Explanatory Answers

Q1. (C) $AB = 0$ does not necessarily imply that $A = 0$ or $B = 0$ or $B = 0$. For example,

If $$A = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \neq 0 \text{ and } B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \neq 0$$

Then $$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Q2. (B) Rank of identity matrix I_n is n.

Q3. (A) In this case If $|A| = 0$, then A possesses non-trivial solution.

Q4. (B) If $|A| = 2$, then $|A^5| = 2^5 \Rightarrow |A^5| = 32$

Q5. (A) The eigen values of the matrix are: 0, 1, 1. The characteristic roots will be, $t(t-1)^2$.

Q6. (C) $\frac{\pi}{3}$

Q7. (C) $\frac{1}{a^2+b^2}\begin{pmatrix} a & b \\ b & a \end{pmatrix}$

Q8. (C) 0

Q9. (D) 3

Q10. (D) For any square matrix, A, $\frac{1}{2}(A - A')$ is skew-symmetric.

Q11. (B) d^{n-1}

Q12. (B) -4

Q13. (B) 121

Q14. (A) [2, 3, 5]

Q15. (C) P is real symmetric and θ real skew-symmetric.

Q16. (B) a, b, c are in Geometric mean.

Q17. (D) $\pm\frac{1}{2}$

Q18. (D) $A^{-1} = \frac{1}{6}(A^2 - 3A + 4I)$

Q19. (A) A

Q20. (C) Greater than or equal to r

Q21. (B) Skew-symmetric

Q22. (C) Rank of AB cannot exceed rank of other matrix.

Q23. (A) There is one case in which a homogeneous system is assured to having nontrivial solutions, namely, whenever the system involves more unknown than equations.

Q24. (C) If a is a scalar and v is a vector in vector space V then

$$av = 0 \text{ iff } a = 0 \text{ or } v = 0$$

Q25. (A) Two bases of V have exactly the same number of elements.

Q26. (A) dim V = O(W)

Q27. (D) Matrix multiplication is associative but not commutative.

Q28. (C) Nilpotent

Q29. (C) Neither A nor B need be zero matrix.

Q30. (A) If Rank $A < n$ (assuming we're starting with a $n \times n$ matrix), then det $A = O$

Q31. Determinant A must be positie.

Q32. (B) If A is a square matrix such that $A^2 = A$. Then matrix A is called indempotent.

Q33. (B) The determinant of a triangular matrix equals the product of the diagonal entries.

Q34. (D) Since, every elementary matrix is invertible.

Q35. (C) The column rank and row rank are indeed equal, this common number is simply called the rank of A.

Q36. (B) Matrix A is nilpotent, iff $A^n = 0$ for some positive integer n.

Q37. (B) A square matrix A is said to be involutary if

$$A^2 = I$$

Q38. (D) The linear equation has a unique solution iff rank $[(AB)]$ = rank $([A]) = n$. Then the solution is $x = A^{-1}B$.

Q39. (C) The determinant of a triangular matrix equals the product of the diagonal entries.

Q40. (B) If M and N are any two $n \times n$ square matrices then det(MN) equals det(M) . det(N).

Q41. (C) If A is a square matrix, then det A^t = det A.

Q42. (C) The rank of the unit matrix, I_n is n.

Q43. (C) Here, order of $A = m \times n$ and order of $B = p \times q$

The two matrices are comfortable to multiply when

$$m \times n = p \times n \quad \Rightarrow \quad n = p$$

Q44. (A) Here,

$$A = \begin{bmatrix} 7 & -2 & -4 \\ -2 & 1 & 4 \\ -2 & 4 & 1 \end{bmatrix}$$

Now characteristic roots of matrix A, is given by

$$|A - \lambda I| = 0 \quad \text{where} \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now eign values are $\begin{bmatrix} a \\ 3 \\ -3 \end{bmatrix}$

Hence, $\lambda_1 = 9 \quad \lambda_2 = 3 \quad$ and $\lambda_3 = -3$

Q45. (D)

$$A'A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 6 \\ 6 & 14 \end{bmatrix}$$

Hence,

$$|A'A| = \begin{vmatrix} 3 & 6 \\ 6 & 14 \end{vmatrix} = 3 \times 14 - 6 \times 6 = 42 - 36 = 6$$

Q46. (A) Cofactors of the elements of second row of the determinant

$$\begin{vmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{vmatrix}$$

are $(-1)^{2+1}(18+21)$, $(-1)^{2+2}(9-6)$, $(-1)^{2+3}(-7-4)$

i.e., $-39, 3, 11$

Q47. (A) -1

Q48. (A) $[a_{ij}]_{m\times n} = [a_{ij}]_{m\times n}$

Q49. (C) $\leq \min[\gamma(A), \gamma(B)]$

Q50. (C) $\lambda . 1 \leq i \leq n$

Q51. (A) Derogatory

Q52. (C) $A^2 = I$

Q53. (C) $A^2 = A^\star$

Q54. (C) P is real symmetric and Q real skew-symmetric

Q55. (A) If A is a skew-symmetric matrix of odd order n, then

$$|A| = |-A'|$$
$$= (-1)^n |A'|$$
$$= -|A|$$
$$\Rightarrow \quad 2|A| = 0 \quad \Rightarrow \quad |A| = 0$$

Q56. (A) $|A| = 2 \Rightarrow |A^{-1}| = 2$

Q57. (C) adj(adj A) = A

Q58. (D) $(AB)^{-1} = B^{-1}A^{-1}$

Q59. (C) Greater than or equal to r.

Q60. (C) $n - r$

Q61. (C) has a unique solution $x = 1, y = 1, z = 1$

Q62. (C) $P(A) <$ number of columns of A.

Q63. (C) Let $A = [a_{ij}]$ be $n \times n$ matrix. Let X be an eigen vector of A corresponding to the eigen value λ. Then by definition.

$$AX = \lambda X$$
$$\Rightarrow \quad AX = \lambda IX$$
$$\Rightarrow \quad AX - \lambda IX = 0$$
$$\Rightarrow \quad X(A - I\lambda) = 0$$
$$\Rightarrow \quad (A - \lambda I)X = 0$$

Q64. (D, C) This property is called commutative property, but matrix multiplication does not hold this property.

Q65. (D) Since A = B

Q66. (A) rank A = n

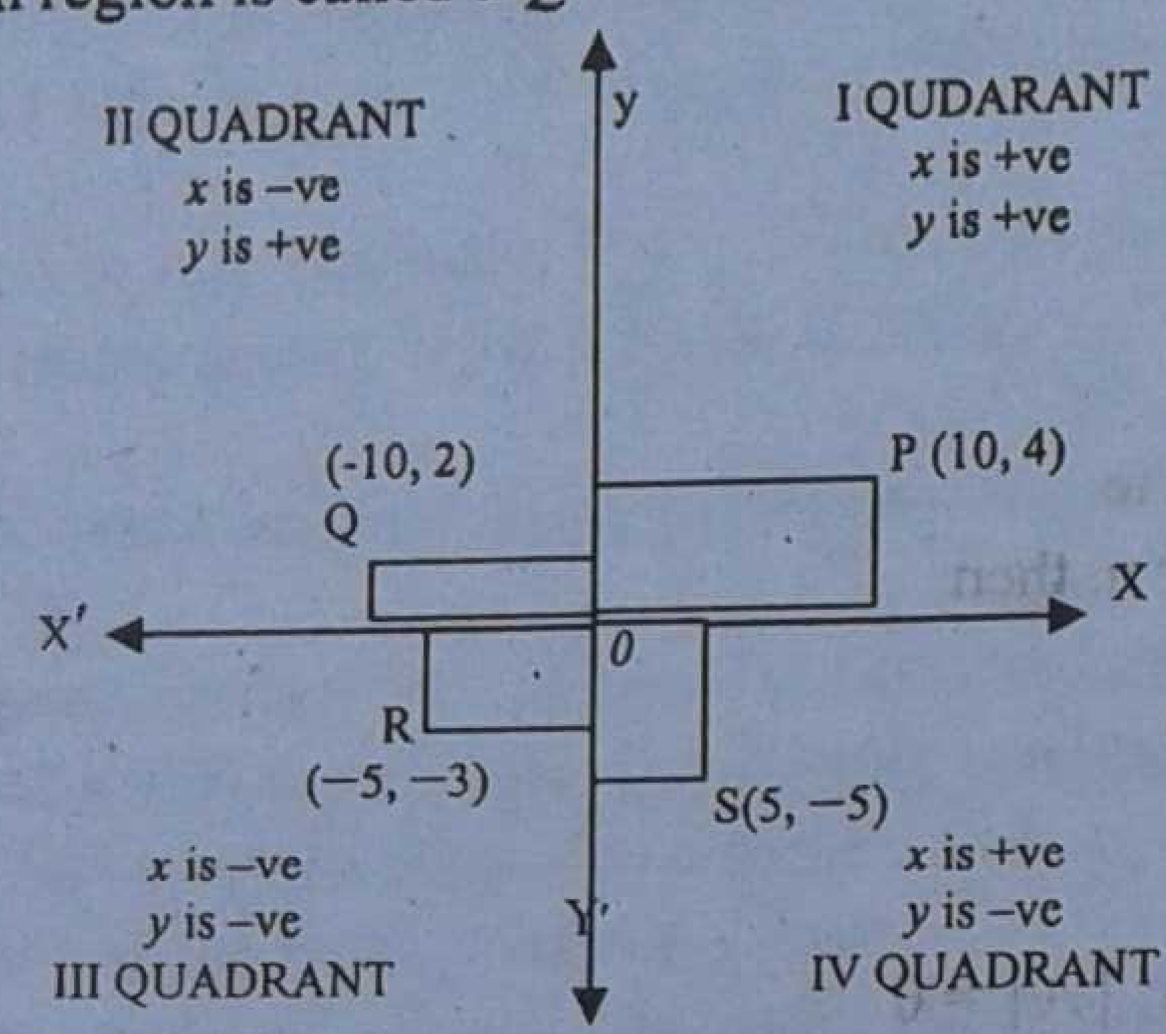
Q67. (C) The inverse of the symmetric matrix is symmetric. That is, if A is an invertible symmetric matrix, then its inverse A^{-1} is symmetric.

Q68. (C) Similar matrices has the same characteristics polynomial.

Q69. (D) 1 or -1

Graph

DEFINITIONS: Let us take two lines XOX′ and YOY′, on a graph paper, right angles to each other. Their point of intersection 'O' is called the *origin*.
The line XOX′ (the horizontal line) and YOY′ (the vertical line) are called the *axes of co-ordinates*. XOX being the *axis* of 'x' and YOY′ the *axis* of 'y'. The line XOX′ and YOY′ divide the graph paper into four regions. Each region is called a *Quadrant.*



Conventionally lengths along OX or parallel to OX are taken to be positive, and the lengths along the OX′ or parallel to OX′ are taken to be negative. Similarly, the lengths along OY or parallel to OY are positive and lengths along OY′ or parallel to OY′ are negative.

Location of the Point.

On the graph paper we can locate any point. For example, to indicate the position of point P, we see it 10 divisions to the right of O the origin, on NP parallel to OX and 4 divisions upward of O an PM parallel to OY. Therefore, its position is 10 units, 4 units or as briefly stated (10, 4) PN and PM are the co-ordinates of point P. PN the x-co-ordinate is called the abscissa and PM, the coordinate is called the ordinate.

Now take another point Q. It is in the second quadrant. QL=10 and QT = 2, that is, the point is 10 divisions left of O and 2 divisions upward. Therefore, its co-ordinates are (–10, 2), that is, its abscissa is –10 and ordinate 2. Consider point R. Its co-ordinates are –5 and –3. Similarly, the co-ordinates of pt. S are (5, –3).

Graph of linear equation

Suppose we want to draw a graph of any first degree equation in x and y.

Let the equation be $x + y = 7$

Tabulate the value of y when $x = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$

As

$$y = 7 - x$$

x	0	1	2	3	4	5	6	7	8	9	10
y	7	6	5	4	3	2	1	0	–1	–2	–3

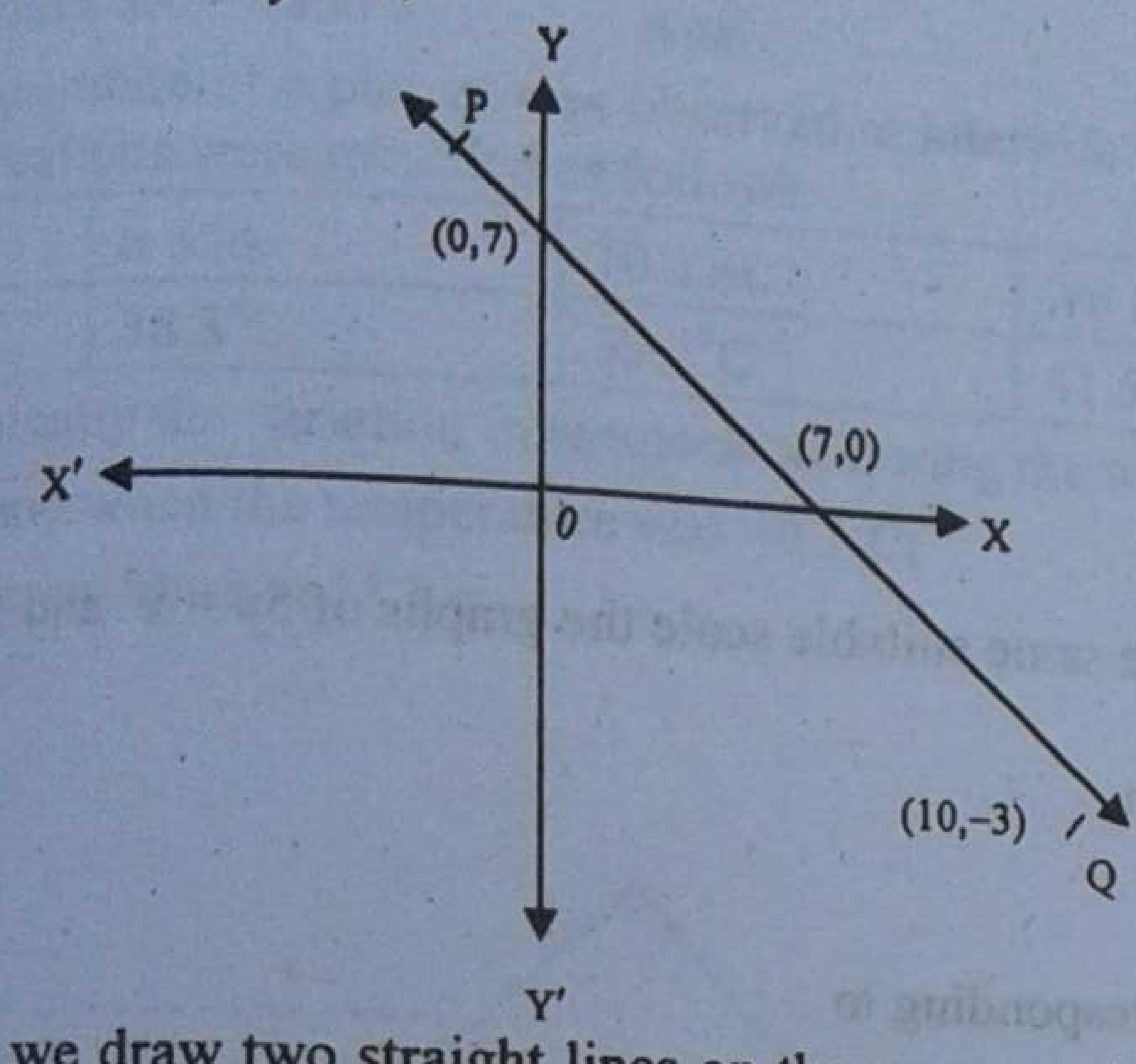
Here 'x' is known as independent variable and 'y' is known as dependent variable as value of y depends on x and now we plot the points (0, 7), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1), (7, 0), (8, – 1), (9, – 2), (10 – 3)

From the graph we see that all these points lie on a straight line.

Similarly, we have more pts. For x = –ve and +ve as well, those points will also lie on the same straight line. So the line tends to infinity to both the sides.

Hence PQ is the required line for the given equation

$$x + y = 7.$$



Similarly, if we draw two straight lines on the same graph paper to proper scale, the co-ordinates of point of intersection of the lines is the solution of the two equations representing those straight lines.

Q.1. Solve graphically

$$x + 2y - 5 = 0 \quad \text{.........(i)}$$
$$3x + 4y - 11 = 0 \quad \text{......(ii)}$$

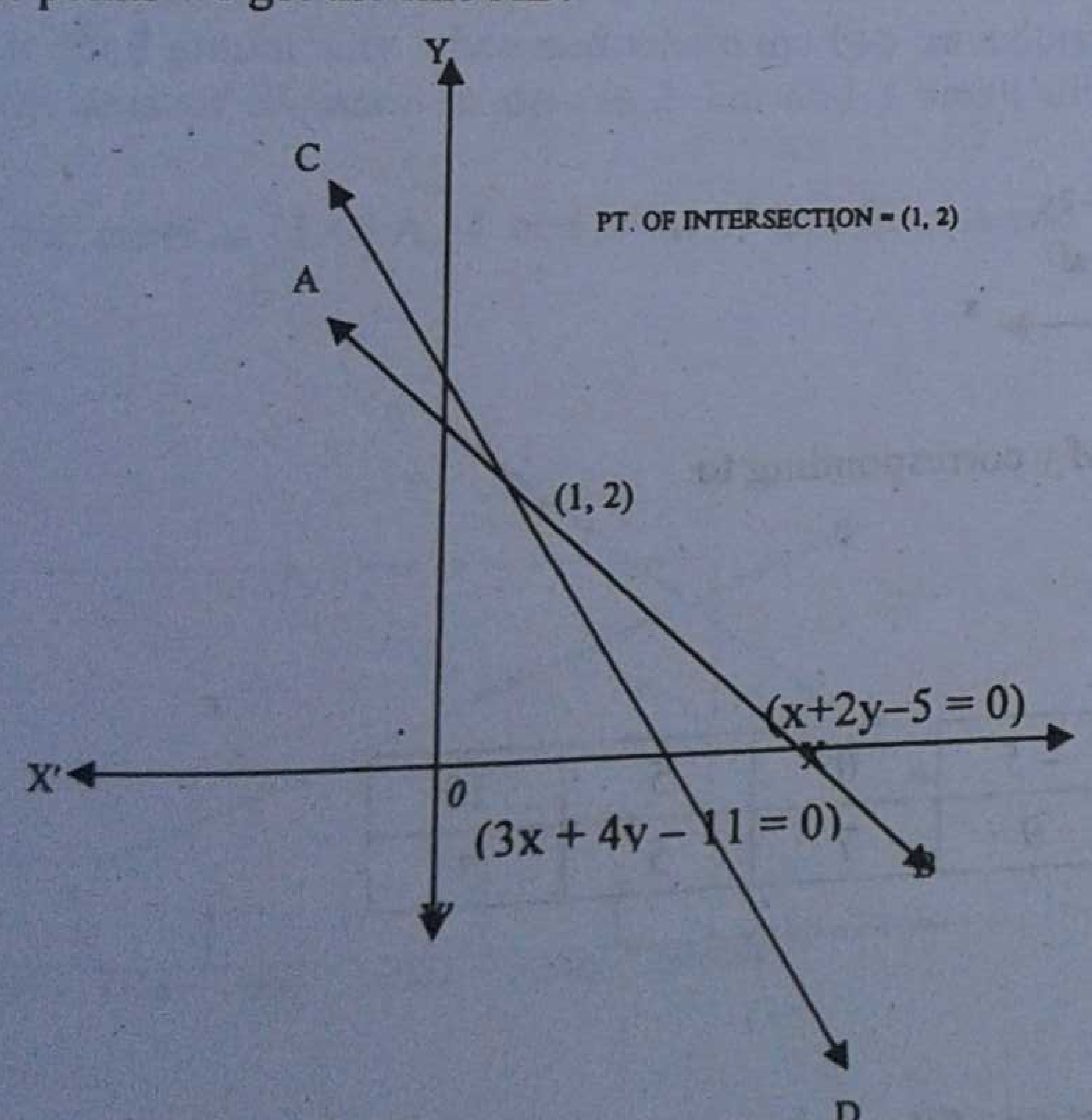
Sol. As (i) is $x + 2y - 5 = 0$

or $$y = \frac{5-x}{2}$$

Tabulating the value of y when $x = -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7$

x	−3	−2	−1	0	1	2	3	4	5	6	7
y	4	7.5	3	2.5	2	2.5	1	0.5	0	−0.5	−1

Plotting these points we get the line AB:



As (ii) is $3x + 4y - 11 = 0$

$$\therefore \quad y = \frac{11-3x}{4}$$

Tabulating the values of y when $x = -3, 1, 3, 5, 9$

x	-3	1	3	5	9
y	5	2	½	-1	-4

Plotting these pts. we get the line CD

The two lines intersect at the point (1, 2).

$x = 1 \quad y = 2$ **Ans.**

Q.2. Draw with the same co-ordinate axes and with the same suitable scale the graphs of $5y = x^2$ and $5y = 35 - 2x$

From these graphs write down the roots of the equation

$$x^2 + 2x - 35 = 0$$

[Take the value of x from – 10 to 10]

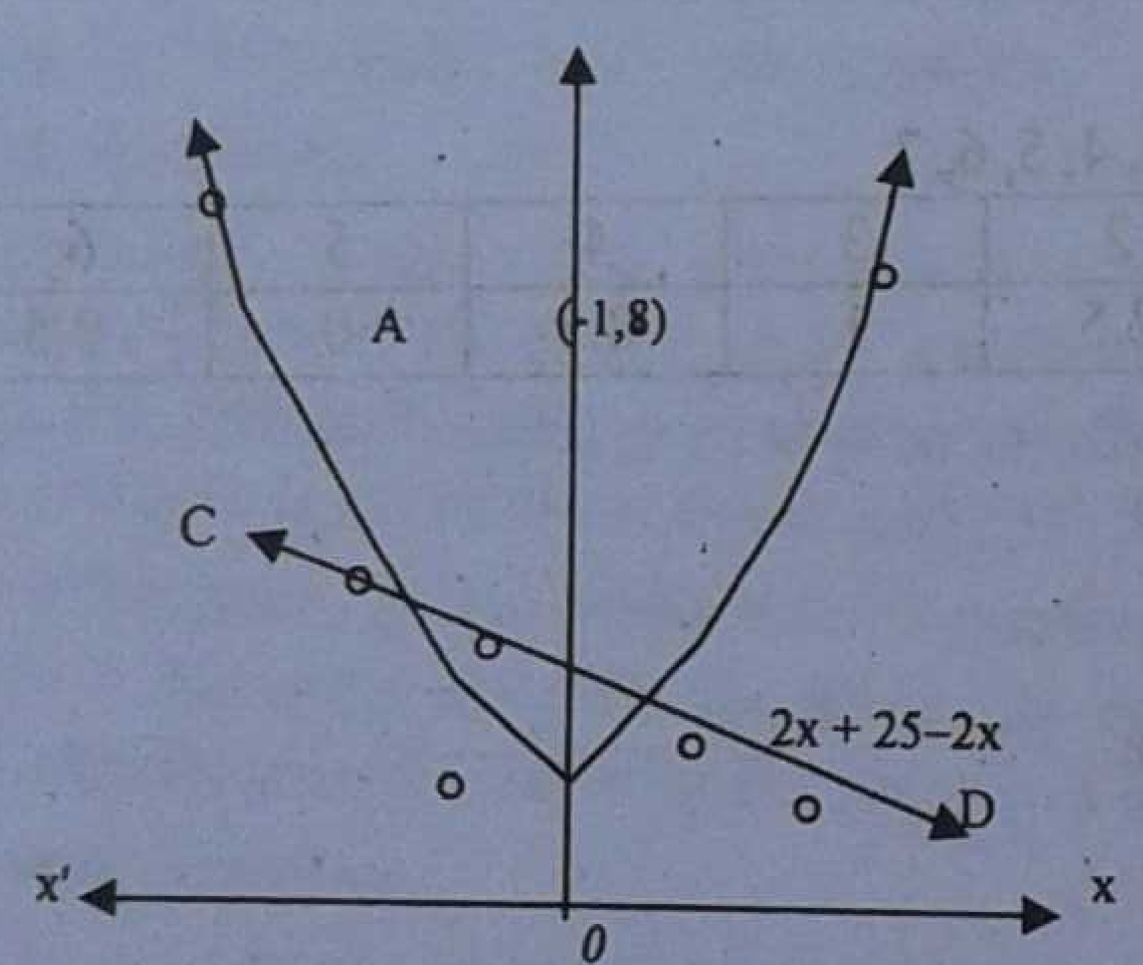
Sol. To draw the graph of $5y = x^2$. The values of y corresponding to

$x = -10, -5, 0, 5, 10, -7$

$$y = \frac{x^2}{5}$$

x	-10	-5	0	5	10	-7
y	20	5	0	5	20	9.8

Plotting these pts. we get the curve.



To draw the graph of $5y = 35 - 2x$. The values of y corresponding to

$x = -10, \quad -5, 0, 5, 10.$

$$y = \frac{35-2x}{5}$$

x	-10	-5	0	5	10
y	11	9	7	5	3

Plotting these pts. we get the line CD

The roots of the equation $x^2 + 2x - 35 = 0$

i.e., $\quad x^2 = 35 - 2x$

will be the x coordinates of the points of intersection of the curve and the straight line.

∴ The roots are −7.1 and 5.

The actual values are −7 and 5. **Ans.**

Q.3. The temperature of a patient was observed at intervals of 4 hours from 6 a.m. to 10 p.m. on a certain day and the observations were recorded as follows:

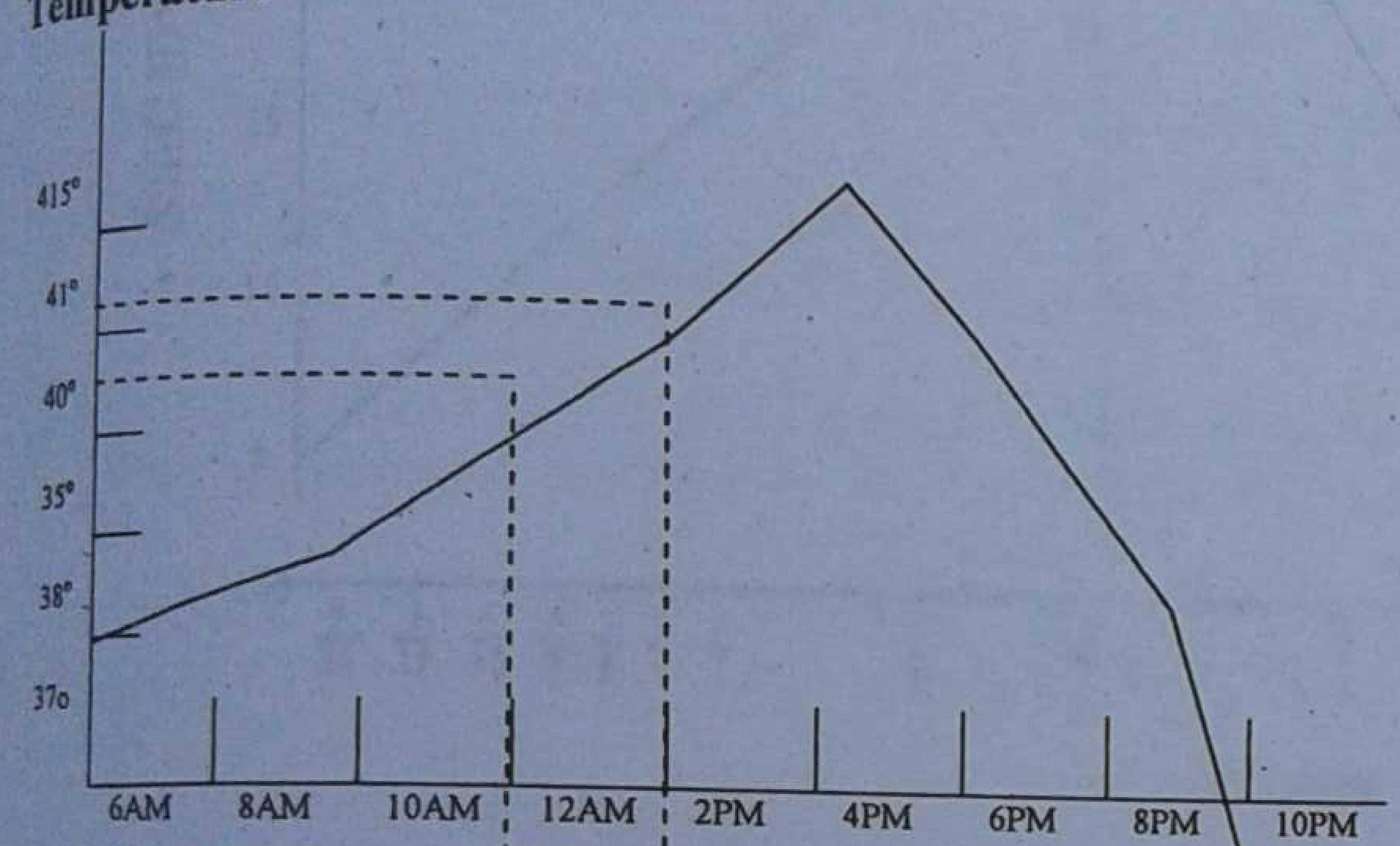
Time	6 a.m.	10 a.m.	2 p.m.	6 p.m.	10 p.m.
Temp.	38.3°C	39.1°C	41.5°C	38.8°C	37.5°C

Exhibit graphically the variation in temperature during the whole interval, and from the graph drawn find:

(i) the time when the temperature was 40.3°C;

(ii) the temperature at 11 a.m.

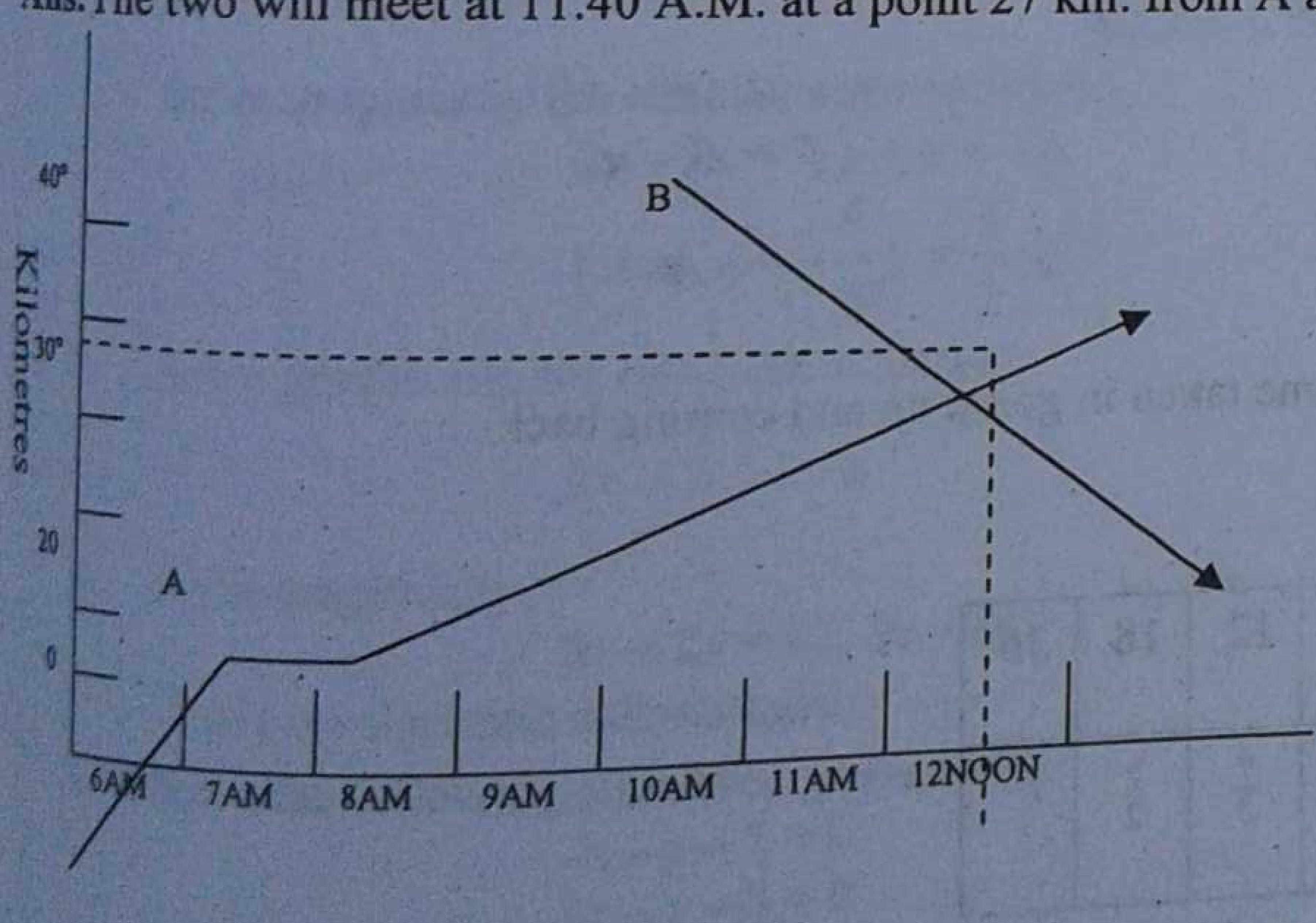
Temperature



Ans. Temperature at 11 AM 39.7°C. [Time when temperature was 40.3°C is 1] 2 noon.

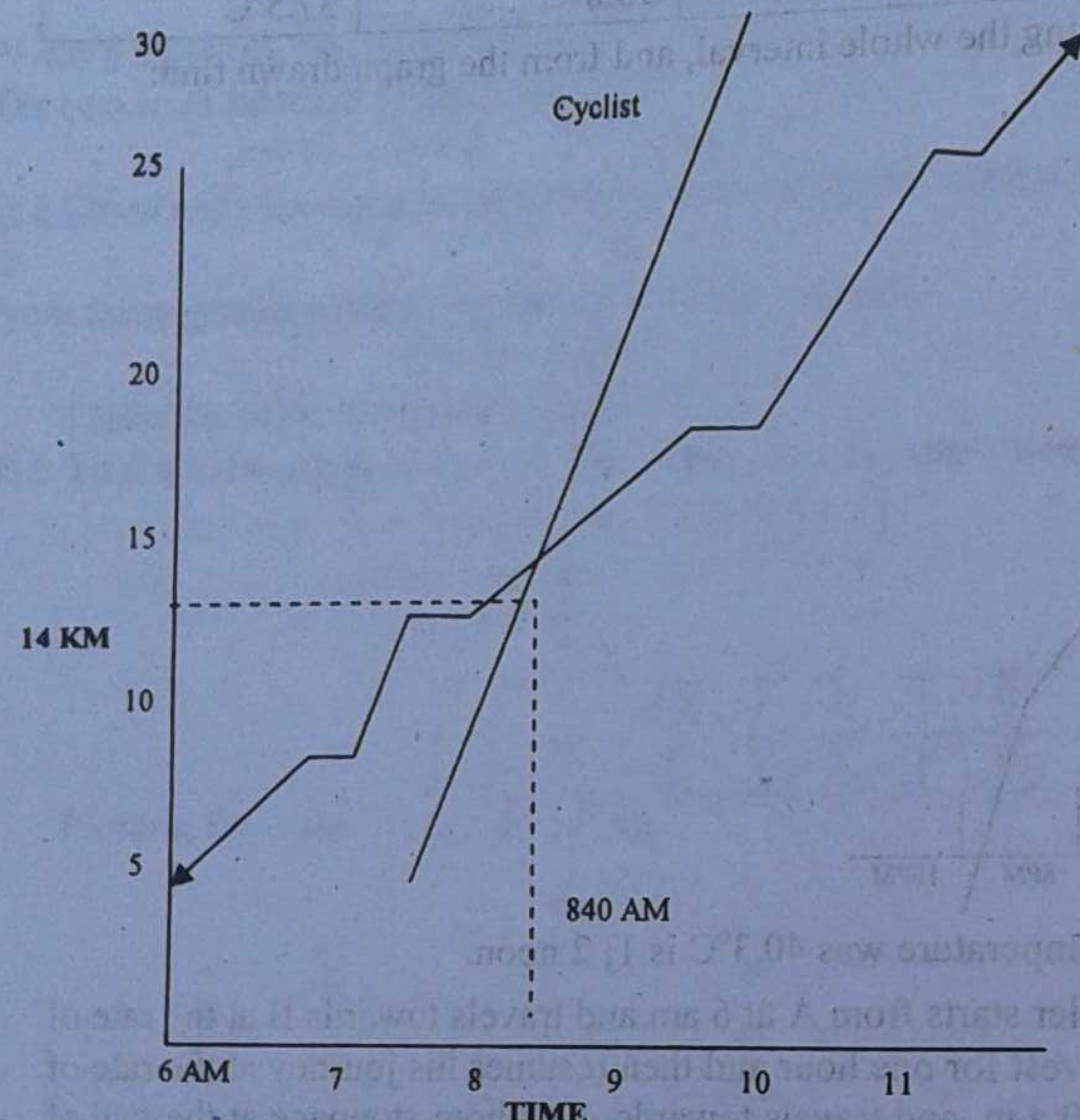
Q.4. A and B are two places 45 km. apart. A traveller starts from A at 6 am and travels towards B at the rate of 7.5 km. per hour ; at the end of two hours he takes rest for one hour and then resumes his journey at the rate of 4.5 km. per hour. Another traveller leaves B at 8.40 a.m. and travels towards A without stoppage at the rate of 6 km. per hour. Find graphically when and where the two travellers meet. [Take 2 small divisions of the graph paper along the axis of distance to denote 1 km and 1 small division along the axis of time to denote 5 minutes.]

Ans. The two will meet at 11.40 A.M. at a point 27 km. from A and 18 km. from B.



TIME

Q.5 A man starting at 6 A.M. walks at the uniform rate of 6 km. per hour resting for 10 minutes at the end of every hour. A cyclist, starting from the same place at 7.30 A.M., travels in the same direction at the uniform rate of 12 km. per hour. Find graphically when and where the cyclist will pass the man.



Ans. The cyclist will pass the man at 8.40 A.M. 14 km. from the start.

Q.6. A boat travels upstream at 9 km. per hour and downstream at 12 km. per hour. If the boat goes from A to B upstream and returns to A in 4 hours 12 minutes, find by graph the distance between A and B.

Sol. The boat travels upstream

9 km. in 1 hr.

∴ 1 km. in $\frac{1}{9}$ hr.

Again it travels downstream

12 km. in 1 hr.

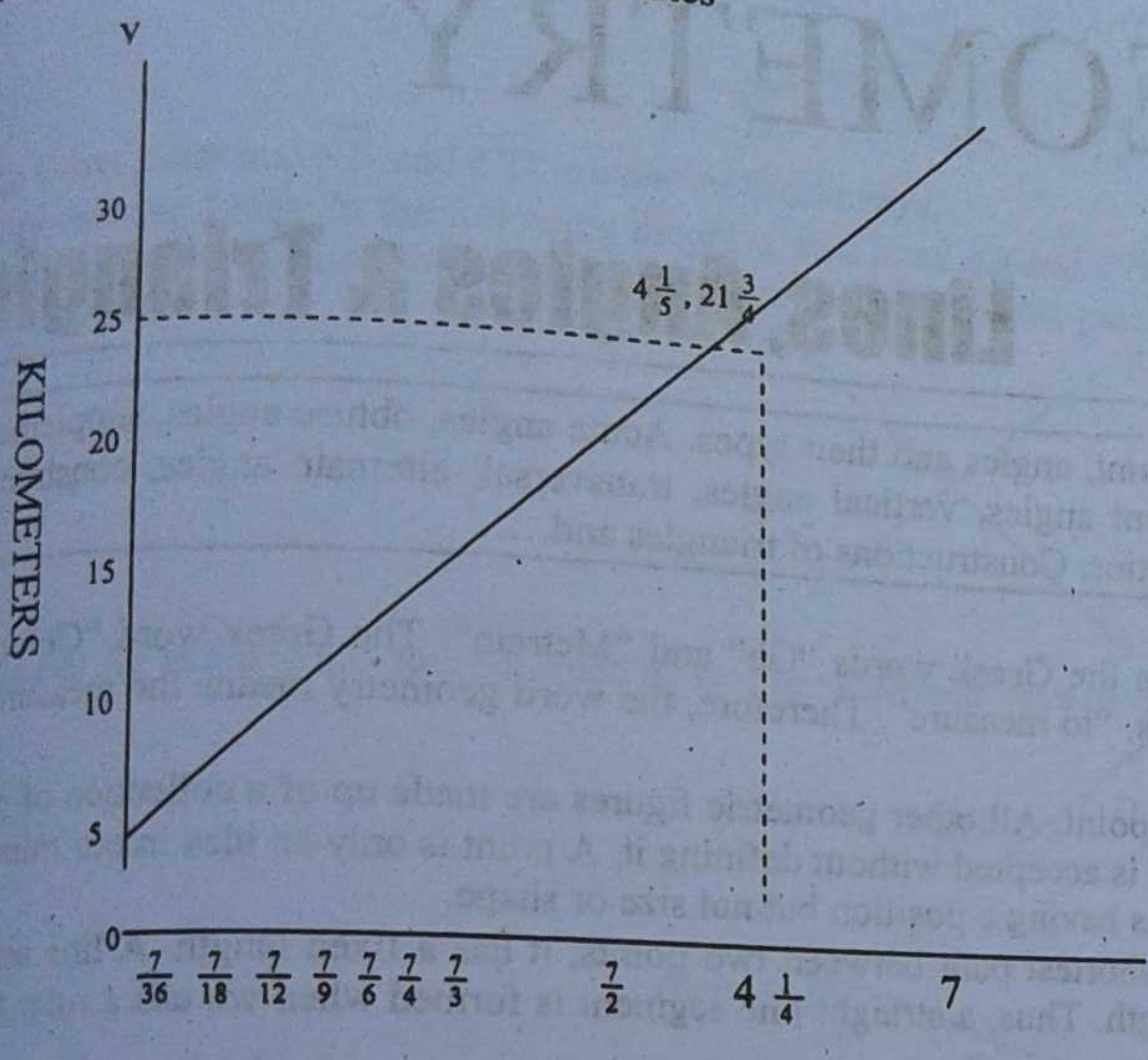
∴ 1 km. in $\frac{1}{12}$ hr.

∴ If distance between A and B is 1 km. the time taken in going up and coming back

$$\frac{1}{9}+\frac{1}{12}=\frac{4+3}{36}=\frac{7}{36} \text{ hr.}$$

Distance (km)	1	2	3	4	6		12	18	36
Time (hr)	$\frac{7}{36}$	$\frac{7}{18}$	$\frac{7}{12}$	$\frac{7}{9}$	$\frac{7}{6}$		$\frac{7}{3}$	$\frac{7}{2}$	7

Now we plot time against X-axis and distance against Y-axis. After plotting the graph we may find the distance corresponding to 4 hrs. 12 minutes



i.e., $4\frac{1}{5}$ hrs. This is $21\frac{3}{4}$ km. from graph. Actual is $21\frac{3}{5}$.

Q.7. Suppose the following is copied from the premium table of a life insurance company for an assurance of Rs. 1000. Draw the graph representing the premia and find the annual premium to be paid by one who intends to insure his life and whose age next birthday is 28.

Age next birthday	20	25	30	35	40	45
Premium	Rs. 21.50	Rs. 24	Rs. 27.25	Rs. 31.25	Rs. 36.75	Rs. 43

Ans. Annual premium to be paid, by one whose age next birthday is 28, will be Rs. 5.15.

EXERCISE

1. Solve graphically the simultaneous equations.

$$2y - 3x = 7, y + x = -1$$

$$[\text{ Ans. } x = -\frac{9}{5} y = \frac{4}{5}]$$

2. Solve graphically the quadratic equation

$$2x^2 - x - 1 = 0 \qquad [\text{Ans. } x = 1, -\frac{1}{2}]$$

3. Solve graphically.

$$3x + 2y = 12;\ 5x - 3y = 1 \qquad [\text{ Ans. } x = 2, y = 3]$$

Verify by algebraic calculations.

4. Solve the following equations graphically:

$$2x - y - 2 = 0$$
$$4x - y - 8 = 0 \qquad [\text{ Ans. } x = 3, y = 4]$$

GEOMETRY

Lines, Angles & Triangles

Highlights: Basic definitions, Line, point, angles and their types. Acute angles, obtuse angles, supplementary angles, complementary angles, adjacent angles, vertical angles, transversal, alternate angles, constructions. Dividing a line segment in the given ratios. Constructions of triangles and......

What is Geometry?

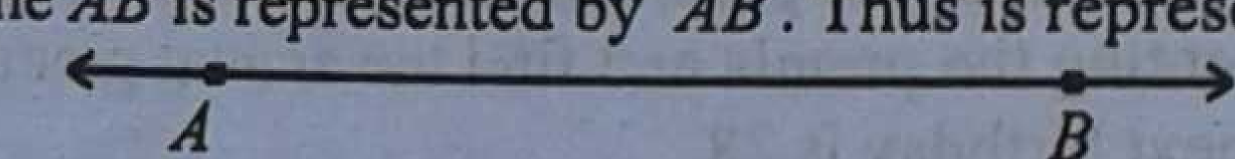
The word "Geometry" is derived from the Greek words "Ge" and "Metrein". The Greek word "Ge" means "Earth" and the word "Metrein" means, "to measure". Therefore, the word geometry means the measurement of Earth.

Point: The basic geometric figure is a point. All other geometric figures are made up of a collection of points. Point is such a geometrical term which is accepted without defining it. A point is only an idea in our mind, it is not a physical object and we regard it as having a position but not size or shape.

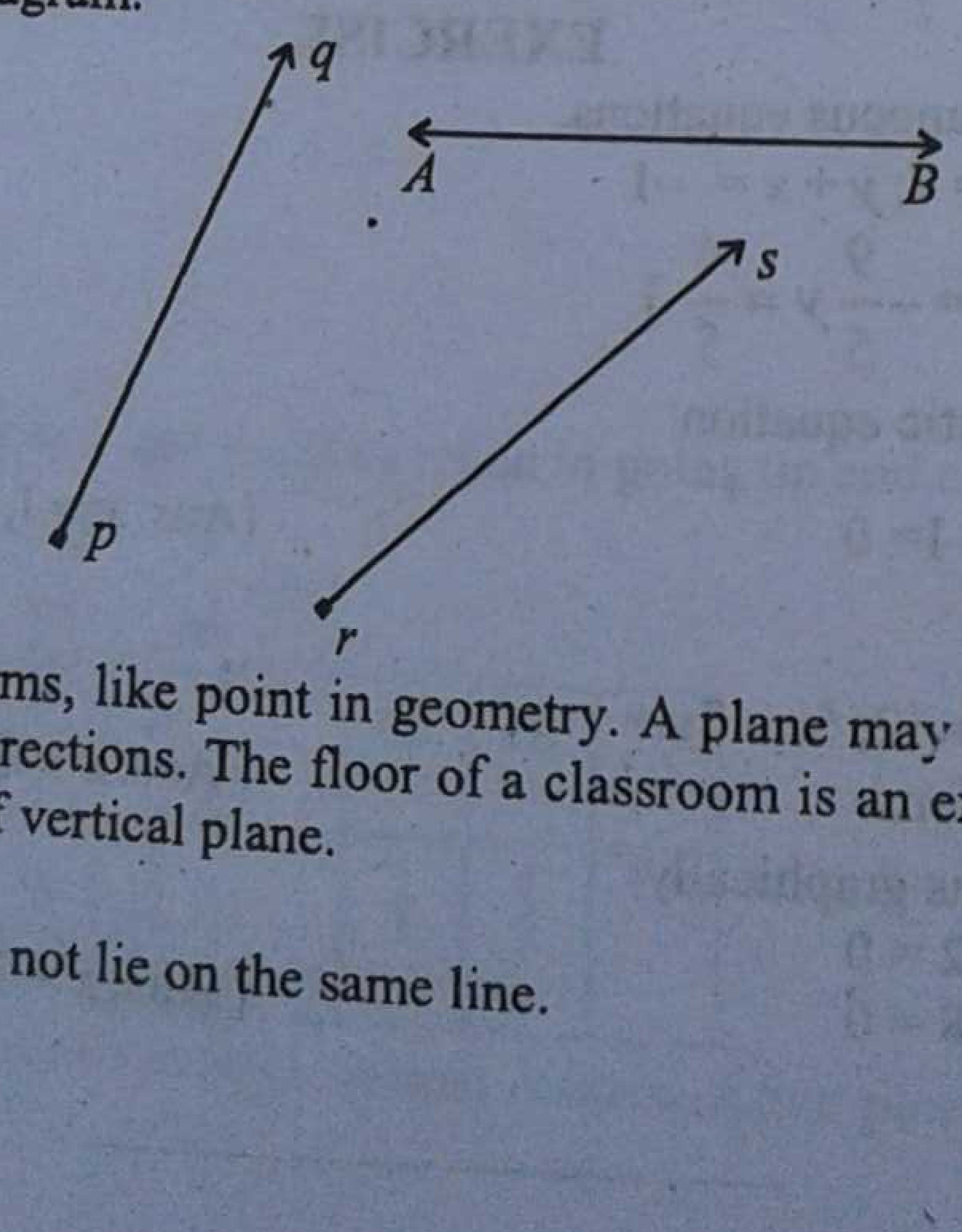
Line Segment: A line segment is the shortest path between two points, it has a fixed length. A line segment thus has a dimensions which is its length. Thus, a straight line segment is formed when we use a ruler to join two points, say A and B.

We call line segment AB or BA. A and B are called the end points.

Line: If a line segment extended beyond its end points indefinitely, we get a line. This endless line is called a straight line or simply a line. Line AB is represented by $\overleftrightarrow{AB}$. Thus is represented in the following diagram.



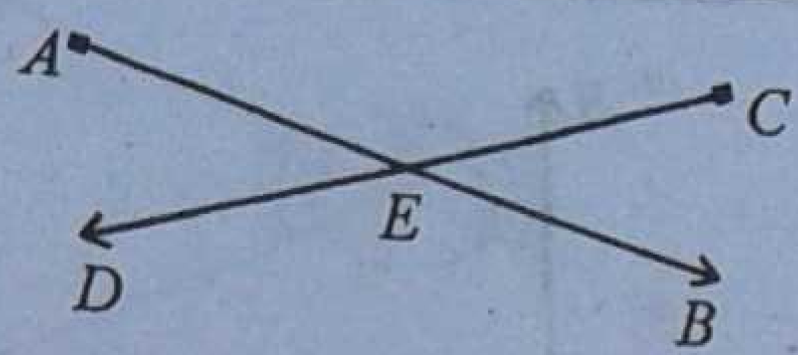
Ray: A ray is simply a line segment with one end ended infinitely. A ray is denoted by the name of a line segment with an arrowhead at one end indicating the direction in which it is extended. The rays, $\overrightarrow{AB}$, $\overrightarrow{pq}$ and $\overrightarrow{rs}$ are shown in the following diagram.



Plane: Plane is also undefined terms, like point in geometry. A plane may be thought of a flat surface which extends indefinitely in all of its directions. The floor of a classroom is an example of horizontal plane and the wall of classroom is an example of vertical plane.

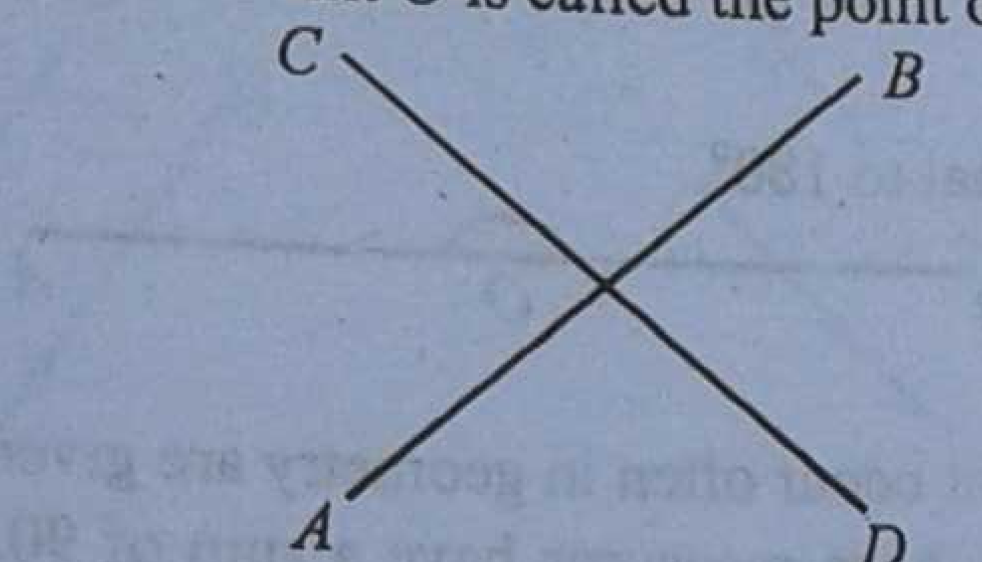
Non-collinear Rays:

Non-collinear rays are rays that do not lie on the same line.

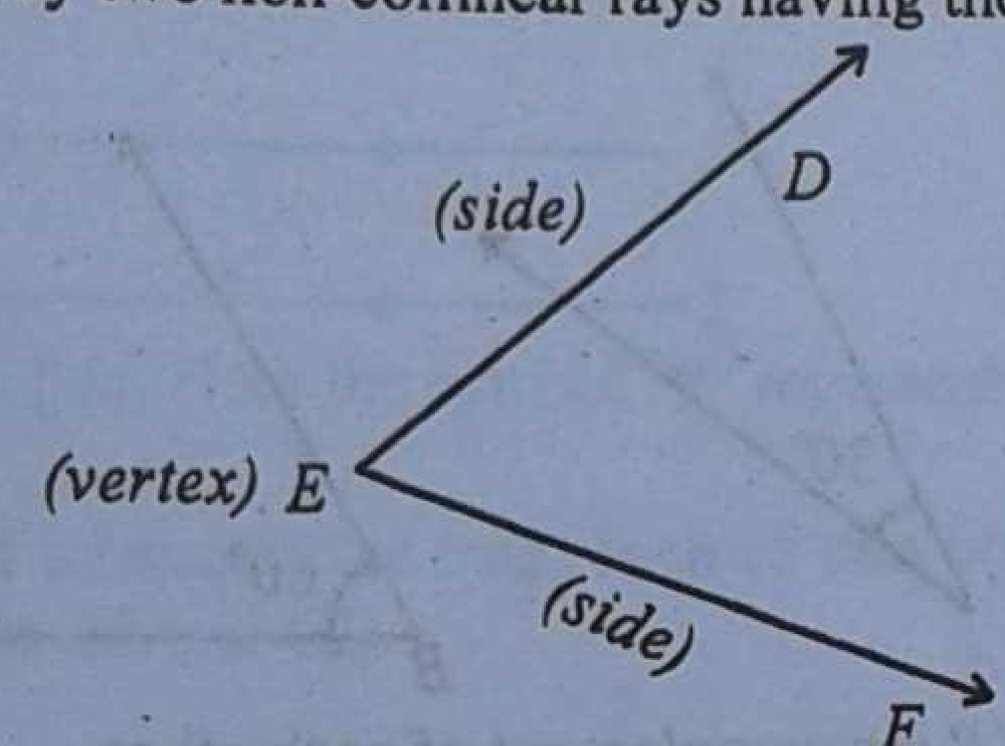


In above diagram, $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are non-collinear rays.

Intersecting Lines: In the following diagram, two lines AB and CD on the same plane having a common point O. We say the two lines intersecting at O. Point O is called the point of intersection.



Angle: An angle is the figure formed by two non-collinear rays having the same end point.

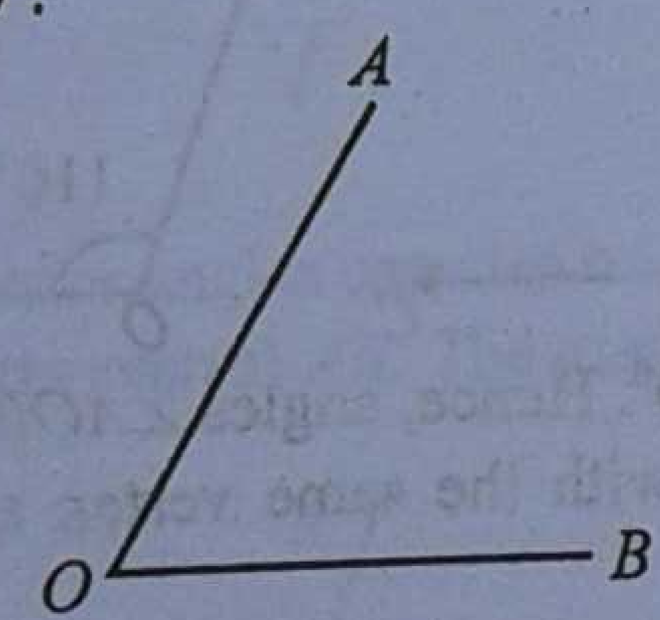


E is human as the vertex of the angle and $\overrightarrow{EF}$ and $\overrightarrow{ED}$ are the sides or arms of the angles.

The angle is called angle DEF or angle FED or angle E and is written as $D\hat{E}F$ or $F\hat{E}D$. Another way of this angle is $\angle DEF$ or $\angle FED$. We may also call it angle E and write $\hat{E}$ or $\angle E$.

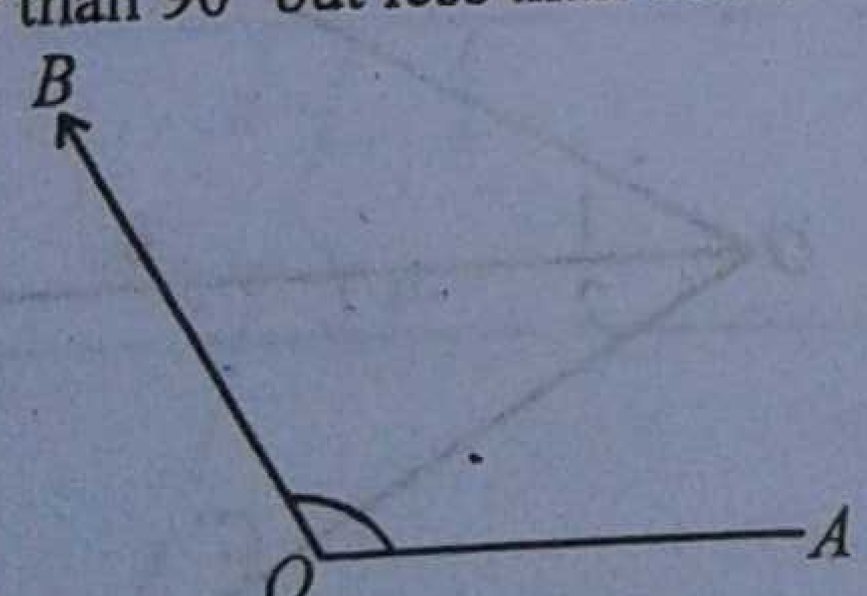
Different Types of Angles:

Acute Angle: An acute angle is less than 90°.



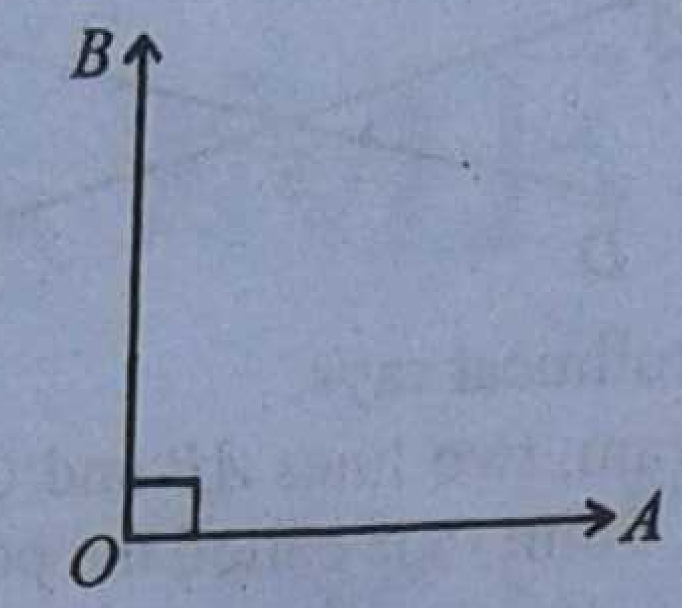
$\angle AOB$ is an acute angle.

Obtuse Angle: An obtuse angle is larger than 90° but less than 180°.



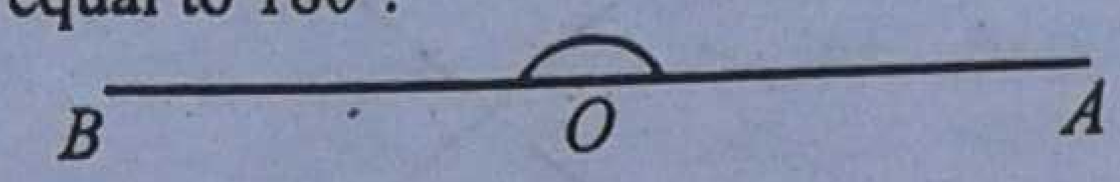
$\angle AOB$ is an obtuse angle.

Right-Angle: A right-angle is equal to 90°.



$\angle AOB$ is a right-angle.

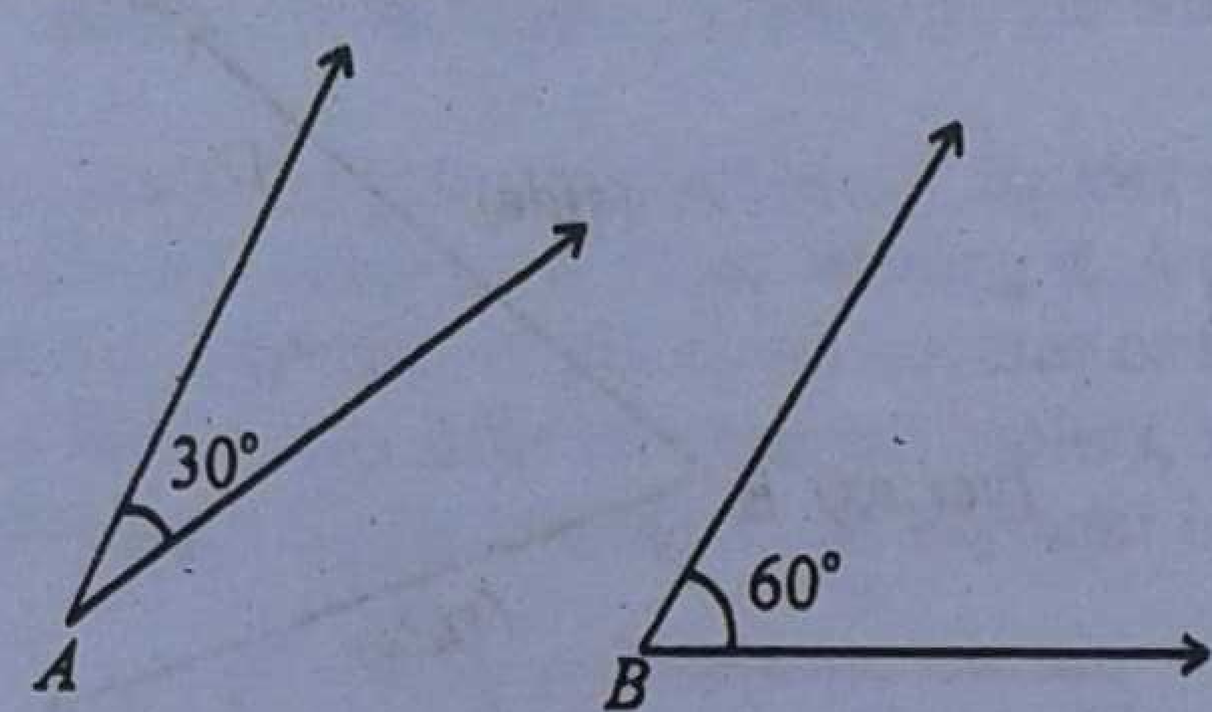
Straight-Angle: A straight-angle is equal to 180°.



$\angle AOB$ is a straight angle.

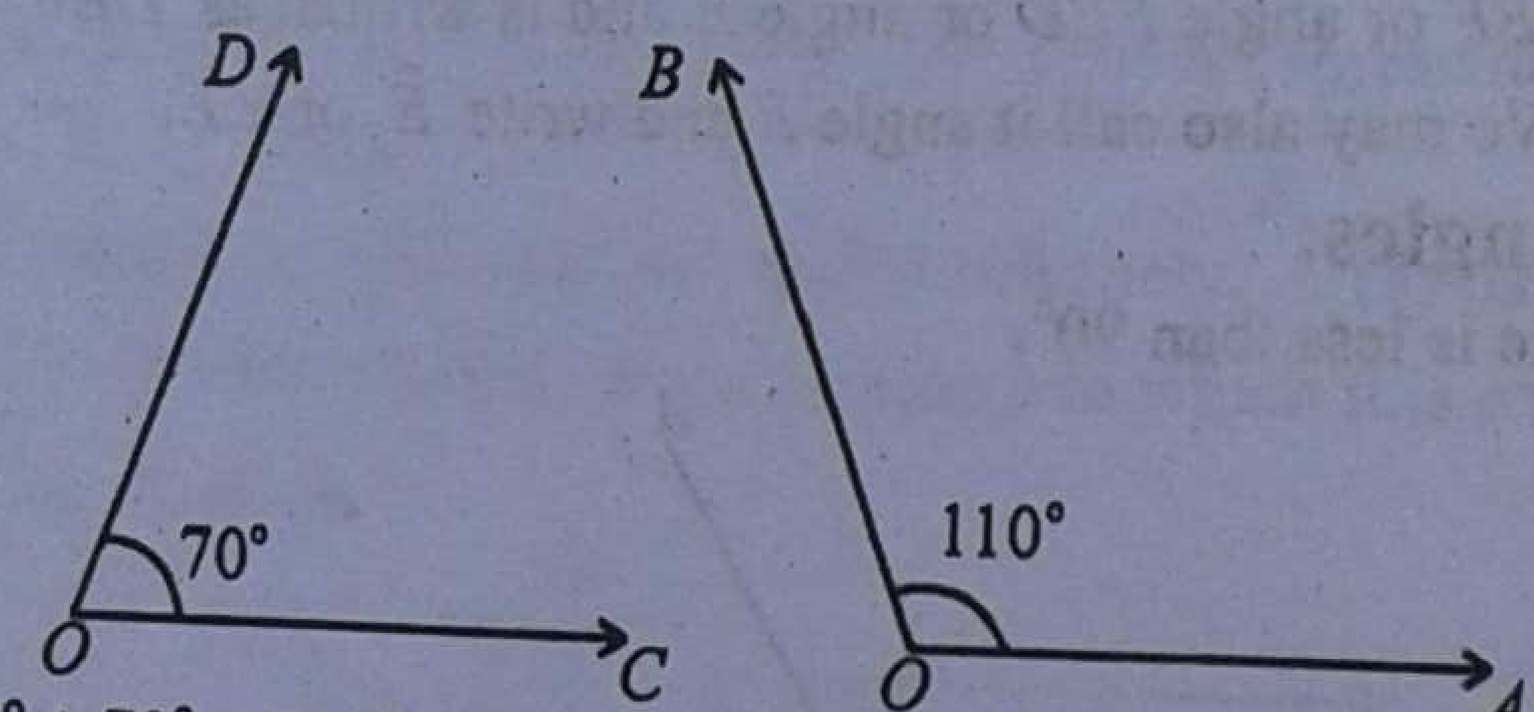
Important Angle Pairs: Angle pairs that occur often in geometry are given special names.

Complementary Angles: Two angles whose measures have a sum of 90. Each angle is a complement of the other.



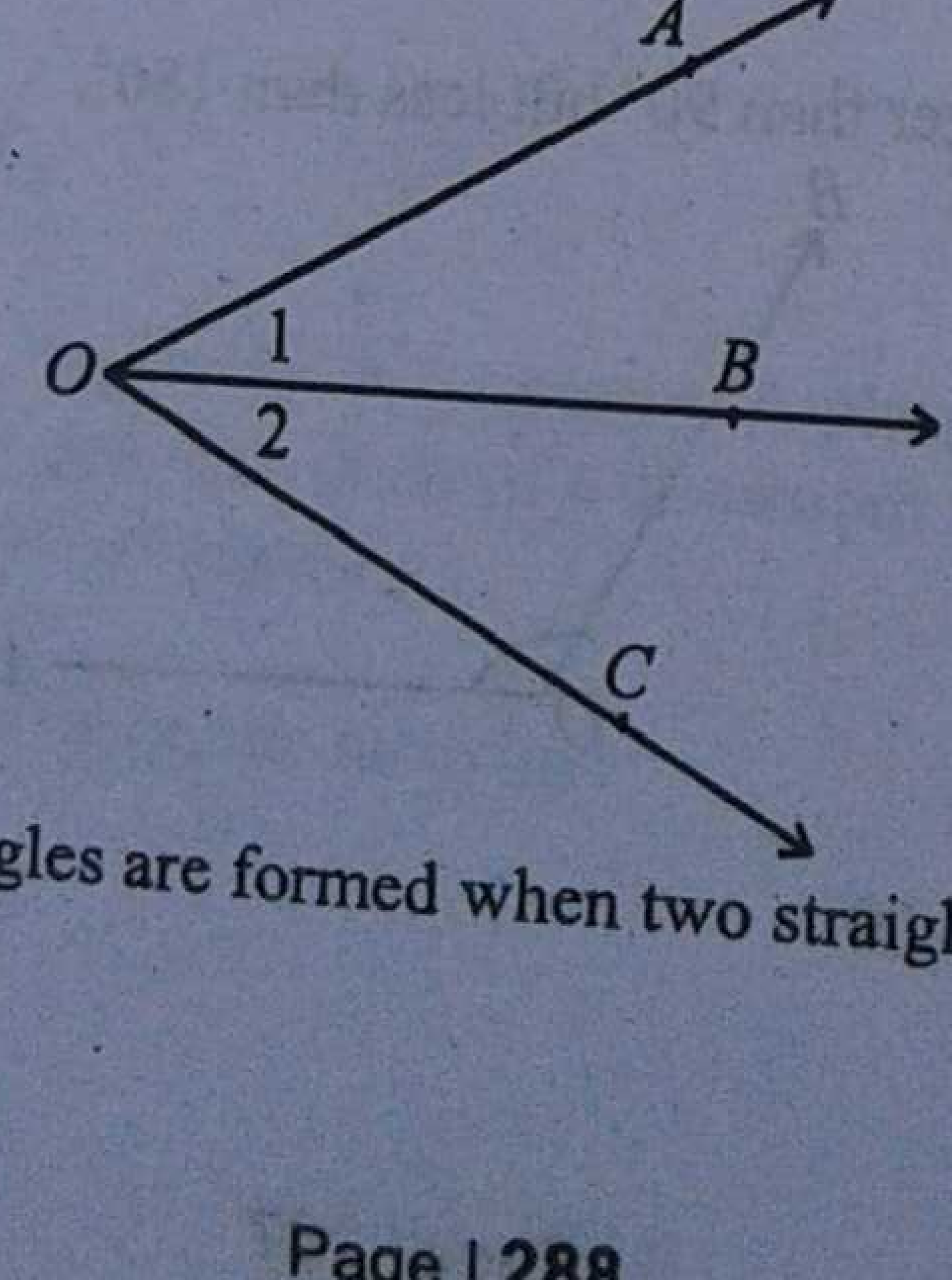
Here, $\angle A + \angle B = 90°$, so $\angle A$ and $\angle B$ are complement of each other.

Supplementary Angles: Two angles whose measures have a sum of 180. Each angle is a supplement of the other.

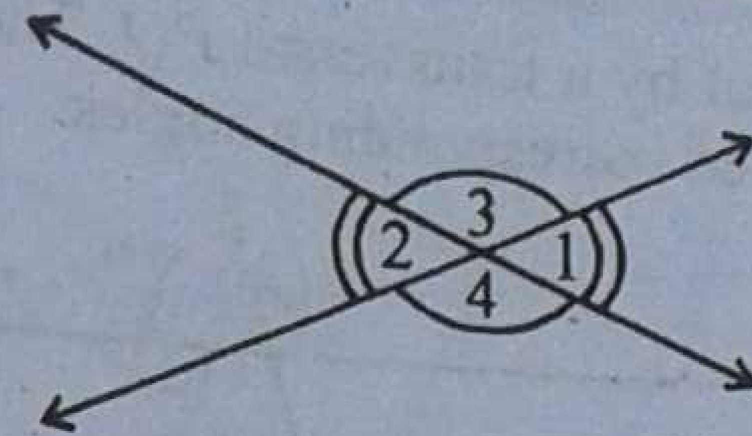


Here, $\angle AOB + \angle COD = 110° + 70° = 180°$. Hence, angles $\angle AOB$ and $\angle COD$ are supplement of each other.

Adjacent Angles: Two coplanar angles with the same vertex and a common side but no interior points in common.

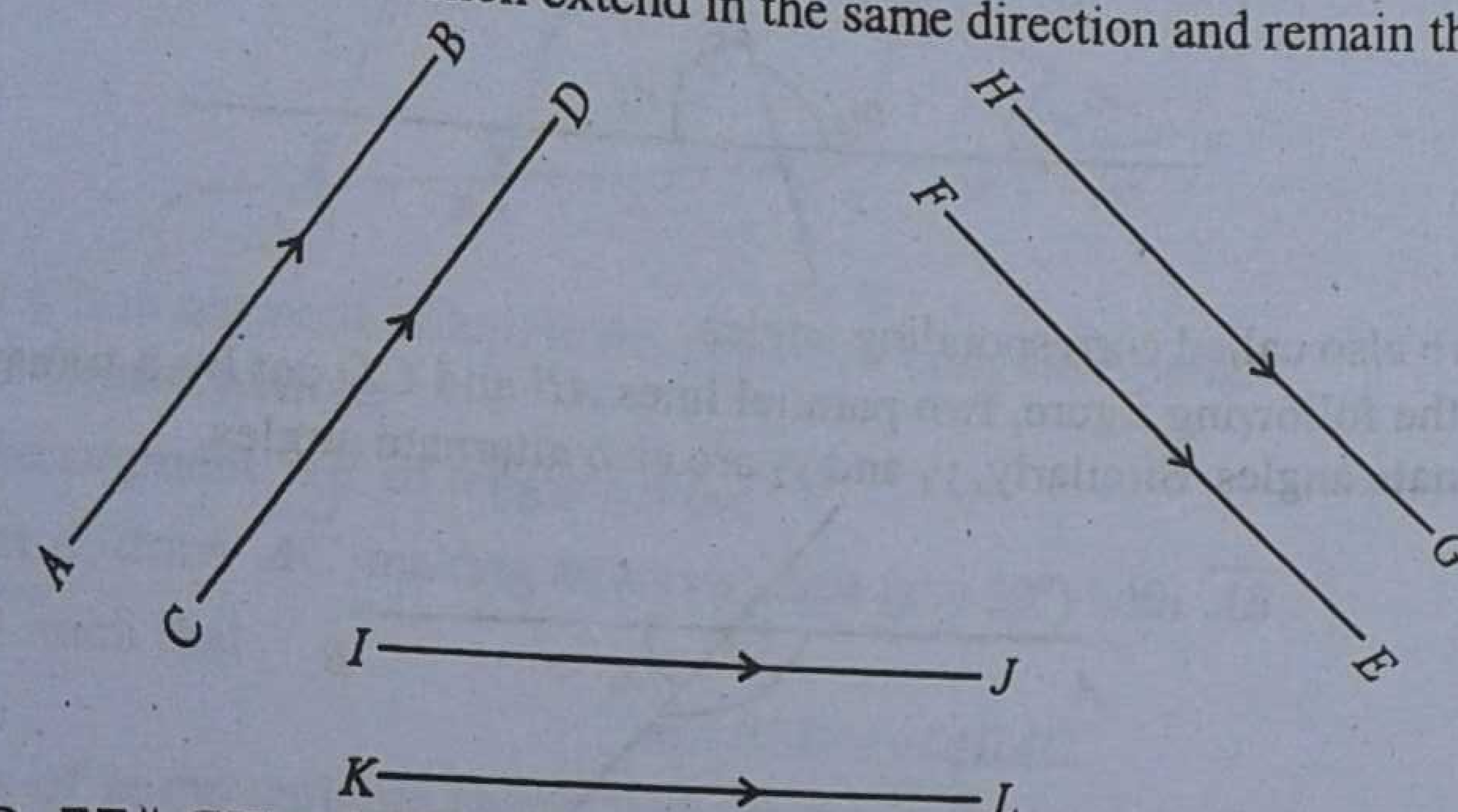


Vertical Angles: Vertically opposite angles are formed when two straight lines intersect each other.



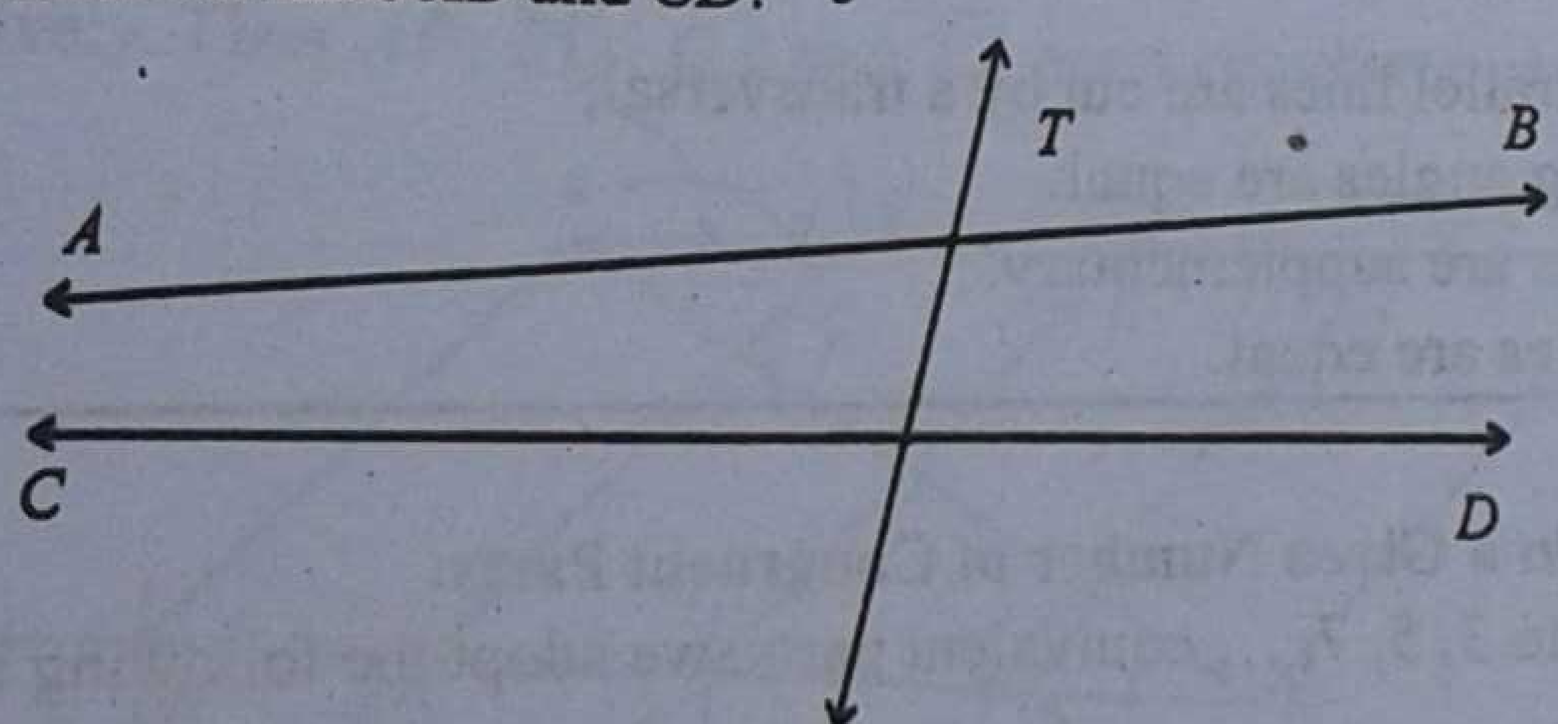
In above fig., $\angle 1$ and $\angle 2$ are vertically opposite angles. Also $\angle 3$ and $\angle 4$ are vertically opposite angles.

Parallel Lines: Parallel lines are lines which extend in the same direction and remain the same distance apart.



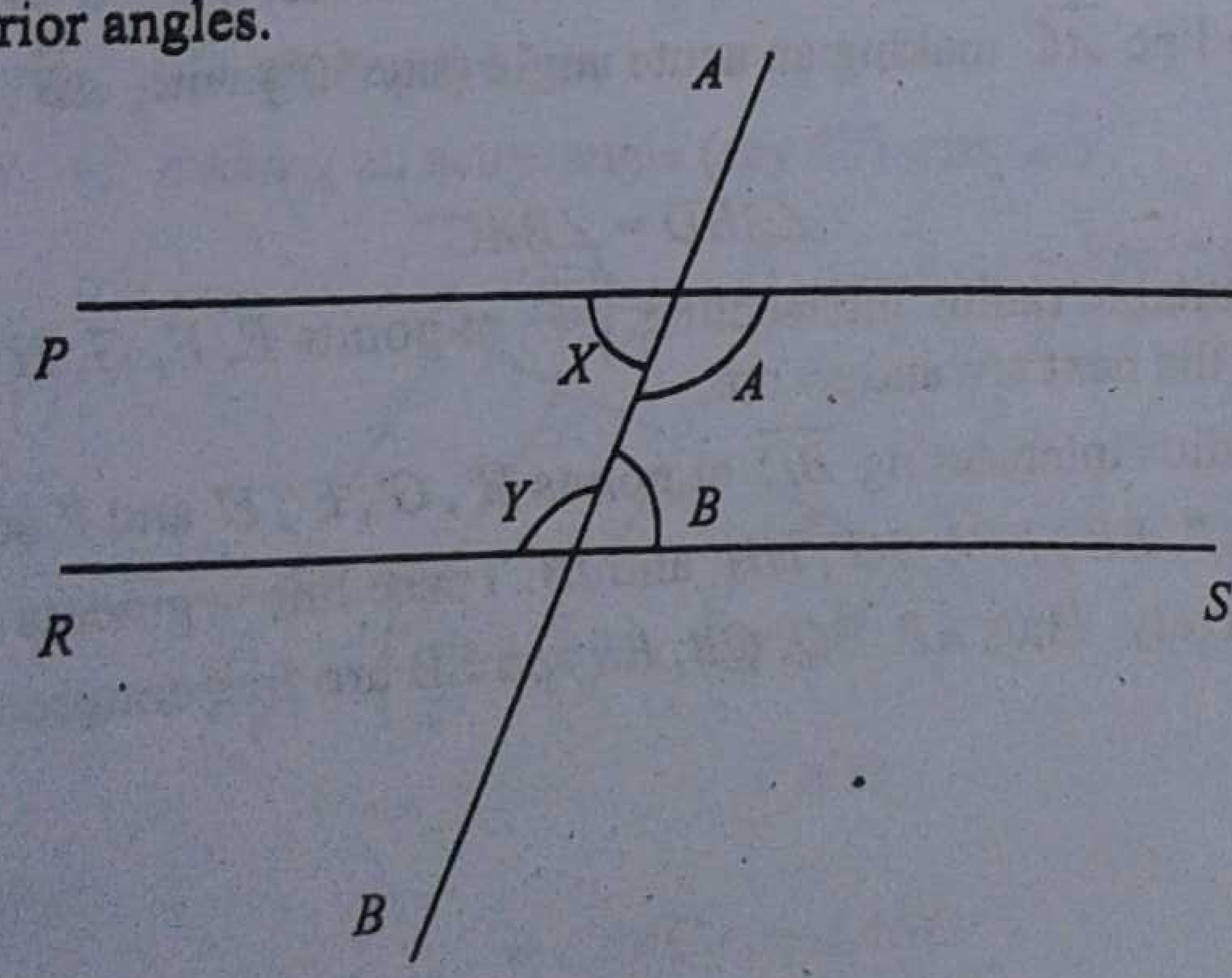
In above fig., $AB \parallel CD$, $EF \parallel GH$ and $KL \parallel IJ$. Here the symbol "||" represent "is parallel to".

Transversal: A transversal is a line that intersects (or cuts) each of two other lines in different points. In the figure below, transversal *t* intersects lines *AB* and *CD*.

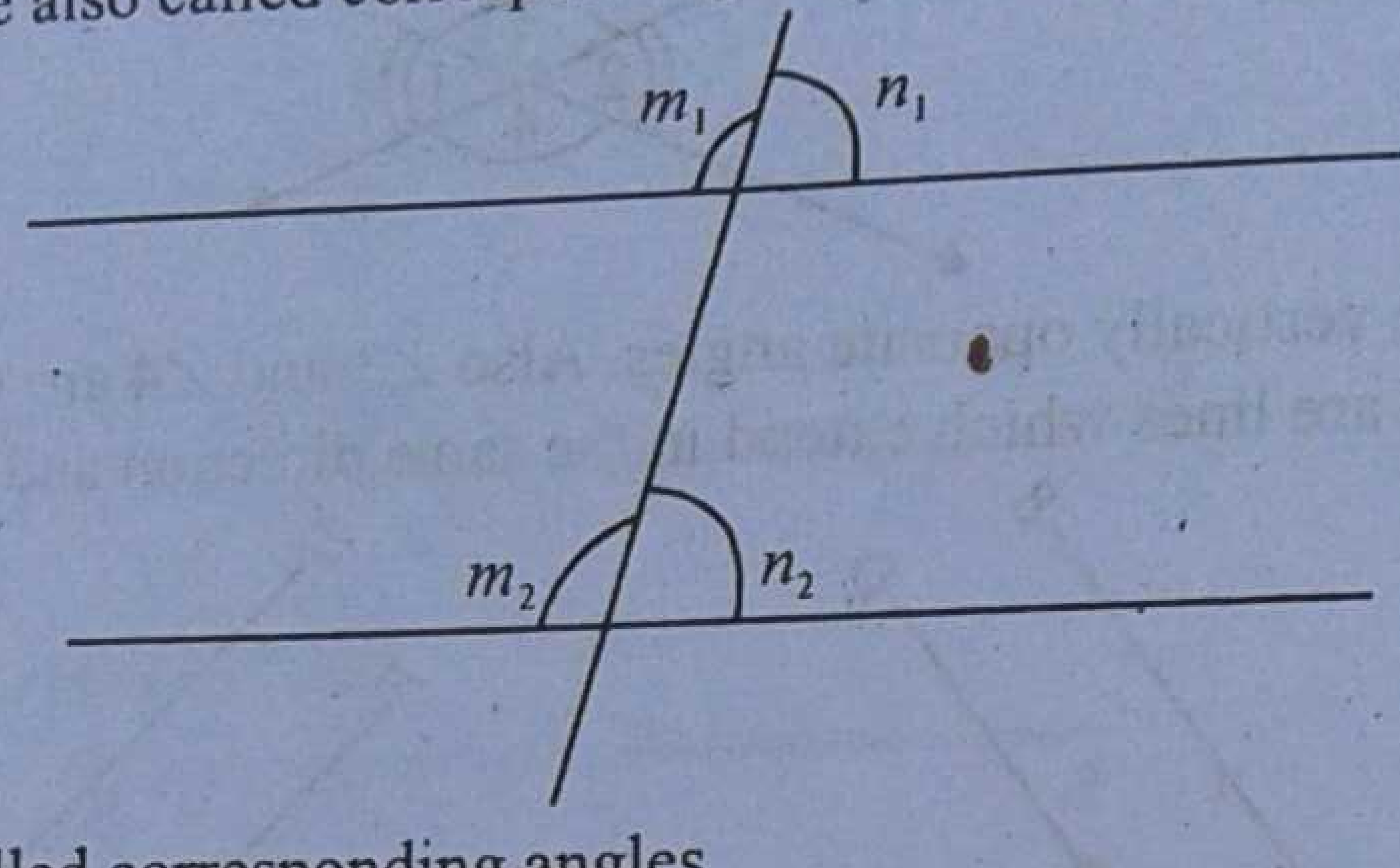


More About Angles:

Interior and Exterior Angles: Interior and exterior angles can also be defined in terms of lines. Such as When two parallel lines *PQ* and *RS* cut by a transversal *AB*. The angles *x* and *y* are called interior angles. The angles *a* and *b* are also interior angles.

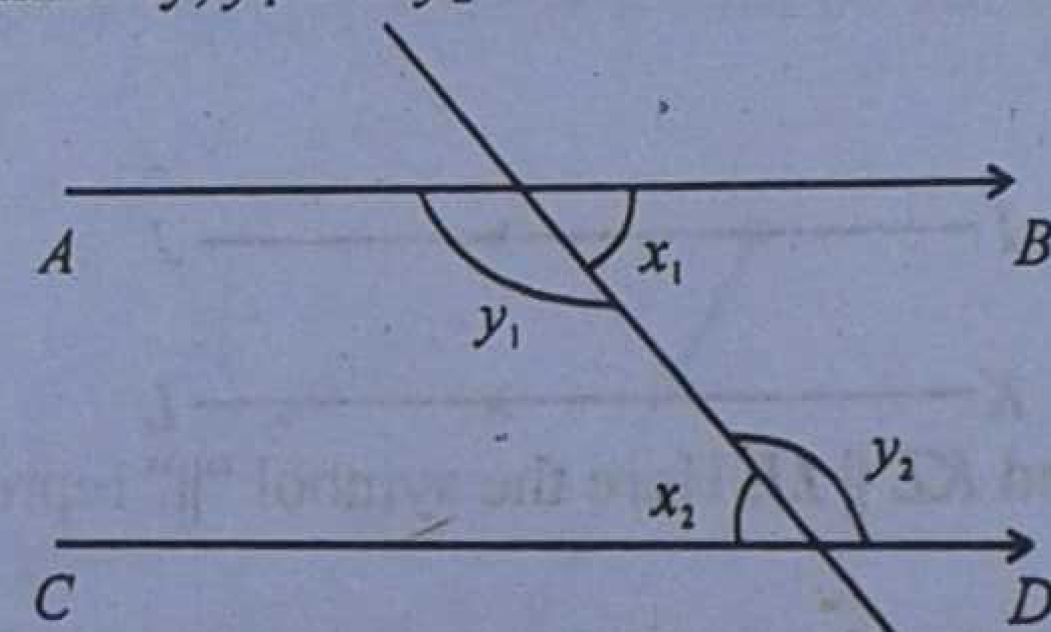


When two parallels lines AB and CD cut by a transversal PQ. The angles m_1 and m_2 are called corresponding angles. Similarly, n_1 and n_2 are also called corresponding angles.



Similarly, n_1 and n_2 are also called corresponding angles.

Alternate Angles: In the following figure, two parallel lines AB and CD cut by a transversal PQ. The angles x_1 and x_2 are called alternate angles. Similarly, y_1 and y_2 are also alternate angles.



Important Results:

From above, when two parallel lines are cut by a transversal;

1. The corresponding angles are equal.
2. The interior angles are supplementary.
3. The alternate angles are equal.

Constructions:

Dividing a Line Segment in a Given Number of Congruent Parts:

In order to divide a line into 3, 5, 7, … equivalent parts, we adopt the following method:

Problem:

Divide a line segment of length 9 cm into five congruent parts.

Method:

Step 1: Draw a line segment $\overline{AB}$ of length 9 cm with the help of scale.

Step 2: From point A, draw a line $\overline{AC}$ making an acute angle (say 30°) with $\overline{AB}$.

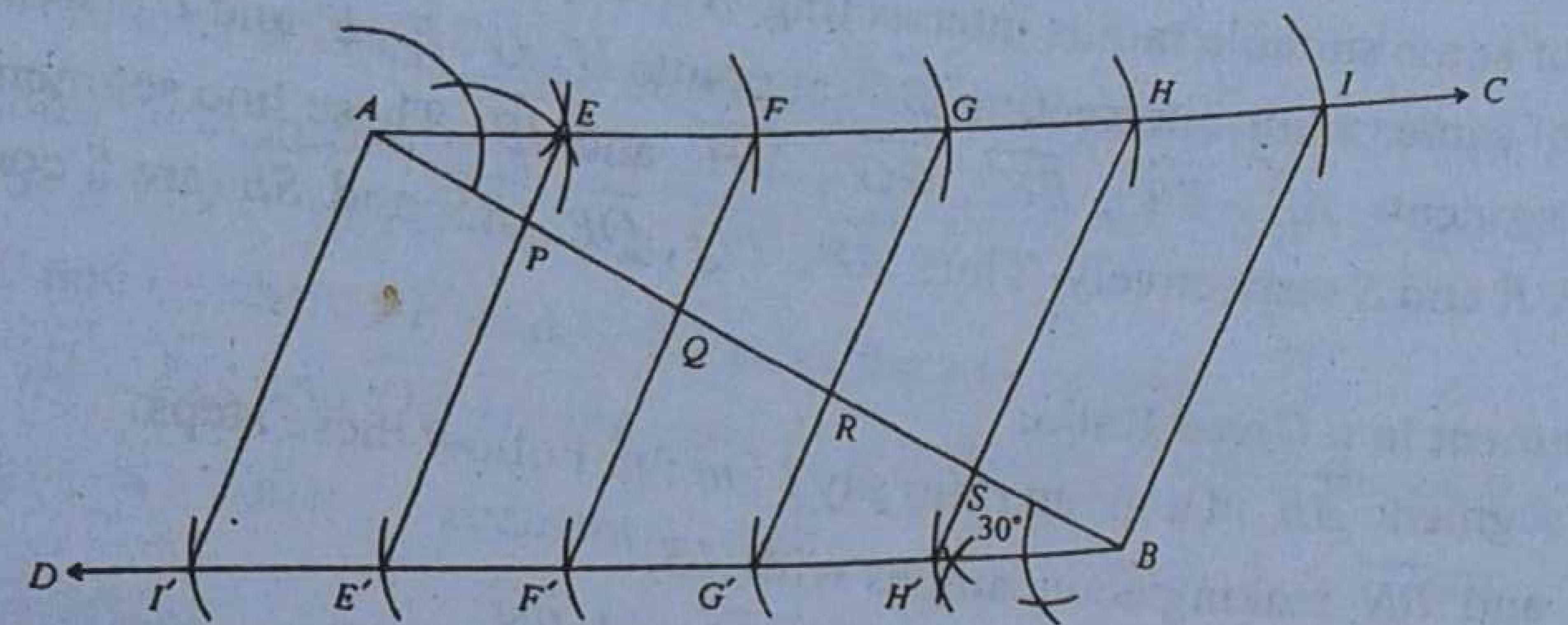
Step 3: Draw $\overline{BD}$, such that

$$\angle ABD = \angle BAC$$

Step 4: Draw arcs of some suitable radius intersecting $\overline{AC}$ at points E, F, G, H and I. For first arc take A as centre and then E for the next arc and so on.

Step 5: Draw arcs of same radius intersecting $\overline{BD}$ at points H', G', F', E' and I' starting from B.

Step 6: Draw line segments AI', EE', FF', GG', HH' and IB. These line segments are intersecting $\overline{AB}$ at points P, Q, R and S respectively. Thus AP, PQ, QR, RS and SB are five congruent parts of $\overline{AB}$.



(Fig. 1)

Example 2: Draw a line segment of length 6.4 cm into 4 congruent parts.

Solution: Steps of Construction:

Step 1: Draw a line segment $\overline{AB}$ of length 6.4 cm.

Step 2: From point A, draw $\overline{AC}$ making an acute angle (say 30°) with $\overline{AB}$.

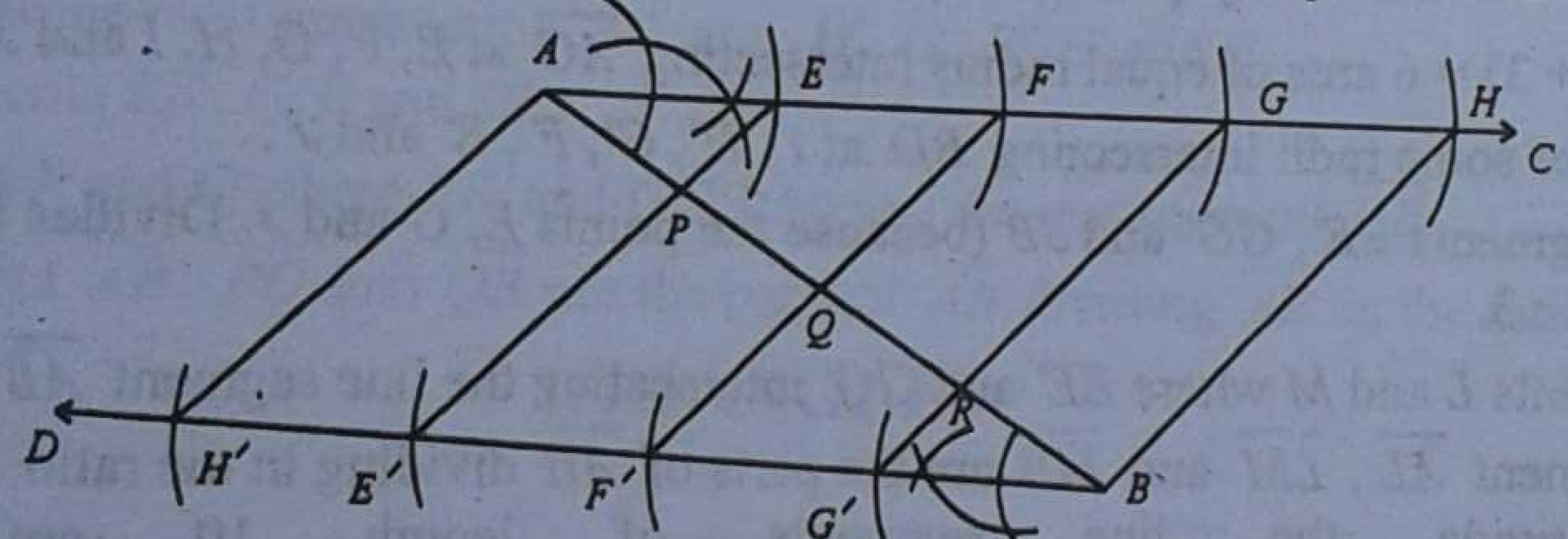
Step 3: Draw $\overline{BD}$ such that

$$m\angle ABD = m\angle BAC$$

Step 4: Draw arcs of some suitable radius intersecting $\overline{AC}$ at points E, F, G and H (For first arc take A as centre and then E for the next arc and soon).

Step 5: Draw arcs of same radius intersecting $\overline{BD}$ at points G', F', E' and H' starting from B.

Step 6: Draw line segments AH', EE', FF', GG' and HB. These line segments are intersecting AB at points P, Q and R respectively. Thus $\overline{AP}$, $\overline{PQ}$, $\overline{QR}$ and $\overline{RS}$ are 4 congruent parts of $\overline{AB}$.



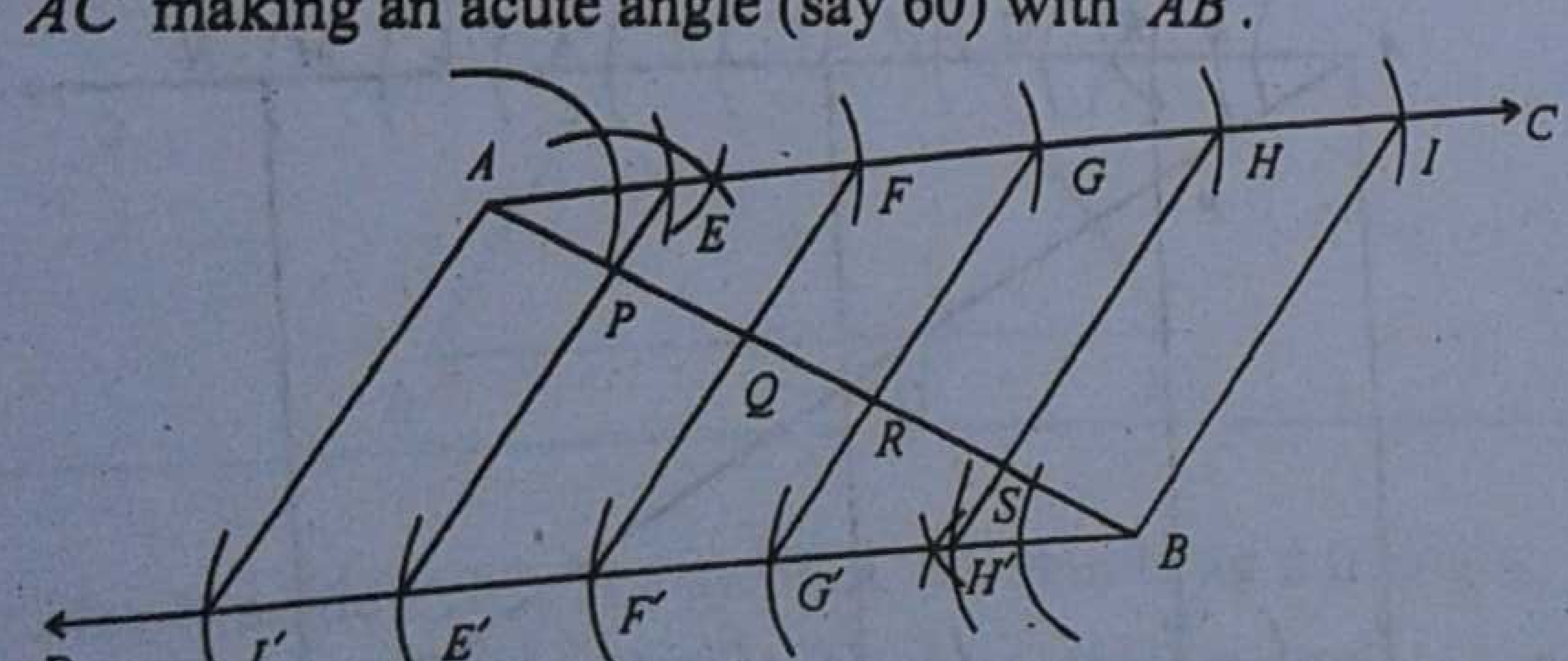
(Fig. 2)

Example 3: Divide a line segment of length 6 cm in 5 congruent parts.

Solution: Steps of Construction:

Step 1: Draw a line segment $\overline{AB}$ of length 6 cm.

Step 2: At point A, draw $\overline{AC}$ making an acute angle (say 60) with $\overline{AB}$.



(Fig. 3)

Step 3: Draw $\overline{BD}$ such that

$$m\angle BAC = m\angle ABD$$

Step 4: Draw arcs of some suitable radius intersecting $\overline{AC}$ at points E, F, G, H and I.

Step 5: Draw arcs of same radius intersecting $\overline{BD}$ at points H', G', F', E' and I' starting from B.

Step 6: Draw line segments $\overline{AI'}$, $\overline{EE'}$, $\overline{FF'}$, $\overline{GG'}$, $\overline{HH'}$ and $\overline{IB}$. These line segments are intersecting $\overline{AB}$ at points P, Q, R and S respectively. Thus $\overline{AP}$, $\overline{PQ}$, $\overline{QR}$, $\overline{RS}$ and $\overline{SB}$ are 5 congruent parts as shown in Fig. 3.

Dividing a Line Segment in a Given Ratio:

For dividing a line segment $\overline{AB}$ in a given ratio say $l : m : n$. Follow these steps:

(i) Draw $\overrightarrow{AM}$ and $\overrightarrow{BN}$ making acute angles with AB.

(ii) Draw $(l + m + n)$ arcs at equal distances on $\overline{AM}$ and $\overline{BN}$.

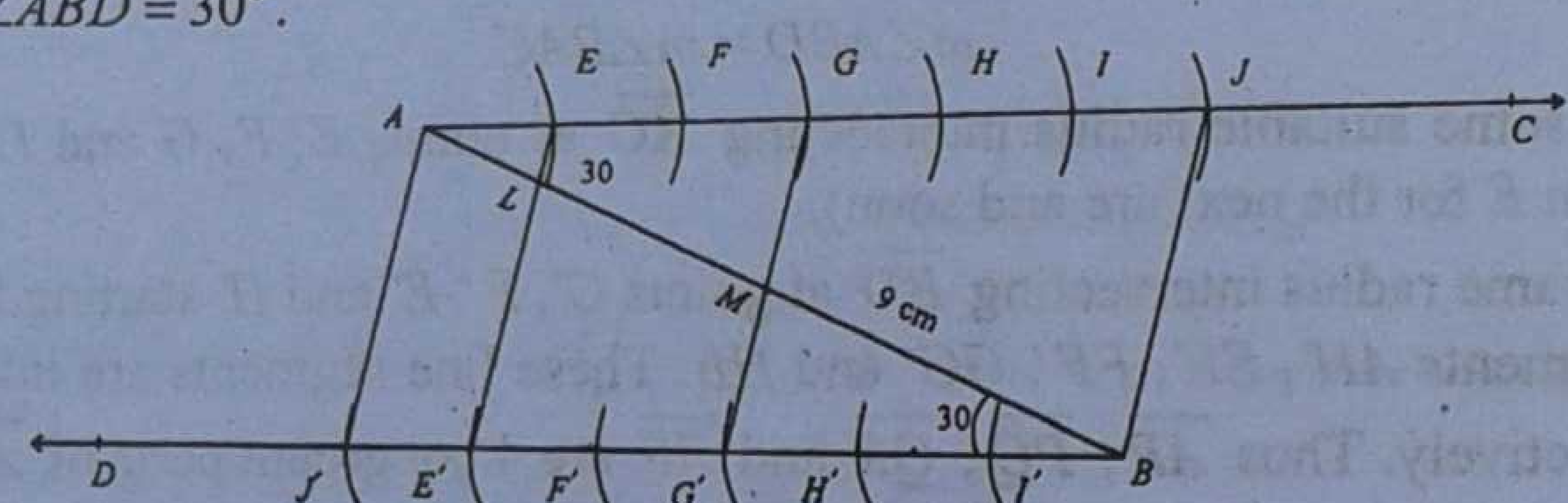
(iii) Join the points corresponding to the ratio $l : m : n$ on $\overline{AM}$ and $\overline{BN}$

Example 4: Divide a line segment $\overline{AB}$ of length 9 cm in the ratio 1 : 2 : 3.

Solution: Steps of Construction:

Step 1: Draw a line segment $\overline{AB}$ such that $m\overline{AB} = 9$ cm.

Step 2: Draw rays $\overline{AC}$ and $\overline{BD}$ making acute angles (say 30°) of equal measurements of A and B *i.e.*, $m\angle B\hat{A}C = m\angle A\hat{B}D = 30°$.



Step 3: Draw $(1 + 2 + 3) = 6$ arcs of equal radius intersecting $\overline{AC}$ at E, F, G, H, I and J.

Step 4: Draw 6 arcs of some radii intersecting $\overline{BD}$ at I', H', G', F', E' and J'.

Step 5: Draw line segments EE', GG' and JB (because the points E, G and J. Divides the line segment $\overline{AC}$ in the ratio 1 : 2 : 3.

Step 6: Name the points L and M where EE' and GG' intersecting the line segment $\overline{AB}$.

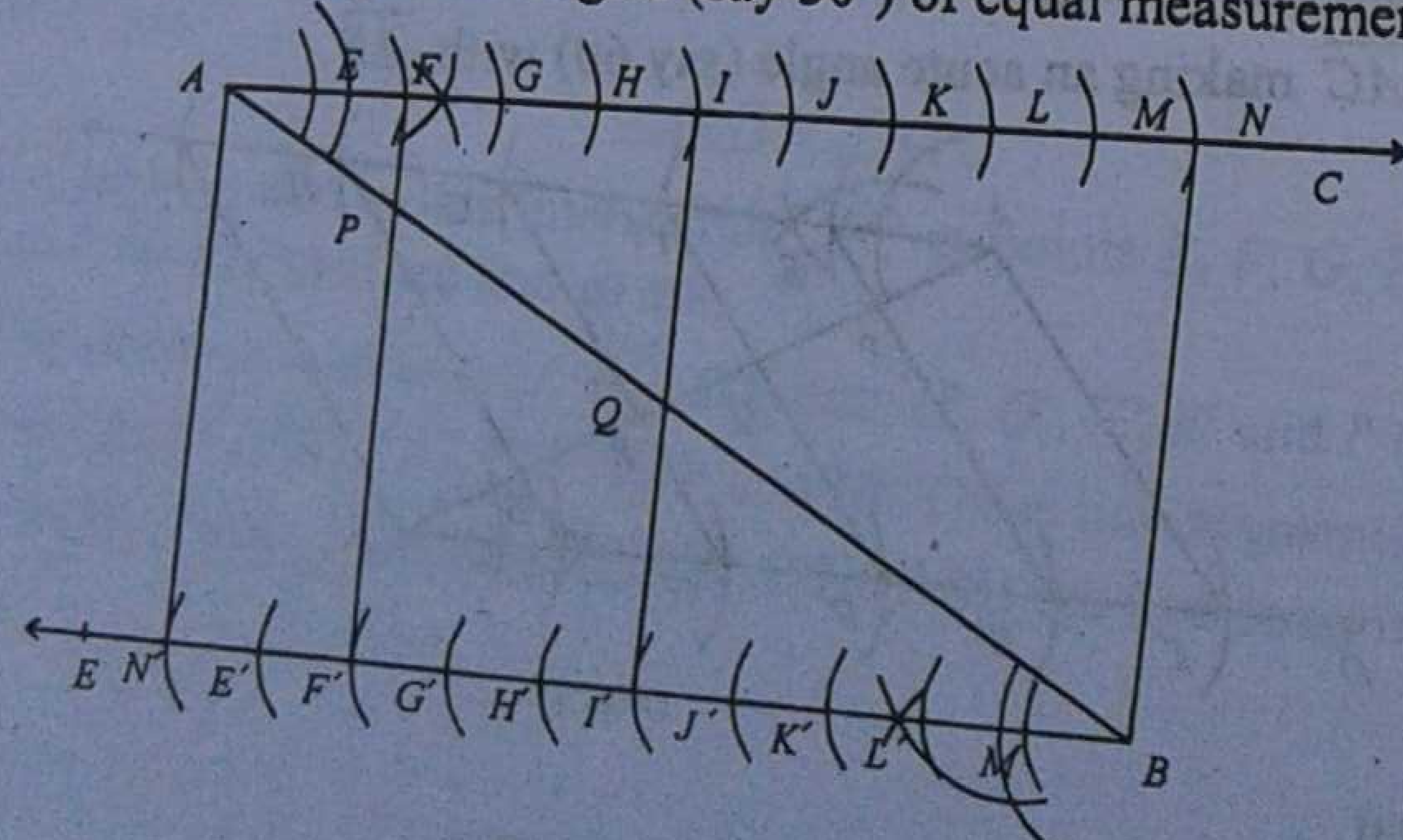
The line segment $\overline{AL}$, $\overline{LM}$ and $\overline{MB}$ are the parts of $\overline{AB}$ dividing in the ratio 1 : 2 : 3.

Example 5: Divide the line segments of length 10 cm into the ratio 2 : 3 : 5.

Solution: Steps of Construction:

Step 1: Draw a line segment $\overline{AB}$ such that $m\overline{AB} = 10$ cm.

Step 2: Draw rays $\overrightarrow{AC}$ and $\overrightarrow{BD}$ making acute angles (say 30°) of equal measurements of A and B.



i.e., $m\angle B\hat{A}C = m\angle A\hat{B}D = 30°$

Step 3: Draw $(2 + 3 + 5) = 10$ arcs of equal radius intersecting $\overline{AC}$ at points $E, F, G, H, I, J, K, L, M$ and N.

Step 4: Draw 10 arcs of some radii intersecting $\overrightarrow{BD}$ at $M', L', K', J', I', H', G', F', E'$ and N'.

Step 5: Draw line segments AN', FF' and II' and NB.

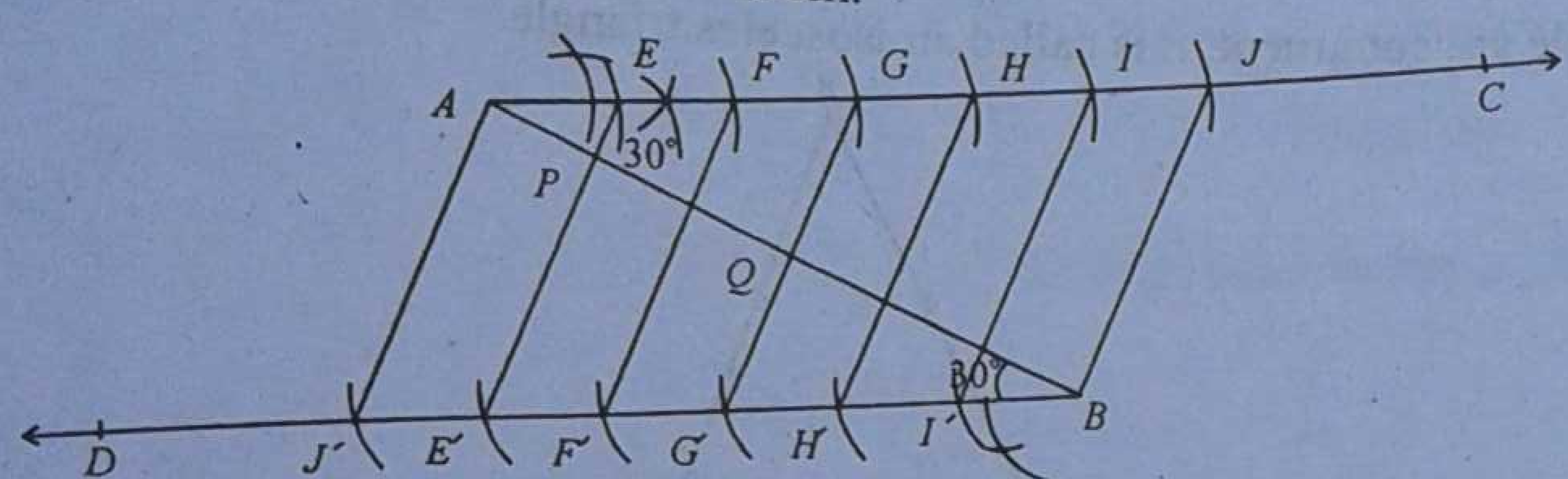
Step 6: Name the points P and Q where FF' and II' intersect the line segment $\overline{AB}$.

The line segment $\overline{AP}$, $\overline{PQ}$ and $\overline{QB}$ are the parts of $\overline{AB}$ dividing $\overline{AB}$ in the ratio 2 : 3 : 5.

Example 6: Divide a line segment $\overline{AB}$ of length 6 cm in the ratio 1 : 2 : 3.

Solution: Steps of Construction:

Step 1: Draw a line segment $\overline{AB}$ such that $m\overline{AB} = 6$ cm.



Step 2: Draw rays $\overrightarrow{AC}$ and $\overrightarrow{BD}$ making acute angles (say 30°) of equal measurements of A and B i.e., $m\angle B\hat{A}C = m\angle A\hat{B}D = 30°$.

Step 3: Draw $(1 + 2 + 3) = 6$ arcs of equal radius intersecting $\overline{AC}$ at points E, F, G, H, I and J.

Step 4: Draw 6 arcs of some radii intersecting $\overrightarrow{BD}$ at I', H', G', F', E' and J'.

Step 5: Draw line segments EE', GG' and JB.

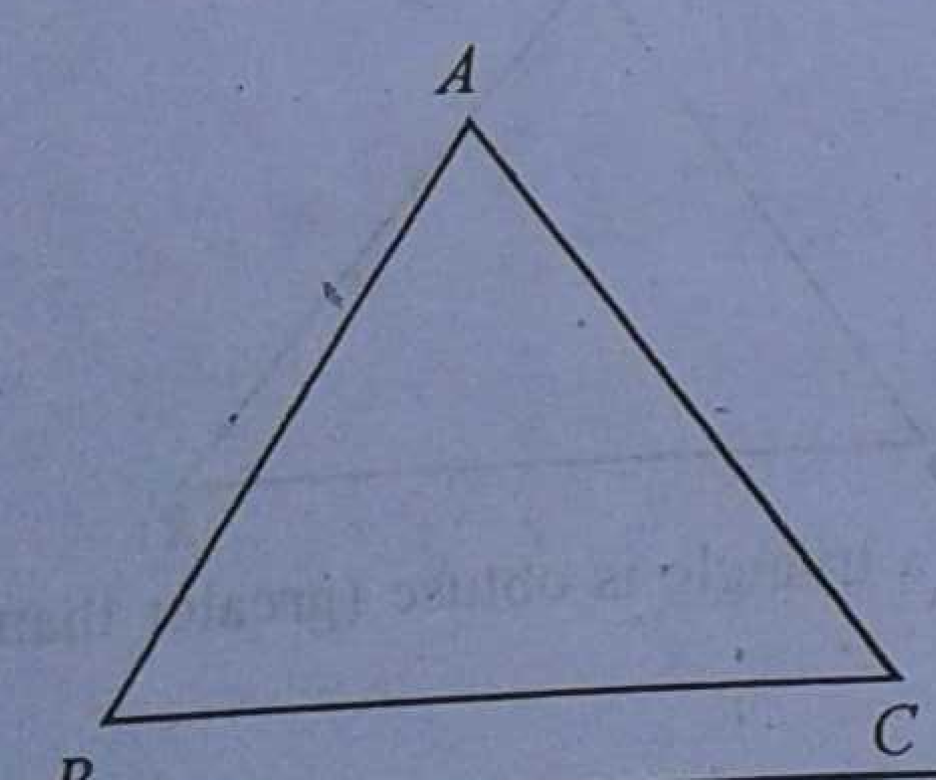
Step 6: Name the points P and Q where EE' and GG' intersect the line segment $\overline{AB}$.

The line segments $\overline{AP}$, $\overline{PQ}$ and $\overline{QB}$ are the parts of $\overline{AB}$ dividing $\overline{AB}$ in the ratio 1 : 2 : 3.

TRIANGLES

Triangle:

If A, B, C are three non-collinear points, the union of the line segments $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ is called the triangle ABC.

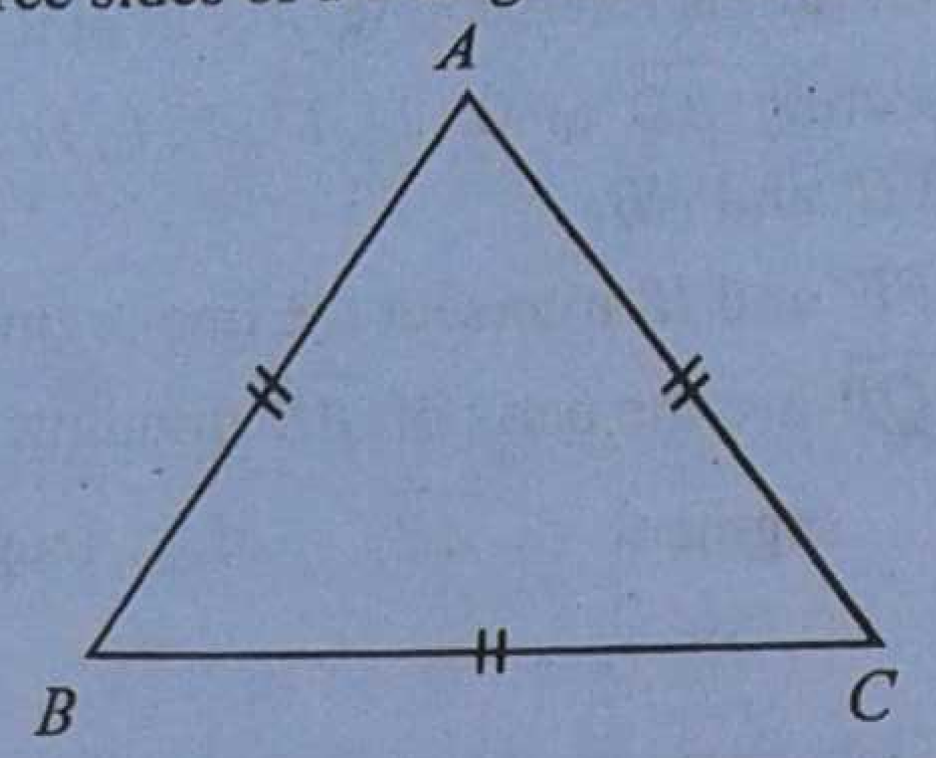


Important Notes:

1. We also write ΔABC for triangle ABC. The symbol "Δ" stands for a triangle.
2. The line segments $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ are called the sides of the triangle.
3. The points A, B and C are the vertices of the ΔABC.

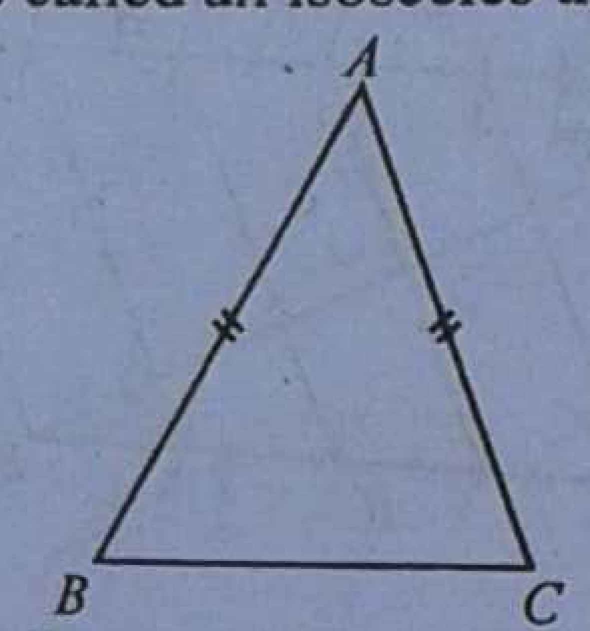
Types of Triangles: Some types of triangles with respect to their sides are explained below:

(i) Equilateral Triangles: If all the three sides of a triangle are congruent, it called an equilateral triangle.

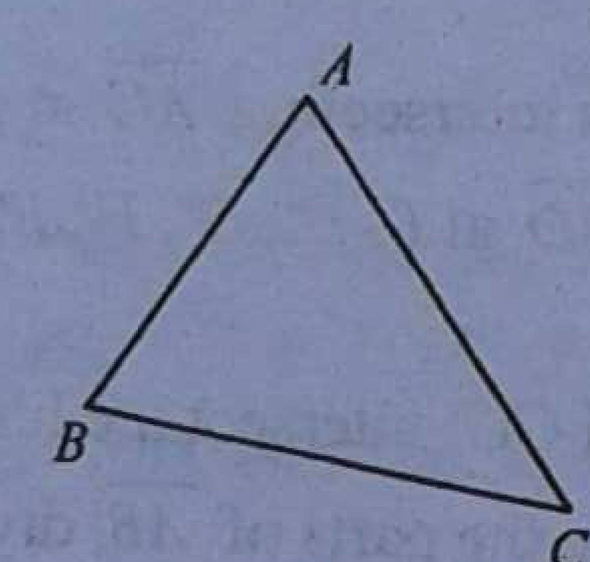


Isosceles Triangle:

If two sides of a triangle are congruent, it is called an isosceles triangle.

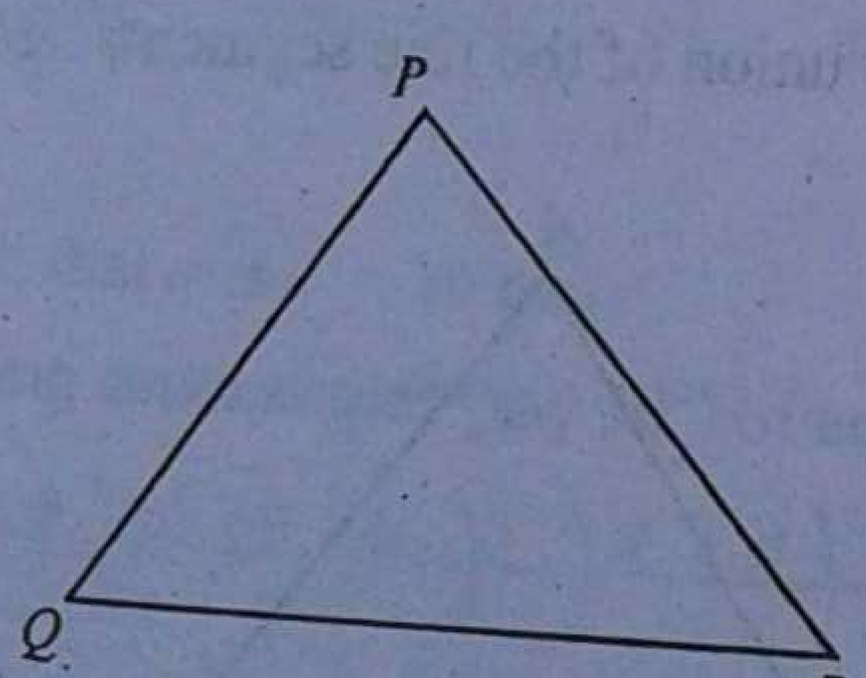


Different Sided Triangle: If the three sides of a triangle are of different sizes, it is called scalence or different sided triangle.

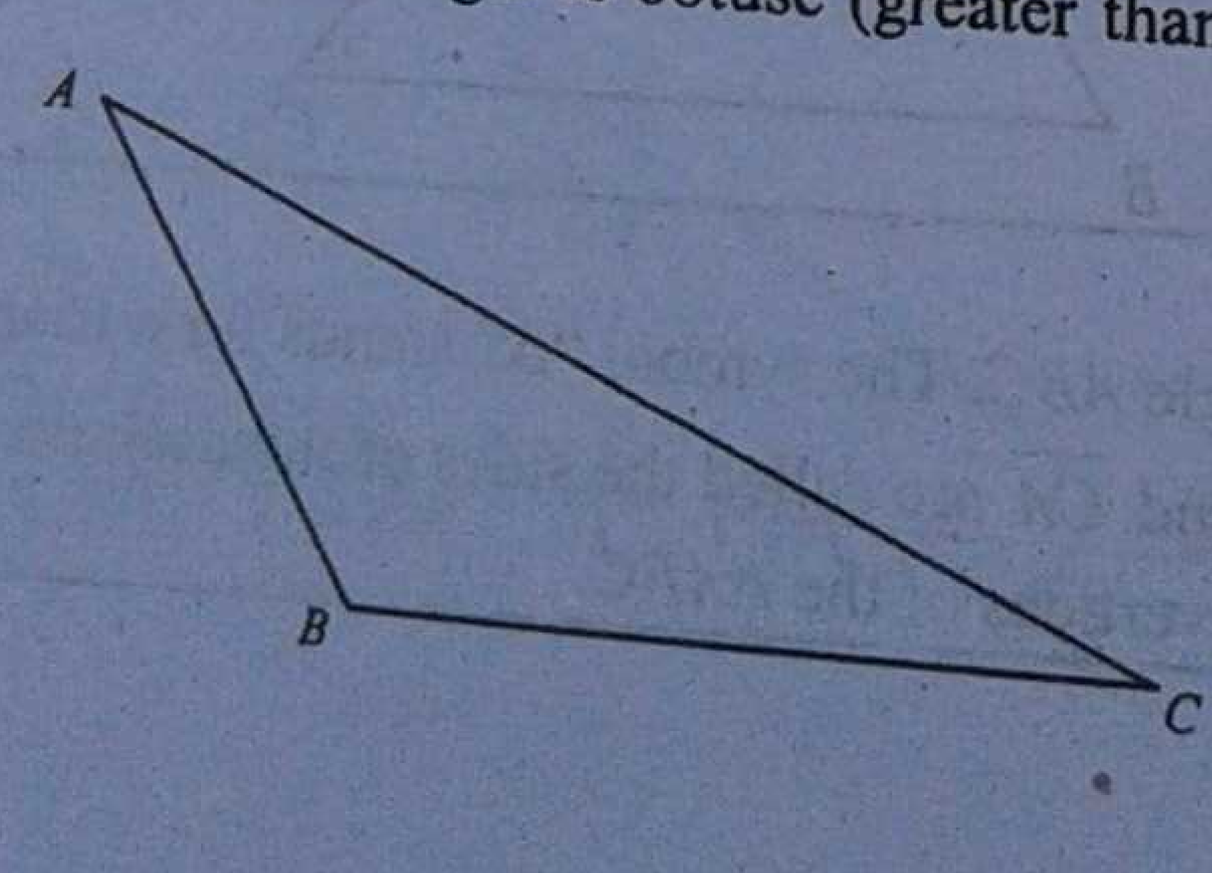


Types of Triangles W.R.T. Angles:

(i) Acute-Angled Triangle: If all the three angles of a triangle are acute (less than 90°), it is called an acute-angled triangle.

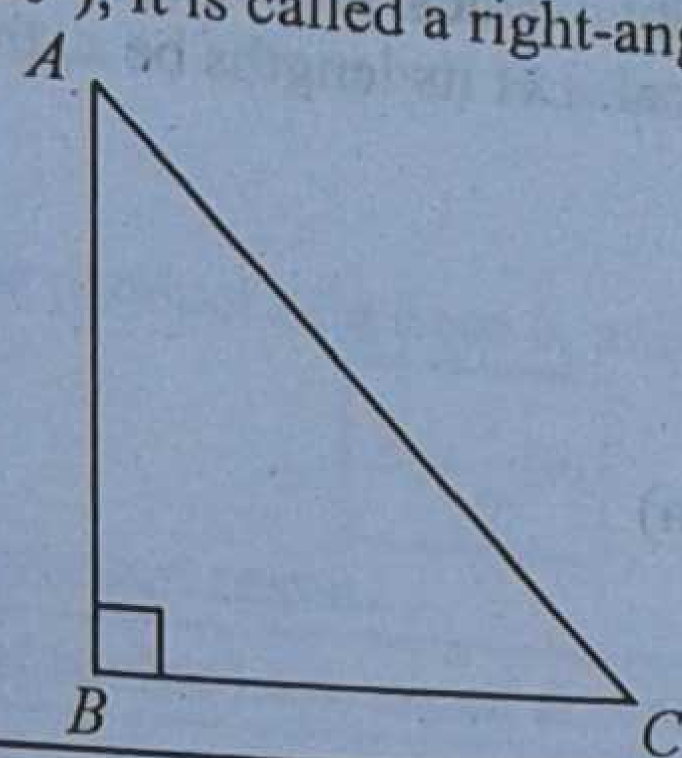


Obtuse-Angled Triangle: If one angle of a triangle is obtuse (greater than 90°), it is called an obtuse-angled triangle.



Right-Angled Triangle:

If one angle of a triangle is a right-angle (90°), it is called a right-angled triangle.



Note: The side opposite to the right-angle is called its hypotenuse.

Pythagoras Theorem:

This important theorem is named after a Greek mathematician Pythagoras 2500 years ago.

Statement of Pythagoras Theorem:

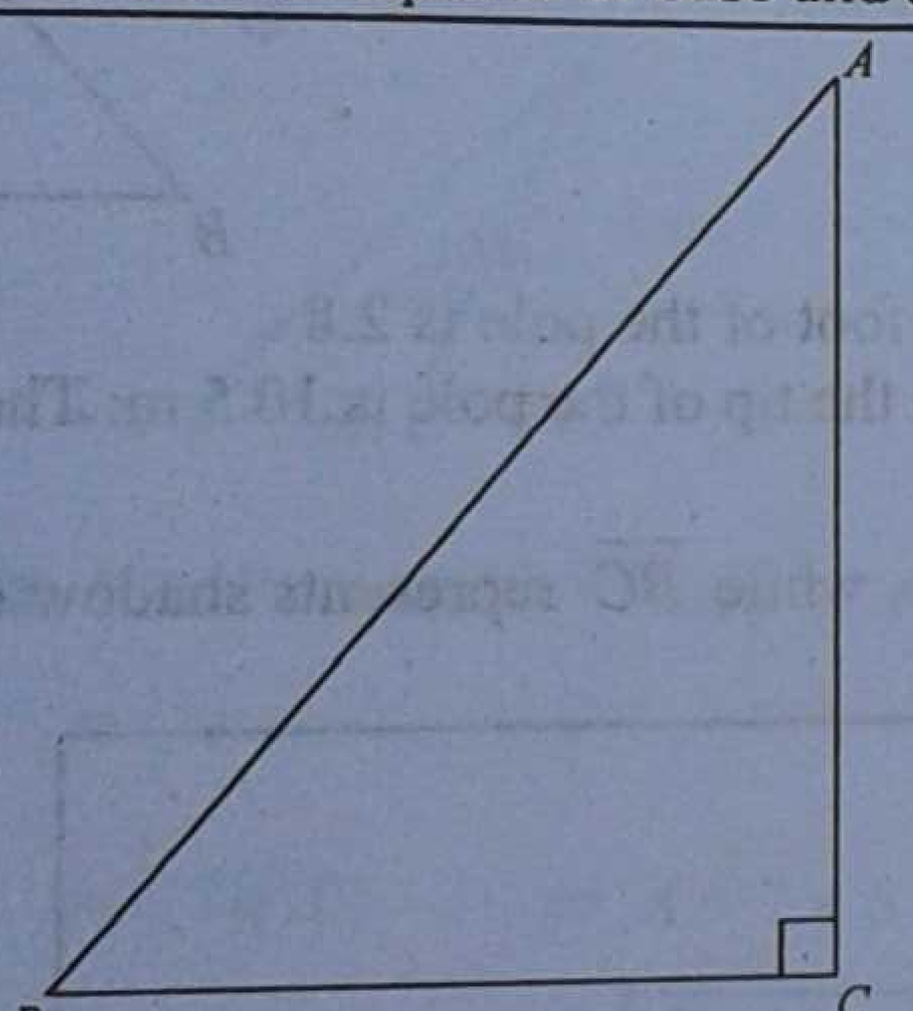
In a right-angled triangle ABC with $m\angle C = 90°$, and a, b, c are opposite sides of the angles $\angle A$, $\angle B$ and $\angle C$ respectively. Then

$$c^2 = a^2 + b^2$$

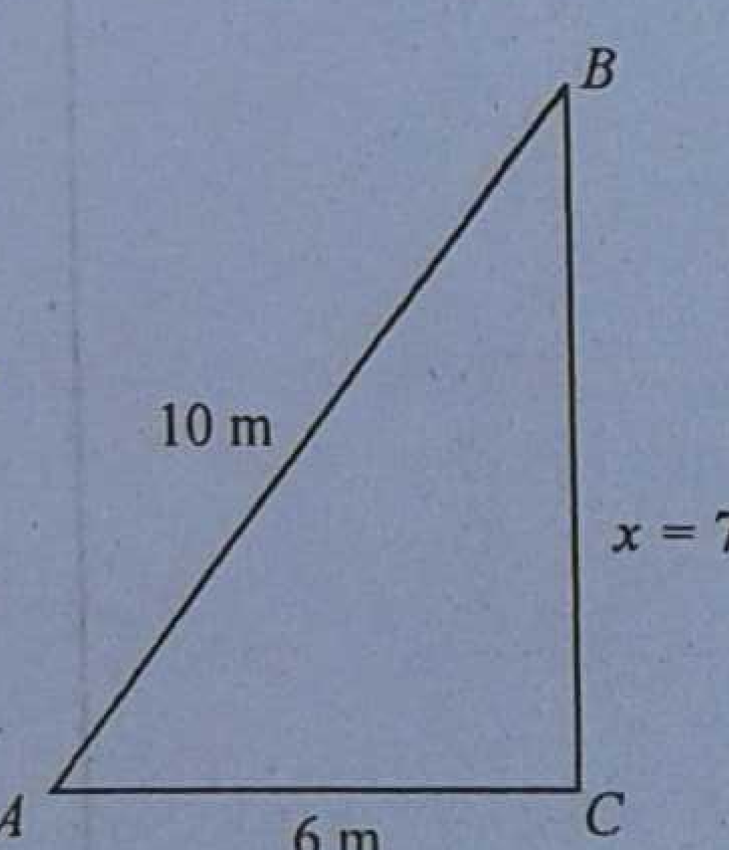
$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Altitude})^2$

In words

Hypotenuse square is the sum of the squares of base and altitude.



Example 1: A ladder 10 m long is made to rest against a wall. Its lower end touches the ground at a distance of 6 *m* from the wall. At what height above the ground the upper end of the ladder rests against the wall?



Solution: In the adjacent diagram, AB represents the ladder and BC represents the wall, where AC represents the distance of the foot of the ladder to the wall. Let x be height of the wall at which the top side rests.

According to the Pythagoras theorem:

$$(AB)^2 = (AC) + (BC)^2$$

$$\Rightarrow (BC)^2 = (AB)^2 - (AC)^2$$

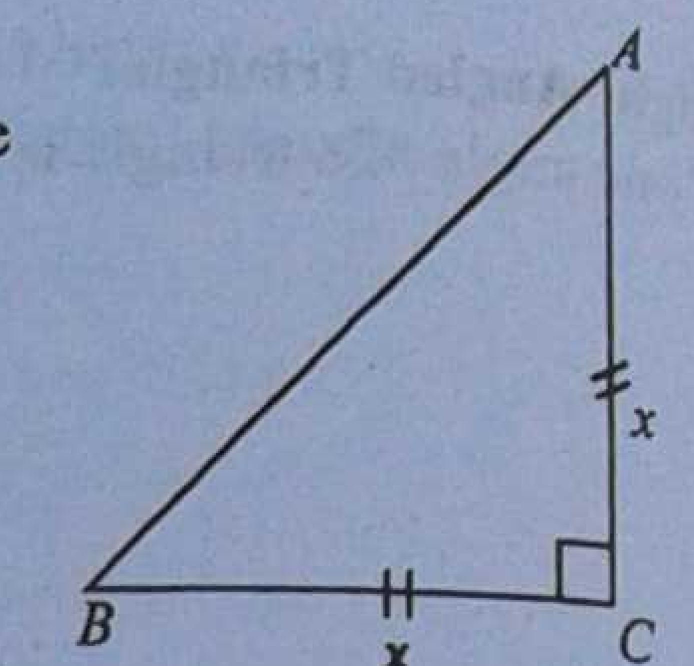
$$x^2 = (10)^2 - (6)^2$$

$$\Rightarrow x^2 = 100 - 36$$

$$\Rightarrow x^2 = 64 \qquad \Rightarrow \sqrt{x^2} = \sqrt{64}$$

$$\Rightarrow \boxed{x = 8}$$

Example 2: In a isosceles right-angle triangle, the square of the hypotenuse is 98 cm^2. Find the length of the



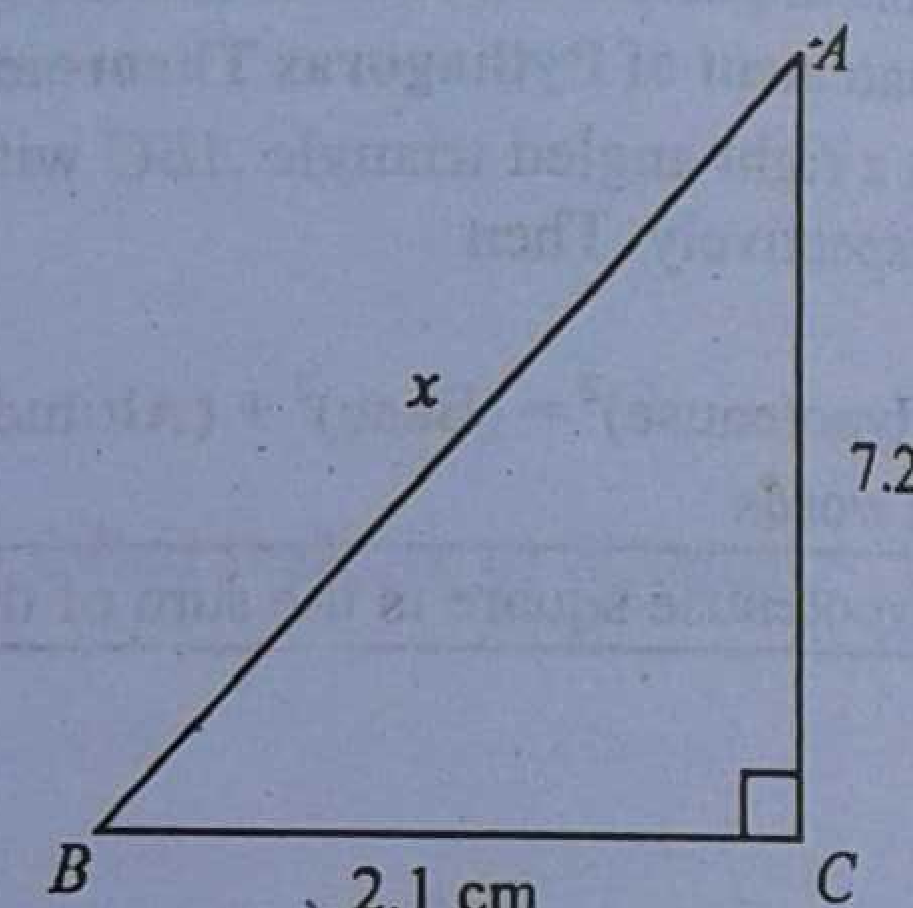
congruent sides.

Solution: Since, in an isosceles triangle, two sides are congruent, therefore the length of the sides BC and AC will be identical. Let its lengths be x, then

$$(AB)^2 = (BC)^2 + (AC)^2$$
$$= (x)^2 + (x)^2$$
$$AB^2 = x^2 + x^2$$
$$AB^2 = 2x^2$$
$$98 = 2x^2 \quad (\because AB^2 = 98 : \text{Given})$$
$$\frac{98}{2} = 2x^2 \qquad \Rightarrow x^2 = 49$$
$$\Rightarrow x = 7$$

Hence the lengths of the congruent sides are 7 cm.

Example 3: In a triangle ABC, right-angled at C, $m\overline{BC} = 2.1$ cm and $m\overline{CA} = 7.2$ cm. What is the length of $\overline{AB}$?



Solution: Given that $m\overline{BC} = 2.1$ cm, $m\overline{CA} = 7.2$ cm

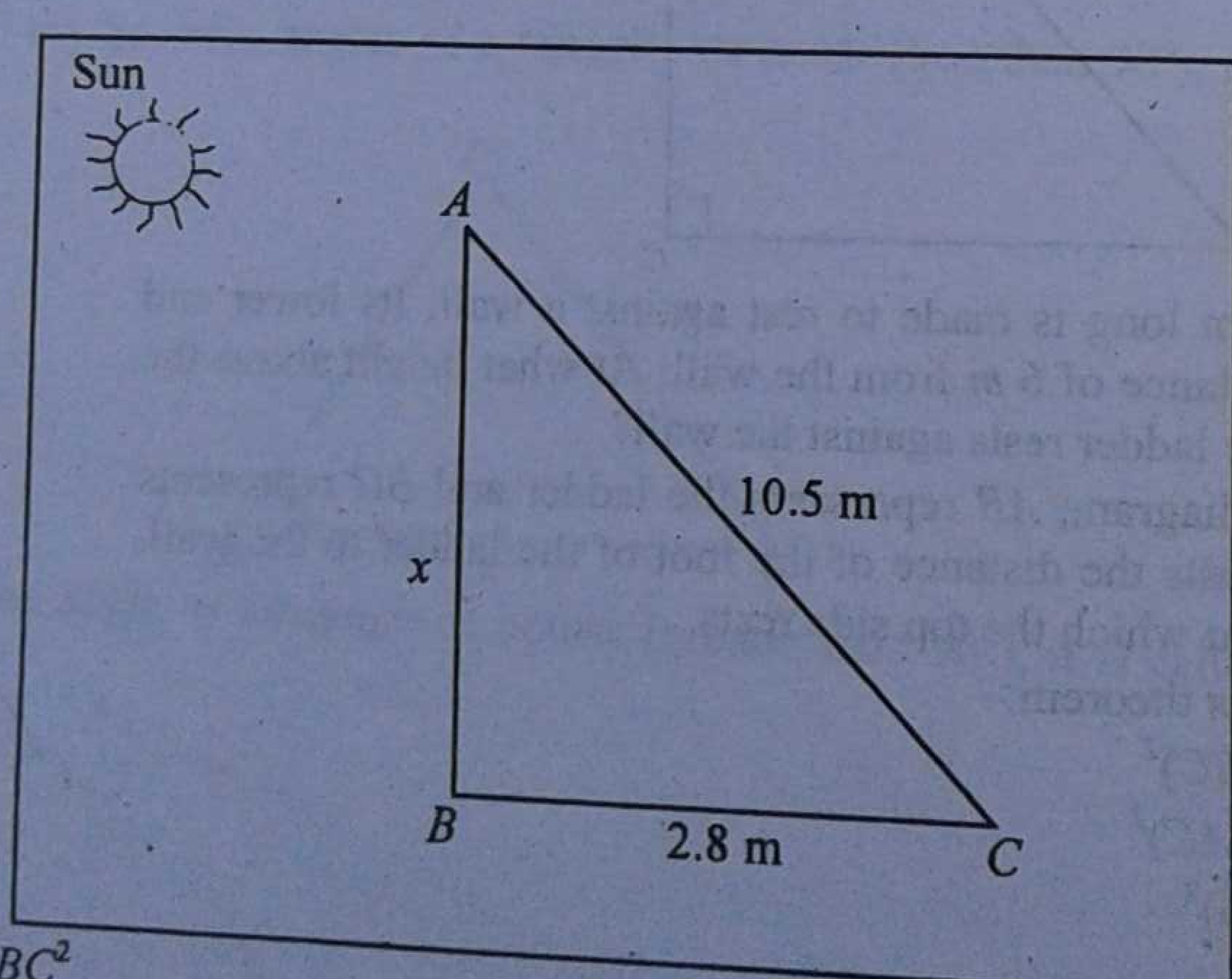
In triangle ABC, let x be the length of AB, then

$$(AB)^2 = (BC)^2 + (AC)^2$$
$$(x)^2 = (2.1)^2 + (7.2)^2$$
$$\Rightarrow x^2 = (2.1)^2 + (7.2)^2$$
$$\Rightarrow x^2 = 4.41 + 51.84 \qquad \Rightarrow x^2 = 56.25$$
$$\Rightarrow \sqrt{x^2} = \sqrt{56.25}$$
$$\Rightarrow \boxed{x = 7.5 \text{ cm}}$$

Hence, length of AB is 7.5 cm.

Example 3: The shadow of a pole measured from the foot of the pole is 2.8 m long, if the distance from the tip of the shadow from the tip of the pole is 10.5 m. Then find the length of the pole.

Solution: In the adjacent diagram, $\overline{AB}$ represents pole, while $\overline{BC}$ represents shadow of the pole. Let x be the length of the pole, then



$$AC^2 = AB^2 + BC^2$$
$$(10.5)^2 = (AB)^2 + (2.8)^2$$
$$110.25 = x^2 + 7.84 \qquad \Rightarrow x^2 = 110.25 - 7.84$$

$$\Rightarrow x^2 = 102.41 \quad \Rightarrow x = \sqrt{102.41}.$$
$$\Rightarrow x = 10 \text{ m (approx.)}$$

Hence, the length of the pole is 10 m.

Area of a Triangle:

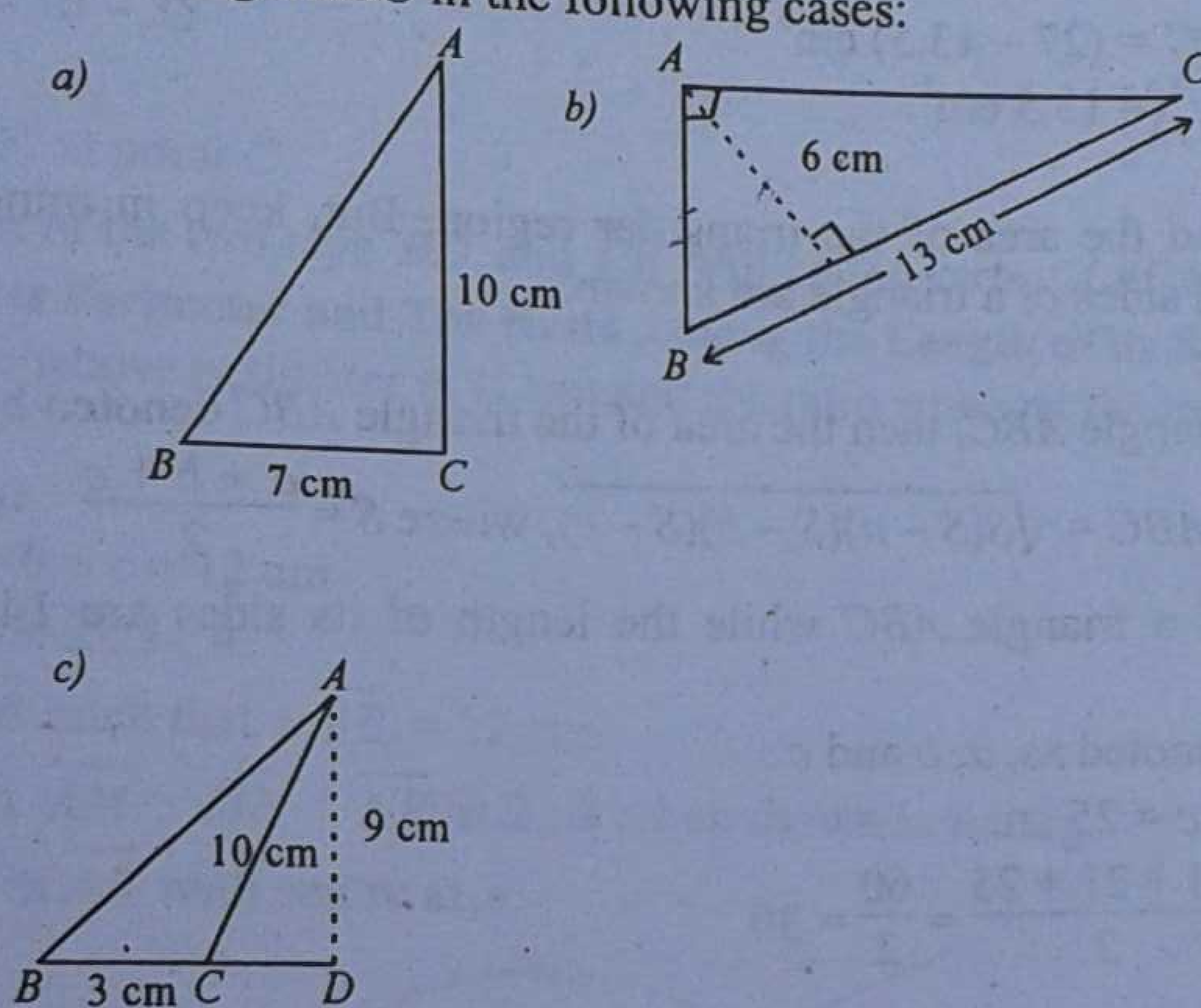
The area A, of a triangle is one-half the product of a base b, and its corresponding altitude, h. That is

$$\boxed{A = \frac{1}{2}bh}$$

From above, we find the following important result.

Triangles with equal bases and equal altitudes have equal areas.

Example 1: Find the area of the triangle ABC in the following cases:



Solution:

(a) In the given fig. $h = 10$ cm and $b = 7$ cm

Since, Area $= \frac{1}{2}bh$

$\Rightarrow \quad A = \frac{1}{2}(7)(10) \quad \Rightarrow A = 7 \times 5$

$\Rightarrow \boxed{A = 35 \text{ cm}^2}$

(b) In the given fig. $b = 13$ cm and $h = 6$ cm, therefore

Area $= \frac{1}{2}bh$

$\Rightarrow \quad A = \frac{1}{2}(6)(13) \quad \Rightarrow \boxed{A = 39 \text{ cm}^2}$

(c) In the given fig., first of all we calculate CD, in ΔABC

$AC^2 = AD^2 + CD^2 \quad \Rightarrow CD^2 = AC^2 - AD^2$

$\Rightarrow CD^2 = (10)^2 - (9)^2$

$\Rightarrow CD^2 = 100 - 81 = 9 \Rightarrow \boxed{CD = 3}$

Now Area of the triangle ACD

$A = \frac{1}{2}bh$

$$= \frac{1}{2} \times 3 \times 9 = \frac{27}{2}\ 13.5\ cm^2$$

Here, the length of the base of the triangle ABD becomes,

$$BD = BC + CD = 3 + 3 = 6\ cm$$

Now area of the triangle ABD is

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2} \times 6 \times 9 = 3 \times 9 = 27\ cm^2$$

To find the area of the triangle ABC, subtract the area of triangle ACD from the area of the triangle ABD.

$\therefore$ Required area of $\Delta ABC = (27 - 13.5)\ cm^2$

$= 13.5\ cm^2$

Hero's Formula:

Hero's formula is used to find the area of the triangular region. But, keep in mind that Hero's formula is applied when the lengths of all sides of a triangle are known.

Statement of Hero's Formula:

If a, b, c are the lengths of a triangle ABC, then the area of the triangle ABC denoted by ΔABC is given by

$$\Delta ABC = \sqrt{S(S-a)(S-b)(S-c)},\ \text{where } S = \frac{a+b+c}{2}$$

Example 1: Find the area of a triangle ABC while the length of its sides are 14 cm, 21 cm and 25 cm respectively.

Solution: Let the lengths be denoted as, a, b and c

So $a = 14$ cm; $b = 21$ cm; $c = 25$ cm

Now $S = \frac{a+b+c}{2} \Rightarrow S = \frac{14+21+25}{2} = \frac{60}{2} = 30$

$\Rightarrow$ $S = 30$

Hero's formula is $\Delta ABC = \sqrt{S(S-a)(S-b)(S-c)}$

$$= \sqrt{30(30-14)(30-21)(30-25)}$$

$$= \sqrt{30(16)(9)(5)}$$

$$= \sqrt{3 \times 2 \times 5 \times 4 \times 4 \times 3 \times 3 \times 5}$$

$$= \sqrt{4 \times 4 \times 5 \times 5 \times 3 \times 3 \times 3 \times 2}$$

$$= 4 \times 5 \times 3\sqrt{6}$$

$$= 60\sqrt{6}\ cm^2$$

Example 2: The lengths of the sides of a triangle are 60 m, 153 m and 11 m. Find the area of the triangle.

Solution: Let the lengths of the given triangle be denoted by a, b and c. Such that, $a = 60$ m, $b = 153$ m and $c = 11$ m.

Then $S = \frac{a+b+c}{2} = \frac{60+153+111}{2} = \frac{324}{2} = 162$ cm

By Hero's formula

$$\Delta ABC = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{162(162-60)(162-153)(162-11)}$$

$$= \sqrt{162(102)(9)(151)}$$

$$= 4739\ m^2$$

Constructions of Triangles:

Construction of a Triangle when Two Angles and Any Side is Given:

Example:

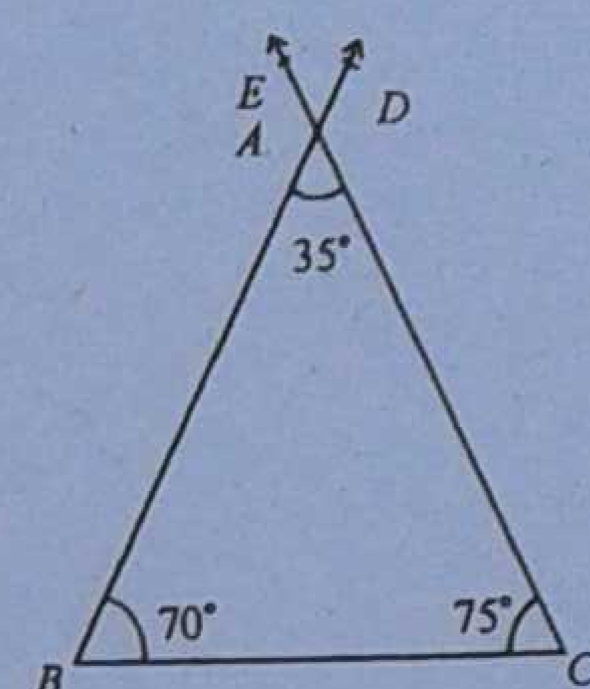
Construct a triangle with $m\angle A = 35^\circ$, $m\angle B = 70^\circ$, $m\overline{BC} = 4$ cm

Steps of Construction:

1. Draw a line segment with length 4 cm.
2. Construct an angle of 70° at B with Protractor.

We have,

$$\angle A + \angle B + \angle C = 180$$
$$35 + 70 + \angle C = 180$$
$$\Rightarrow \angle C = 180 - 70 - 35$$
$$\Rightarrow \angle C = 75^\circ$$



3. Construct an angle of 75° at point C.
4. The point of intersection of the two rays $\overline{BD}$ and $\overline{CE}$ will be the vertex A of the ΔABC.

Constructing a Triangle when Its Perimeter and The Ratio Among the Length of Its Sides is Given:

Example 1: Construct a triangle whose perimeter is 12 cm and the ratio amongst the lengths of its sides is 2 : 3 : 4.

Solution: Steps of Construction:

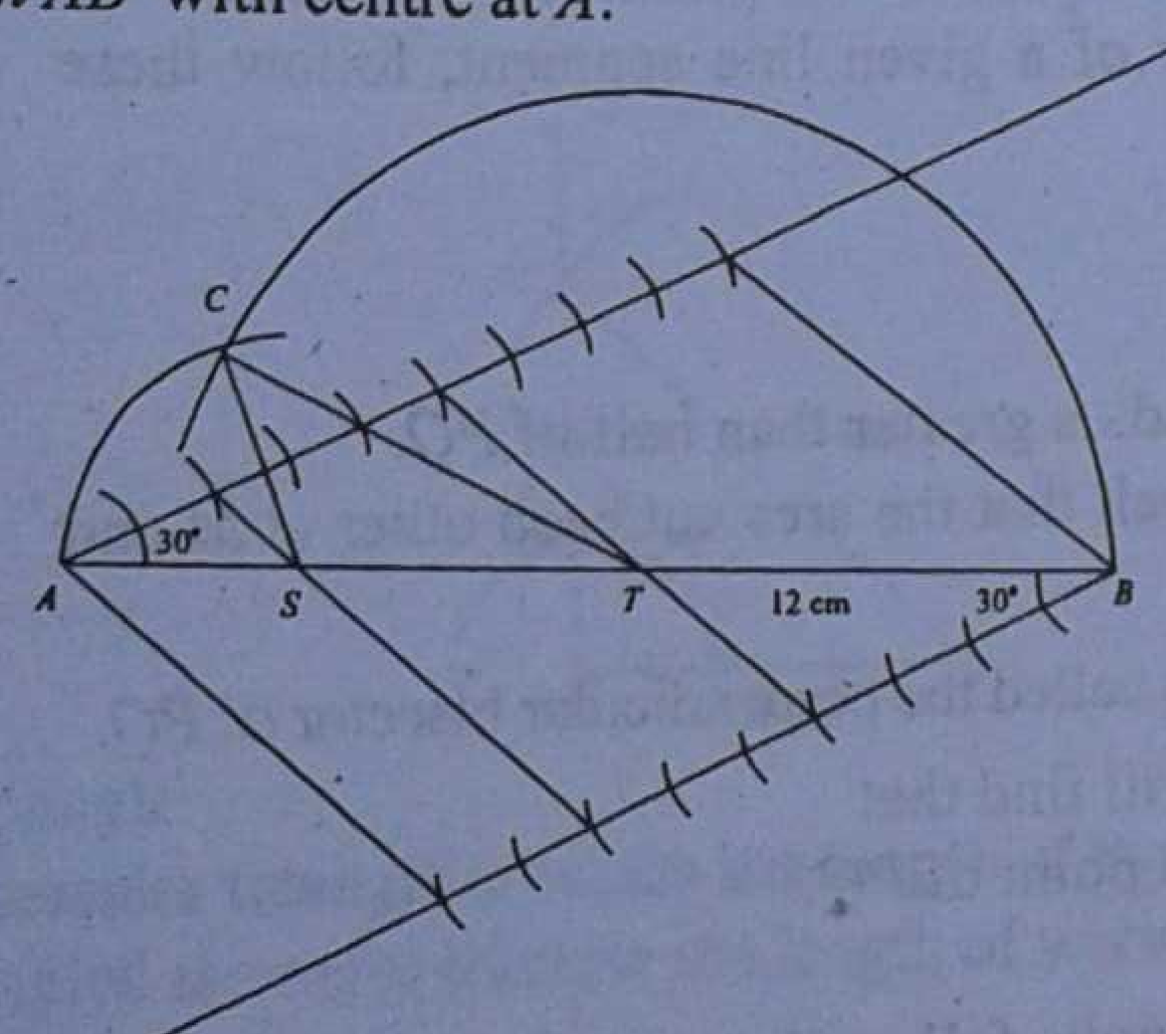
Given that: (i) $P = a + b + c = 12$ cm

(ii) $a : b : c = 2 : 3 : 4$

Step 1: Draw a line segment $\overline{AB}$ such that $m\overline{AB} = 12$ cm.

Step 2: Draw rays $\overline{AB}$ such that $\overline{AM} : \overline{MN} : \overline{NB}$ is 2 : 3 : 4 as shown in the fig.

Step 3: Draw an arc of radius $= m\overline{AB}$ with centre at A.



Step 4: Draw an arc of radius $m\overline{TB}$ with centre at T.

Step 5: Name, the point of intersection of the two arcs as C.

Step 6: Draw $\overline{CS}$ and $\overline{CT}$, ΔCST is the required triangle.

Example 2: Construct triangle whose perimeter is 9 cm and the ratio amongst the lengths of its sides is 2 : 4 : 3.

Solution: Steps of Construction:

Given that: (i) $a + b + c = 9$ cm

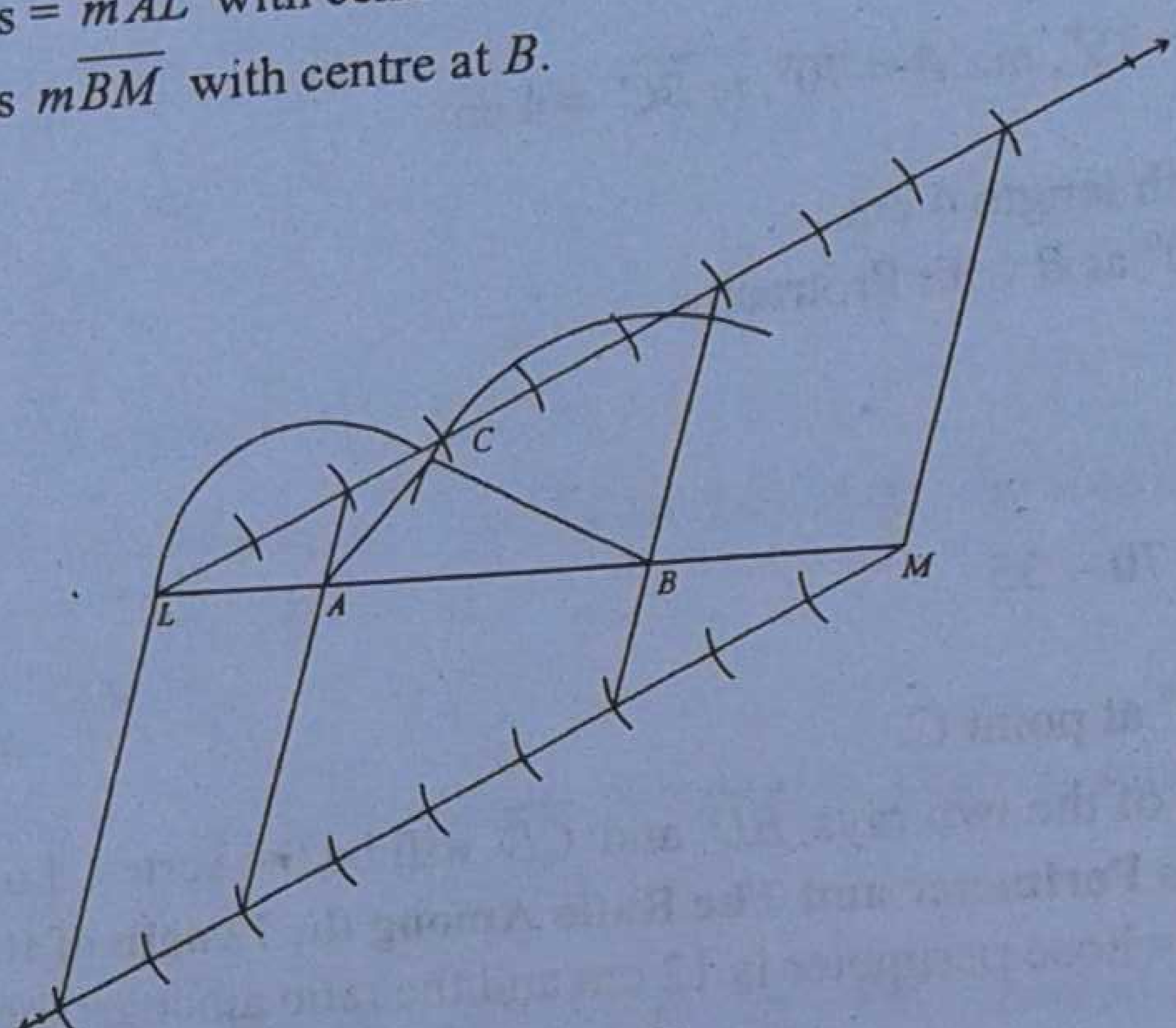
(ii) $a : b : c = 2 : 4 : 3$

Step 1: Draw a line segment $\overline{LM}$ such that $m\overline{LM} = 9$ cm.

Step 2: Divide $\overline{LM}$ such that $\overline{LA} : \overline{LB} : \overline{BM}$ is $2 : 4 : 3$ as shown in the fig.

Step 3: Draw an arc of radius = $m\overline{AL}$ with centre at A.

Step 4: Draw an arc of radius $m\overline{BM}$ with centre at B.



Step 5: Name, the point of intersection of the two arcs as C.

Step 6: Draw $\overline{AC}$ and $\overline{BC}$

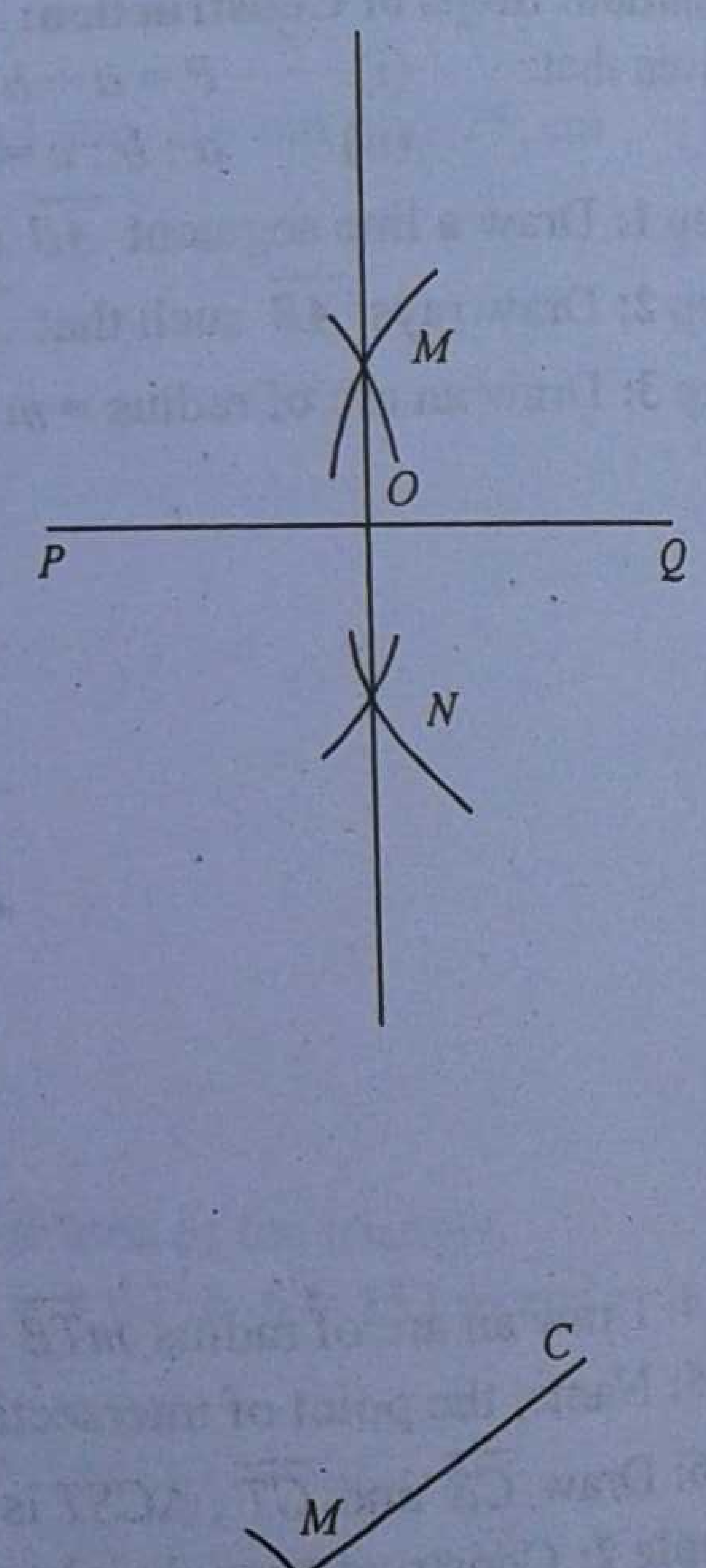
$\therefore$ ΔABC is the required triangle.

Use of Compass:

Compass is a mathematical instrument often used for drawing circles, making of lengths, making angles, measuring the length of a line segment.

Perpendicular Bisector of A Line:

To construct a perpendicular bisector of a given line segment, follow these steps.



Steps of Construction:

1. Draw a line segment $\overline{PQ}$.
2. Taking P and Q as centres and a radius greater than half of PQ.
3. Draw arcs above and below PQ such that the arcs cut each other at M and N.
4. Join M and N to cut $\overline{PQ}$ at O. MN is called the perpendicular bisector of PQ.

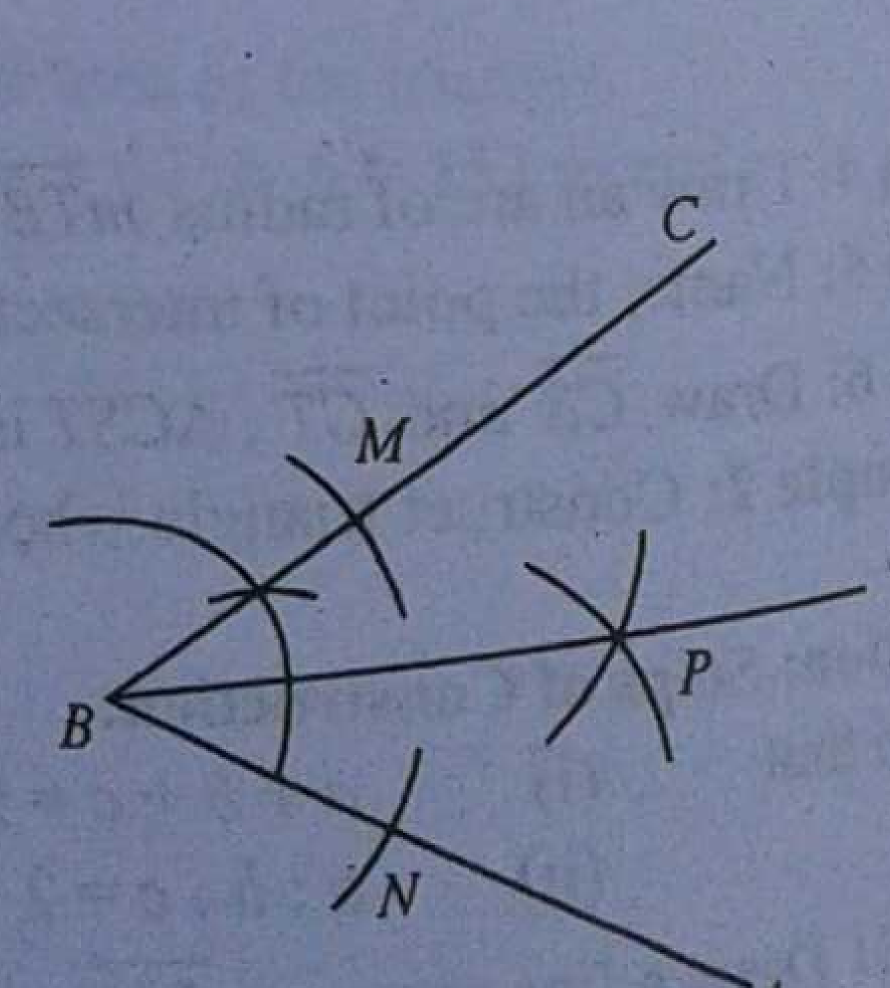
   If you measure PO and QO. You will find that
   $PO = OQ$. Thus O is called the mid point of PQ.

Angle Bisector:

To construct angle bisector of a given angle, follow these steps.

Steps of Construction:

1. Make an angle of any measurement (here we take 60°).
   Thus the angle $\angle ABC = 60°$.
2. Taking B as a centre and a fixed radius.
3. Cut BC at M and BA at N.
4. Taking M and N as centres and the same radius.
5. Draw arcs to cut each other at P.
6. Join BP.

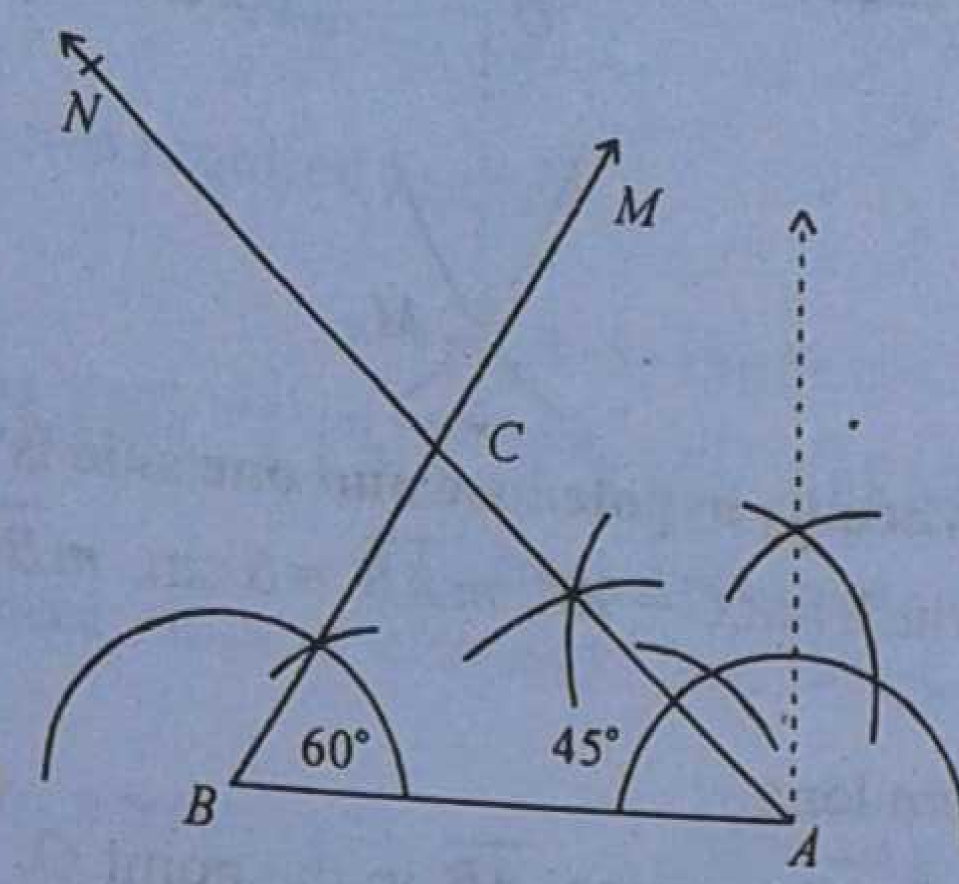


Example 1: Construct the triangle ABC in which $m\overline{AB} = 4$ cm,

$m\angle A = 45^\circ$, $m\angle B = 60^\circ$

Solution: Steps of Construction:

(i) Draw a line segment $\overline{AB} = 4$ cm.
(ii) At the point B, draw an angle of measure 60°.
(iii) At point A, draw an angle of 45°.
(iv) The Ray $\overrightarrow{AN}$ intersects ray $\overrightarrow{BM}$ at point C.
(v) Joint point C to A and B.



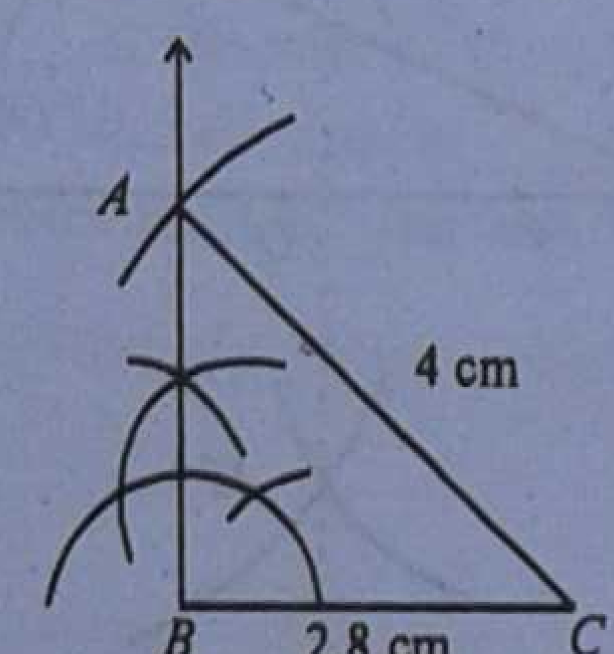
$\therefore$ ABC is the required triangle.

To construct a right-angled triangle when the measure of its hypotenuse and one side is given:

Example: Construct a right-angled triangle ABC in which $m\overline{AC} = 4$ cm, $m\angle B = 90^\circ$, $m\overline{BC} = 2.8$ cm

Solution: Steps of Construction:

(i) Draw a line segment $\overline{BC} = 2.8$ cm long.
(ii) At the point B draw an angle of measure 90°.
(iii) With centre C draw an arc of radius 4 cm cutting $\overrightarrow{BP}$ at the point A.
(iv) Join A with C.



$\therefore$ ABC is the required triangle.

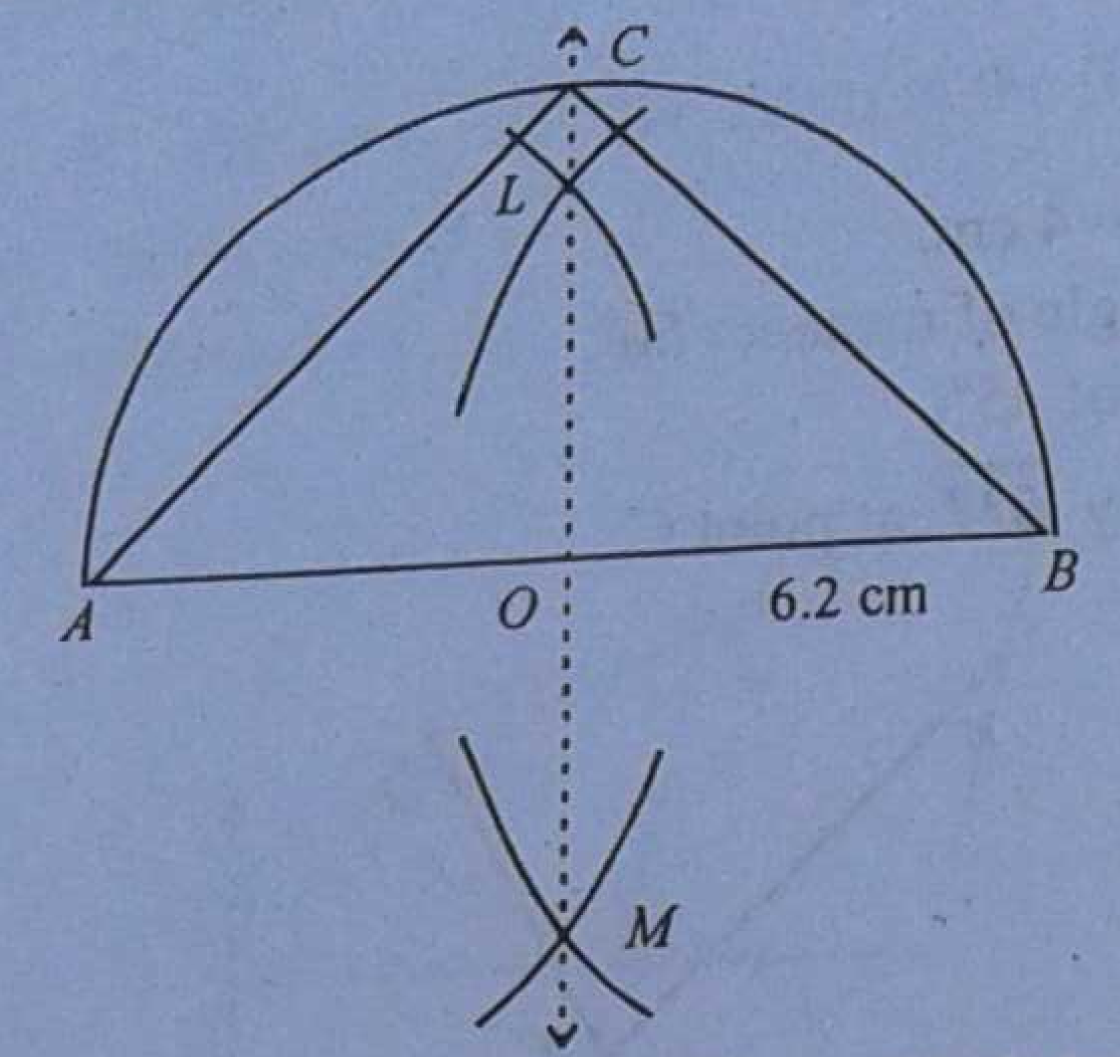
To construct a right-angled isosceles triangle in which the length of its hypotenuse is given:

Example: Construct a right-angled isosceles triangle the length of whose hypotenuse is 6.2 cm.

Solution: Steps of Construction:

(i) Draw a line segment $\overline{AB}$ of length 6.2 cm.
(ii) Draw $\overline{LM}$ the right bisector of $\overline{AB}$.
(iii) Draw a semicircle with centre O and a radius $m\overline{OA}$ or $m\overline{OB}$, cutting $\overline{LM}$ in the point C.
(iv) Join C with A and B.

$\therefore$ ABC is the required triangle.



To construct a right-angled triangle when its hypotenuse and one side is known:

Example: Construct a triangle ABC in which $m\angle C = 90^\circ$, $m\overline{AB} = 5$ cm, $m\overline{BC} = 2.1$ cm.

Solution: Steps of Construction:

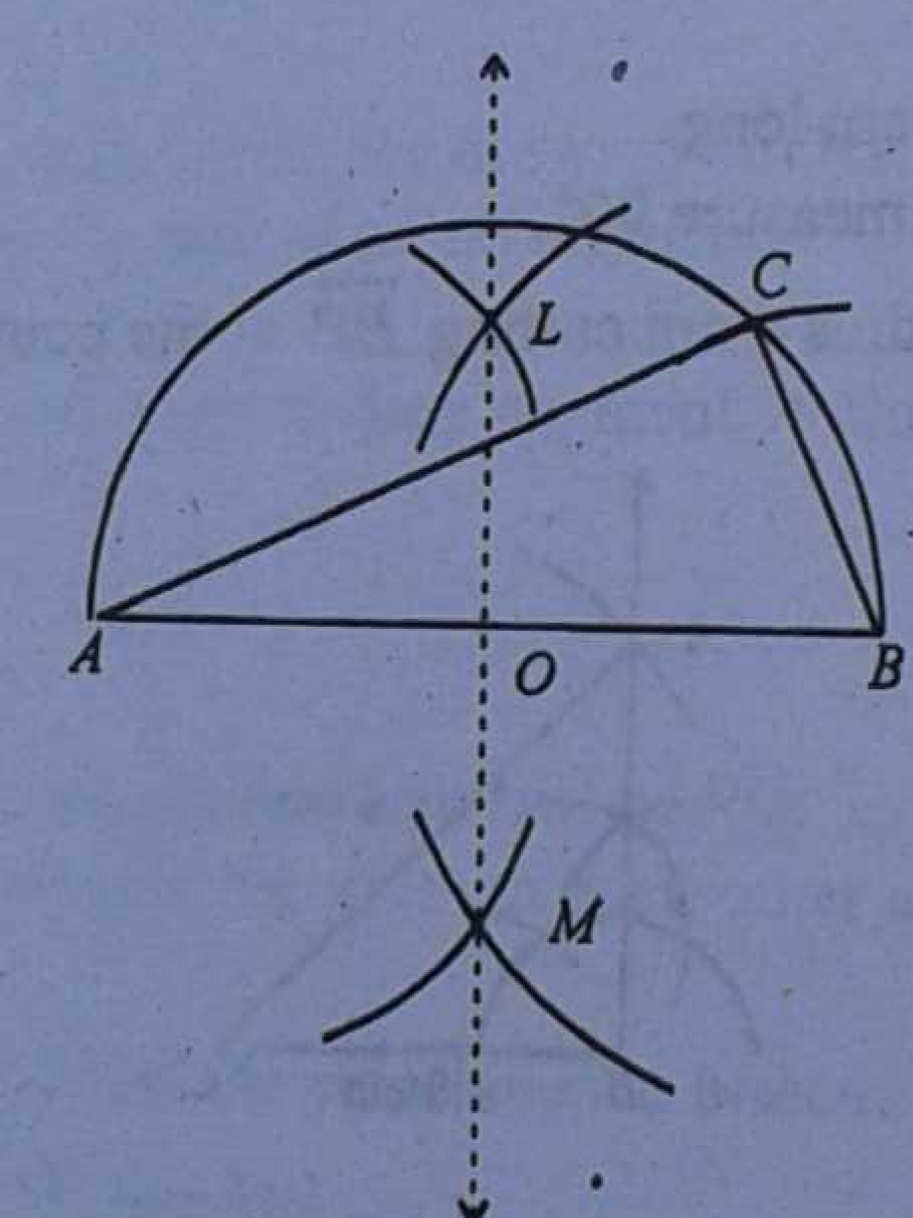
(i) Take a line segment $\overline{AB} = 5$ cm long.

(ii) Draw $\overleftrightarrow{LM}$ the right bisector of $\overline{AB}$ meeting $\overline{AB}$ in the point O.

(iii) Draw a semicircle with centre O and radius equal to $m\overline{OA}$ or $m\overline{OB}$.

(iv) Draw an arc with B as centre and radius 2.1 cm meeting the semi-circle in the point C.

(v) Join C with A and B.



$\therefore$ ABC is the required triangle.

To construct a triangle when the measure of its two sides and the measure of the angle opposite to one of them is given:

Example: Construct the triangle ABC when $m\angle B = 60^\circ$, $m\overline{AC} = 4.2$ cm, $m\overline{AB} = 5$ cm.

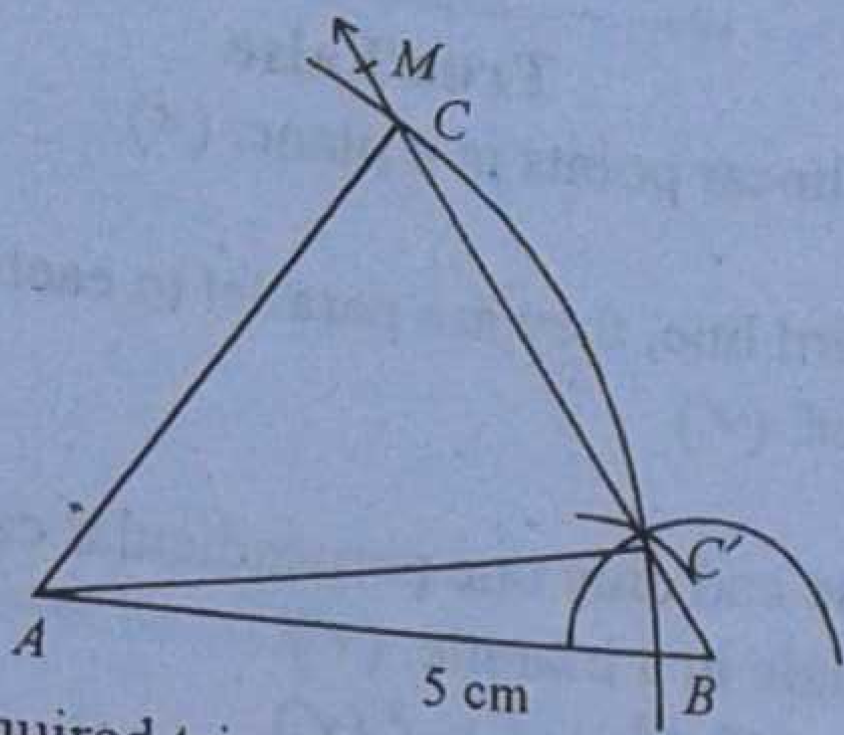
Solution: Steps of Construction:

(i) Take the line segment 5 cm long.

(ii) Taking B as vertex draw the angle $\angle ABM$.

(iii) Draw an arc with centre A and radius equal to 4.2 cm, meeting $\overline{BM}$ at the point C and C.

(iv) Join A with C and C'.



∴ ΔABC and ABC' are the two required triangles.

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OBJECTIVE TYPE QUESTIONS

Fill in the Blanks

Q1. If the ratio among the three sides of a triangle is 3 : 4 : 5, then triangle is always a ______ triangle. (right)

Q2. Hero's formula for area of a triangle is ______. $\sqrt{S(S-a)(S-b)(S-c)}$

Q3. If two parallel lines are cut by a transversal, the corresponding angles thus formed are ______ in measure. (equal)

Q4. A line that cuts two or more lines at different points is called a ______. (Transversal)

Q5. If a transversal cuts two parallel lines, then the alternate angles are ______ in measure. (equal)

Q6. If a transversal cuts two parallel lines, the sum of the interior angles on the same side of the transversal is equal to ______. (180°)

Q7. Two angles are said to be ______ if the sum of their measure is 90°. (Complementary)

Q8. Vertically opposite are ______ in measure. (equal)

Q9. Two angles are said to be ______ if the sum of their measure is 180°. (Supplementary)

Q10. An angle measuring less than 90° is called a/an ______ angle. (Obtuse)

Q11. An angle whose measure is greater than 90° but less than 180° is called ______ angle. (Obtuse)

Q12. $\frac{1}{2}$ revolution = degrees. (180°)

Q13. $\frac{1}{12}$ revolution = degrees. (30°)

Q14. $\frac{2}{3}$ revolution = degrees. (120°)

Q15. An angle between 180° and 360° is called ______ angle. (reflex)

Q16. The supplement of 171° is ______. (9°)

Q17. Adjacent angles are ______ of straight line. (180°)

Q18. Perpendicular lines intersect at ______ angles. (90°)

Q19. The complement of 87° is ______. (3°)

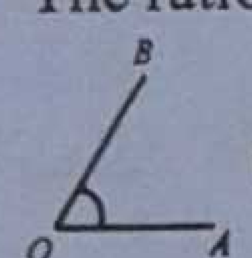
Q20. $\frac{1}{8}$ turn = degrees. (45°)

Q21. If the adjacent angles are supplementary, their outer arms are ______. (opposite)

Q22. Sum of angles of Δ = ______ (180°)

Q23. The common end point of the rays whose union is an angle is called the ______ of the angle. (union)

True/False

Q1. There are at least four non-collinear points in a plane. (✗)
Q2. A ray has two end points. (✓)
Q3. If two lines are parallel to a third line, they are parallel to each other. (✓)
Q4. ≡ is the notation for congruence. (✓)
Q5. Opposite rays are parallel. (✗)
Q6. From a point outside a line, one and only one perpendicular can be drawn. (✓)
Q7. There can be only one right-angle in a triangle. (✓)
Q8. In any triangle ABC with $\angle B = 90^\circ$, $b^2 = a^2 + c^2$ (✓)
Q9. If the lengths of the sides of a triangle are 3, 4 and 5 then $S = 6$. (✓)
Q10. In a right-angle triangle the side opposite to 90° is called base. (✗)
Q11. Hero's formula for area of triangle is $\Delta = \sqrt{S(S-a)(S-b)(S-c)}$ (✓)
Q12. The ratio among the three sides of a right-angle triangle is always 3 : 4 : 5. (✓)
Q13. 

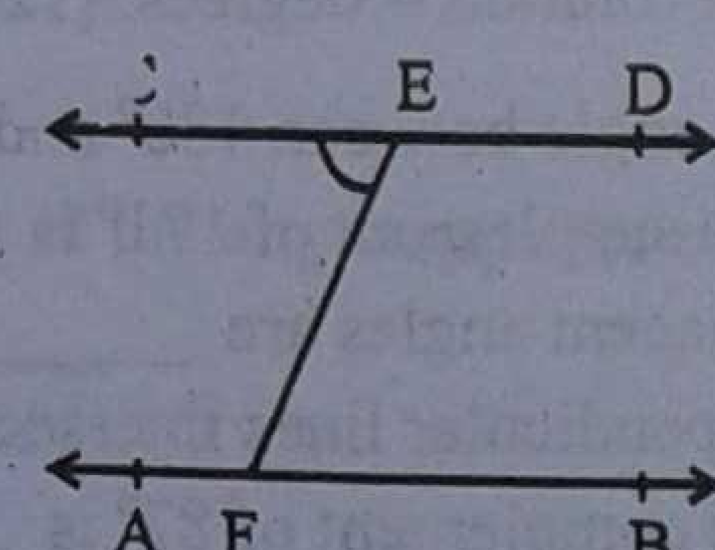
is an acute angle. (✓)
Q14. A right-angle is equal to 90°. (✓)
Q15. An obtuse angle lies between 90° and 180°. (✓)
Q16. Complementary angles are angles whose sum is 90°. (✓)
Q17. Supplementary angles are angles whose sum is 180°. (✓)
Q18. When two straight lines intersect the opposite angles are equal. (✓)
Q19. Two corresponding angles are equal. (✓)
Q20. Two alternate angles are not equal. (✗)
Q21. The interior angles are supplementary. (✗)
Q22. The total angle on a straight line is 180°. (✓)
Q23. The interior angles are supplementary. (✓)
Q24. A reflex angle is greater than 180°. (✗)

Construction of Parallel Lines

Example: Draw a line parallel to $\overline{AB}$

Steps of Construction:

(i) Draw a line segment $\overline{AB}$.
(ii) Take point E which is not on $\overline{AB}$.
(iii) Take F on $\overline{AB}$.
(iv) Draw a line segment $\overline{EF}$.
(v) Draw $\angle BFE$ on F which is congruent to $\angle FEC$.
(vi) Take D on $\overline{CE}$.
(vii) $\overline{CD}$ is parallel to $\overline{AB}$.



Exercise

1. Draw a line $\overline{AB}$ which is parallel to $\overline{CD}$.

Steps of Construction:

(i) Draw a line segment $\overline{AB}$.

(ii) Take E which is not on $\overline{AB}$.

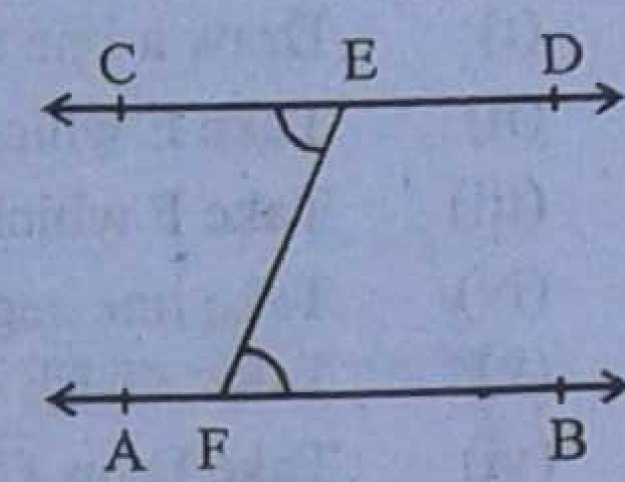
(iii) Take F which is on the $\overline{AB}$.

(iv) Take line segment $\overline{EF}$.

(v) Take $\angle BFE$ on F which congruent to $\angle FEC$.

(vi) Take D on $\overline{CE}$.

(vii) $\overline{CD}$ is required line which parallel to $\overline{AB}$.



2. Draw a line which is parallel to $\overline{AB}$, E is not on $\overline{AB}$.

Steps of Construction:

(i) Draw a line segment $\overline{AB}$.

(ii) Take E which is not on $\overline{AB}$.

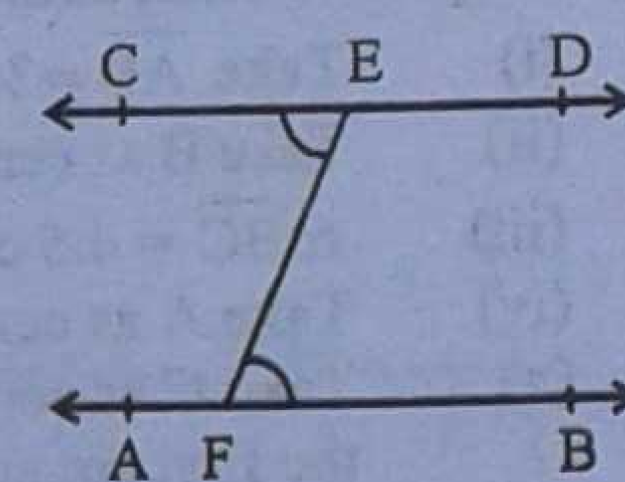
(iii) Take F which is on the $\overline{AB}$.

(iv) Take line segment $\overline{EF}$.

(v) Take $\angle BFE$ on F which congruent to $\angle FEC$.

(vi) Take D on $\overline{CE}$.

(vii) $\overline{CD}$ is required line which parallel to $\overline{AB}$.



3. Take a line segment $\overline{CD}$. $\overline{CD}$ is 4 cm away. $\overline{CD}$ is line which passes through A.

Steps of Construction:

(i) Draw a line segment $\overline{CD}$.

(ii) Take A which is not on $\overline{EF}$.

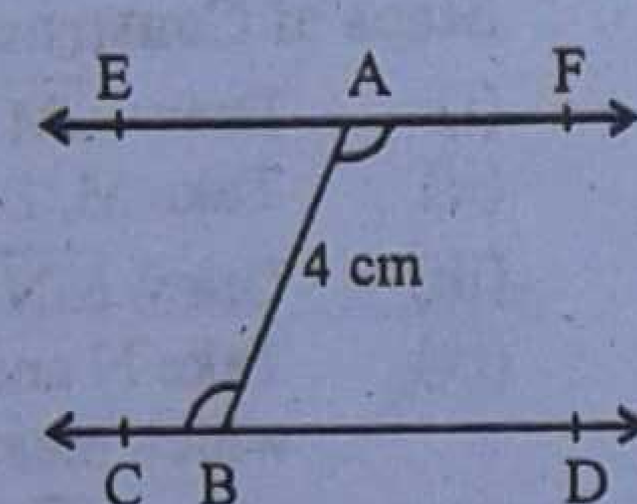
(iii) Draw a line segment $\overline{AB}$.

(iv) Take $\angle DBA$ on B which is congruent to $\angle BAE$.

(v) Take B on $\overline{CD}$.

(vi) Take F on $\overline{EA}$.

(vii) $\overline{EF}$ is required line parallel to $\overline{CD}$.



4. A line $\overline{LM}$ which 3 cm away from N. Draw a parallel to $\overline{LM}$.

Steps of Construction:

(i) Draw a line segment $\overline{LM}$.

(ii) Take N which is not $\overline{LM}$.

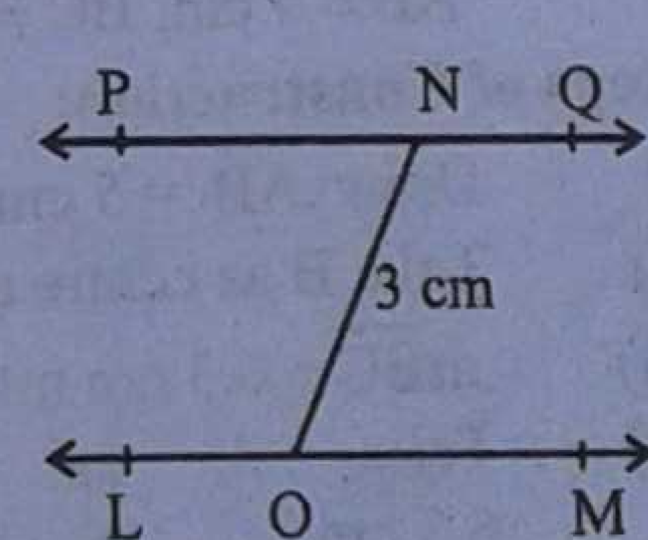
(iii) Take O on $\overline{LM}$.

(iv) Take $\angle MON$ on O which is congruent to $\angle ONP$.

(v) Draw a line segment $\overline{NO}$.

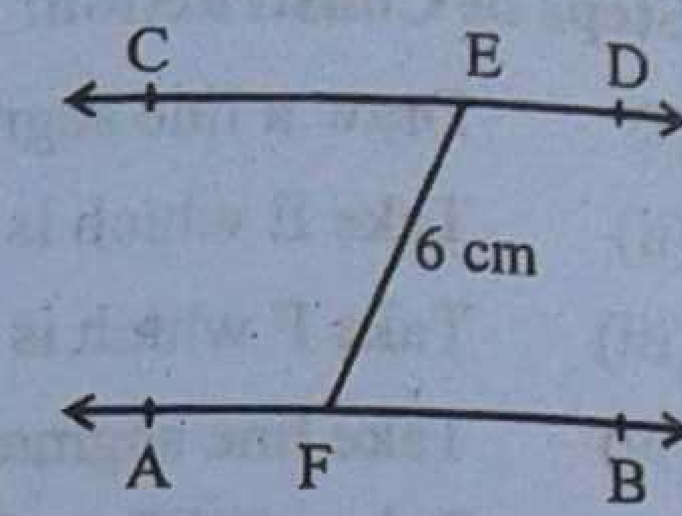
(vi) Take Q on $\overline{PN}$.

(vii) $\overline{PQ}$ is required line parallel to $\overline{LM}$.



5. Take 6 cm line which is parallel to $\overline{CD}$.

Steps of Construction:

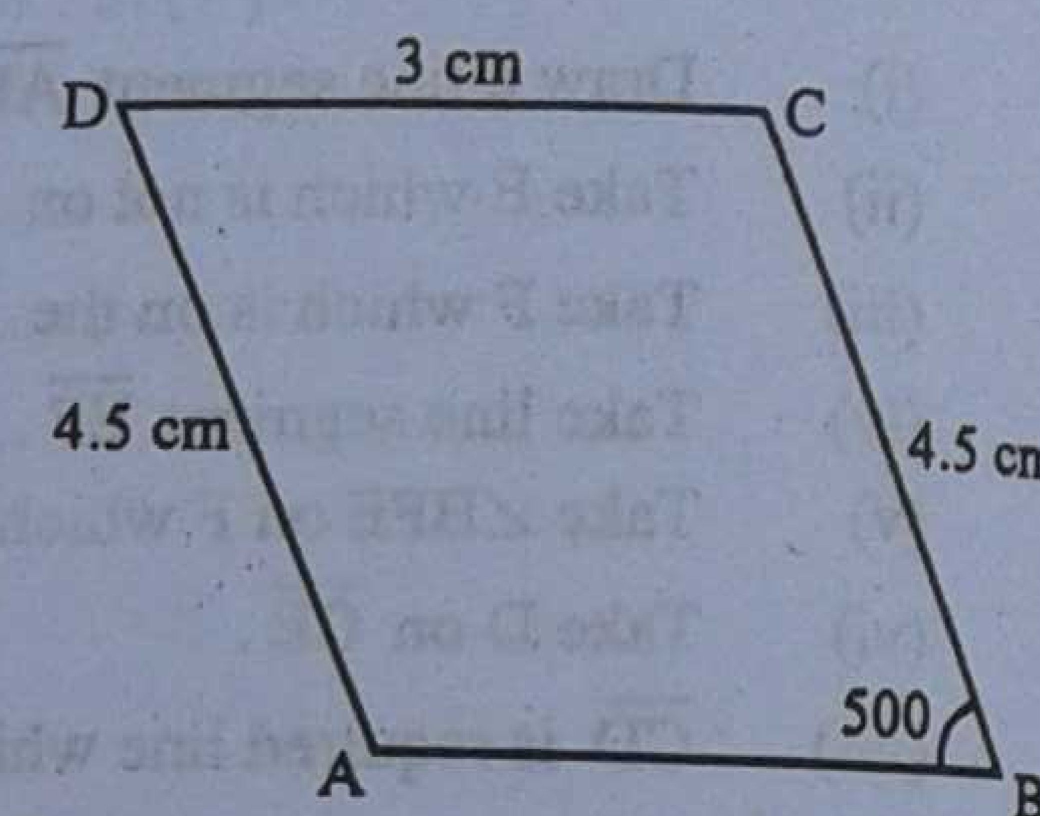


(i) Draw a line segment $\overline{AB}$.
(ii) Take E which is not on $\overline{AB}$.
(iii) Take F which is on the $\overline{AB}$.
(iv) Take line segment $\overline{EF}$.
(v) Take $\angle BFE$ on F which congruent to $\angle FEC$.
(vi) Take D on $\overline{CE}$.
(vii) $\overline{CD}$ is required line which parallel to $\overline{AB}$.

Construction of Parallelogram:

- **Example 1:** Construct a ABCD whose sides are $\overline{AB} = 3.5$ cm, $m\overline{BC} = 4.5$ cm, $m\overline{CD} = 3$ cm, $m\overline{DA} = 4.5$ cm, $m\angle B = 50°$

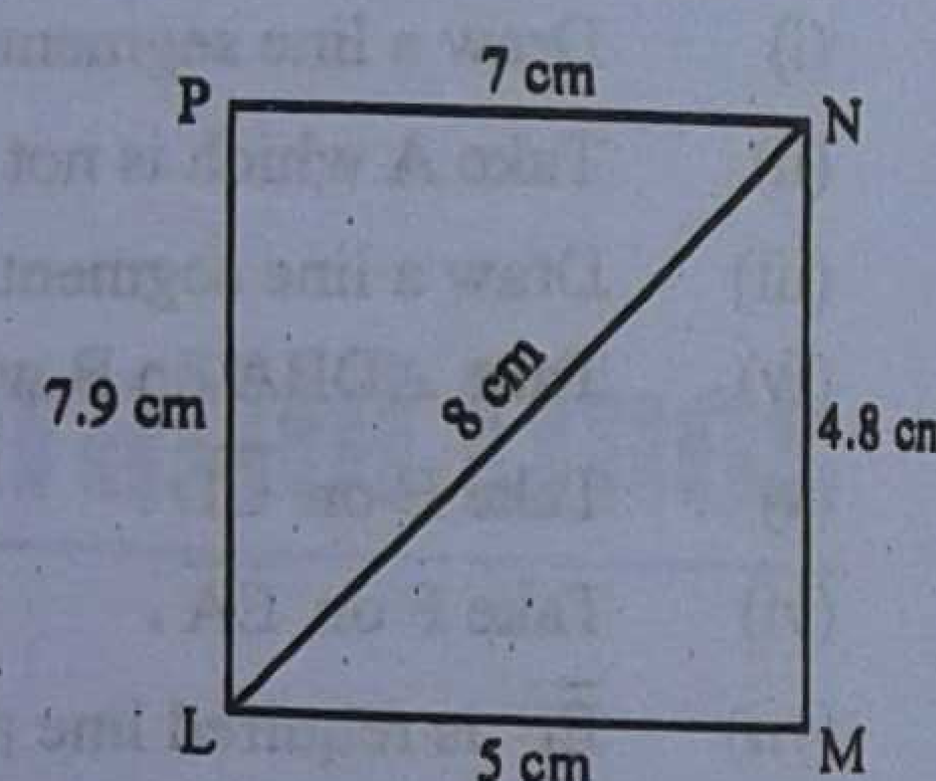
Steps of Construction:



(i) Take $\overline{AB} = 3.5$ cm.
(ii) Take B as centre and draw an angle of 50°.
(iii) $m\overline{BC} = 4.5$ cm intersect
(iv) Take A as centre and draw an arc of radius 4.5
(v) Take C as centre and draw an arc of radius 3 cm which intersect the first arc at D.
(vi) Draw $\overline{AD}$ and $\overline{CD}$.
(vii) ABCD is required parallelogram.

Example 2: Construct $\angle MNP$, where $\overline{LM} = 5$ cm, $\overline{MN} = 4.8$ cm, $\overline{NP} = 7$ cm, $\overline{PL} = 7.9$ cm, $\overline{NL} = 8$ cm

Steps of Construction:



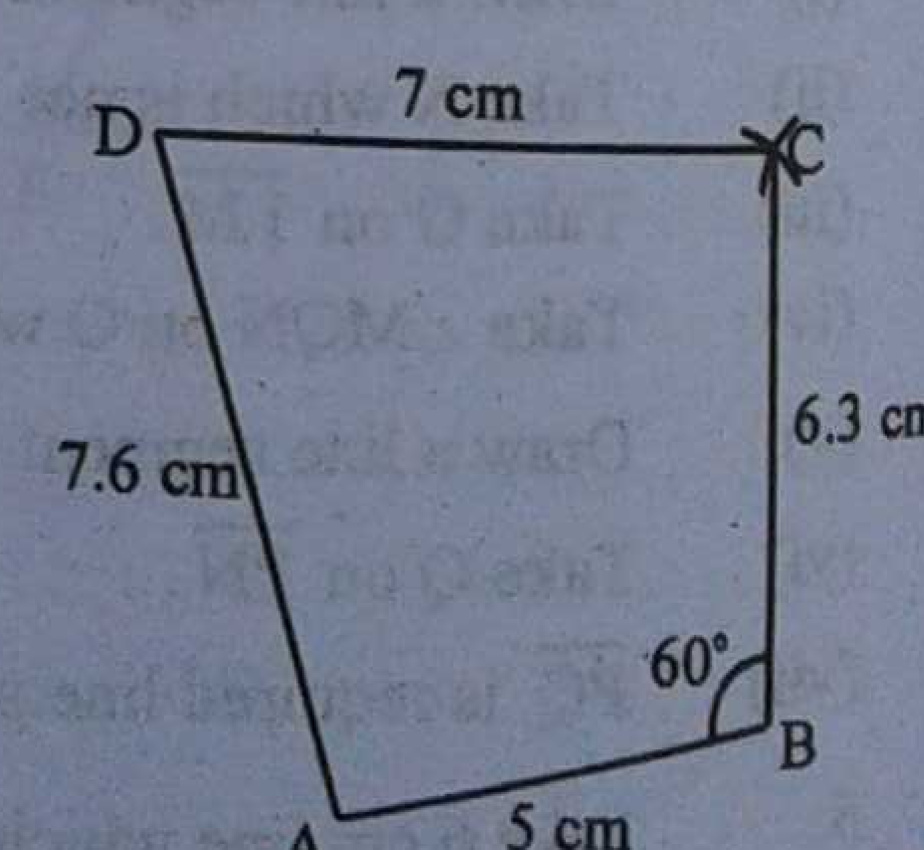
(i) Draw $\overline{LM} = 5$ cm
(ii) Take M, L as centre and draw an arc of radius 4.8 cm and 8 cm.
(iii) Draw $\overline{MN}$ and $\overline{NL}$.
(iv) Take N and L as centre and draw an arcs of radius 7 cm and 7.9 cm which meet at P.
(v) Draw $\overline{NP}$ and $\overline{PL}$.
$\angle MNP$ is required parallelogram.

Exercise

Construct ABCD in which

1. $\overline{AB} = 5$ cm, $\overline{BC} = 6.3$ cm, $\overline{CD} = 7$ cm, $\overline{DA} = 7.6$ cm, $m\angle B = 60°$

Steps of Construction:



(i) Draw $\overline{AB} = 5$ cm
(ii) Take B as centre and draw an angle of 60°.
(iii) $m\overline{BC} = 6.3$ cm intersect.
(iv) Take A as centre and draw an arc of radius 6.3 cm.
(v) Take C as centre and draw an arc of radius 7 cm which intersect first arc at D.
(vi) Draw $\overline{AD}$ and $\overline{CD}$.
(vii) ABCD is required parallelogram.

2. $\overline{AB} = 3.5$ cm, $m\overline{BC} = 6.5$ cm, $\overline{CD} = 5$ cm, $m\overline{DA} = 6.5$ cm, $m\angle A = 48°$

Steps of Construction:

(i) Draw $\overline{AB} = 3.5$ cm

(ii) Take B as centre and draw an angle of 48°.

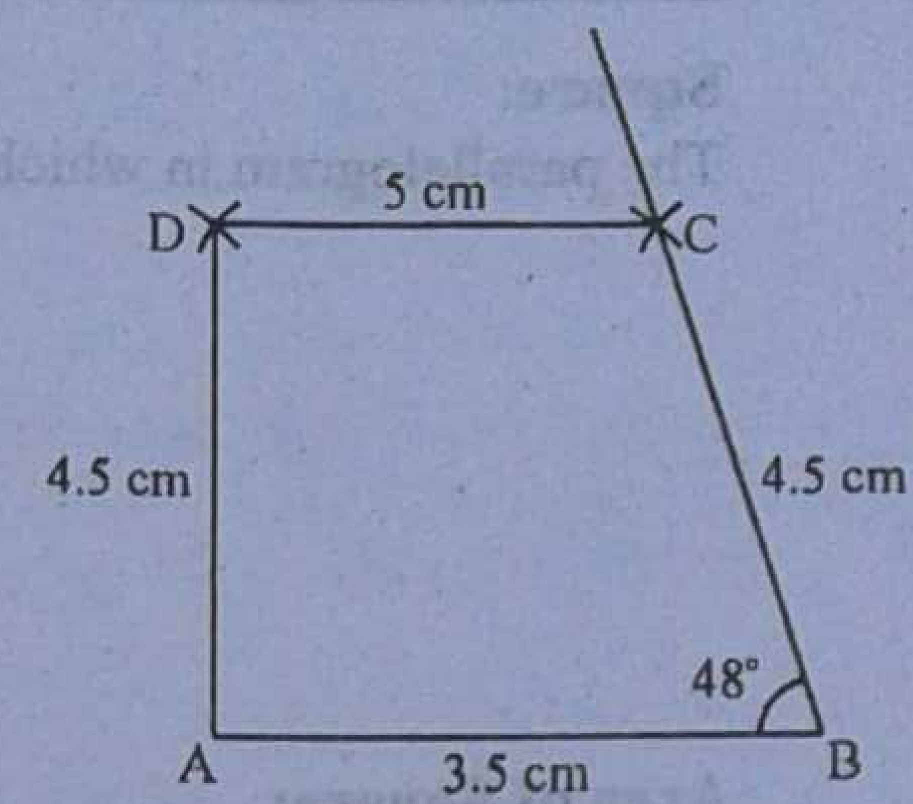
(iii) $m\overline{BC} = 4.5$ cm intersect.

(iv) Take A as centre and draw an arc of radius 6.5 cm.

(v) Take C as centre and draw an arc of radius 5 cm. Which intersect first arc at D.

(vi) Draw $\overline{AD}$ and $\overline{CD}$.

(vii) ABCD is required parallelogram.



3. $\overline{AB} = 4.6$ cm, $m\overline{BC} = 5.7$ cm, $m\overline{CD} = 6.2$ cm, $m\overline{DA} = 5.7$ cm, $m\angle C = 70°$

Steps of Construction:

(i) Draw $\overline{AB} = 4.6$ cm

(ii) Take B as centre and draw an angle of 70°.

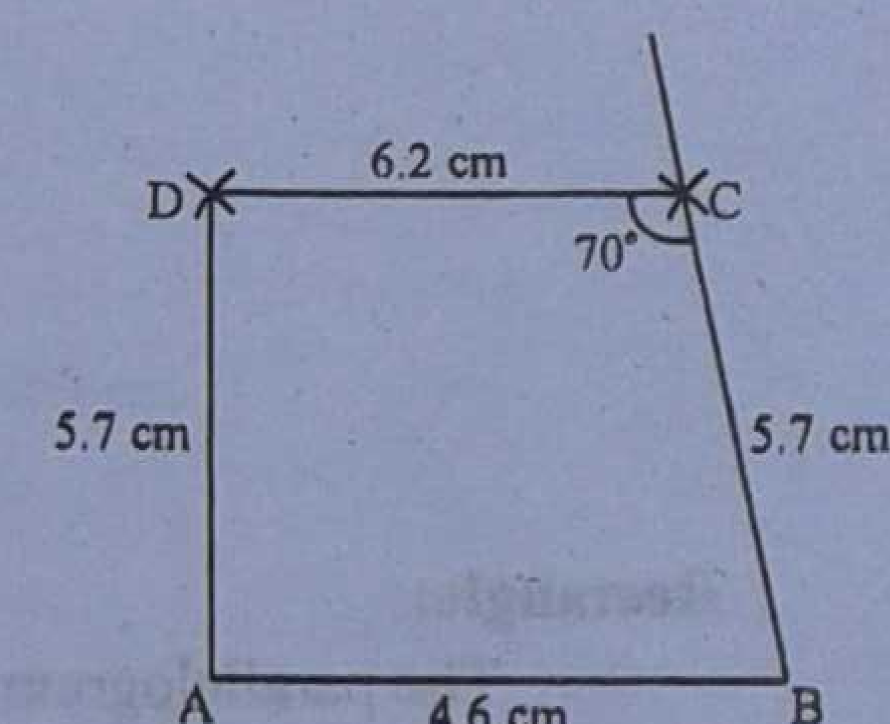
(iii) Take A as centre and draw an arc of radius 5.7 cm.

(iv) Take C as centre and draw an arc of radius 6.2 cm which intersect at D first arc.

(v) $m\overline{BC} = 5.7$ cm intersect.

(vi) Draw $\overline{AD}$ and $\overline{CD}$.

(vii) ABCD is required parallelogram.



4. $m\overline{AB} = 4.5$ cm, $m\overline{BC} = 5.7$ cm, $\overline{CD} = 6.3$ cm, $m\overline{DA} = 5.5$ cm, $m\overline{AC} \doteq 7.4$ cm

Steps of Construction:

i) Draw $\overline{AB} = 4.5$ cm

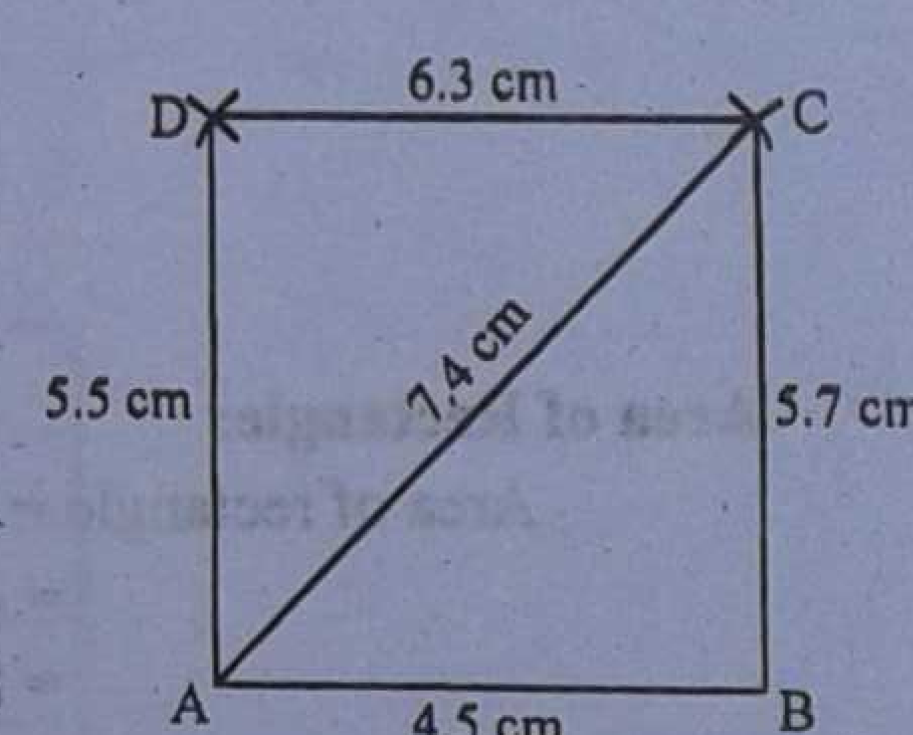
(ii) Take B as centre of radius 5.7 cm and A as centre of radius 7.4 cm which intersect each other at C.

(iii) Draw $\overline{BC}$ and $\overline{CA}$

(iv) Take B as centre of radius 6.3 cm and A as centre and draw an arc of radius 5.5 cm which intersect at D.

(v) Draw $\overline{CD}$ and $\overline{DA}$.

ABCD is required parallelogram.



5. $m\overline{AB} = 4$ cm, $m\overline{BC} = 4.5$ cm, $m\overline{CD} = 5$ cm, $m\overline{DA} = 6$ cm, $m\overline{AC} = 7$ cm

Steps of Construction:

(i) Draw $\overline{AB} = 4$ cm

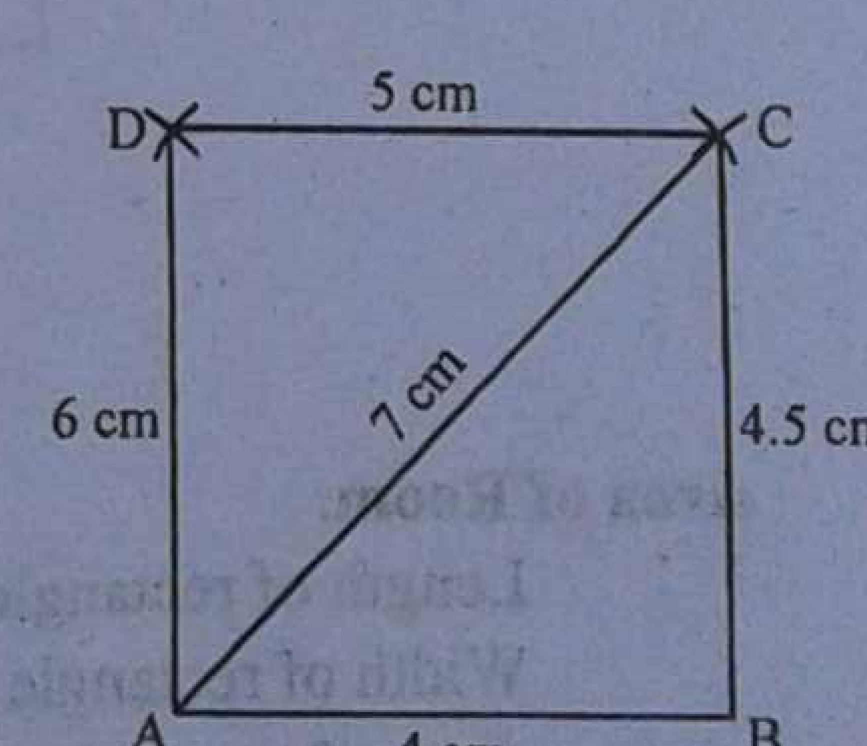
(ii) Take B as centre and draw an arc of radius 4.5 cm, take A as centre and draw an arc of radius 7 cm.

(iii) Draw $\overline{BC}$ and $\overline{CA}$.

(iv) Take C and A as centre and draw an arcs of radii 5 cm, 6 cm. Both intersect at D.

(v) Draw $\overline{CD}$ and $\overline{DA}$.

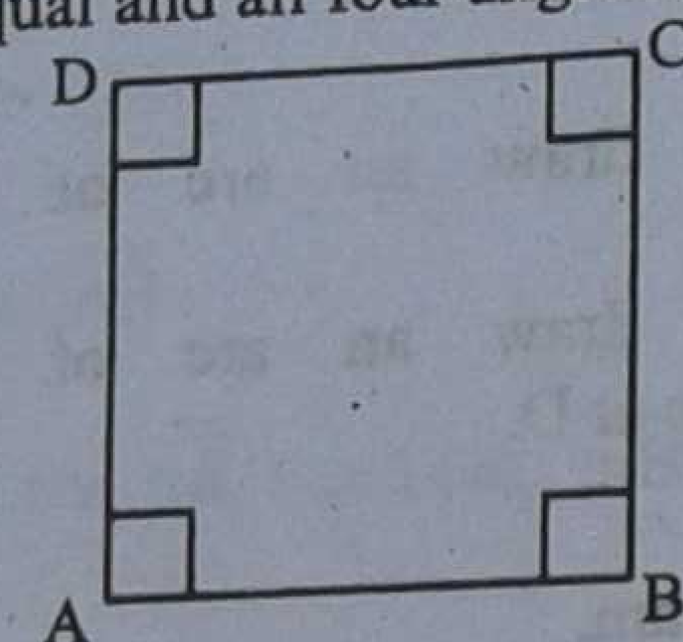
ABCD is required parallelogram.



Areas

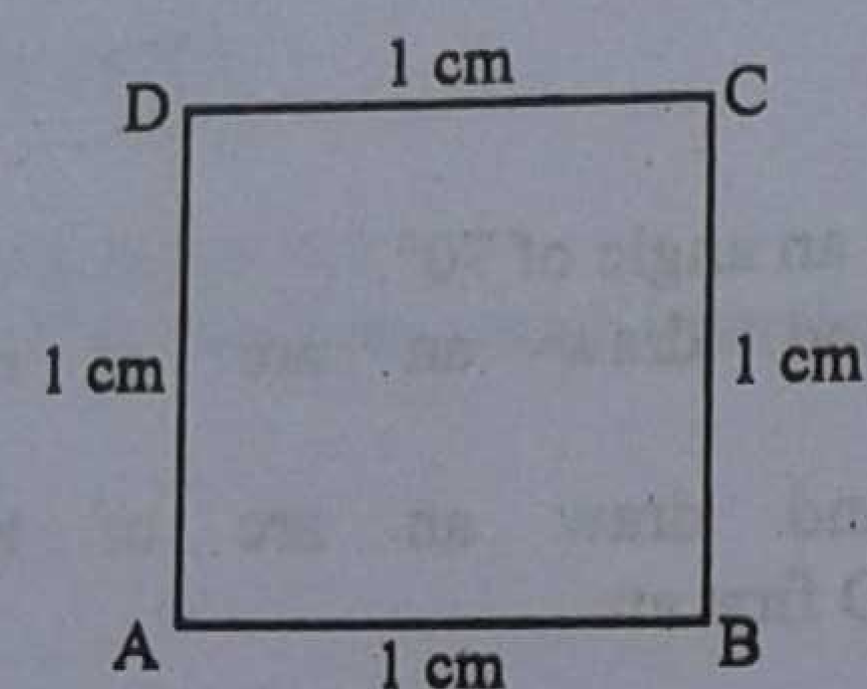
Square:

The parallelogram in which all sides are equal and all four angles are 90° each.



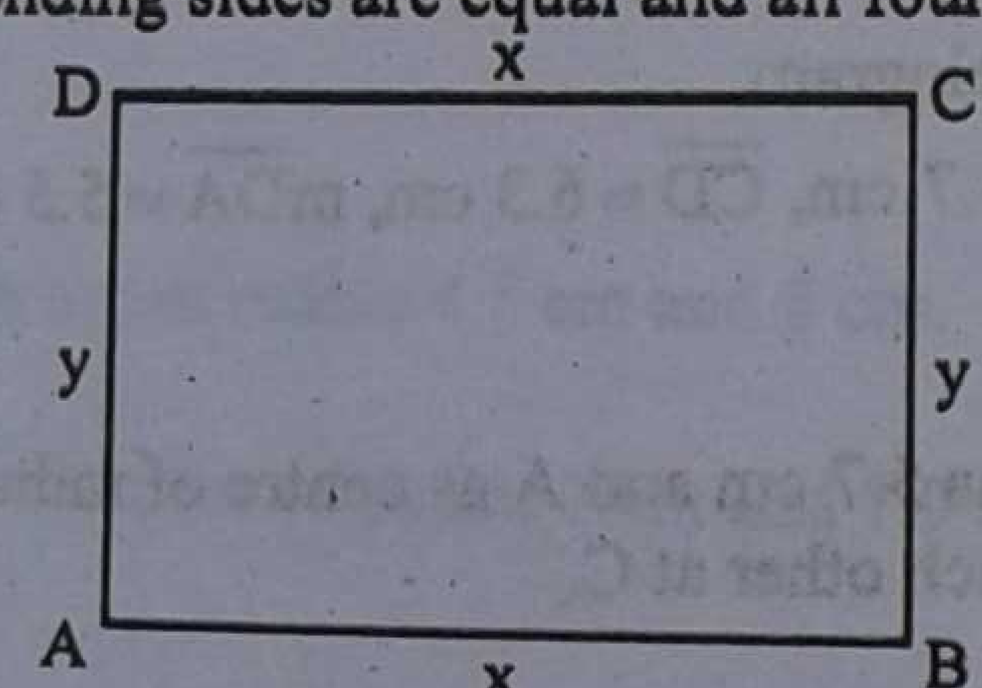
Area of Square:

Area of square = 1 × 1 = 1 cm^2



Rectangle:

The parallelogram in corresponding sides are equal and all four angles are 90° each.

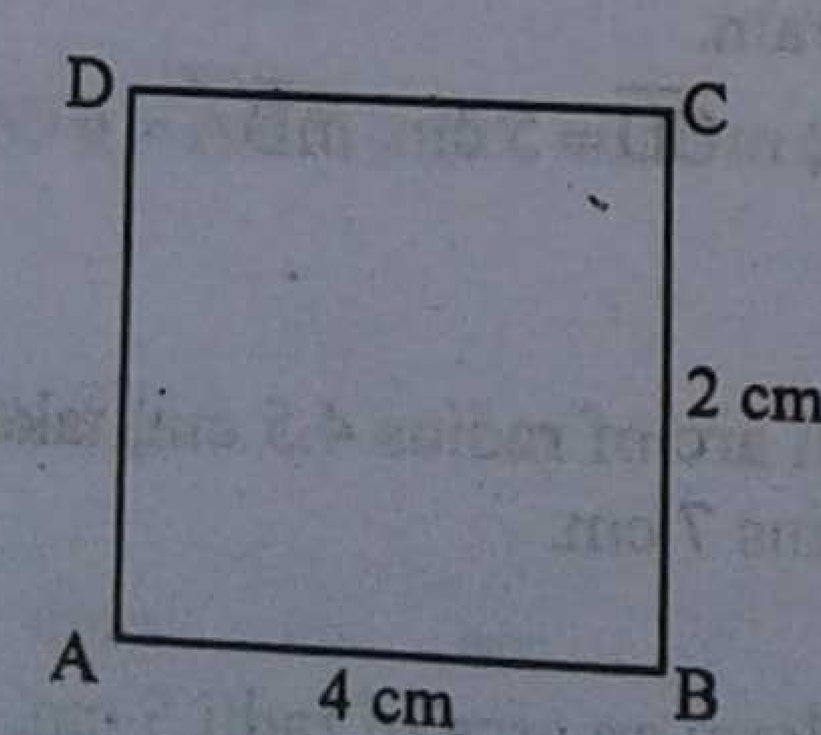


Area of Rectangle:

Area of rectangle = length × width

= 4 × 2

= 8 cm^2



Area of Room:

Length of rectangle = 2(length + width)

Width of rectangle = height of room

Area of rectangle = length × width

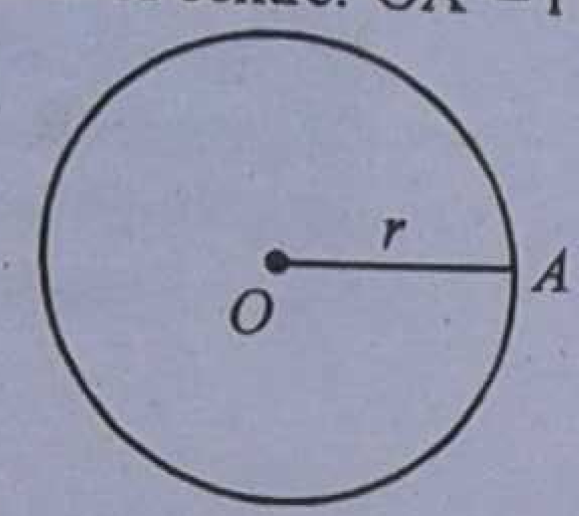
= 2(length + width) × height

Area of rectangle = Area of room

Area of room = 2(length + width) × height

Circle:

The figure is of circle. Here O is radius of centre. $\overline{OA} = r$ is radius of circle.



Area of circle = Area of rectangle

$= \text{length} \times \text{width}$

$= r \times \frac{2\pi r}{2}$

$= \frac{2\pi r^2}{2}$

$= \pi r^2$

Exercise

1. **Find the area of square, where sides are given below:**
(i) 2 cm (ii) 3 cm (iii) 5 cm (iv) 7 cm (v) 9 cm
(vi) 11 cm (vii) 10 cm (viii) 15 cm (ix) 20 cm

2. **Find the area of rectangle:**

	Width	Length
(i)	2 cm	5 cm
(ii)	6 cm	20 cm
(iii)	10 cm	20 cm
(iv)	4 m	6 m
(v)	5 m	8 m

3. **Find the area of room:**

	Length	Width	Height
(i)	5 m	4 m	3 m
(ii)	9 m	7 m	5 m
(iii)	12 m	10 m	9 m
(iv)	18 m	15 m	10 m
(v)	0.5 m	0.4 m	0.3 m

4. **Find the area of circle whose radius is** $\left(\pi = \frac{22}{7}\right)$
(i) 21 cm (ii) 35 cm (iii) 49 cm
(iv) 5.6 cm (v) 2.8 cm (vi) 6.3 m

Answers

1. (i) 2 cm
Side of square = 2 cm
Area of square $= 2 \times 2 = 4\ cm^2$

(ii) 3 cm
Side of square = 3 cm
Area of square $= 3 \times 3 = 9\ cm^2$

(iii) 5 cm
Side of square = 5 cm
Area of square $= 5 \times 5 = 25\ cm^2$

(iv) 7 cm
Side of square = 5 cm
Area of square $= 5 \times 5 = 25\ cm^2$

(v) 9 cm
Side of square = 9 cm
Area of square $= 9 \times 9 = 81\ cm^2$

(vi) 11 cm
Side of square = 11 cm
Area of square $= 11 \times 11 = 121\ cm^2$

(vii) 10 cm
Side of square = 10 cm
Area of square $= 10 \times 10 = 100\ cm^2$

(viii) 15 cm
Side of square = 15 m
Area of square $= 15 \times 15 = 225\ m^2$

(ix) 20 m
Side of square = 20 m
Area of square $= 20 \times 20 = 400\ m^2$

2.

(i) Area of rectangle = length × width
$= 5 \times 2$
$= 10\ cm^2$

(ii) Area of rectangle = length × width
$= 12 \times 6$
$= 72\ cm^2$

(iii) Area of rectangle = length × width
$= 20 \times 10$
$= 200\ cm^2$

(iv) Area of rectangle = length × width
$= 4 \times 6$
$= 24\ cm^2$

(v) Area of rectangle = length × width
$= 5 \times 8$
$= 40\ cm^2$

3.

(i) Area of room = 2(length × width) × height

$= 2(4 + 5) \times 3 = 54\ cm^2$

(ii) Area of room $= 2(\text{length} \times \text{width}) \times \text{height}$
$= 2(9 + 7) \times 5 = 150\ cm^2$

(iii) Area of room $= 2(\text{length} \times \text{width}) \times \text{height}$
$= 2(10 + 12) \times 9 = 406\ cm^2$

(iv) Area of room $= 2(\text{length} \times \text{width}) \times \text{height}$
$= 2(15 + 18) \times 10 = 660\ cm^2$

(v) Area of room $= 2(\text{length} \times \text{width}) \times \text{height}$
$= 2(0.5 + 0.9) \times 0.3 = 0.84\ cm^2$

4.
(i) r $= 21$ cm
Area of circle $= \pi r^2$
$= \left(\frac{22}{7}\right)(21)^2 = 1386\ cm^2$

(ii) r $= 35$ cm
Area of circle $= \pi r^2$
$= \left(\frac{22}{7}\right)(35)^2 = 3850\ cm^2$

(iii) r $= 49$ cm
Area of circle $= \pi r^2$
$= \left(\frac{22}{7}\right)(49)^2 = 7546\ cm^2$

(iv) r $= 5.6$ m
Area of circle $= \pi r^2$
$= \left(\frac{22}{7}\right)(5.6)^2 = 98.56\ m^2$

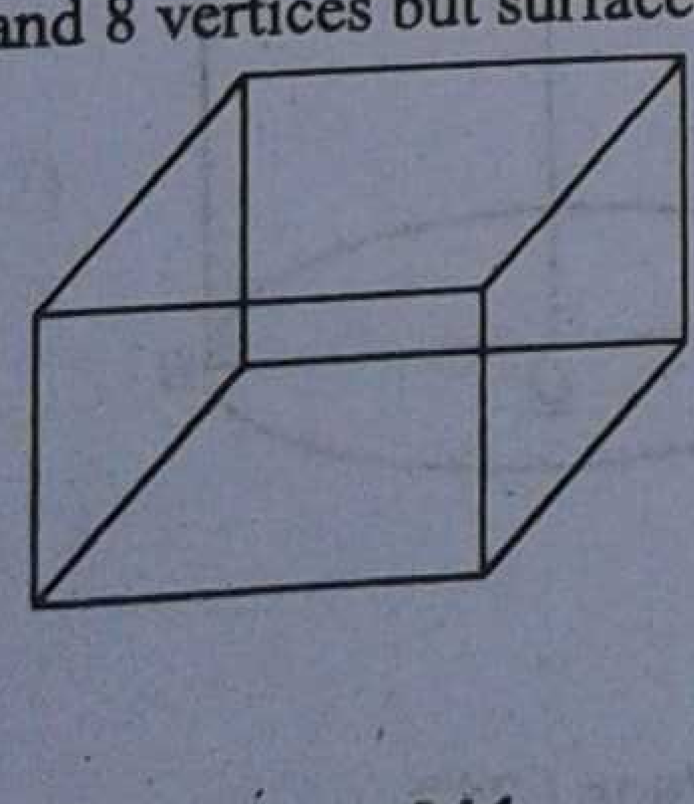
(v) r $= 2.8$ m
Area of circle $= \pi r^2$
$= \left(\frac{22}{7}\right)(2.8)^2 = 24.64\ m^2$

(vi) r $= 6.3$ m
Area of circle $= \pi r^2$
$= \left(\frac{22}{7}\right)(6.3)^2 = 124.74\ m^2$

Volume:

Cube: One cube has 6 surfaces, 12 edges and 8 vertices. Length of all edges are same in cube.

Cuboid: Cuboid has 6 surfaces, 12 edges and 8 vertices but surfaces of cuboid are not square.



Example:

Find the volume of cube whose length of edge is 14 cm.

Sol.

Volume of cube $= 4 \times 4 \times 4 = 64\ cm^3$

Exercise

1. **Find the volume of cube if length of edge is**
   (i) 2 cm (ii) 5 cm (iii) 9 cm (iv) 10 m (v) 12 m
2. **Find the area of cuboid if:**

	Width	Height	Length
(i)	2 cm	3 cm	4 cm
(ii)	6 cm	5 cm	7 cm
(iii)	7 cm	8 cm	9 cm
(iv)	5 cm	3 cm	6 cm
(v)	8 cm	12 cm	15 cm

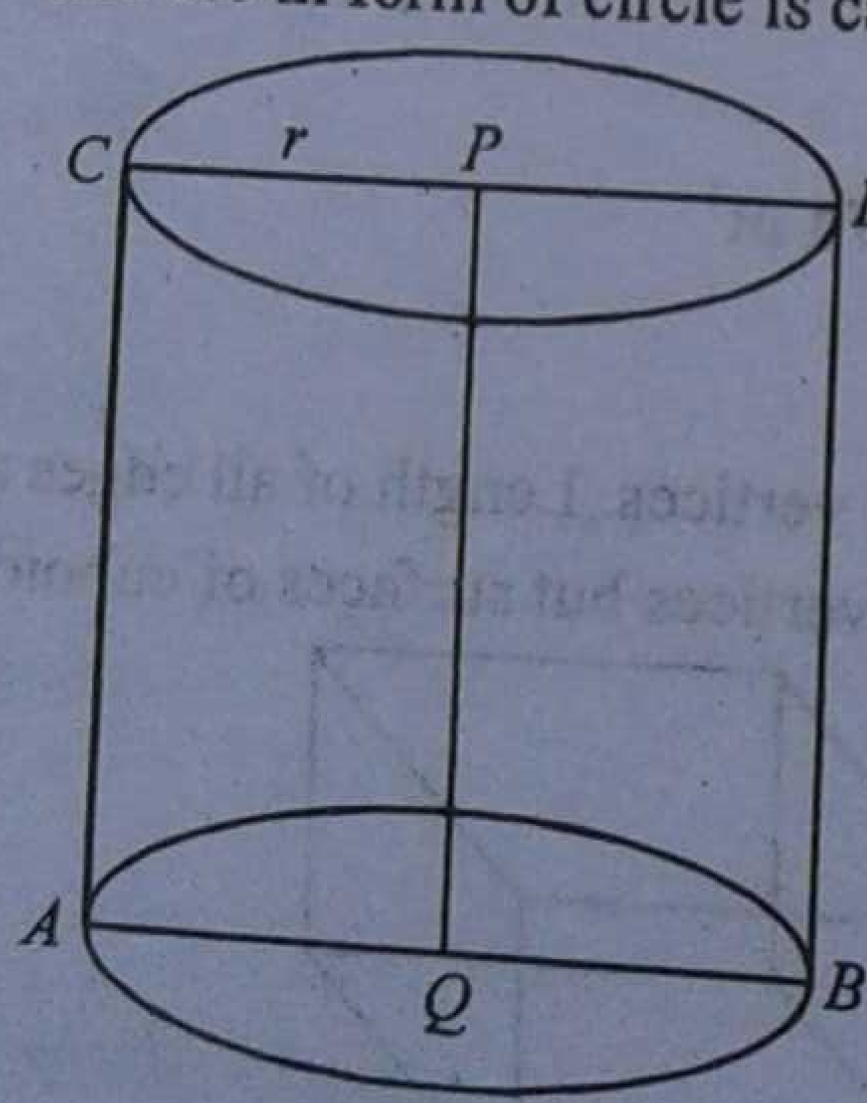
Answers

1.
(i) Volume of cube $= 2 \times 2 \times 2 = 8\ cm^3$
(ii) Volume of cube $= 5 \times 5 \times 5 = 125\ cm^3$
(iii) Volume of cube $= 9 \times 9 \times 9 = 729\ cm^3$
(iv) Volume of cube $= 10 \times 10 \times 10 = 1000\ cm^3$
(v) Volume of cube $= 12 \times 12 \times 12 = 1728\ cm^3$

2.
(i) Volume of cuboid $= 2 \times 3 \times 4 = 24\ cm^3$
(ii) Volume of cuboid $= 6 \times 5 \times 7 = 210\ cm^3$
(iii) Volume of cuboid $= 7 \times 8 \times 9 = 504\ cm^3$
(iv) Volume of cuboid $= 5 \times 3 \times 6 = 90\ cm^3$
(v) Volume of cuboid $= 8 \times 12 \times 15 = 1440\ cm^3$

Cylinder:

These bodies whose lits and bottom are in form of circle is called cylinder.



Volume of cylinder= Volume of cuboid

Base of cuboid $= \frac{2\pi r}{2} = \pi r$

So,

Volume of cuboid $= \pi r \times r \times h$

V $= \pi r^2 h$

Example:

Length of cylinder is 40 cm and radius is 5 cm. Find the volume of cylinder.

Sol.

Radius of cylinder = 5 cm

Height of cylinder = 40 cm

Volume of cylinder= $\pi r^2 h$

$= \frac{22}{7} \times (5)^2 (40)$

$= 3142.86 \text{ cm}^3$

Exercise

1. Find the volume of cylinder $\left(\pi = \frac{22}{7}\right)$

	Radius	Height
(i)	2 cm	10 cm
(ii)	5 cm	12 cm
(iii)	8 cm	15 cm
(iv)	10 cm	20 cm

2. A cylinder's radius is 3 cm and height is 5.5 cm. Find the volume of cylinder.
3. The length of pipe is 4 cm and diameter is 5 cm. Find the volume.
4. The radius of well is 2.5 m and height is 14 m. Find the volume.
5. The radius of drum is 1 m and height is 2 m. Find the volume.

Answers

1. (i) Volume of cylinder $= \pi r^2 h$

$= \left(\frac{22}{7}\right)(2)^2(10)$

$= 125.714 \text{ cm}^3$

(ii) Volume of cylinder $= \pi r^2 h$

$= \left(\frac{22}{7}\right)(5)^2(12)$

$= 942.86 \text{ cm}^3$

(iii) Volume of cylinder $= \pi r^2 h$

$= \left(\frac{22}{7}\right)(8)^2(15)$

$= 30171.14\ cm^3$

(iv) Volume of cylinder $= \pi r^2 h$

$= \left(\frac{22}{7}\right)(10)^2(20)$

$= 6285.714\ cm^3$

2. Radius of cylinder $= 5$ cm

Height of cylinder $= 5.5$ cm

Volume $= \pi r^2 h$

$= \left(\frac{22}{7}\right)(5)^2(5.5)$

$= 432.142\ cm^3$

3. Length of pipe $= 4$ cm

Diameter of pipe $= 5$ cm

Radius of pipe $= \frac{5}{2} = 2.5$ cm

Volume $= \pi r^2 h$

$= \left(\frac{22}{7}\right)(2.5)^2(4) = 78.57\ cm^3$

4. Radius of well $= 2.5$ m

Height of well $= 14$ m

Volume $= \pi r^2 h$

$= \left(\frac{22}{7}\right)(2.5)^2(14) = 242\ m^3$

5. Radius of drum $= 1$ m

Height of drum $= 2$ m

Volume $= \pi r^2 h$

$= \left(\frac{22}{7}\right)(1)^2(2) = 6.286\ m^3$

Area

Some Important Formulae

(1) Area of a rectangle or square
$= \text{length} \times \text{breadth}$

(2) Perimeter of a rectangle or square
$= 2\,(\text{length} + \text{breadth})$

(3) Area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

(4) Area of a triangle of sides a, b, and c
$= \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{(a+b+c)}{2}$

(5) Area of a circle $= \pi r^2$

(6) Circumference of a circle
$= 2\pi r$

(7) Area of four walls of a room
$= 2(\text{length} + \text{breadth}) \times \text{height}$

(8) Area of a parallelogram
$= \text{base} \times \text{height}$

(9) Area of a trapezium
$= \frac{1}{2} \times \text{sum of two parallel sides} \times \text{width}$

(10) Area of a regular hexagon of side a
$= 6a^2\sqrt{\frac{3}{4}}.$

Model Examples

Q.1. The difference between the circumference of a circle and its diameter is 135 ft. Find the area of the circle.

$$\left(\text{Take } \pi = \frac{22}{7}\right)$$

Sol. Let r be the radius of the circle, then circumference of circle

$= 2\pi r$

Diameter of circle $= 2r$

By the given condition

$= 2\pi r - 2r = 135$

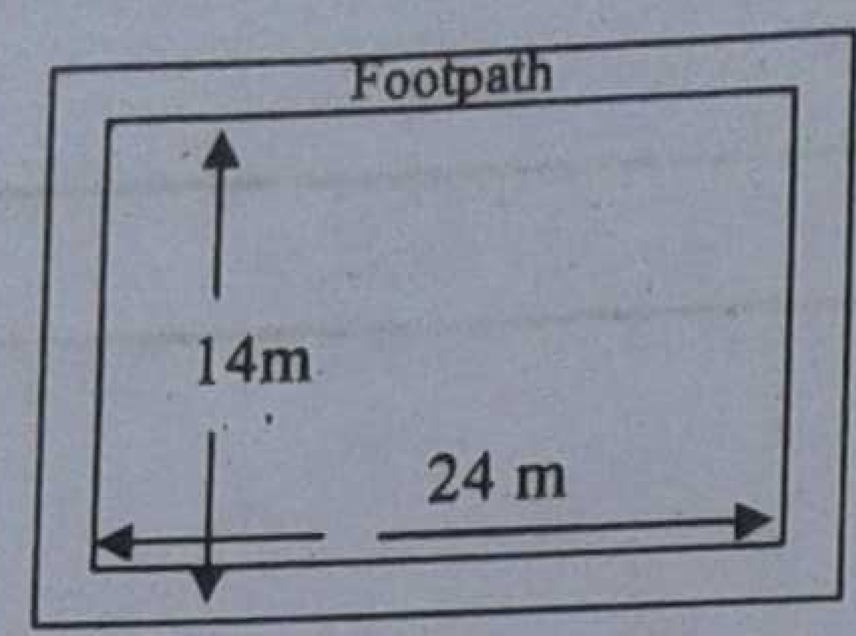
$r = \frac{63}{2}$ ft.

Area of circle $= \pi r^2$

$= \frac{22}{7} \times \frac{63}{2} \times \frac{63}{2}$ sq. ft.

$= 3118.5$ sq. ft. **Ans.**

Q.2. How many tiles 20 cm. square will be required to have a foot path 1 metre wide carried round the outside of grass plot 24 meter long by 14 metres broad?



Sol. Area of the grass plot $= 24 \times 14$
$= 336$ sq. m.

Length and breadth of the plot including the path is $= 26$ m and 16 m

Area of plot including path $= 26 \times 16$
$= 416$ sq. m.

$\therefore$ Area of path $= 416 - 336 = 80$ sq. m.

Area of one tile $= \dfrac{20}{100} \times \dfrac{20}{100} = \dfrac{1}{25}$ sq. m.

$\therefore$ No. of tiles required $= \dfrac{80}{\frac{1}{25}} = 80 \times 25$
$= 2000$ **Ans.**

Q.3. If the length of a rectangular piece of land were 5 metres less and the breadth 2 metres more, the area would be 10 sq. m. less; but if the length were 10 metres more and breadth 5 metres more, the area would have been 275 sq. m. more. Find its length and breadth.

Sol. Let the length be 'l' breadth 'b'

$\therefore$ area $= l \times b$

Then $(l - 5)(b + 2) = lb - 10$..............(i)

And $(l + 10)(b + 5) = lb + 275$...........(ii)

From (*i*)

$lb + 2l - 5b - 10 = lb - 10$

$2l = 5b \Rightarrow l = \dfrac{5}{2}b$

From (*ii*)

$lb + 5l + 10b + 50 = lb + 275 \Rightarrow 5l + 10b = 225$

Putting $l = \dfrac{5}{2}b$

$\dfrac{25}{2}b + 10b = 225 \Rightarrow \dfrac{45}{2}b = 225$

$b = \dfrac{2 \times 225}{45} = 10$ and $l = 25$ m

Q.4. The perimeter of one square exceeds the perimeter of another square by 120 metres and the area of the larger square exceeds twice the area of the smaller square by 900 square metres. Find the length of the sides of the squares,

Sol. Let the length of one side of larger square $= x$ sq. m

Let the length of one side of smaller square = y mts.

Now by the given condition the primeter ($4x$) of larger exceeds the perimeter of smaller ($4y$) by 120 sq. m

i.e., $4x - 4y = 120$

or $x = 30 + y$ ..... (*i*)

Again by the second condition

$x^2 - 2y^2 = 900$ ..... (*ii*)

Putting the value of x from (*i*) in (*ii*), we get

$(30 + y)^2 - 2y^2 = 900 \Rightarrow -y^2 + 60y = 0$

or $y = 60$

Putting the value in (*i*) we get $x = 90$

$\therefore$ Larger square side length is = 90 sq. m

Smaller square side length is = 60 sq. m **Ans.**

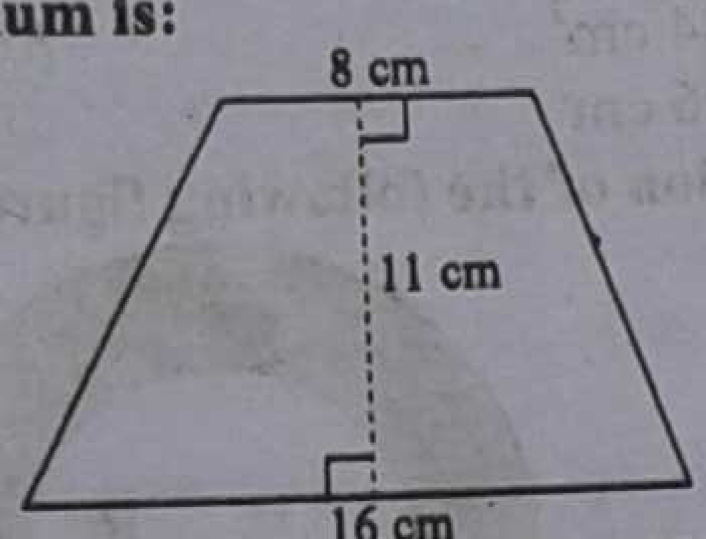
MULTIPLE CHOICE QUESTIONS (MCQs)

Q1. **The surface area of sphere of radius $3\frac{1}{2}$ cm is:**

a) 130 sq.cm *b)* 69 sq.cm

c) 154 sq.cm *d)* 98 sq.cm

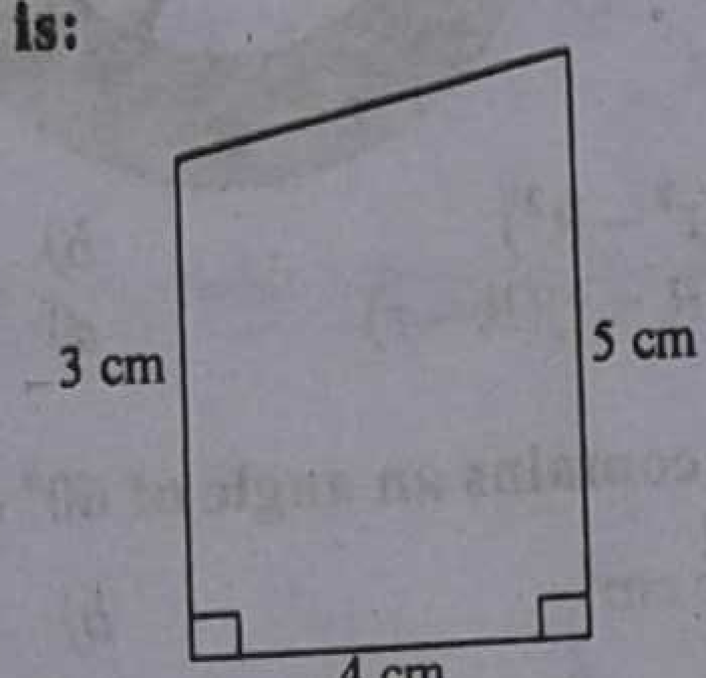
Q2. **The area of the following trapezium is:**



a) 125 sq.cm *b)* 132 sq.cm

c) 139 sq.cm *d)* 97 sq.cm

Q3. **The area of the following figure is:**



a) 16 sq.cm *b)* 15 sq.cm

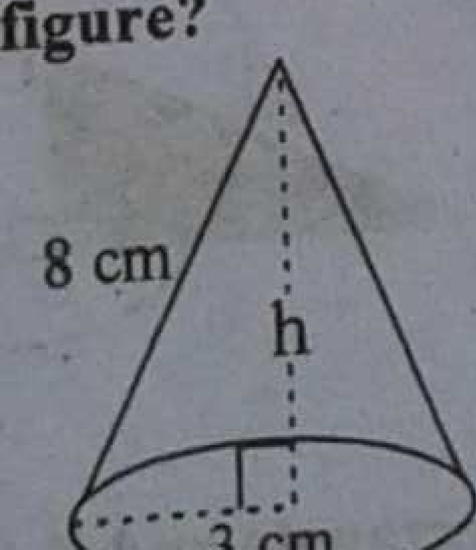
c) 60 sq.cm *d)* None of these

Q4. **The height of a triangle of base 3 cm and area 9 cm is:**

a) 6 cm *b)* 9 cm

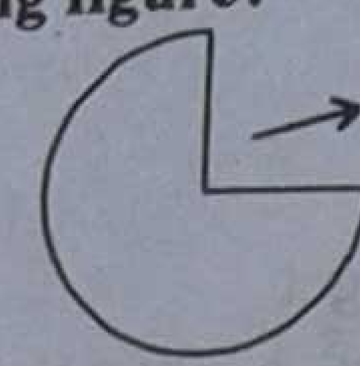
c) 18 cm *d)* 22 cm

Q5. **What is the surface of the following figure?**



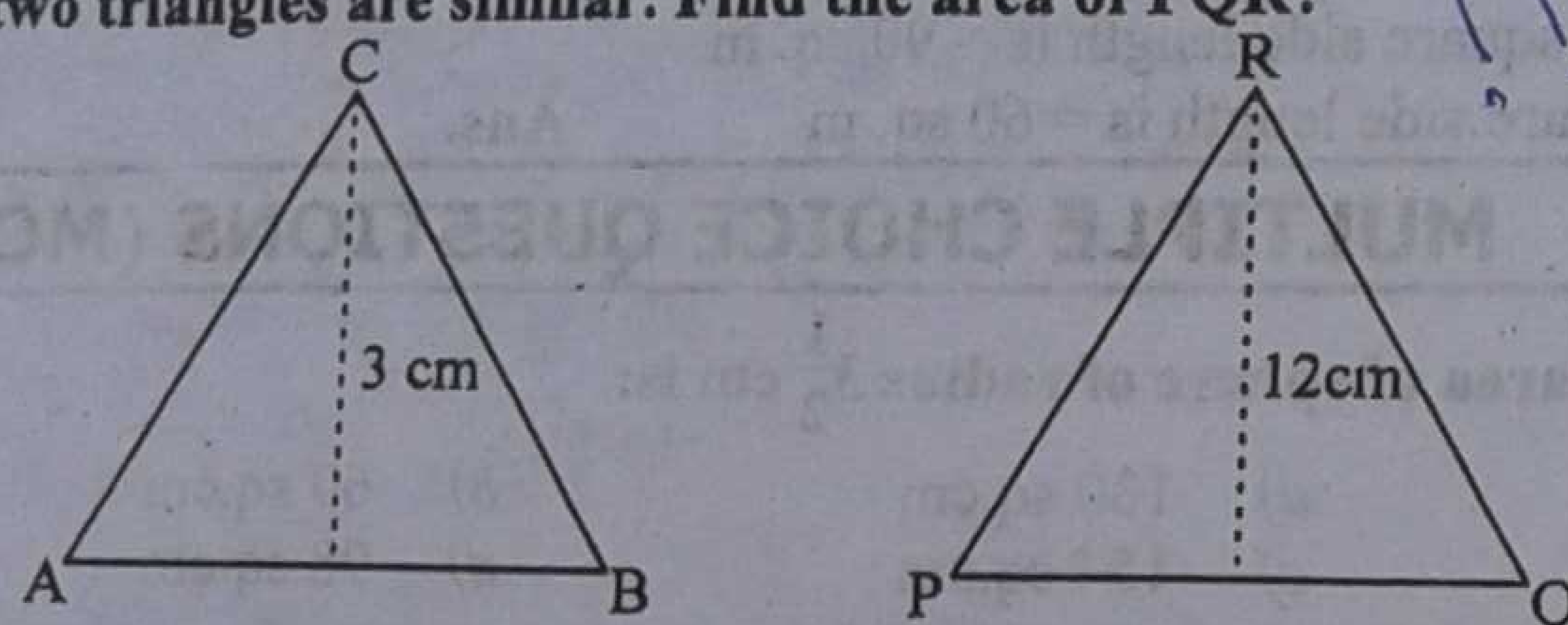
a) 33π
b) 24π
c) 25π
d) 11π

Q6. What is the area of the following figure?



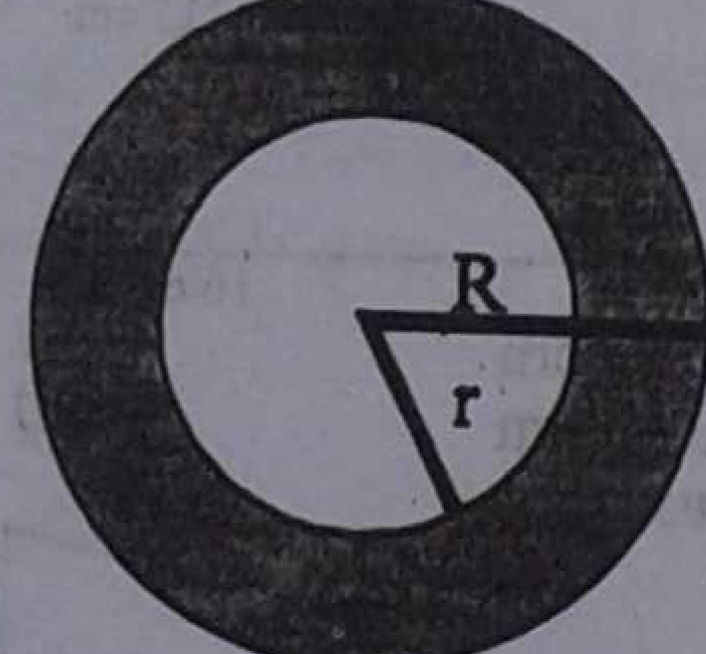
a) 125 sq.cm
b) 150.72 sq.cm
c) 64 sq.cm
d) 56 sq.cm

Q7. The following two triangles are similar. Find the area of PQR?



a) 54 cm^2
b) 24 cm^2
c) 96 cm^2
d) 59 cm^2

Q8. The area of the shaded region of the following figure is?

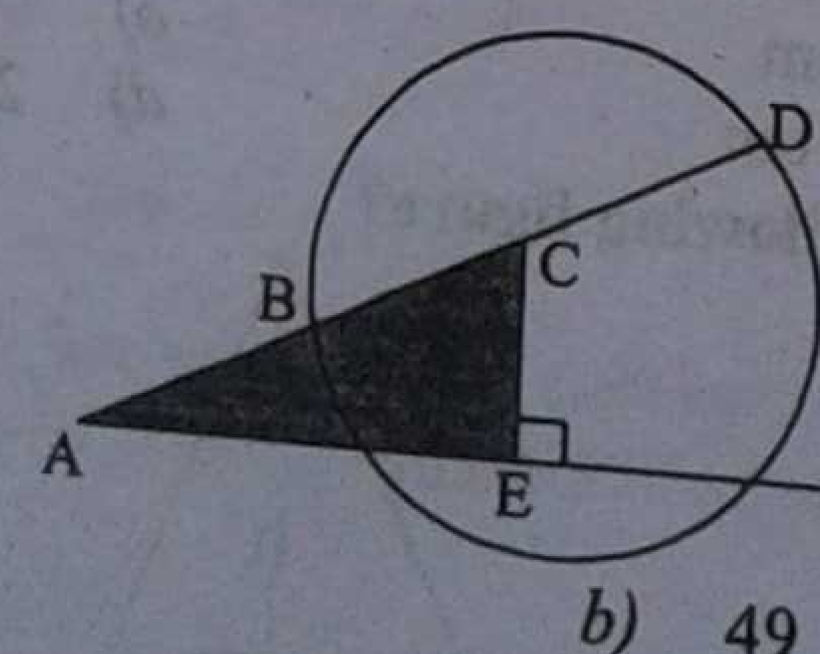


a) $\pi^2(r^2 - R^2)$
b) $\pi r^2 + \pi R^2$
c) $\pi^2(R + r)(R - r)$
d) $\pi(R + r)(R - r)$

Q9. The area of the sector which contains an angle of 60° of circle of radius 7 cm, is:

a) $25\frac{2}{3} \text{ cm}^2$
b) $27\frac{2}{3} \text{ cm}^2$
c) 41 cm^2
d) $\sqrt{3}\frac{5}{28} \text{ cm}^2$

Q10. In the following figure, what is the area of the right triangle? If $\overline{AE} = 16$ cm and $\overline{BD} = 12$ cm

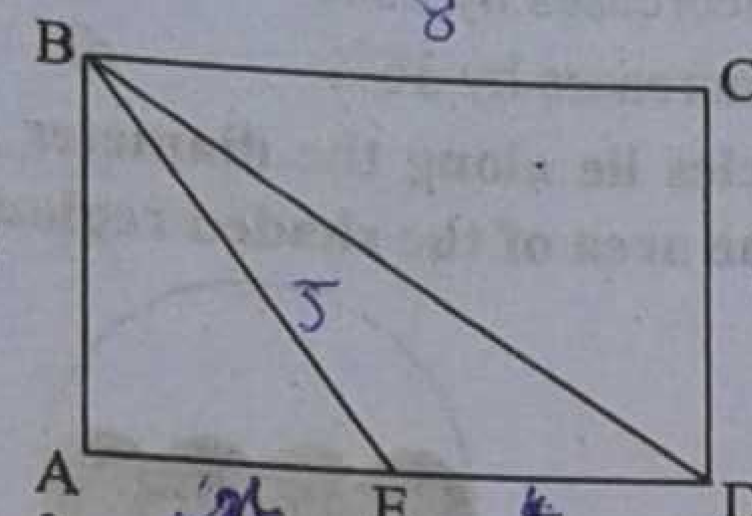


a) 6
b) 49
c) 8
d) Not possible

Q11. In the figure below, ABCD is a rectangle and E is the mid point of one side, what is the area of

triangle BCD?

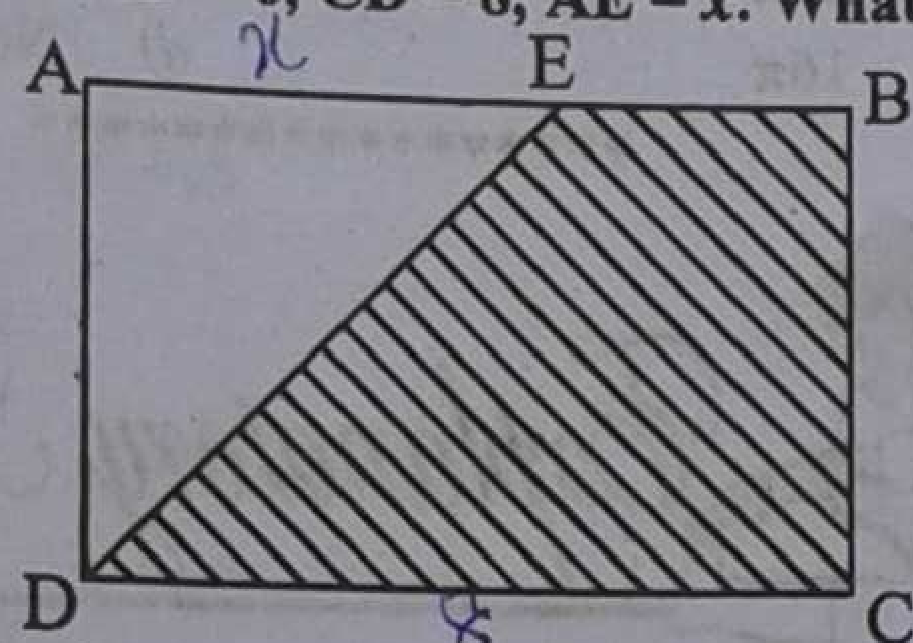
If $\overline{BE} = 5$ cm and $\overline{CD} = 3$cm



$3^2 = x^2 + 5^2$ $\frac{1}{2}(8)(3) = 12$

a) 12 cm²   b) 25 cm²
c) 9 cm²   d) 17.23 cm²

Q12. In the figure below, given that AD = 6, CD = 8, AE = x. What is the area of the shaded region?



Whole area of rectangle $= 6 \times 8 = 48$

area of $\Delta ADE = \frac{1}{2}(AD)(AE) = \frac{1}{2}(6)(x) = 3x$

area of shaded region $= 48 - 3x$

a) $28 - 2x$ b) $4(14 - 4x)$
c) $48 - 3x$ d) $7(6 - 3x)$

20% ↓ 0.8 remaing

$\pi(0.8)^2 = 1 - 0.64$

Q13. If the radius of the circle is decreased by 20%, what happens the area?

a) 10% in crease   b) 20% decrease
c) 80% increase   d) 36% decrease

Q14. A square, with perimeter 16, is inscribed in a circle, what is the area of the circle?

a) 3π b) $2\sqrt{2}\pi$
c) 32π d) 8π

$\to 4 \to \pi(2)^2$

$(4)^2 + (4)^2 = d^2$

$d = 4\sqrt{2}$

$r = \frac{4\sqrt{2}}{2}$

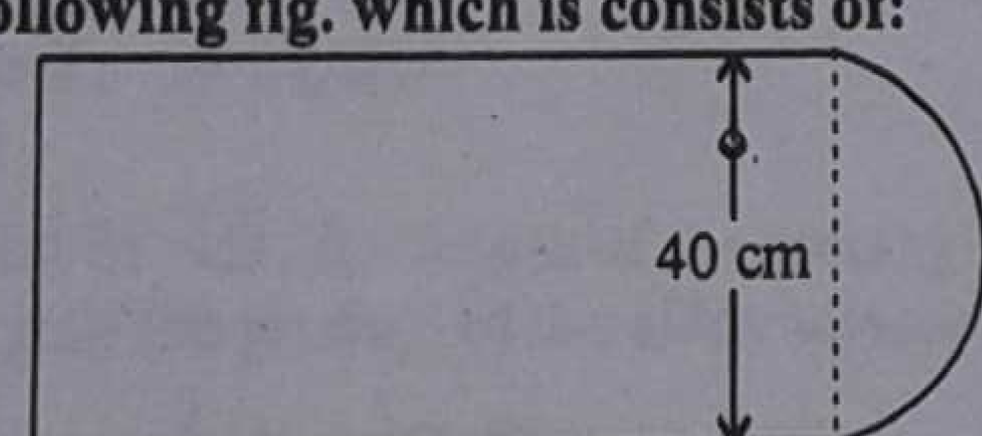
$r = 2\sqrt{2}$

$A = \pi(2\sqrt{2})^2$

$\sqrt{2^2 \cdot 2^2 \cdot 2}$

$4\sqrt{2}$

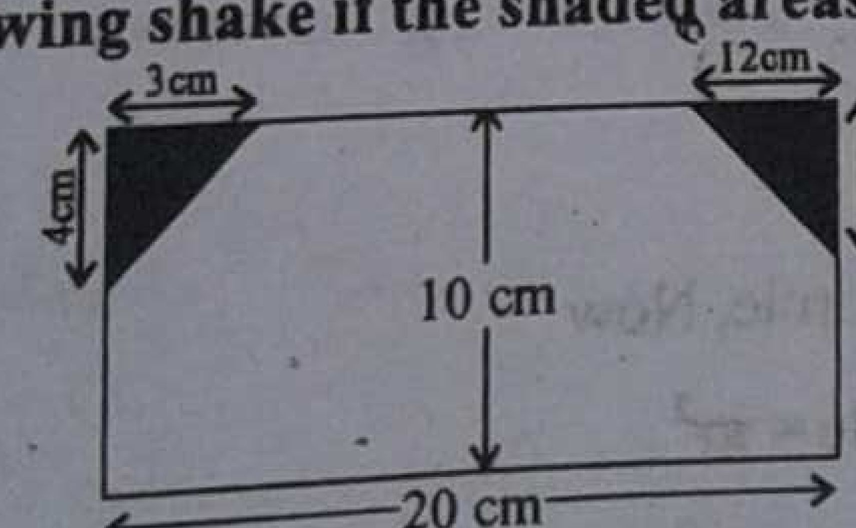
Q15. Calculate the area of the following fig. which is consists of:



The rectangle is of 40 cm and 68 cm and half a circle of diameter 40 cm

a) 2535 cm²   b) 2720 cm²
c) 3348 cm²   d) 628 cm²

Q16. What is the area of the following shake if the shaded areas are cut away?

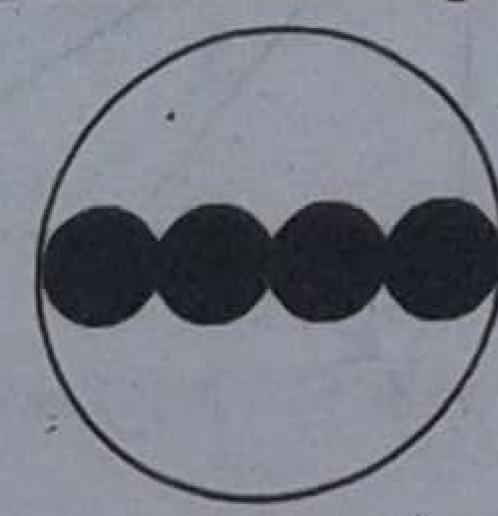


a) 100 cm²   b) 125 cm²
c) 120 cm²   d) 164 cm²

Q17. **The length of rectangle is decreased by 15% and its width is increased by 40%. The area is:**

a) decreases by 25%　　b) no effect
c) increases by 36%　　d) increases by 19%

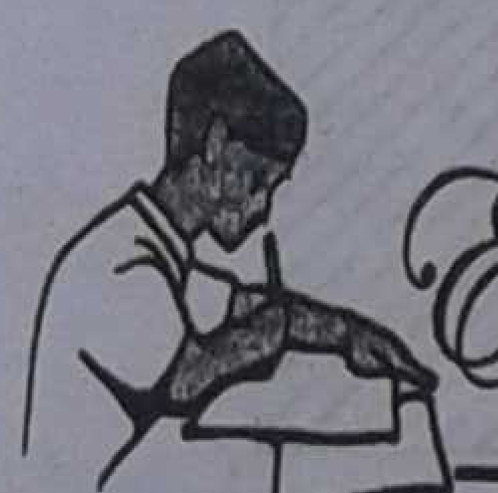
Q18. **In following fig. equal circles lie along the diameter of the large circle. If the circumference of the circle is 64π. What is the area of the shaded region?**



$2\pi r = 64\pi$
$r = 32$
$\frac{32}{4} = r = 8$
Area of small circle $= (8)^2\pi = 64\pi$
$= \pi 64 \times 4$
4 circle $= 256\pi$

a) 64π　　b) 256π
c) 16π　　d) None of these

****



Explanatory Answers

Q1. *c)* $A = 4\pi r^2$

$= \frac{4}{1} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$ sq.cm

$= 154$ sq.cm

Q2. *b)* A = Average width × Height

$= \frac{(8+16)}{2} \times 11$ sq.cm

$= 132$ sq.cm

Q3. *a)* $A = \frac{(3+5)}{2} \times 4$

$= 16$ sq.cm

Q4. *a)* $9 = \frac{1}{2} \times 3 \times h$

$\therefore h = \frac{2 \times 9}{3}$

$= 6$ cm

Q5. *a)* $S = \pi rs + \pi r^2$

$= (\pi \times 3 \times 8) + \pi(3 \times 3)$

$= 24\pi + 9\pi = 33\pi$

Q6. *b)* The figure is $\frac{3}{4}$ of the circle, Now

Area of the whole circle $= \pi r^2$

$\frac{3}{4}$ of the circle $= \frac{3}{4}(3.14)(18)^2$

$= \frac{3}{4} \times 200.96$

$= 150.72\ cm^2$

Q7. *a)* $\frac{AB}{DE} = \frac{\text{height of the triangle ABC}}{\text{height of the triangle DCP}}$

$\text{height} = \frac{36\ cm}{4} = 9cm$

$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 12 \times 9$

$= 54\ cm^2$

Q8. *d)* Area= Whole – hole

$= \pi R^2 - \pi r^2$

$= \pi(R^2 - r^2) = \pi(R - r)(R + r)$

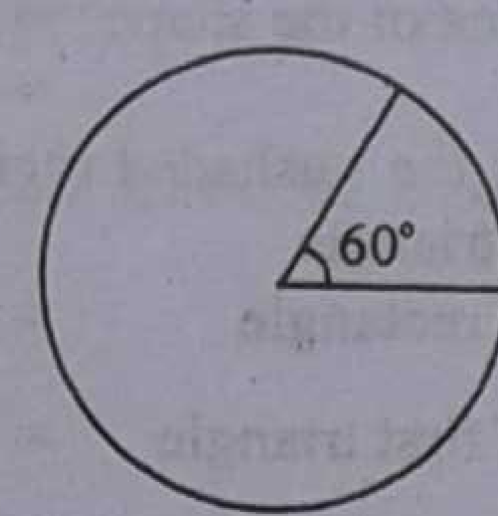
$\frac{\text{angle}}{360} = \frac{\text{Sector}}{\text{area}}$

$\frac{\text{angle}}{360} =$

Q9. *a)* $\text{Sector} = \frac{\theta^o}{360^o} \times \pi r^2$

$= \frac{60}{360} \times \frac{22}{7} \times \frac{7}{1} \times \frac{7}{1}\ cm^2$

$= 25\frac{2}{3}\ cm^2$



Q10. *d)* The area of the height triangle equals half the product of two legs. The point E is not on the circle, the length CE is given. We cannot find the exact value of CE. So lacking the length of CE, we cannot calculate the area.

Q11. *a)* In the given rectangle, $\overline{AB} = \overline{CD} = 3cm$. The length of the side AE of triangle BAE can be found with the Pythagorean theorem as:

$(BF)^2 = (AB)^2 + (AE)^2 \Rightarrow 25 = 9 + (AE)^2$

$\Rightarrow (AE)^2 = 16 \Rightarrow AE = 4$

As point E is the mid-point of $\overline{AD}$, the length of the rectangle is twice $\overline{AE}$ or 8 cm. The area of the right triangle BCD is half the product of the sides adjacent to the right angle

$$A = \frac{1}{2}bh = \frac{1}{2}(8)(3) = 12\ cm^2$$

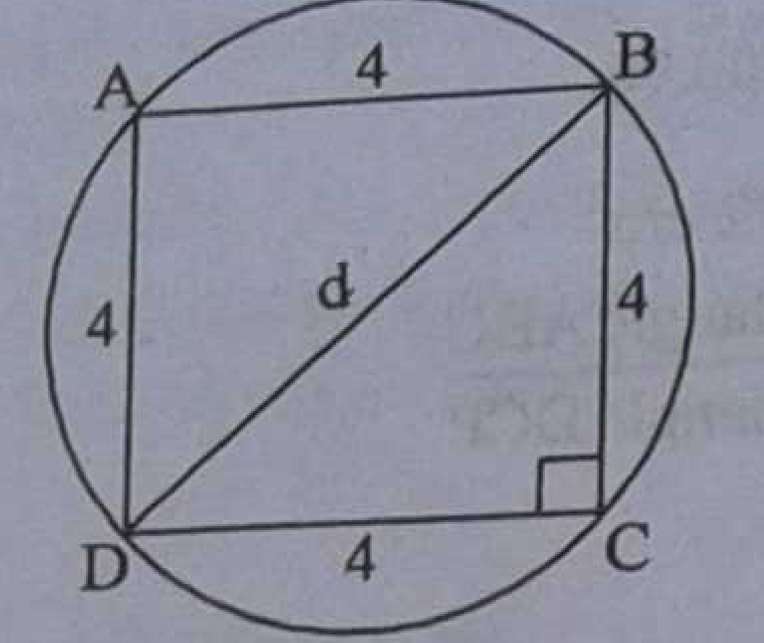
Q12. *c)* The difference between the entire rectangle and the white triangle is the shaded area. Thus

$(6 \times 8 = 48) - \frac{6 \times x}{2} = 48 - 3x$

Q13. *d)* If the radius of a circle is 1, after reducing 20% it becomes 0.8. Since Area = $0.8 \times 0.8 = 0.64 =$ $1 - 0.64 = 0.36 = \%$

Q14. *d)* Since the area of the square is 16.

$\therefore$ each side of the square is 4. In ΔBCD



$(4)^2 + (4)^2 = (BD)^2 \Rightarrow BD = \sqrt{32} = 4\sqrt{2}$

The radius of the circle $= \frac{d}{2} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

Area of the circle $= \pi r^2 = \pi(2\sqrt{2})^2 = \pi(4)(2) = 8\pi$

Q15. *c)* Area of the work surface = Area of the rectangle + area inside half a circle

Area of the rectangle $= 68\text{ cm} \times 40\text{ cm} = 2720\text{ cm}^2$

Area of the circle $= \pi r^2 = 3.14 \times \left(\frac{40}{2}\right)^2 = 3.14 \times (20)^2$

$= 628\text{ cm}^2$

Area of the shape $= 2720\text{ cm}^2 + 628\text{ cm}^2$

$= 3348\text{ cm}^2$

Q16. *d)* Area of the unshaded region = Whole Area of the rectangle − Area of first triangle – Area of second triangle

Area of rectangle $= 20 \times 10 = 200\text{ cm}^2$

Area of first triangle $= \frac{1}{2}(3)(4) = 6\text{ cm}^2$

Area of second triangle $= \frac{1}{2}(12)(5) = 30\text{ cm}^2$

Area of unshaded region $= 200 - 6 - 30 = 164\text{ cm}^2$

Q17. *d)* The 85%L shows a 15% decrease in length 140% W represents a 40% increase in width. The news rectangle will be

Area = (new length)(new width)

$= (80\%\ L)(40\%\ W) = \frac{85}{100}L \times \frac{40}{100}W$

$= \frac{119}{100}LW$

$= 119\%\ LW$

$\therefore$ The area of new rectangle will increase $(119\% - 100\%) = 19\%$

Q18. *b)* The circumference of big circle $= 64\pi$

We know circumference $= 2\pi r$

$\Rightarrow 64\pi = 2\pi r \Rightarrow r = 32$

From given radius of the small circle = 8

Area of the small circle $= \pi r^2 = \pi(8)^2$

$= 64\pi$

Area of 4 small circles = 4(area of 1 circle)

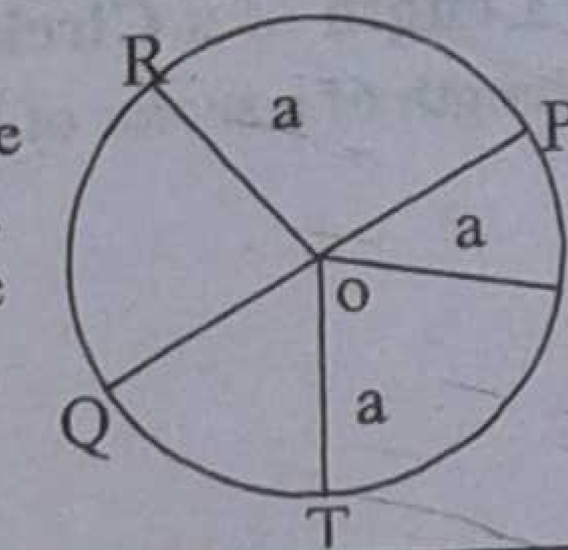
$= 4(64\pi)$

$= 256\pi$

Circles

Circle:

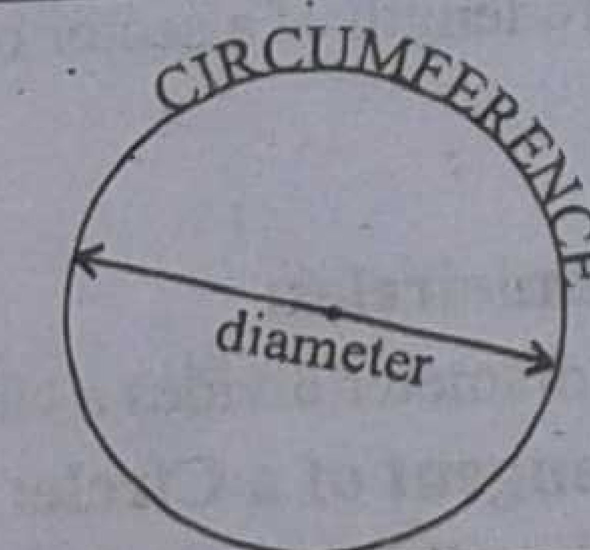
A circle is a set of all points in a plane at a given distance from a fixed point of the plane. The fixed point is the centre of the circle and the given distance is the radius. The adjacent figure is a circle of radius ***a*** unit whose centre is at the point O. The point P, Q, R, S and T lies on the circle, each ***a*** unit from O. Therefore, the following statement follows from the definition of a circle.



All radii (plural of radius) of the same circle are congruent.

Circumference:

The perimeter of a circle is called its circumference.



Note:

If "c" stands for the circumference of the circle and "d" is the diameter of the circle, then $\frac{c}{d}$ *(circumference ÷ diameter)* is the same for all circles. Its value cannot stated exactly.

The Greek letter π*(pi)* is used to stand for it.

$\therefore \quad \frac{c}{d} = \pi$

$c = \pi d$

or $\quad c = \pi \times 2 \times r$

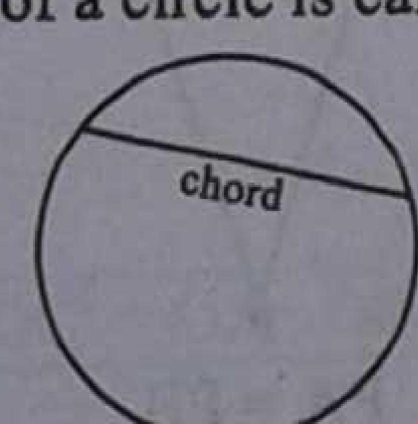
$c = 2\pi r$

π is a special number and is equal to 3.14159....... or $\frac{22}{7}$

Angles and Circles:

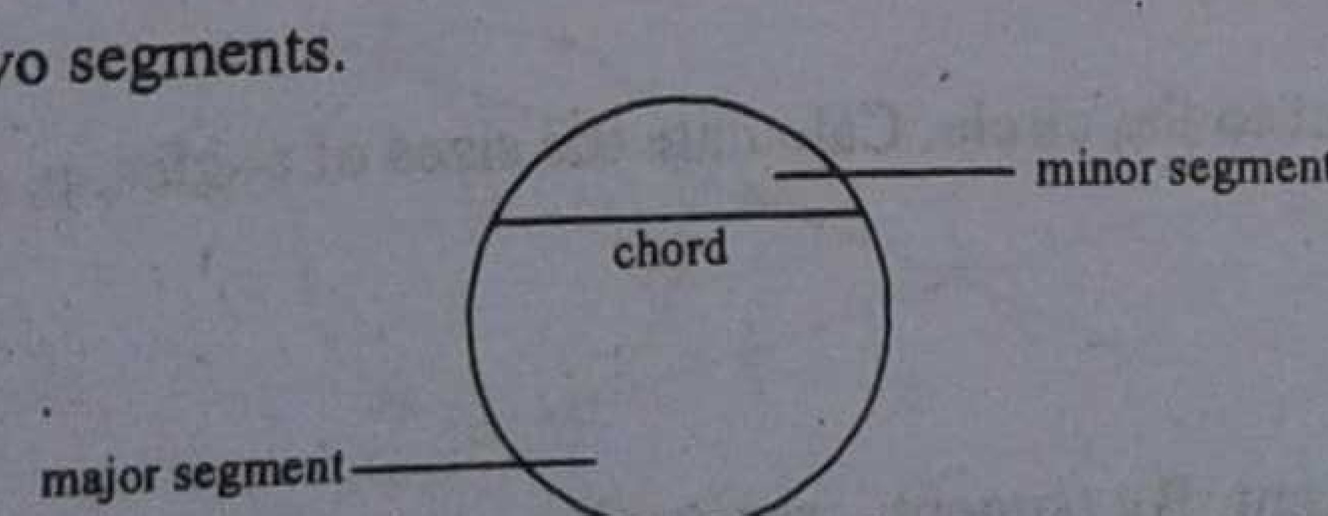
Chord:

A line joining two points on the circumference of a circle is called a chord.



Note:

A chord divides a circle into two segments.

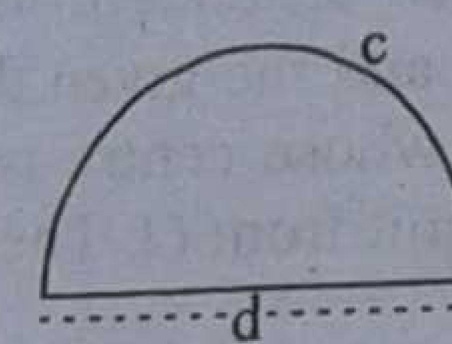


Diameter:

A chord which passes through the centre of the circle is a diameter.

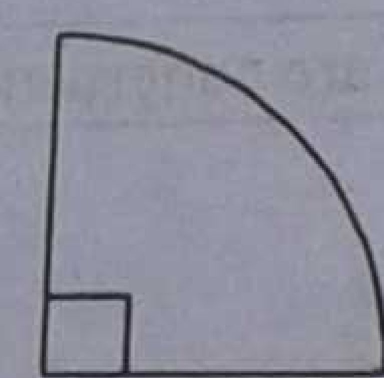
Arc Length of a Circle:

Arc length of a sector of half a circle is



$$c = \frac{\pi d}{2}$$

Arc length of a sector of quarter of a circle is



$$c = \frac{\pi d}{4}$$

Semicircles:

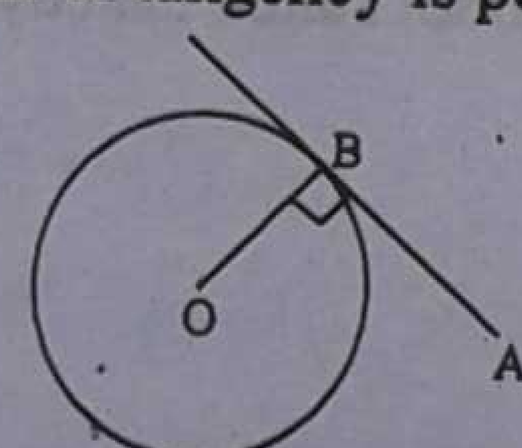
A diameter divides a circle into two congruent halves which are called semicircles.

Tangent of a Circle:

A line that intersects the circle in exactly one point is a tangent to the circle. The point of intersection is called the point of tangency.

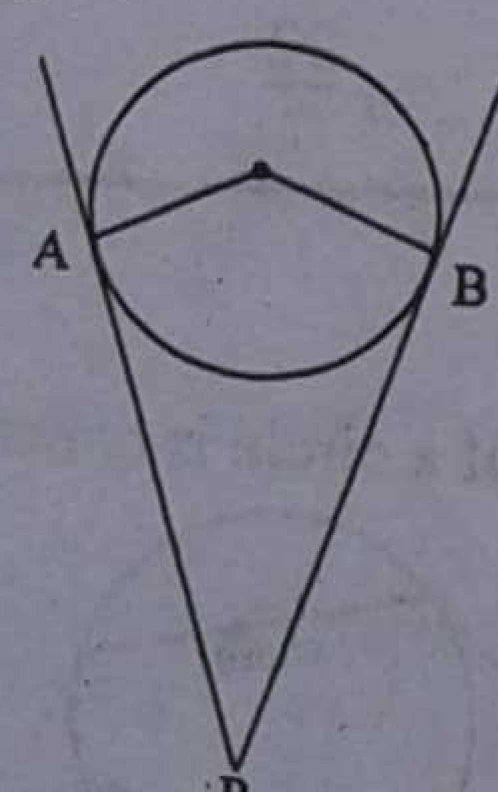
Note:

1. The radius from the centre to the point of tangency is perpendicular to the tangent.



AB is tangent to the circle with centre O. OB is perpendicular to BA.

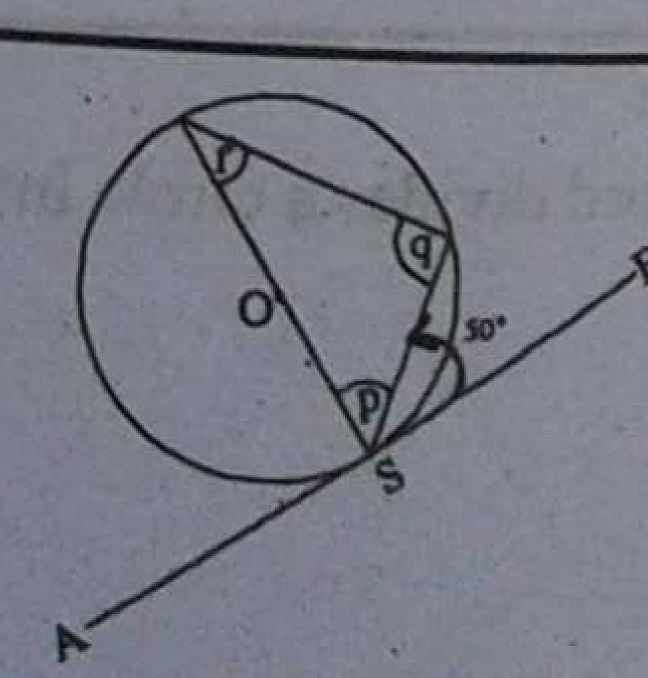
2. Tangents from the same point are equal



PA = PB

Example 1:

In the given figure, if AB is tangent to the circle. Calculate the sizes of angles, p, q, r.



Solution:

OS is the radius and AB is the tangent. By tangent – radius theorem $\angle OSB = 90^\circ$

$\angle p = 90° - 50° = 40°$

Now $\angle q = 90°$ because angle in a semicircle is right. Now angle p, q and r the angles of triangle.

$\therefore \angle p + \angle q + \angle r = 180°$

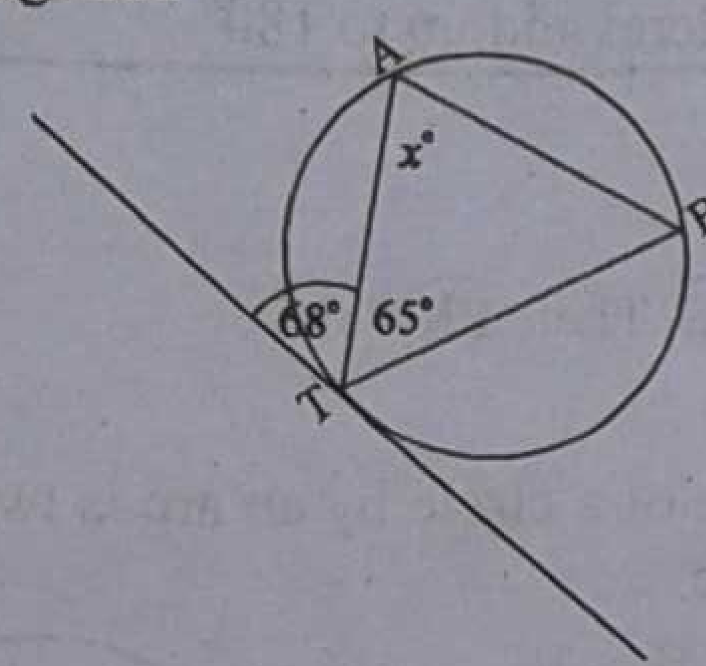
$\therefore 40° + 90° + r = 180°$

$\therefore r = 180° - 130°$

$r = 50°$

eXample 2:

What is the value of x in the following diagram?



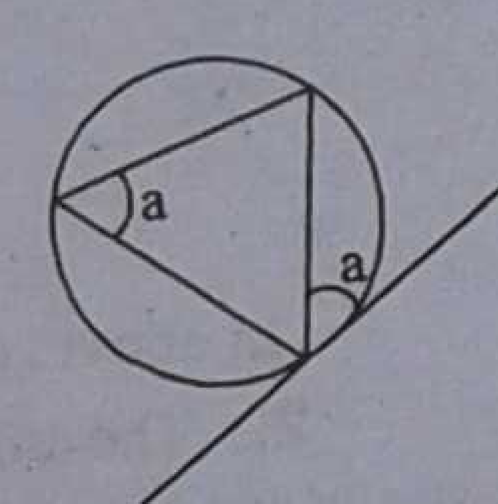
Solution:

In above diagram $\angle TBA = 68°$ because angles in alternate segment are equal

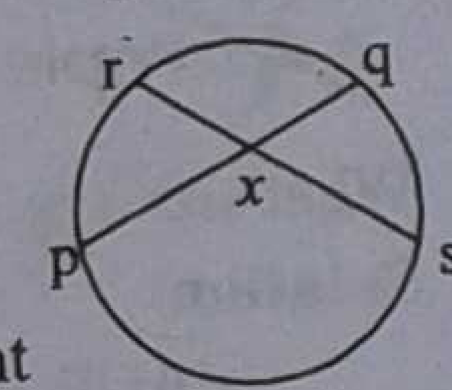
$\therefore x° + 65° + 68° = 180° \Rightarrow x = 47°$

Theorems: Tangents and Secants

1. The angle between a tangent and a chord drawn to the point of contact is equal to the angles in the alternate segment.

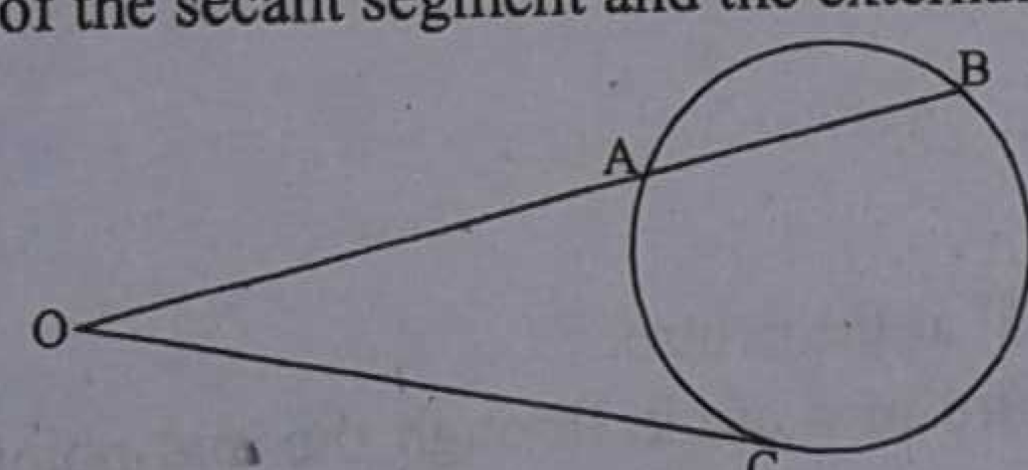


2. The products of the intercepts of two intersecting chords of a circle are equal.



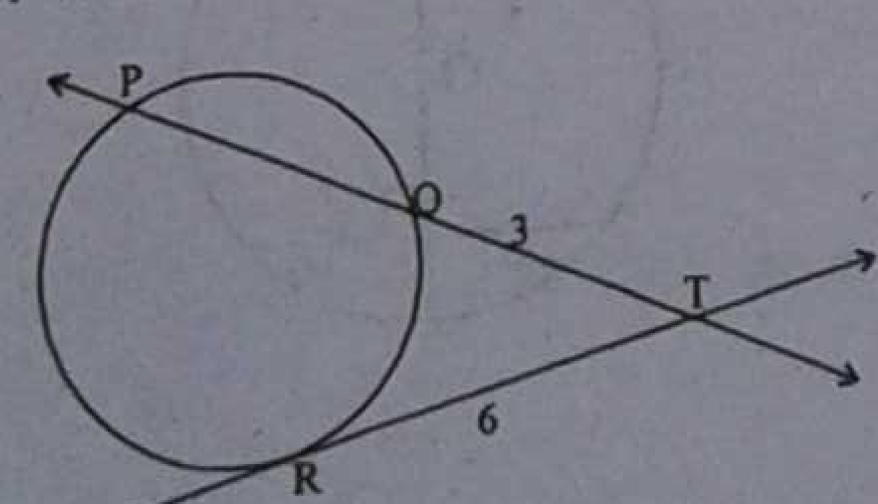
That is: $px \cdot qx = rx \cdot sx$

3. If a tangent and a secant intersect in the exterior of a circle, the square of the tangent segment equals the product of the secant segment and the external secant segment.



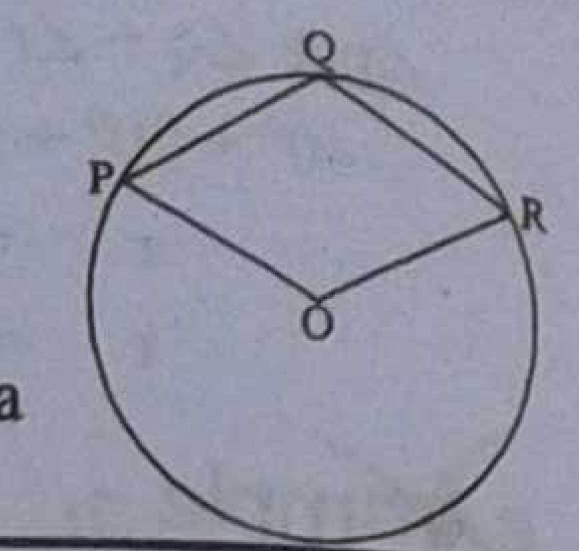
$OC^2 = OA \cdot OB$

eXample: In the following figure. What is the value of TP?



Solution:

$$(TR)^2 = TQ \cdot TP$$
$$(6)^2 = 3 \cdot TP$$
$$36 = 3 \cdot TP$$
$$\Rightarrow \quad TP = 12$$



Cyclic Quadrilateral:

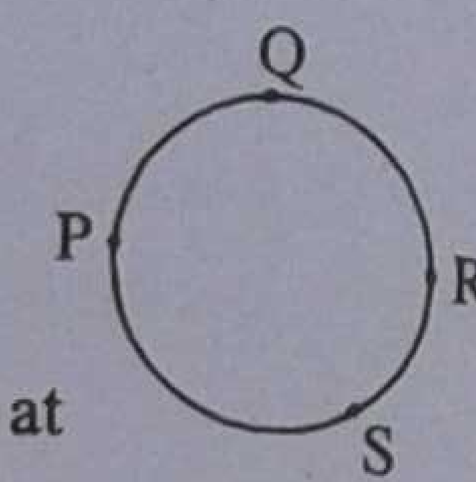
A quadrilateral which has all its vertices laying on the circumference of a circle is called a cyclic quadrilateral.

> **Note:**
> Opposite angles of a cyclic quadrilateral add up to 180°.

Common Arc Theorem:

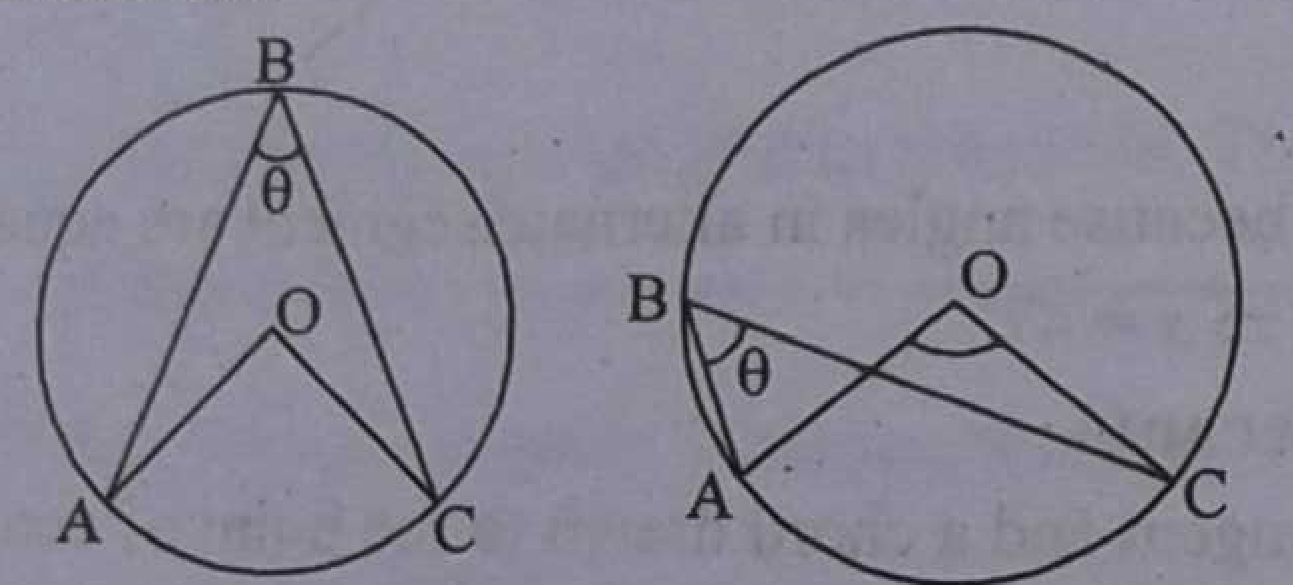
If four points on a circle are, in order and

$\widehat{PQ} = \widehat{RS}$ Then $\widehat{PR} = \widehat{QS}$



Central Angle Theorems:

1. The angle subtended at the centre of a circle by an arc is twice the angle subtended at the circumference by the same arc.



In above, angle AOC = 2 × angle ABC.

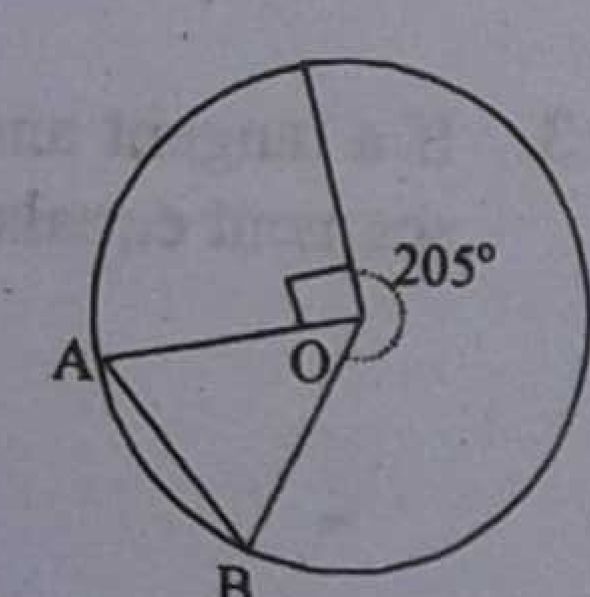
2. The angle measured in degree to complete one revolution in a circle is 360°.

eXample:

What size angle is subtended at the centre (O) by chord AB?

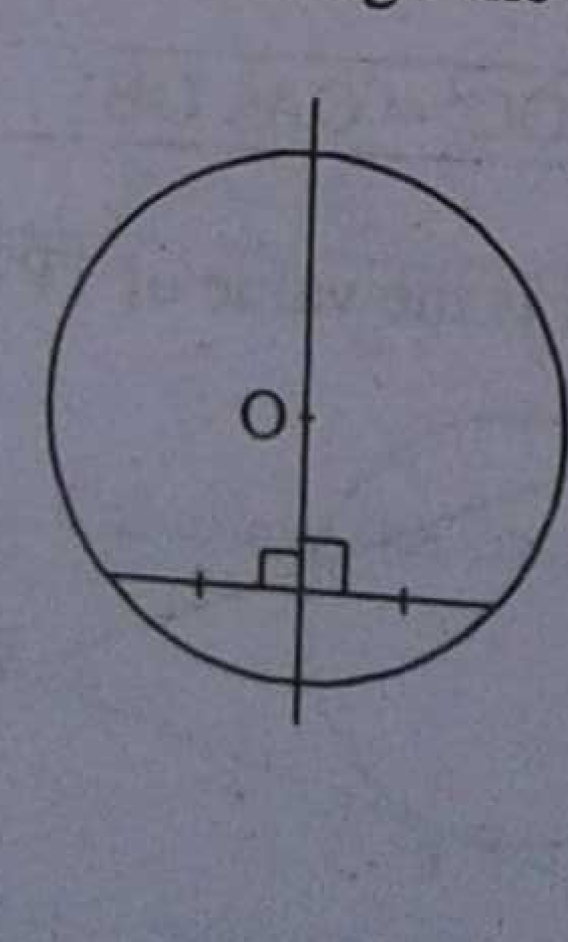
Solution:

$$\angle AOB + 90^\circ + 205^\circ = 360^\circ$$
$$\angle AOB + 295^\circ = 360^\circ$$
$$\angle AOB = 360^\circ - 295^\circ$$
$$\Rightarrow \quad \angle AOB = 65^\circ$$

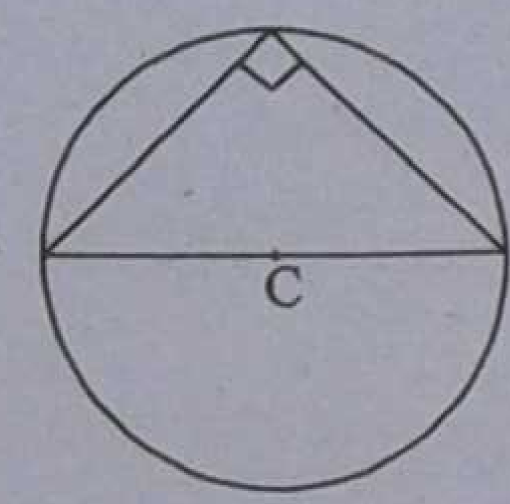


∴ Chord AB subtends an angle of 65° at the centre.

Theorem: A line from the centre of a circle through the mid-point of a chord meets the chord at right angles.



Theorem: Angle In a Semicircle:
An angle in a semicircle is a right angle.

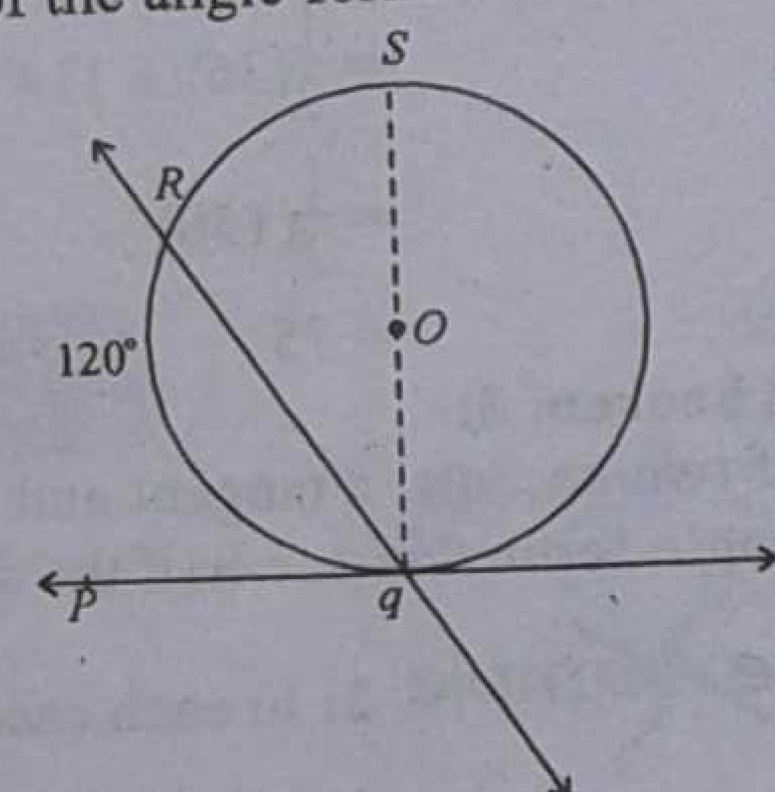


Converse of Theorem:
If a circle passes through the ventricles of a right-angled triangle, then the hypotenuse of the triangle is a diameter of the circle.

More Angles and Arcs:

Theorem 1:
If a tangent and a secant (or chord) intersect in a point on a circle, the measure of the angle formed is one half the measure of the intercepted arc.

Explanation:
In circle O as shown in the figure, secant QR and tangent PQ intersect at point Q on the circle, forming angle PQR. The above theorem focuses upon the relationship between the measure of this angle and the degree measure of the intercepted arc, $\widehat{QR}$.

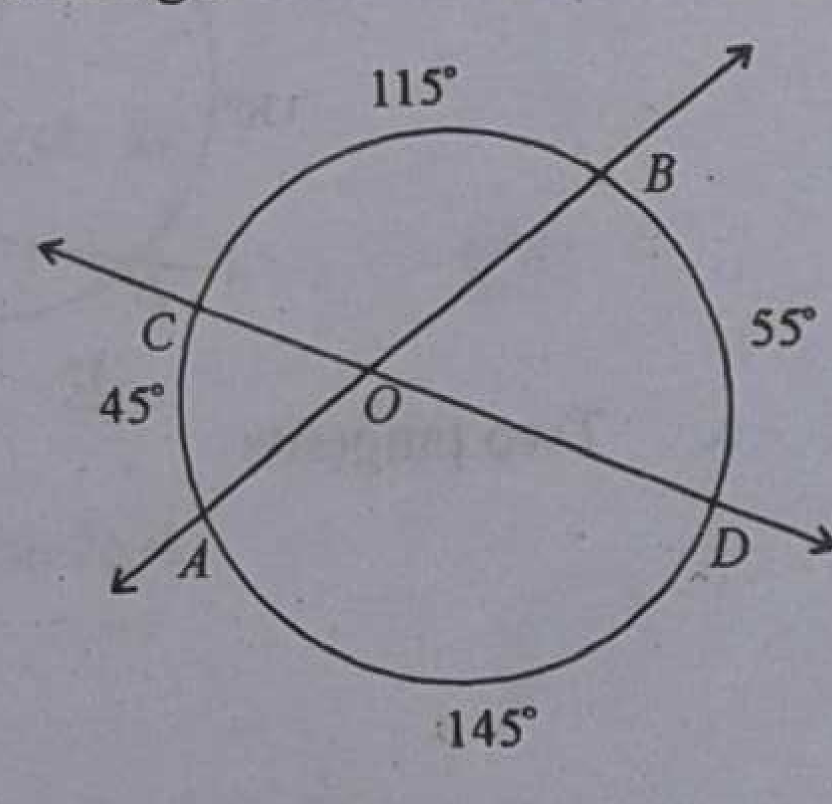


According to this theorem, $\angle PQR = \frac{1}{2}(120)$

$$= \frac{1}{2}(\text{Arc } QR)$$
$$= 60^\circ$$

Theorem 2:
If two secants (or chords) intersect in the interior of a circle, the measure of an angle formed is one half the sum of the measures of he arcs intercepted by the angle and its vertical angle.

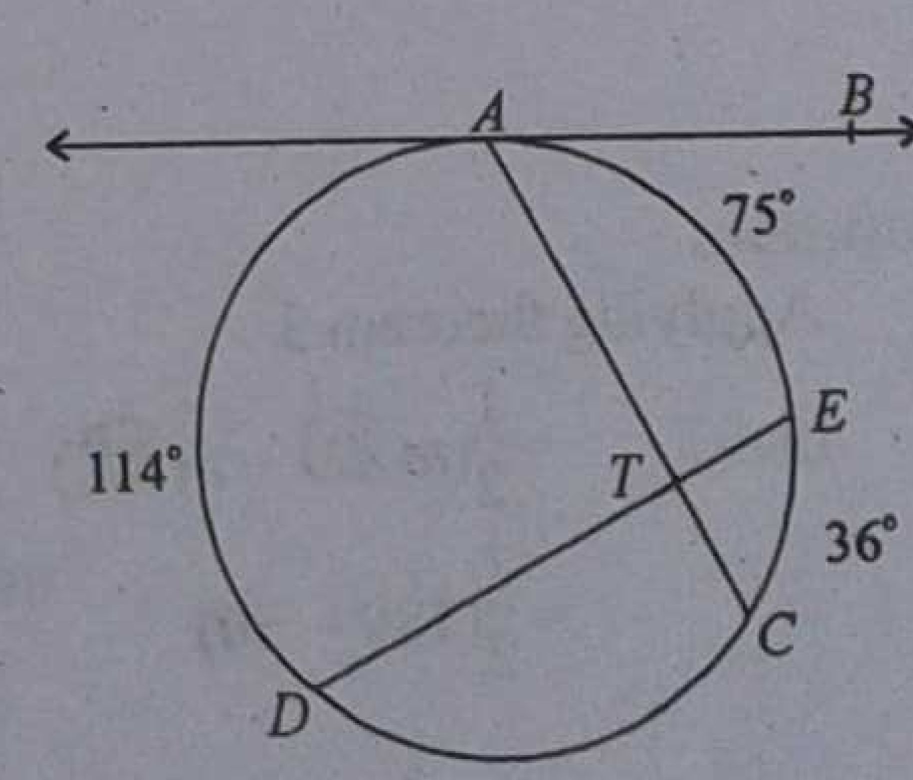
Explanation:
When two secants AB and CD intersect in the interior of a circle, as circle O shows to the right, two pair of vertical angles are formed. According to the given theorem



$$\angle AOC = \angle DOB = \frac{1}{2}(55^\circ + 45^\circ)$$
$$= \frac{1}{2}(100) = 50^\circ$$

and
$$\angle AOD = \angle COB = \frac{1}{2}(145^\circ + 115^\circ)$$
$$= \frac{1}{2}(260)$$
$$= 130^\circ$$

example 1:



Chord AC and DE intersect at T, $\overleftrightarrow{AB}$ is tangent to the circle at A.
$m\widehat{AD} = 114$, $m\widehat{EC} = 36^\circ$ and $m\widehat{AE} = 75$

a. Find $m\angle CAB$

b. Find $m\angle ATD$

Solution: a. By theorem 1

$$\begin{aligned} m\angle CAB &= \frac{1}{2} m\widehat{AC} \\ &= \frac{1}{2}(m\widehat{SR} + m\widehat{RT}) \\ &= \frac{1}{2}(75^\circ + 36^\circ) \\ &= \frac{1}{2}(111) \\ &= 55.5 \end{aligned}$$

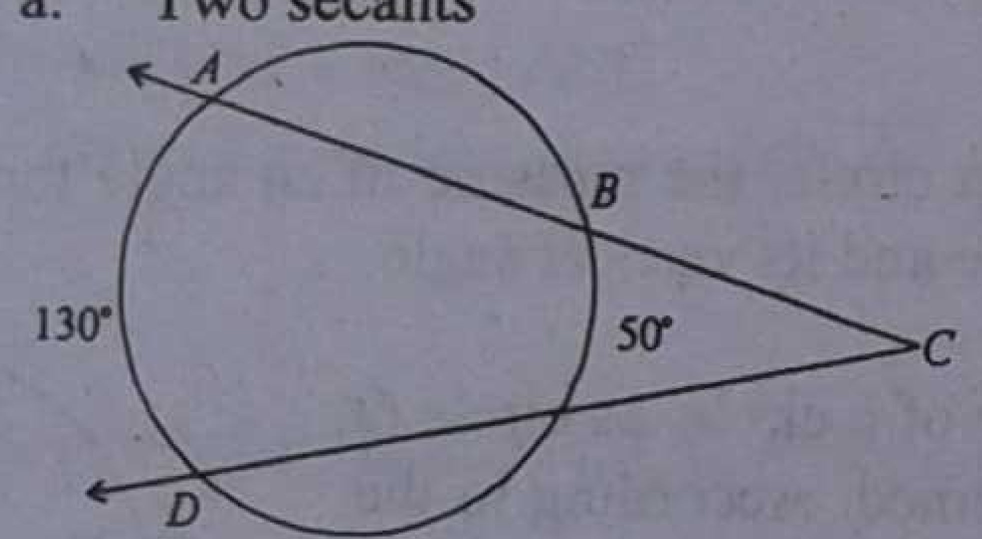
b. By theorem 2

$$\begin{aligned} m\angle ATD &= \frac{1}{2}(m\widehat{EC} + m\widehat{AD}) \\ &= \frac{1}{2}(36^\circ + 114^\circ) \\ &= \frac{1}{2}(150) \\ \Rightarrow \quad &= 75 \end{aligned}$$

Theorem 3:
If two secants, a tangent and a secant, or two tangents intersects in the exterior of a circle, the measure of the angle formed is one-half the difference of the measures of the intercepted arcs.

example **2: In each case of the following figure, find m∠C.**

a. Two secants



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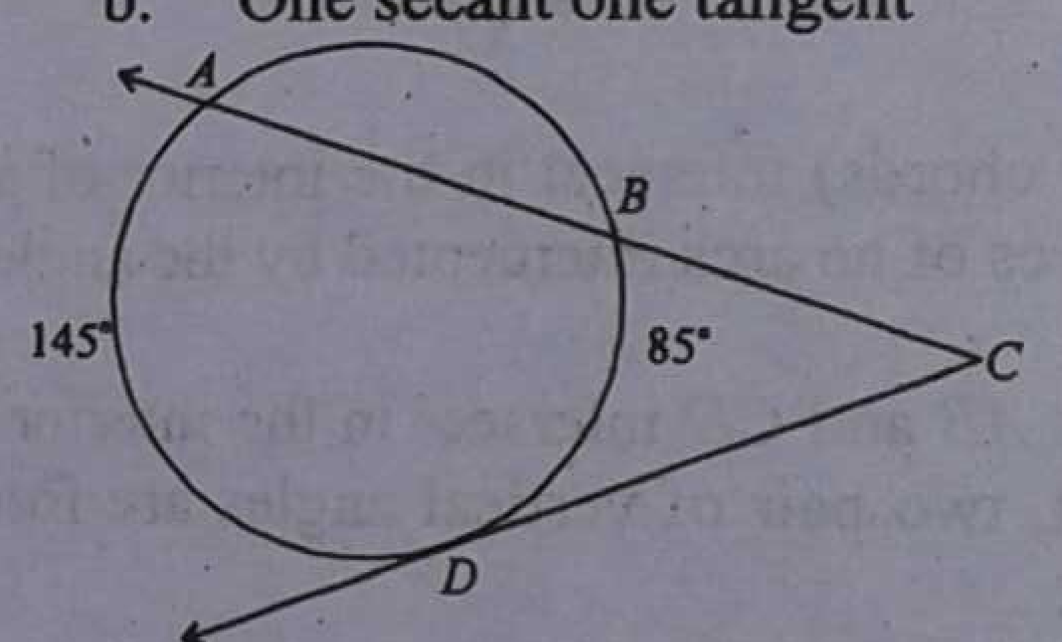
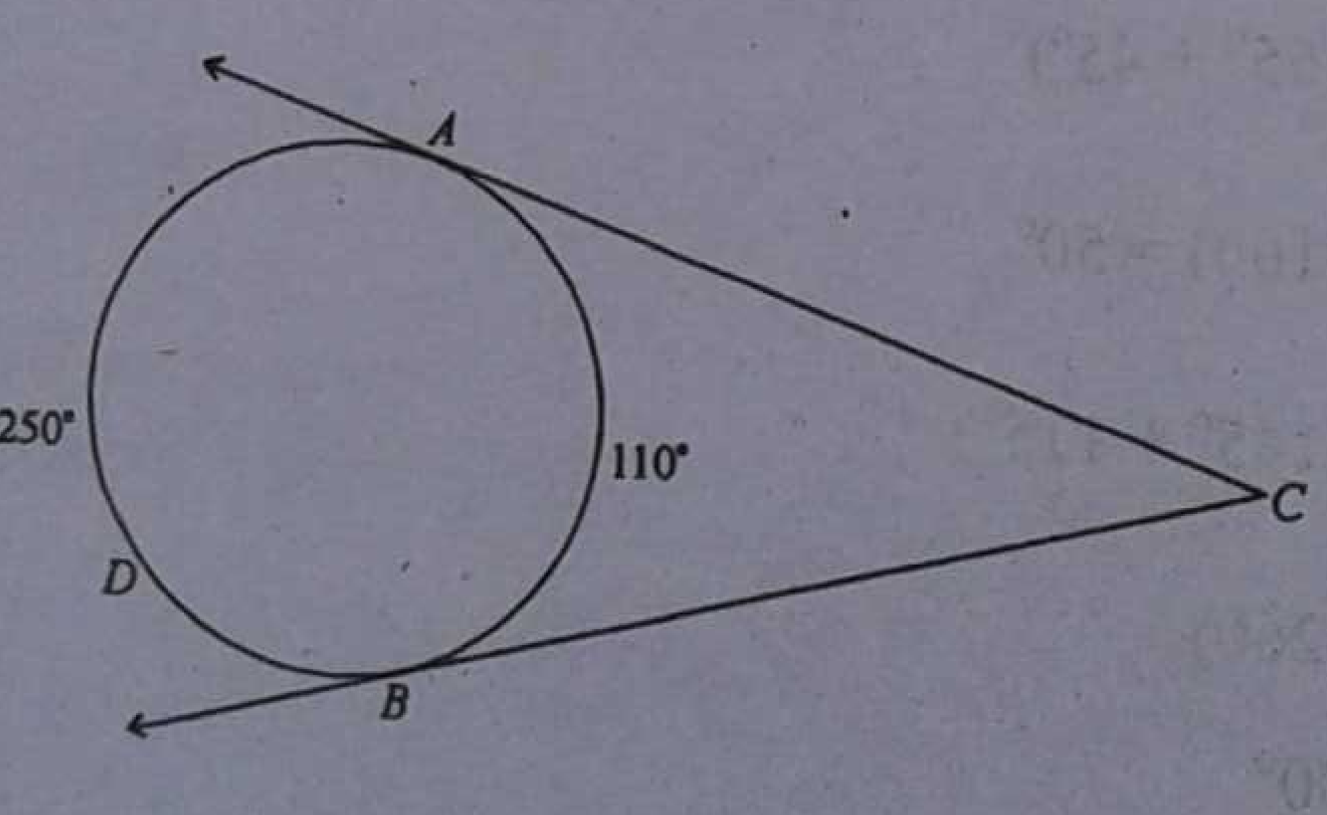
b. One secant one tangent



c. Two tangents



Solution:

a. Applying theorem 3

$$\begin{aligned} m\angle C &= \frac{1}{2}(m\widehat{AD} - m\widehat{BE}) \\ &= \frac{1}{2}(130 - 50) \end{aligned}$$

$= \frac{1}{2}(80) = 40$

b. Applying theorem 3

$m\angle C = \frac{1}{2}(m\widehat{AD} - m\widehat{BD})$

$= \frac{1}{2}(250 - 110)$

$= \frac{1}{2}(140)$

$= 70$

c. Applying theorem 3

$m\angle C = \frac{1}{2}(m\widehat{ADB} - m\widehat{AB})$

$= \frac{1}{2}(250 - 110)$

$= \frac{1}{2}(150)$

$\Rightarrow m\angle C = 75$

Multiple Choice Questions (MCQs)

Q1. If the area of a circle is 81π, then its circumference is:

(A) 61π (B) 20π

(C) 18π (D) 16π

$81\pi = \pi r^2$
$r = 9$
$2\pi(9)$

Q2. If circumference of a circle is 3π, then its area is:

(A) $\frac{7\pi}{2}$ (B) $9\pi^2$

(C) $4\pi^2$ (D) $\frac{9\pi}{4}$

$3\pi = 2\pi r$
$r = \frac{3}{2}$
$\pi \frac{9}{4}$

Q3. If a circle is inscribed in a square of area 4, then the area of the circle is:

(A) π (B) $\frac{\pi}{2}$

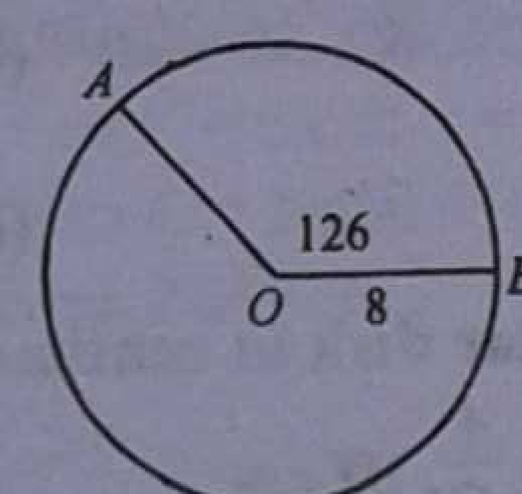
(C) $\frac{\pi}{4}$ (D) $\frac{3\pi}{4}$

Q4. If a square of area 3 is inscribed in a circle, then the area of the circle is:

(A) $\frac{9}{4}\pi$ (B) $9\pi^2$

(C) 3π (D) $\sqrt{3}\pi$

Questions 5 – 6 refer to the following figure



Q5. What is the length of arc AB?

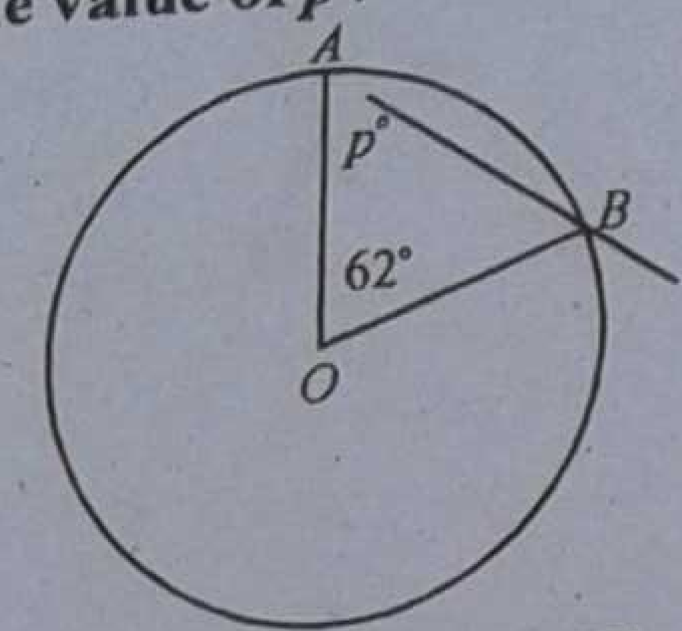
(A) 2.6π (B) 5.6π

(C) 7.6π (D) $\frac{1}{2}\pi$

Q6. What is the area of the shaded sector?

(A) 22.9π (B) 22.4π
(C) 60π (D) 62.3π

Q7. In the following figure, what is the value of p?



(A) 49 (B) 39
(C) 59 (D) 69

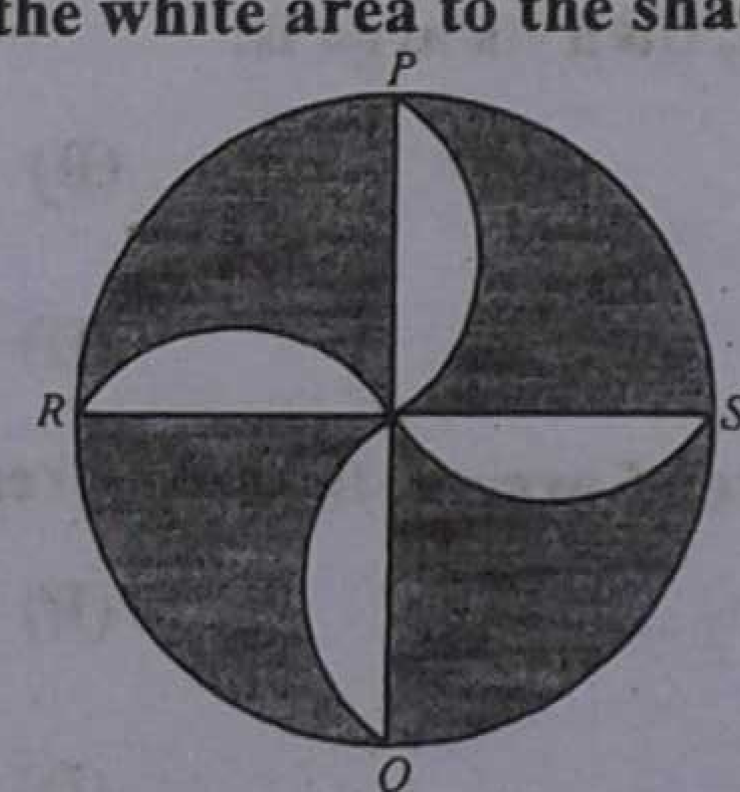
Q8. If P represents the area and W represents the circumference of the circle, then P in terms of W is:

(A) $\frac{2\pi}{W}$ (B) $\frac{4\pi^2}{W}$
(C) $\frac{2\pi^2}{W^2}$ (D) $\frac{W^2}{4\pi}$

Q9. What is the area of a circle whose radius is the diagonal of a square whose area is 9?

(A) $\sqrt{3}\pi$ (B) 12π
(C) 4π (D) 13π

Q10. In the following figure, PQ and RS are perpendicular, and each of the unshaded regions is a semicircle. What is the ratio of the white area to the shaded area?



(A) $\frac{4}{\pi}$ (B) $\frac{1}{1}$
(C) $\frac{2}{3}$ (D) $\frac{1}{2}$

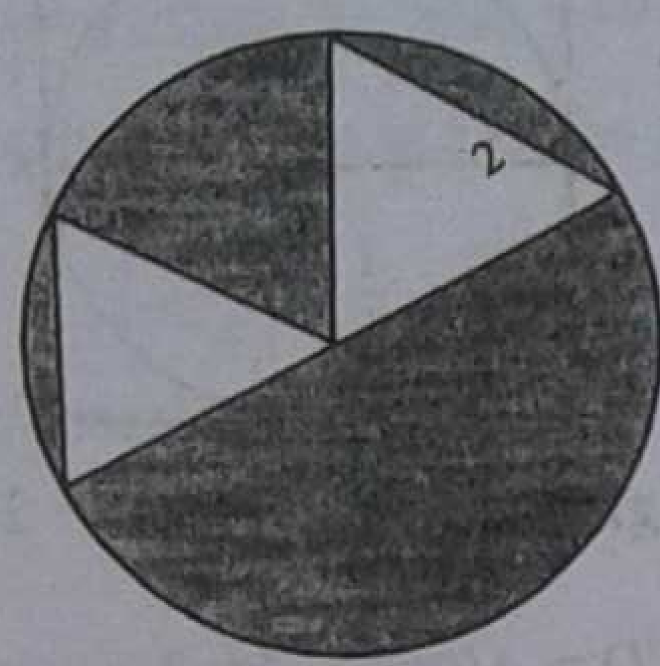
Q11. If C is the circumference of a circle of radius r, then which of the following statement is true?

(A) $\frac{C}{r} < 6$ (B) $\frac{C}{r} = 6$
(C) $\frac{C}{r} > 6$ (D) $\frac{C}{r} = \pi$

Q12. If C is the circumference of a circular disk in centimeters, and A is the area of the same circular disk in square centimeter. Then $\frac{C}{A} = \frac{A}{C}$, iff $r =$

(A) 1 (B) 2
(C) 3 (D) 4

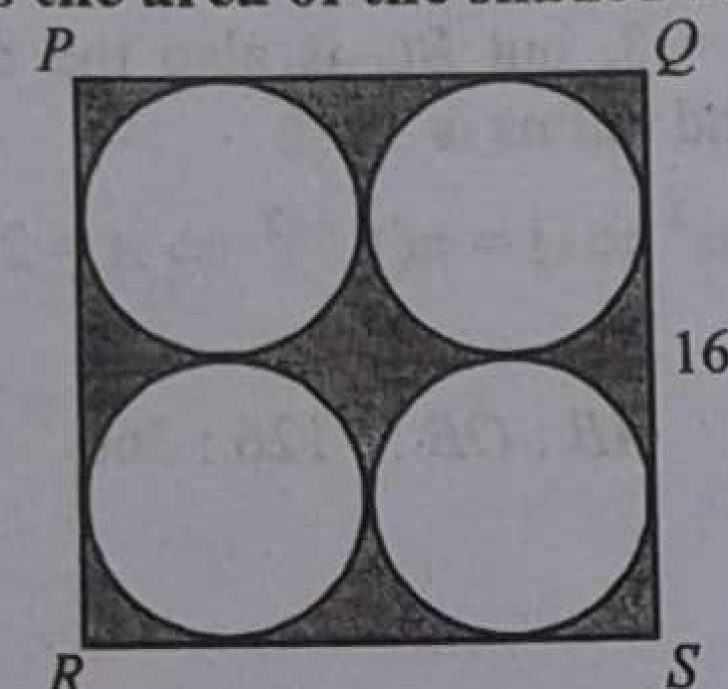
Q13. **In the following figure, what is the area of the shaded region, if each of the triangle is equilateral?**



(A) 8π (B) $\frac{8}{3}\pi$

(C) 3π (D) 6π

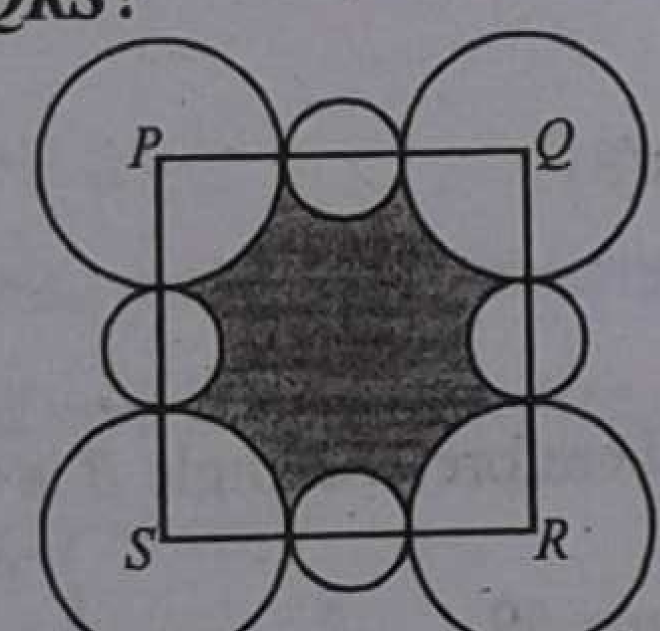
Q14. **In the following figure, *PQRS* is a square, and all the circles are tangent to one another and to the sides of the squares. What is the area of the shaded region?**



(A) 256 (B) 64π

(C) 256π (D) $64(4 - \pi)$

Q15. **In the following figure, the large circles have radius 4, and the small circles have diameter 4. What is the area of the square *PQRS*?**



(A) 144 (B) 169

(C) 100 (D) 64

Explanatory Answers

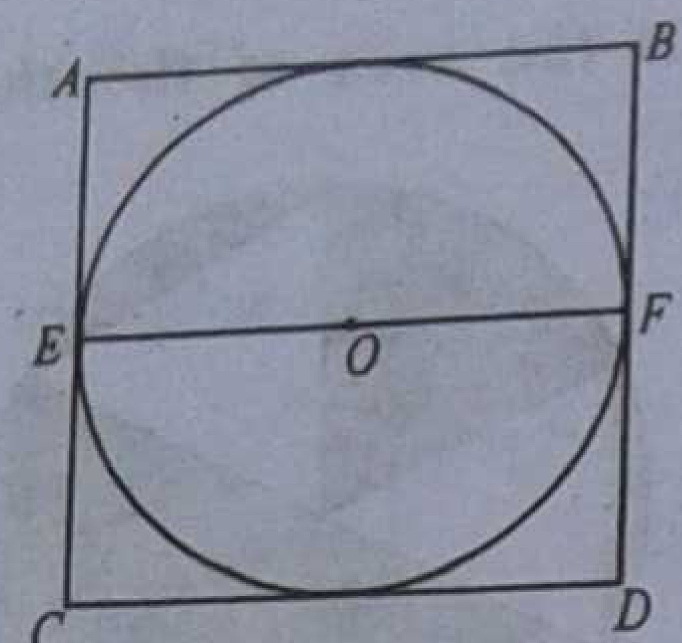
Q1. (C) Area of a circle: $A = \pi r^2 = 81\pi \quad \Rightarrow r^2 = 81 \Rightarrow r = 9$
Circumference of a circle: $C = 2\pi r \Rightarrow C = 2\pi(9) = 18\pi$

Q2. (D) Circumference of a circle: $C = 2\pi r \Rightarrow 2\pi r = 3\pi$

$$\Rightarrow r = \frac{3}{2}$$

Area of a circle: $A = \pi r^2 \Rightarrow A = \pi \cdot \left(\frac{3}{2}\right)^2 \Rightarrow A = \frac{9\pi}{4}$

Q3. (A) First of all, we draw the diagram



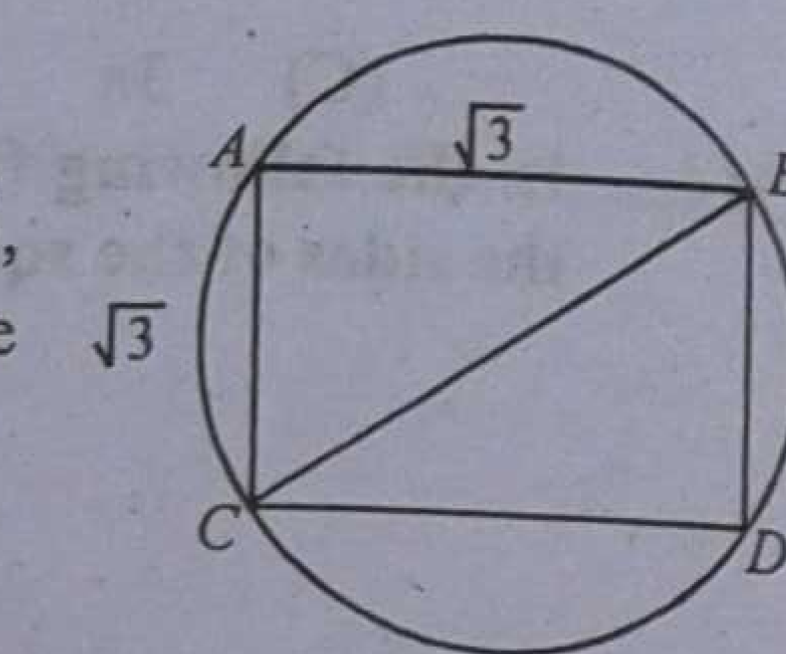
Since the area of the square is 4 (given), therefore $AC = 2$, as in a square all sides are equal, therefore $AC = AB = BD = CD = 2$.

$\because$ Diameter of the circle, $EF = CD = AB \Rightarrow EF = 2$

and radius of the circle $= r = OF = OE = 1$ (half the diameter)

Hence the area of the circle with radius 1 is

$$\pi(1)^2 = \pi$$



Q4. (A) First we draw a diagram, because area of the square is 3, thus $AC = \sqrt{3}$, then diagonal $BC = \sqrt{3} \times \sqrt{3} = 3$, but BC is also the diameter of the circle, hence the diameter is 3 and radius is 1.5

Now, the area of the circle $A = \pi r^2 \Rightarrow A = \pi(1.5)^2 \Rightarrow A = 2.25\pi = \frac{9}{4}\pi$

Q5. (B) Setting a proportion

$$AB : OB :: 126 : 360$$

$$\frac{\overline{AB}}{2\pi r} = \frac{126}{360}$$

$$\overline{AB} = \left(\frac{126}{360}\right) \times 2\pi r$$

$$\overline{AB} = \left(\frac{126}{360}\right) \times 2\pi \times 8$$

$$\overline{AB} = 5.6\pi$$

Q6. (B) The area of shaded sector is $\frac{126}{360}\pi(8)^2$

$= (0.35)\pi(64)$

$= 22.4\pi$

Q7. (C) Because, the triangle is isosceles therefore, the angle B is also p, thus

$$p^\circ + p^\circ + 62^\circ = 180^\circ$$

$$\Rightarrow \quad 2p^\circ = 118 \Rightarrow p = 59$$

Q8. D Since, P is the area, so $P = \pi r^2$, and

W is the perimeter, thus $W = 2\pi r \Rightarrow r = \frac{W}{2\pi}$

$$\Rightarrow \quad P = \pi\left(\frac{W}{2\pi}\right)^2$$

$$P = \pi \frac{W^2}{4\pi^2}$$

$$\Rightarrow \quad P = \frac{W^2}{4\pi}$$

Q9. (B) Since the area of the square is 9, so its each side is 3, and the length of the diagonal will be $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$

Thus area of the circle of radius $2\sqrt{3}$ is

$$A = \pi r^2 \Rightarrow A = \pi(2\sqrt{3})^2 \Rightarrow A = 12\pi$$

Q10. (B) Let the radius of the big circle be r, then its area will be πr^2, also the radius of the semicircle becomes $\frac{r}{2}$, so area of the small circle will be $\pi \frac{r^2}{2}$ and the area of the each semicircle is

$$\left(\pi \frac{r^2}{4}\right) \times \frac{1}{2} = \frac{\pi r^2}{8}$$

Then the area of the four small semicircles is

$$= 4\left(\frac{\pi r^2}{8}\right) = \frac{\pi r^2}{2}$$

So, shaded area = Total area – White area

$$= \pi r^2 - \frac{\pi r^2}{2} = \frac{\pi r^2}{2}$$

Therefore the ratio of shaded area is

$$\frac{\frac{\pi r^2}{2}}{\frac{\pi r^2}{2}} = \frac{1}{1}$$

Q11. (C) Since $C = 2\pi r \Rightarrow C = \pi(2r)$, but $2r = d$

Hence $C = \pi d \Rightarrow \pi = \frac{C}{d} \Rightarrow \frac{C}{r} = 2\pi$

$$\Rightarrow \frac{C}{r} = 2\left(\frac{22}{7}\right) > 6$$

Q12. (B) As $C = 2\pi r$ and $A = \pi r^2$, so

$$\frac{C}{A} = \frac{2\pi r}{\pi r^2} = \frac{2}{r}$$

and $\frac{A}{C} = \frac{\pi r^2}{2\pi r} = \frac{r}{2}$

Thus $\frac{C}{A} = \frac{A}{C}$, only possible, when $r = 2$

Q13. (B) Because the triangles are equilateral, then the white central angles each measure $60°$, so their sum = $60 + 60 = 120$. Then, the unshaded area is $\frac{120}{360} = \frac{1}{3}$ of the circle, so the shaded area of $\frac{2}{3}$ of the circle.

As the area of the circle $= \pi r^2 = \pi(2)^2 = 4\pi$

and the area of the shaded region $= \frac{2}{3} \times 4\pi = \frac{8}{3}\pi$

Q14. (D) Since $QS = 16$, thus the diameter of each circle is 8, and radius of each circle is 4.

$\therefore$ The area of each circle $= \pi r^2 = \pi(4)^2 = 16\pi$

Thus, the area of four circles $= 4(16\pi) = 64\pi$

Now, the area of the square $= 16 \times 16 = 256$

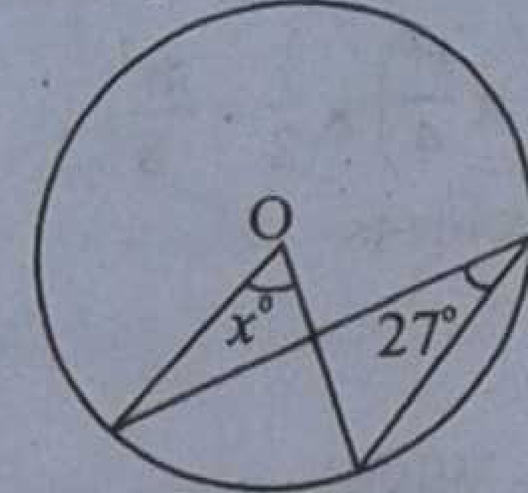
Area of the shaded region = Area of the square – Area of the circle

$$= 256 - 64\pi$$
$$= 64(4 - \pi)$$

Q15. (A) Since the radius of the large circle is 4, and diameter of the small circle is 4, so each side of the square is $4 + 4 + 4 = 12$, so area of the rectangle $= 12 \times 12 = 144$

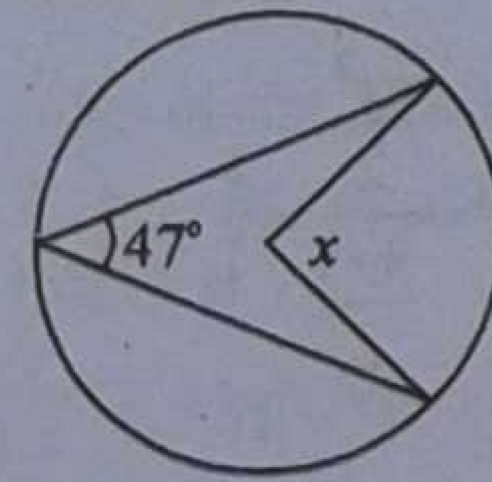
MULTIPLE CHOICE QUESTIONS (MCQs)

Q1. In the following figure, the value of the pronumeral is:



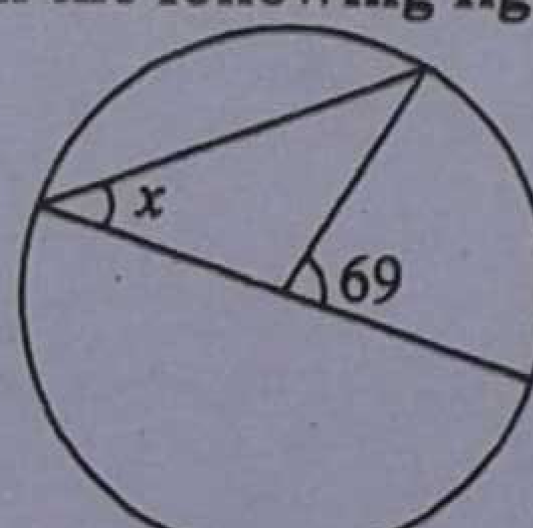
a) 63°
b) 27°
c) 90°
d) 54°

Q2. In the following figure, what is the value of x?



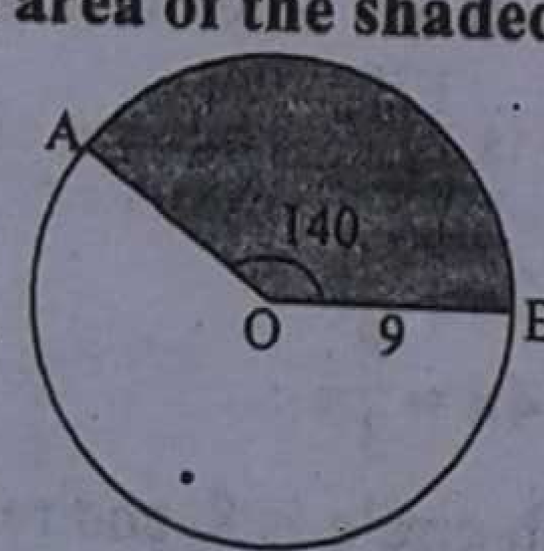
a) 43
b) 94
c) 137
d) 90

Q3. What is the value of pronumeral, in the following figure?



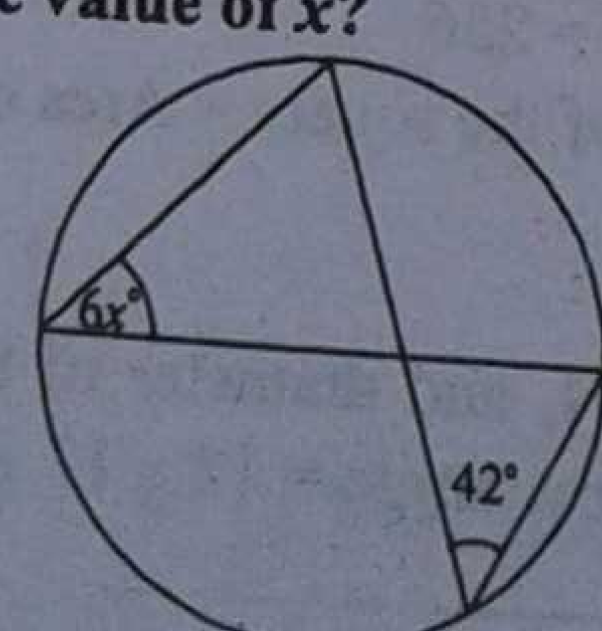
a) 69
b) 34.5
c) 60
d) 111

Q4. In the following figure, what is the area of the shaded region?



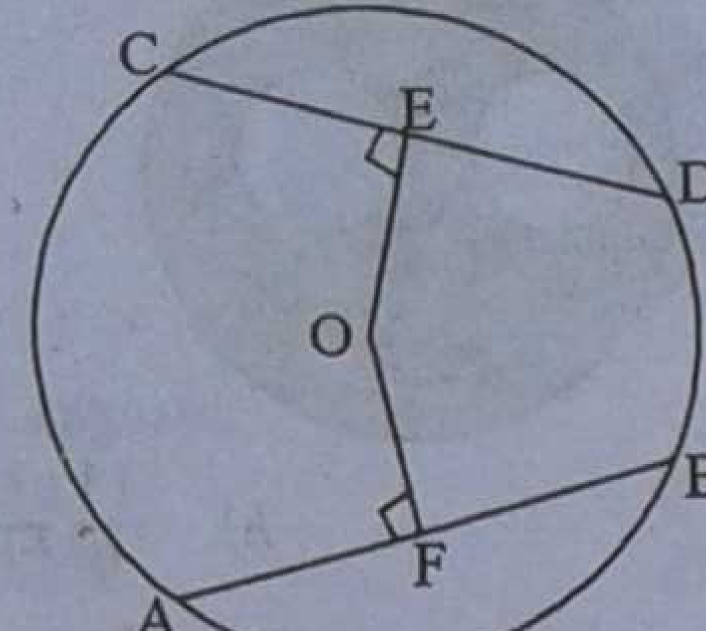
a) 31.5 π
b) 15.56 π
c) 24.4 π
d) 59 π

Q5. In the following figure, what is the value of x?



a) 6
b) 7
c) 15
d) $\frac{1}{42}$

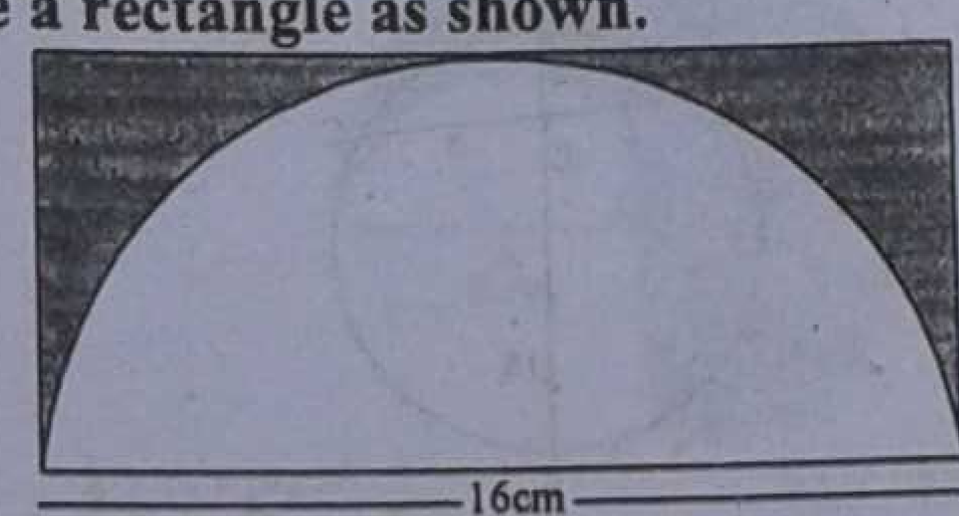
Q6. **In the following figure, if AB = CD and OE = 2.5, what is the value of OF?**



a) 6.25 *b)* 5
c) 1.58 *d)* 2.5

Q7. **A semicircle is drawn inside a rectangle as shown.**

!!)



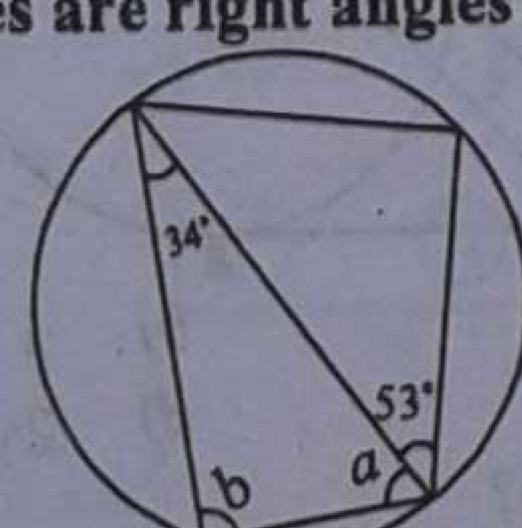
The shaded area is closest to:

a) 50 *b)* 40
c) 30 *d)* 45

Q8. **What is the circumference of the earth at the equator if the diameter is taken as 1.27×10^4 km?**

a) 39878 km *b)* 12700 km
c) 25400 km *d)* 79756 km

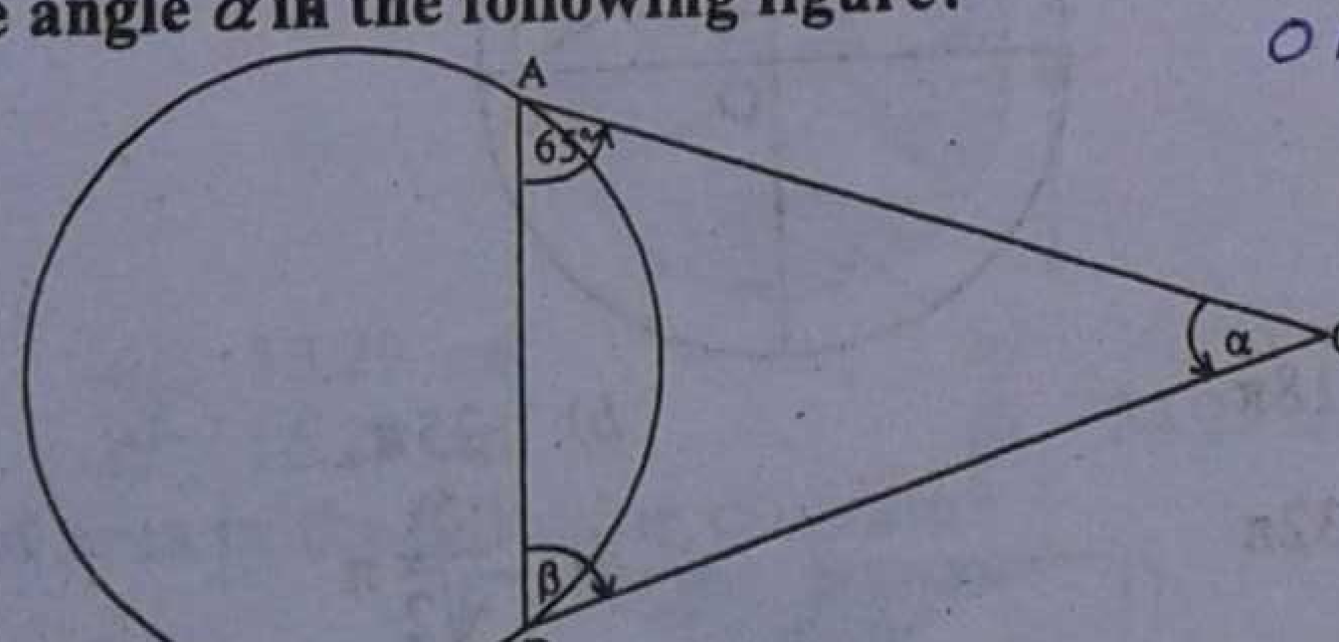
Q9. **In the following figure, which angles are right angles?**



a) *a* *b)* *b*
c) *a* and *b* *d)* none of these

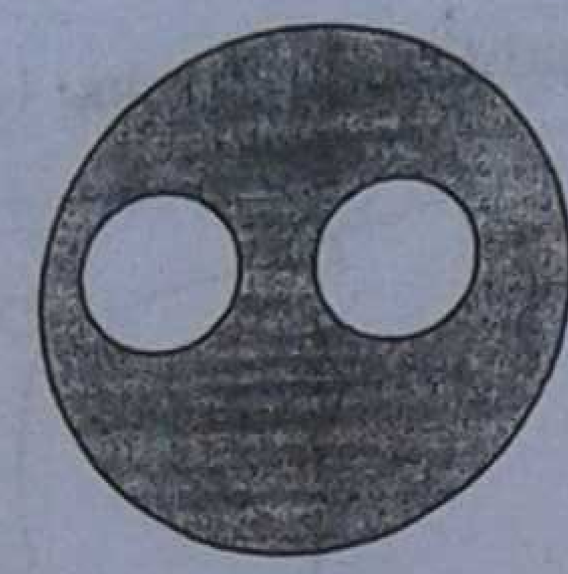
Q10. **What is the size of the angle α in the following figure?**

OA = OB equal tangets as from the same Point O



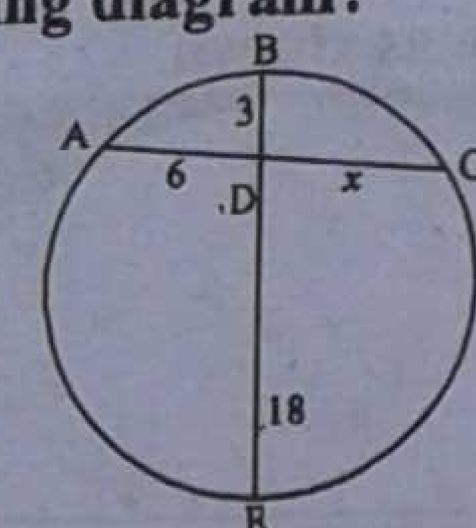
a) 25° *b)* 50°
c) 115° *d)* 90°

Q11. **In the following figure, if the radius of the outer circle is p and the radius of each of the circles inside the larger circle is $\frac{p}{3}$, then what is the area of the shaded region?**



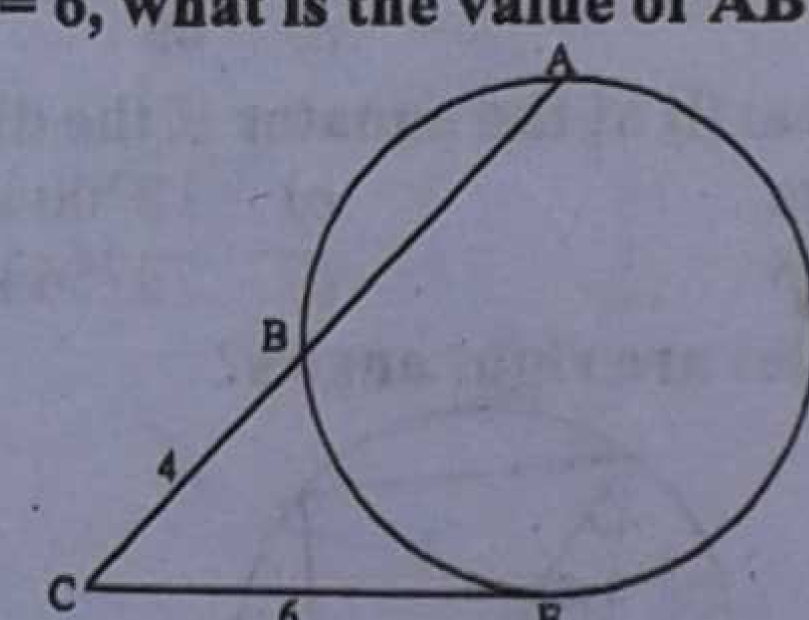
a) $\frac{2}{9}\pi p^2$
b) $\frac{11}{9}\pi p^2$
c) $\frac{22}{9}\pi p^2$
d) $\frac{7}{9}\pi p^2$

Q12. **What is the value of x in the following diagram?**



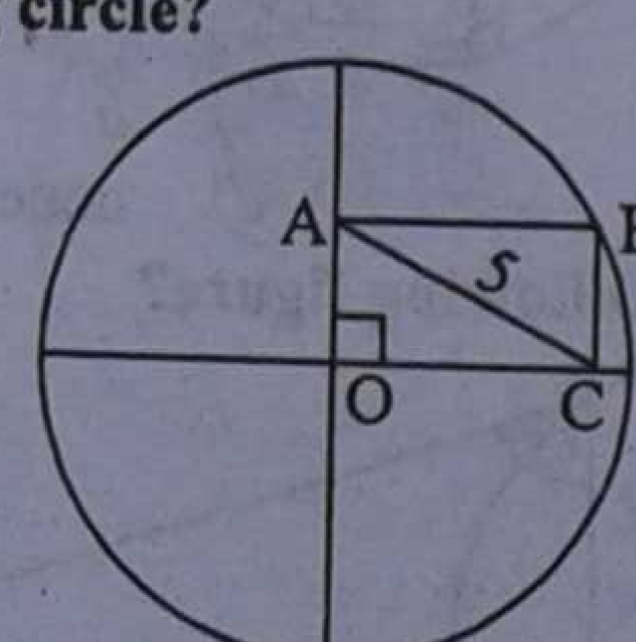
a) 9
b) 3
c) 6
d) 12

Q13. **In the figure, CB = 4 and CE = 6, what is the value of AB:**



a) 9
b) 5
c) 2
d) $2\sqrt{5}$

Q14. **What is the area of the following circle?**

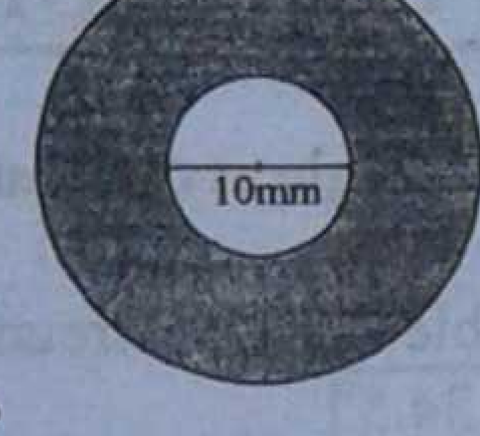


a) 18π
b) 25π
c) 32π
d) $\frac{5}{\sqrt{2}}\pi$

Q15. **A circle is inscribed in a square of area $\sqrt{6}$. What is the area of the circle?**

a) $\frac{3}{2}\pi$
b) 6π
c) π
d) 9π

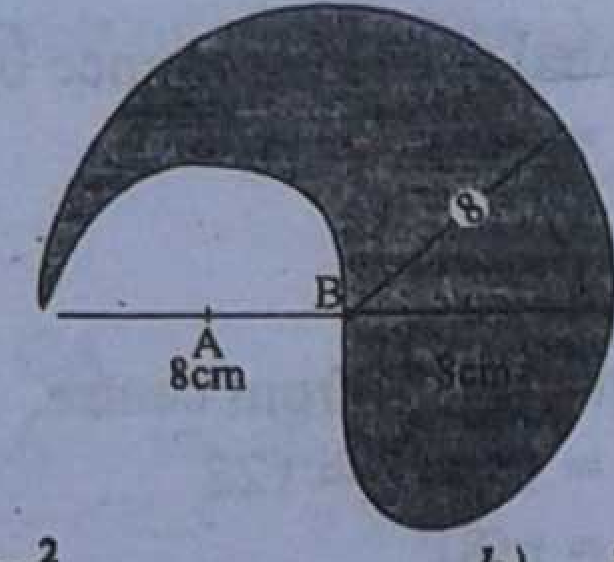
Q16. **A circle of radius 5 mm is removed from the centre of a circular piece of metal of radius 7 mm to make a washer as shown below:**



What is the area of the shaded region?

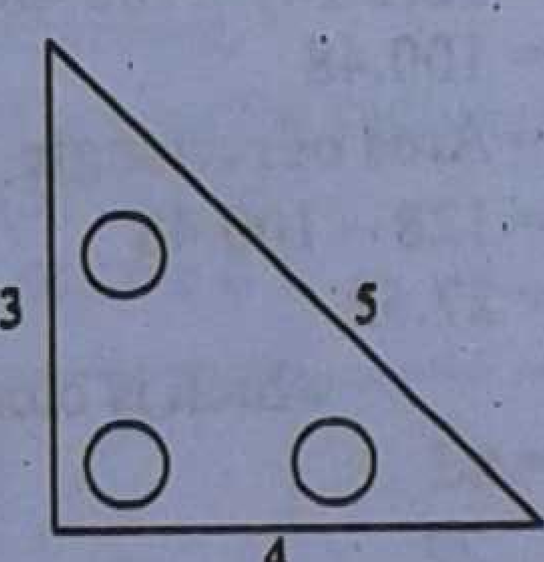
a) 25π b) 49π
c) $35\pi^2$ d) 24π

Q17. **In the following metal cam, A, B and C are the centres of the semicircles shown. What is the area of the cam?**



a) 32.07 cm^2 b) 157.08 cm^2
c) 101.53 cm^2 d) 201.06 cm^2

Q18. **The sketch below shows a triangular copper plate with sides of 3cm, 4cm and 5cm. It has three small circular holes cut out of it. The radius of each circle is 3mm. What is the area of the copper triangle?**

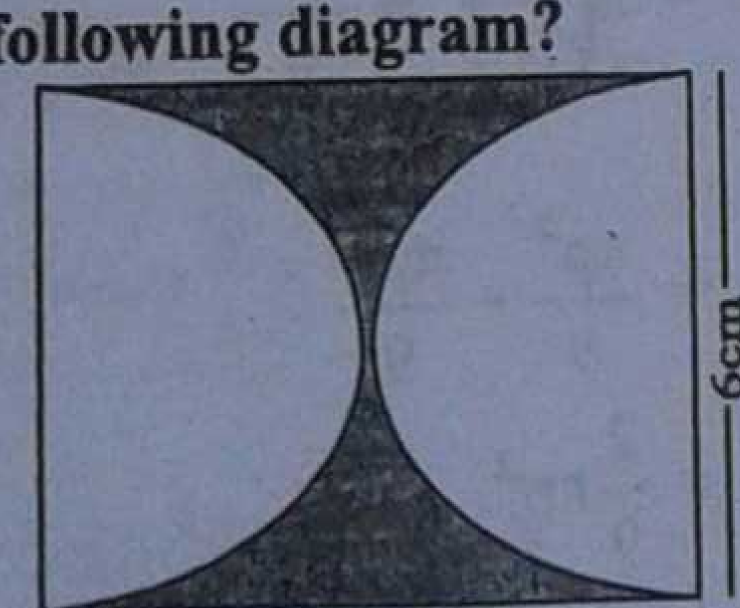


a) 6 cm^2 b) 5.16 cm^2
c) 5.9973 cm^2 d) 5.71 cm^2

Q19. **What is the shaded area in the following diagram?**

a) 50.29 cm^2 b) 16 cm^2
c) 16.29 cm^2 d) 34.29 cm^2

Q20. **What is the shaded area in the following diagram?**



a) 36 cm^2 b) 7.72 cm^2
c) 28.3 cm^2 d) 14.14 cm^2

EXPLANATORY ANSWERS

Q1. (d) $\boxed{x = 54^\circ}$

Because angle at centre is twice angle at circumference.

Q2. (b) $\boxed{x = 94^\circ}$

Because angle at centre is double angle at circumference.

Q3. (b) $\boxed{x = 34.5^\circ}$

Because angle at centre is double angle at circumference.

Q4. (a) The area of the shaded region is

$$\left(\frac{140}{360}\right)\pi(9)^2 = \boxed{31.5\pi}$$

Q5. (b) Since angles subtended at circumference by same arc are same

$\therefore$ $6x = 42$

$\Rightarrow$ $\boxed{x = 7}$

Q6. (d) $\boxed{OF = 2.5}$

Because equal chords are equidistant from centre.

Q7. (c) Area of the rectangle $= 16 \times 8 = 128$

Area of the circle of radius 8cm $= \pi(8)^2$

$= 64\pi$

Area of the semicircle $= \frac{64}{2}\pi = 32\pi$

$= 32(3.14) = 100.48$

$= 100.48$

Area of the shaded region = Area of rectangle – Area of the semicircle

$= 128 - 100.48$

$= 27.52$

which is closest to 30

Q8. (a) Circumference $= \pi d$

$= \frac{22}{7} \times 1.27 \times 10^4$

$= 3.1428 \times 1.27 \times 10^4$

$= 39878$ km

Q9. (b)

Q10. (b)

$\therefore$ OA = OB Because equal tangents from O

$\angle B = 65^\circ$ isosceles triangle

$\angle a + 65^\circ + 65^\circ = 180^\circ$

$\boxed{\angle a = 50^\circ}$

Q11. (d) Area of the outer circle $= \pi(p)^2 = \pi p^2$

Area of the inner circle $= \pi\left(\frac{p}{3}\right)^2 = \frac{\pi p^2}{9}$

Total area of the inner circles

$= \frac{\pi p^2}{9} + \frac{\pi p^2}{9}$

$= \frac{2}{9}\pi p^2$

Area of the shaded region $= \pi p^2 - \frac{2\pi p^2}{9}$

$= \frac{9\pi p^2 - 2\pi p^2}{9}$

$\boxed{= \frac{7}{9}\pi p^2}$

Q12. (a) Given

AD = 6   BD = 3   DE = 18

CD = x

Since the products of intercepts of intersecting chords of a circle are equal.

$\therefore$ AD . CD = BD . DE

$\Rightarrow$ $6 \times x = 3 \times 18$

$\Rightarrow$ $\boxed{x = 9}$

Q13. (b)

$(CE)^2 = CB \cdot CA$

$(6)^2 = 4 \cdot CA \Rightarrow 4CA = 36$

$CA = 9$

Now $AB = CA - BC$

$= 9 - 4 = 5$

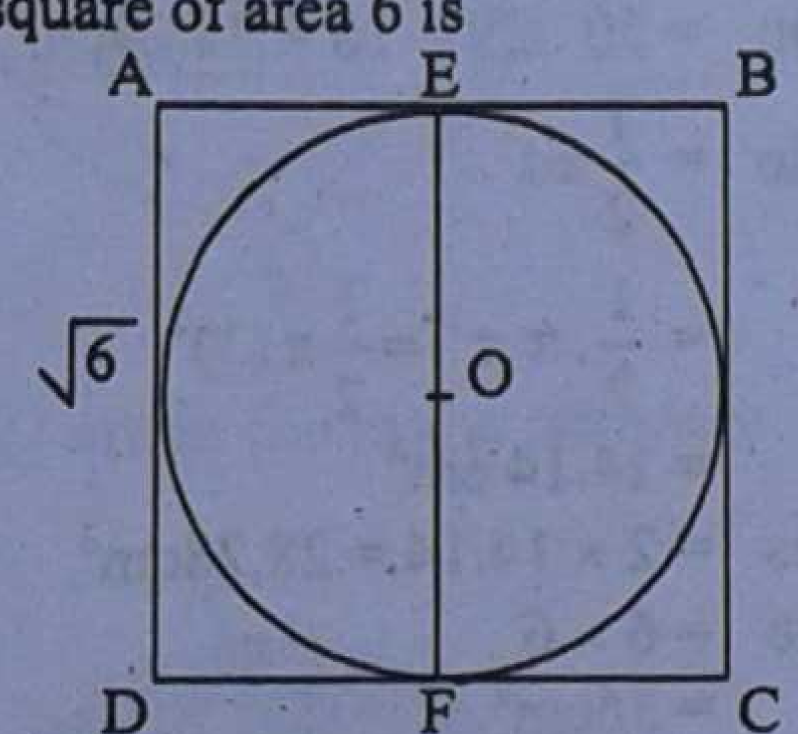
$\Rightarrow$ $\boxed{AB = 5}$

Q14. (b) In the figure OABC is a rectangle and AC = 5 is the diagonal of the rectangle. Since the diagonals of a rectangle are congruent.

$\therefore$ AC = OB = 5, But OB is also the radius of the circle

$\therefore$ Area $= \pi r^2 = \pi(5)^2 = 25\pi$

Q15. (a) The inscribed circle in a square of area 6 is



The side AD $= \sqrt{6}$ and also the diameter $= \sqrt{6}$. The radius of the circle O is OF $= \frac{\sqrt{6}}{2} \Rightarrow$ OF $= \sqrt{\frac{3}{2}}$

The area of the circle of radius $\sqrt{\frac{3}{2}}$ is

Area $= \pi\left(\sqrt{\frac{3}{2}}\right)^2 \Rightarrow \boxed{\text{Area} = \frac{3}{2}\pi}$

Q16. (d)

Area of the washer removed $= \pi r^2 = \pi(5)^2 = 25\pi$

Area of the metal $= \pi r^2 = \pi(7)^2 = 49\pi$

Area of the shaded region $= 49\pi - 25\pi = 24\pi$

Q17. (c) When we shift the semicircle of diameter 8cm in the space the shape becomes semicircle of diameter 16cm

$\therefore$ Area $= \pi r^2$

$= \pi \times (8)^2 = 64\pi$

Area of semicircle $= \frac{64\pi}{2} = 32\pi = \boxed{101.53cm^2}$

Q18. (b) Area of the triangle $= \frac{1}{2} \times \text{base} \times \text{Altitude}$

$= \frac{1}{2} \times 4 \times 3 = 6cm^2$

Now 3mm $= 0.3cm$

Area of the circle $= \pi r^2 = 0.28cm^2$

Area of 3 circles $= 3 \times 0.28 = 0.84cm^2$

Area of copper in plate $= 6 - 0.84 = \boxed{5.16cm^2}$

Q19. (d) Diameter of the circle $= 8cm$

Radius $= 4cm$

Area of the circle $= \pi r^2 = \frac{22}{7} \times 16 = 50.29cm^2$

Area of triangle $= \frac{1}{2} \text{bases} \times \text{altitude}$

$= \frac{1}{2} \times 4 \times 4$

$= 8cm^2$

Area of 2 triangles $= 2 \times 8 = 16cm^2$

Area of the shaded region $= 50.29 - 16 = \boxed{34.29cm^2}$

Q20. (b) Area of the semicircle $= \frac{1}{2} \pi d$

$= \frac{1}{2} . \pi . r^2 = \frac{1}{2} \pi (3)^2$

$= 14.14\ cm^2$

Area of the 2 same semicircles $= 2 \times 14.14 = 28.28cm^2$

Area of the square $= 6 \times 6$

$= 36cm^2$

Area of the shaded region $= 36 - 28.28 = \boxed{7.72cm^2}$

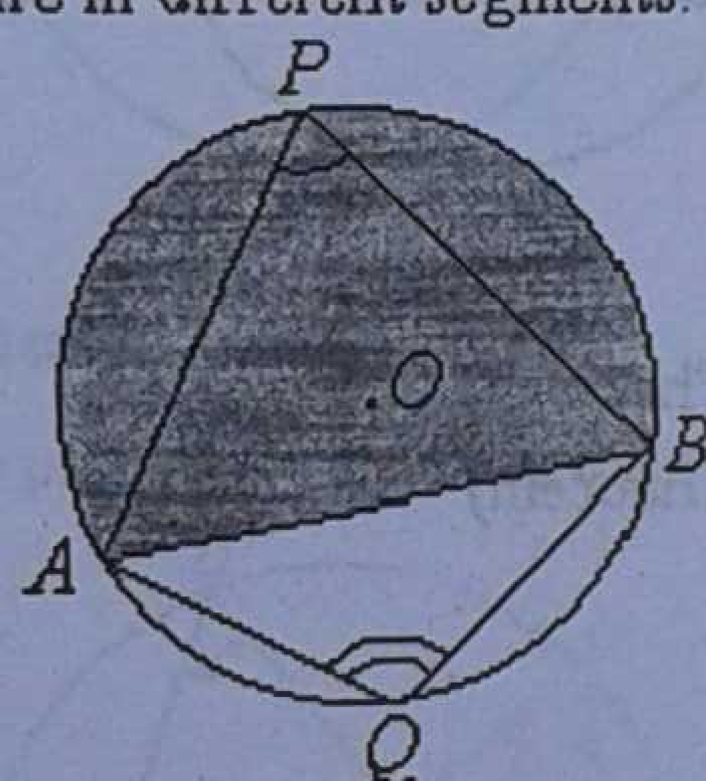
****

Angles in a Segment of a Circle

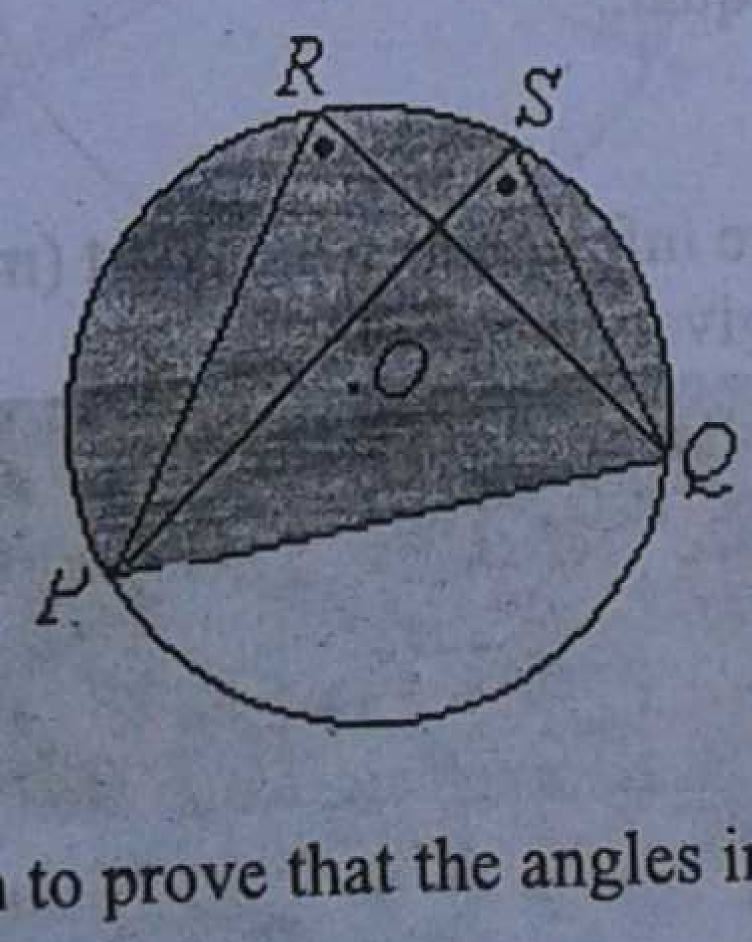
A **chord** of a circle divides the circle into two regions, which are called the **segments** of the circle. The **minor segment** is the region bounded by the chord and the minor arc intercepted by the chord.

The **major segment** is the region bounded by the chord and the major arc intercepted by the chord.

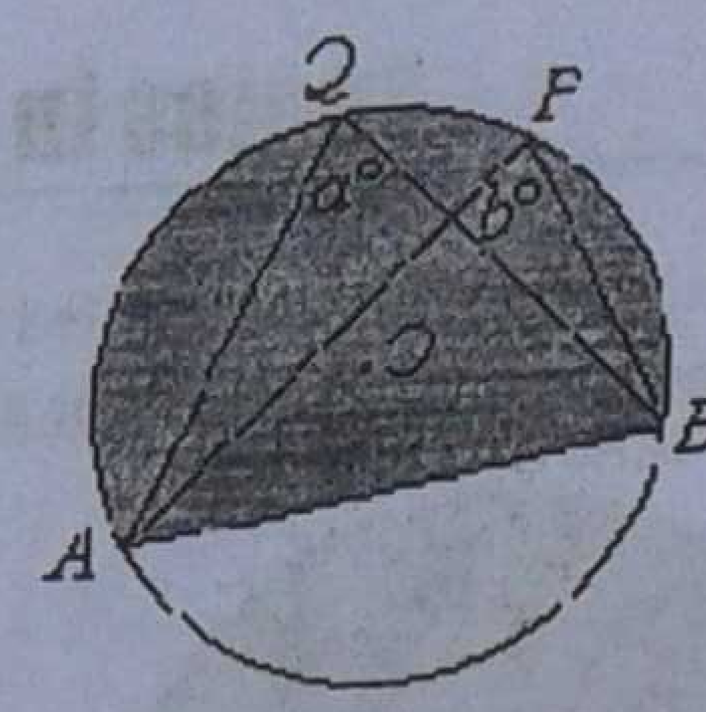
Angles in Different Segments
In the following diagram, $\angle APB$ is in the major segment and $\angle AQB$ is in the minor segment. So, we say that angle APB and angle AQB are in different segments.



Angles in the Same Segment
In the following diagram, $\angle PRQ$ and $\angle PSQ$ are in the major segment. So, we say that angle PRQ and angle PSQ are in the same segment.



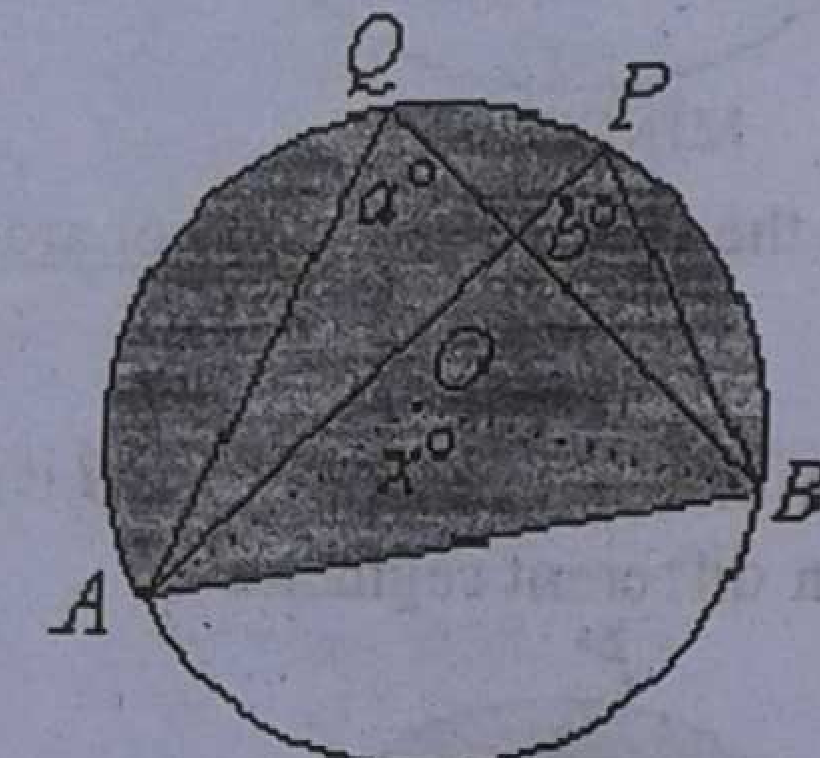
Theorem
Use the information given in the diagram to prove that the angles in the same segment of a circle are equal. That is, $a = b$.



Given:
$\angle APB$ and $\angle AQB$ are in the same segment; and O is the centre of the circle.

To prove:
$\angle APB = \angle AQB$

Construction:
Join O to A and B.



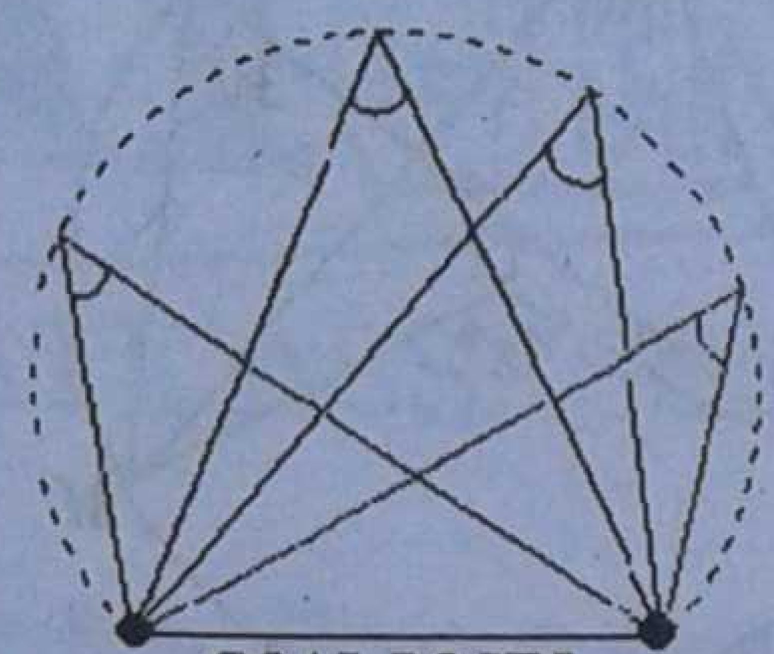
Proof:
Let $\angle AOB = x^\circ$.

Clearly, $x = 2a$ (Angle at Centre Theorem)
$x = 2b$ (Angle at Centre Theorem)
$\therefore 2a = 2b$ (Transitive)
$\therefore a = b$

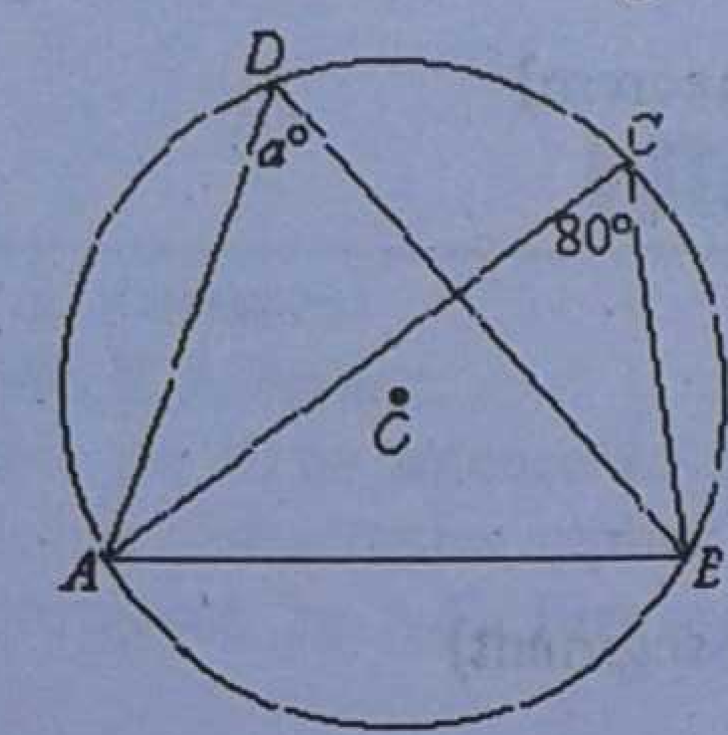
As required.

In general:
Angles in the same segment of a circle are equal.
Practical applications
Danger Angle
If there are rocks near the shore, then boats are informed by the chart (map) to keep the angle subtended by two land marks, A and B, smaller than the given danger angle.

Angle for Scoring a Goal in Soccer: All positions on the same arc of a circle give the same angle for scoring a goal in soccer. Note that the distance of the shot changes but the angle of possible shots remains constant.



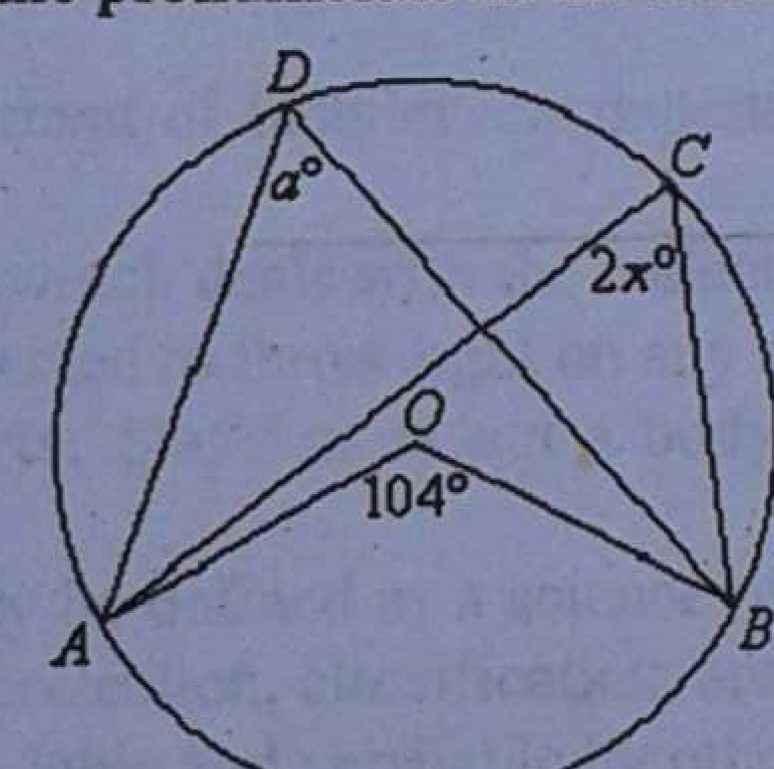
Example 25: Find the value of the pronumeral in the following circle centred at *O*.



Solution:

$a = 80$ (Angles in the same segment)

Example 26: Find the value of each of the pronumerals in the following circle centred at *O*.



Solution:

$2a = 104$ (Angle at Centre Theorem)

$\frac{2a}{2} = \frac{104}{2}$

$a = 52$

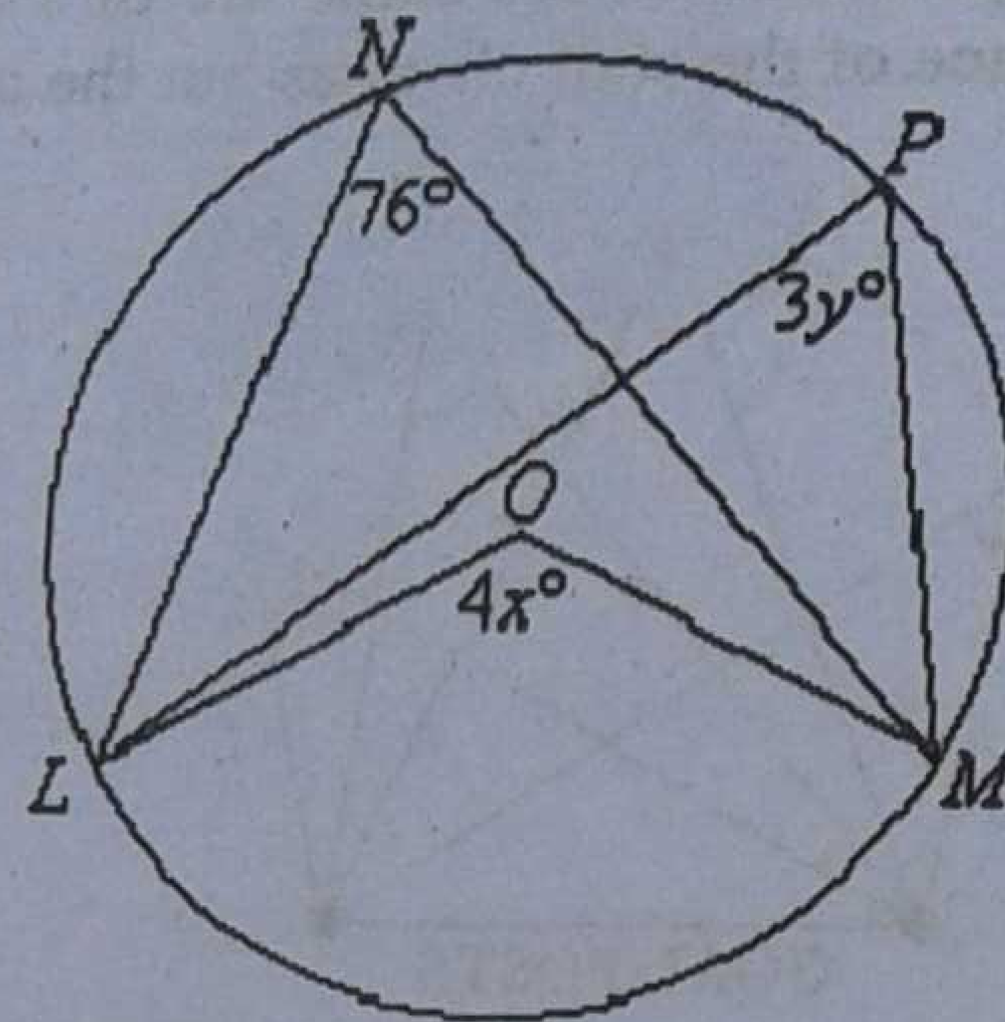
Also, $2x = a$ (Angles in the same segment)

$\therefore 2x = 52$

$\frac{2x}{2} = \frac{52}{2}$

$x = 26$

Example 27: Find the value of each of the pronumerals in the following circle centred at O:



Solution:

$4x = 2 \times 76$ {Angle at Centre Theorem}

$4x = 152$

$\therefore \frac{4x}{4} = \frac{152}{4}$

$x = 38$

Also, $3y = 76$ {Angles in the same segment}

$\therefore \frac{3y}{3} = \frac{76}{3}$

$y = 25\frac{1}{3}$