)

- Give simultaneous equations to students (y+2x=-3 and 3y-x=5).
- Ask students to write equations with the help of matrices.

$$\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \end{bmatrix} \text{ i.e in } AX = B$$

or $X=A^{-1}B$

- Ask students to find determinant of A. *i.e* $|A|$ or D
- Ask them to find D_1 and D_2
- Ask students to find $x = D_1/|A|$ and $y= D_2/|A|$.

Note:

The determinant D, is the determinant of matrix A so it can also be written as $|A|$

i.e $|A|=D$.

Similarly D_x and D_y can also be written as D_1 or D_2 i.e $D_x=D_1$, $D_y=D_2$.

Activity 2

- Take a box and put strips containing simultaneous equation on each of them.
- Ask students to draw two strips to get two equations.
- Ask them to use Cramer's rule to find value of two unknowns.
- After solving equations ask students to interchange their solved equations with fellow students.
- Involve students in peer checking and address mistakes if required.

Activity 3

- Arrange students in groups.
- Give some word problems to students to form two equations. E.g. 2 chair and 3 tables cost Rs. 2500 where as 4 chair and 1 table cost Rs. 3000. Find the cost of chair and table.
- Ask each group to find solution of problem using elimination, substitution methods and Cramer's Rule.
- Ask students to share their work with rest of the groups.

Conclusion/Sum up

- With the help of work produce by students in activities discuss with students stepwise process to apply Cramer's rule for solving simultaneous equations.
- Steps of Cramer's rule will be, conversion of equations into matrices, finding out determinant, D_1 and D_2, and value of x and y.
- Word problem related to simultaneous equation can be solved through Cramer's rule.

Assessment

- Give some simultaneous equations to students and ask them to use Cramer's rule to find out their solution e.g

  $y+2x=1$ and $2y-5x=7$

  or

  $45x-23y=21$ and $21x-24y=17$

- Give word problems and ask students to form simultaneous equation and with the help of Cramer's rule to find solution. e.g price of 15 notebooks and 12 books is Rs.1500/- and price of 18 notebooks and 15 books is Rs.2200. Find the price of a notebook and a book.

Follow-up

Ask students to develop some word problems which they can convert into matrices and apply Cramer's rule to find solution.

- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.