Lecture 16 Exercises: Auctions and Other Mechanisms

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Unit V: Mechanism Design

Ex 16.1: Room allocation

Three Stanford undergraduates, Alex Vickrey, Billy Clarke, and Casey Groves have grown tired of the modest housing options available on Stanford campus. By asking very nicely, they convince their parents to buy a 3-bedroom apartment off-campus for them to live in.

The students now face the problem of how to allocate the 3 bedrooms amongst themselves. Let's call the rooms Room 1, Room 2, and Room 3. We will use the following notation to describe the allocations: {A,B,C} means Alex gets Room 1, Billy gets Room 2, and Casey gets Room 3. $v_A(\{A, B, C\})$ is the value Alex gets from the allocation {A,B,C} and so forth.

Each student has a different valuation of the potential bedroom allocations, and these valuations are **private information**. In particular, Room 1 is bigger than Room 2 and Room 3, so the students generally attach the highest valuation to Room 1. However, each student cares not only about which room he or she gets, but about the rooms the other students get. This is because Casey suffers from night terrors, occasionally waking up screaming in the middle of the night. Hence, Alex and Billy prefer rooms further away from Casey, all else equal.

In the matrix below, each column shows the valuations of every student (in hundreds of dollars) for a given room allocation. For example, column 1 says that if the allocation is {A,B,C}, Alex gets a value of \$800, Billy gets a value of \$300, and Casey gets a value of \$500.

	{A,B,C}	{A,C,B}	{B,A,C}	{B,C,A}	{C,A,B}	{C,B,A}
v_A	8	7	5	5	5	6
v_B	3	3	9	5	7	3
v_C	5	5	5	5	8	8

(a) Which room allocation maximizes total welfare?

(b) Suppose Alex were to say he didn't care who got which room: that is, he declared his valuation to be $\phi = [0, 0, 0, 0, 0, 0]$. Would that change which room maximized total welfare? In other words, is Alex **pivotal**?

(c) Repeat part (b) for Billy and Casey. Are they pivotal?

(d) The Vickrey-Clarke-Groves pivot mechanism in this setting states that given the three students' vectors of reported valuations, the outcome that maximizes reported value will be chosen, and each player n should make a payment equal to the sum of all the other players' values from the chosen outcome, minus the sum of all the other players' values from the outcome that would be chosen if player n professed no preference: mathematically,

$$p_n(k^*(v), v_{-n}) = \sum_{j \neq n} v_j(k^*(v)) - \sum_{j \neq n} v_j(k^*(\phi, v_{-n}))$$

In practice, this means that for each of the players that's pivotal, you should find the difference between the value to the other players from the outcome you found in part (a), and the value those other players would get from the outcomes you found in parts (b) and/or (c). (For non-pivotal players, the two terms are identical since the outcome doesn't change as a result of their report; so their payment is zero.)

What transfers would this mechanism entail?

(e) Verify that Casey has no incentive to misreport her valuation of any room allocation.