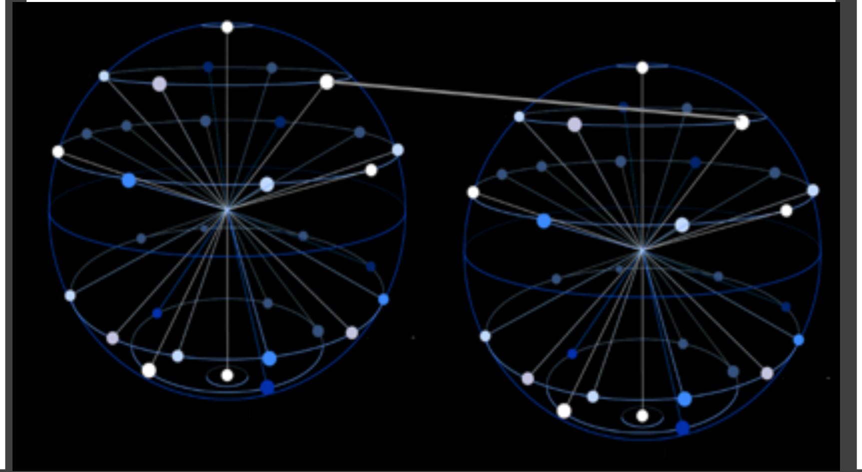

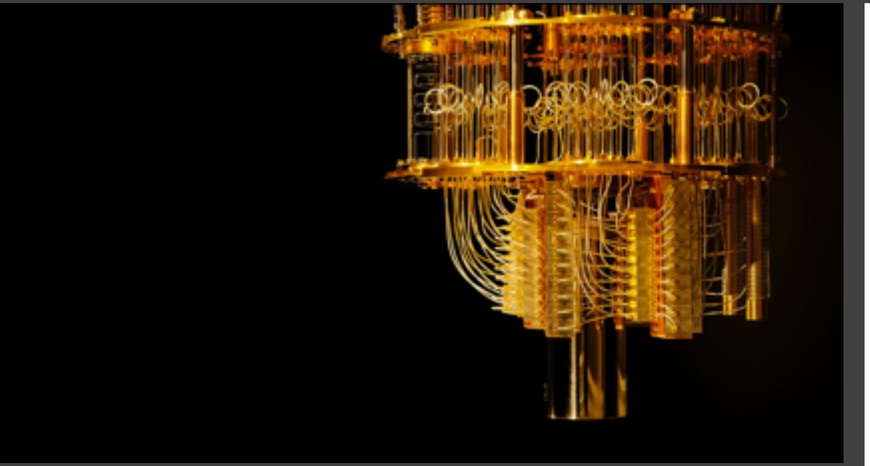

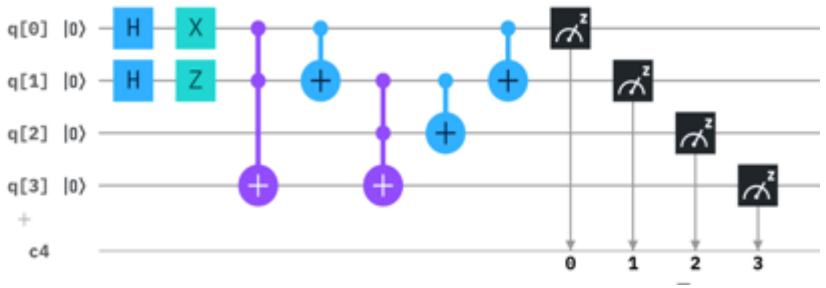

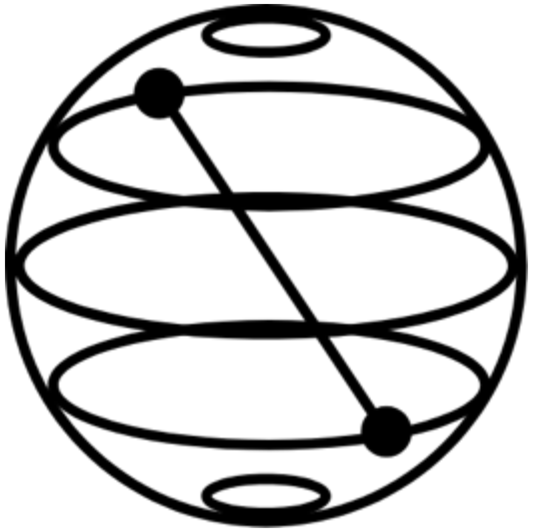

Qubit 101

IBM Client Center Montpellier

JM Torres | torresjm@fr.ibm.com

June 30, 2020

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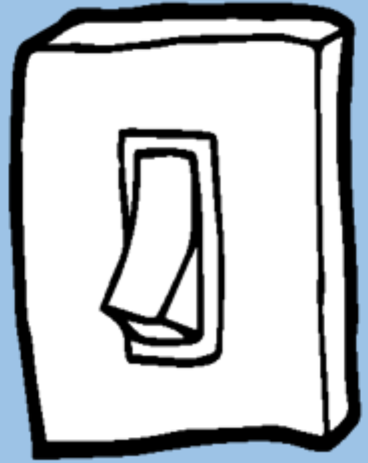

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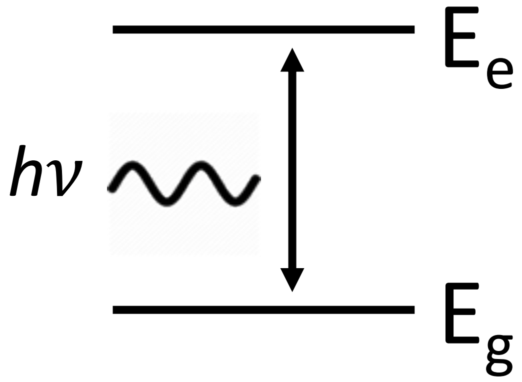
« classical bit »

Definition of a quantum bit



$$|e\rangle \sim |1\rangle$$

$$|g\rangle \sim |0\rangle$$

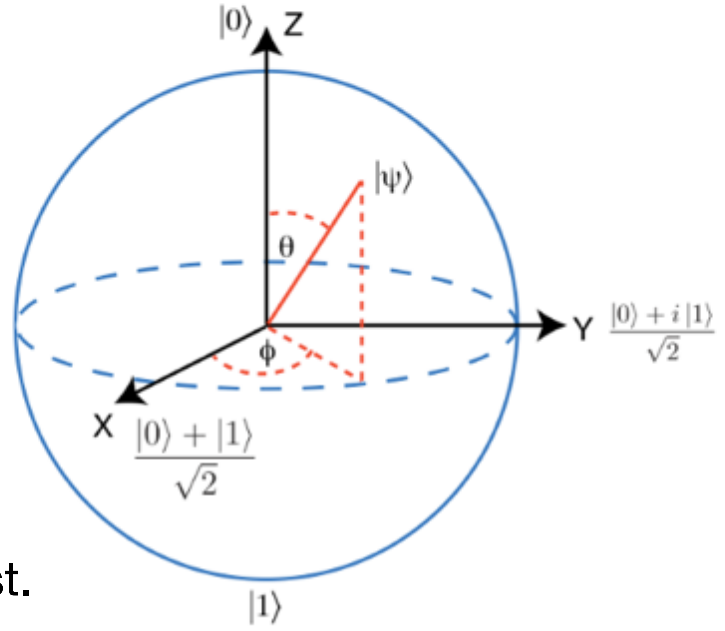
$$|\psi\rangle = \alpha|g\rangle + \beta|e\rangle$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

1. For any possible state:
   the measurement can only result in: $|0\rangle$ **or** $|1\rangle$

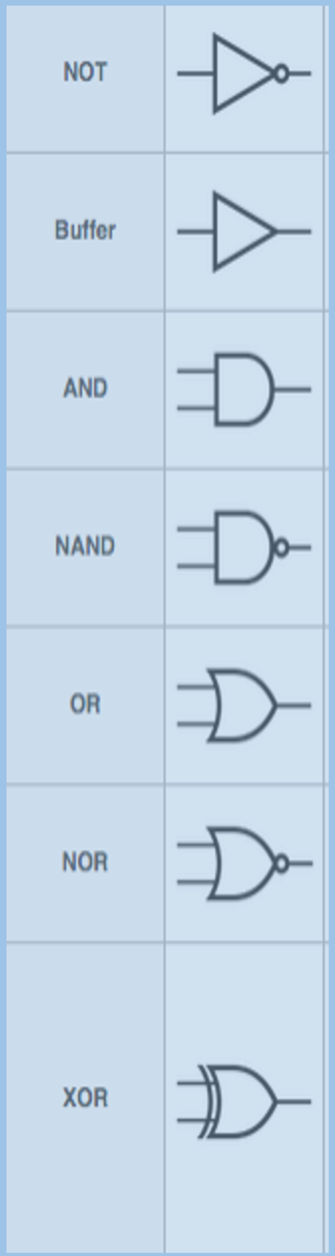
2. Probability of measuring $|0\rangle$ is $|\alpha|^2$,
   probability of measuring $|1\rangle$ is $|\beta|^2$

3. When the measurement is done, the superposition is lost.



The Bloch sphere

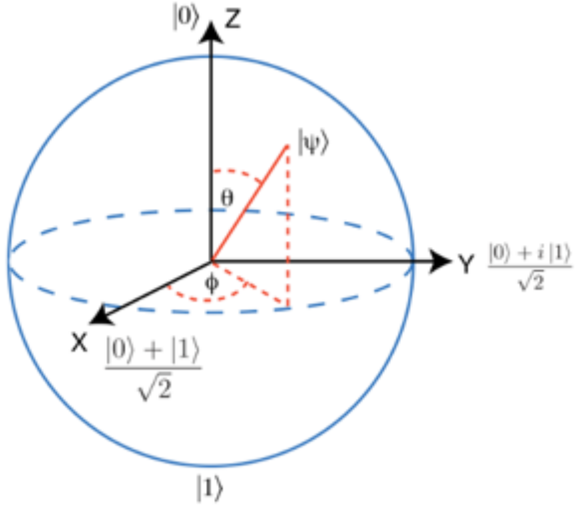
with two qubits : $|\phi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$



Controlling a qubit

« PAULI » Operators

rotation around x axis	X	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	qc.x(qr[n])	R_x	$\begin{pmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$
rotation around y axis	Y	$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	qc.y(qr[n])	R_y	$\begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$
rotation around z axis	Z	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	qc.z(qr[n])	R_z	$\begin{pmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{pmatrix}$
Identity	Id	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	qc.id(qr[n])		

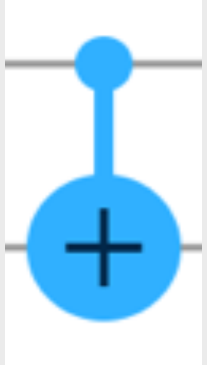


U_1	$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$	U1 gate is known as the phase gate and is essentially the same as Rz(θ).
U_2	$\begin{pmatrix} 1 & -e^{i\lambda} \\ e^{i\phi} & e^{i\lambda+i\phi} \end{pmatrix}$	From this gate, the Hadamard is done by H=U2(0,π)
U_3	$\begin{pmatrix} \cos\frac{\theta}{2} & -e^{i\lambda}\sin\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} & e^{i\lambda+i\phi}\cos\frac{\theta}{2} \end{pmatrix}$	U3 has the effect of rotating a qubit to a state with an arbitrary superposition and relative phase

superposition
(X+Z)
Hadamard gate

H $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ qc.h(qr[n])

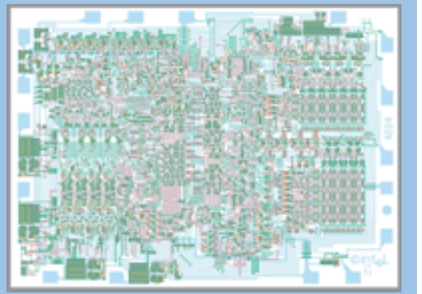
More operators are available from qiskit (S, T, swap, cswap, ccx, cz, …)

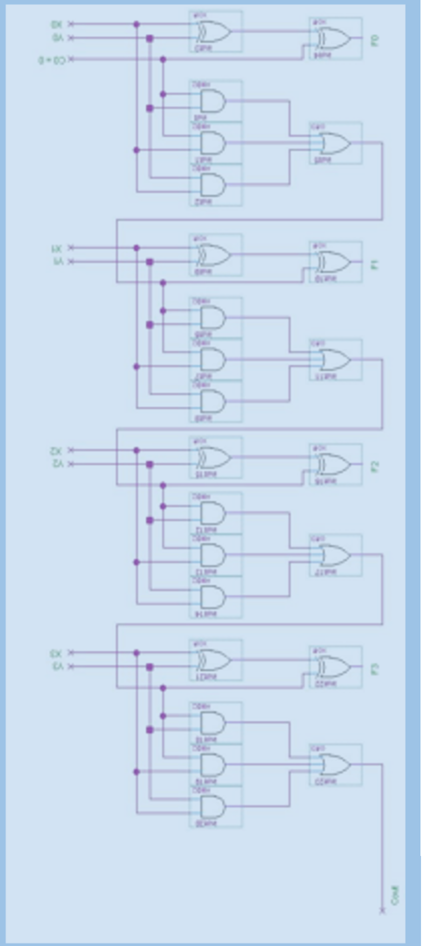


CNOT : flips target qubit according to control qubit state.

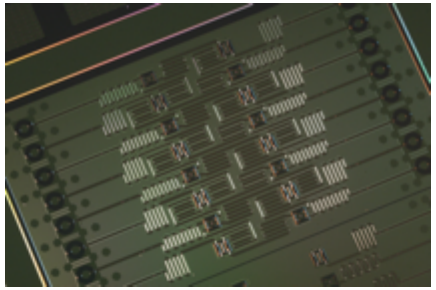
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

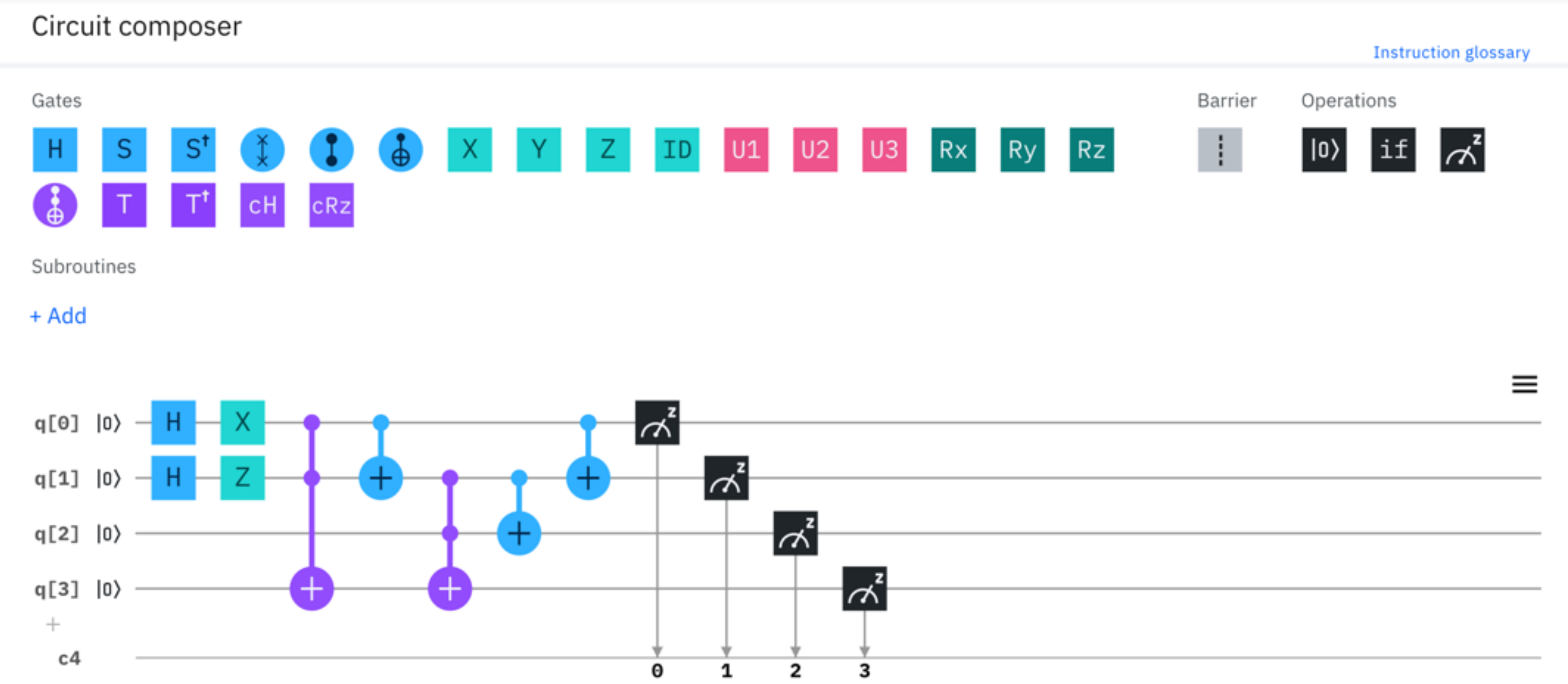
measurement measures quantum state in quantum register into classical register (0/1)





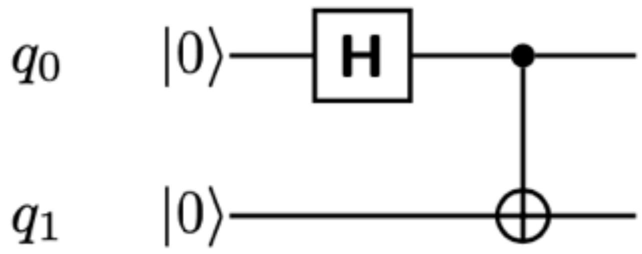
Quantum Circuit





Hello World! example

Hadamard gate applied to q_0, then Control-Not applied to q_1, controlled by q_0



This produces the « Bell-State »

With words :

System starts in $|00\rangle$ (both q_0 and q_1 in state $|0\rangle$).
Then q_0 goes through Hadamard and gets into equal superposition of $|0\rangle$ and $|1\rangle$.
After q_0 controls q_1, the state of q_1 is in a superposition of $|0\rangle$ & $|1\rangle$, (q_1 stays at $|0\rangle$ when q_0 is $|0\rangle$, and q_1 goes $|1\rangle$ when q_0 is $|1\rangle$).
So : both q_0 and q_1 are in $|0\rangle$ (state $|00\rangle$) or both q_0 and q_1 are in $|1\rangle$ (state $|11\rangle$).
Our system is in equal superposition of $|00\rangle$ and $|11\rangle$.
The two qubits are entangled: if you measure one of the qubits, you immediately know the state of the other.

In between :

System starts in $|00\rangle$ state ,then :

$$H|00\rangle = \frac{1}{\sqrt{2}}(|00\rangle+|10\rangle)$$

Applying CNOT: left part of the sum stays as is, right term goes to $|11\rangle$ resulting state is $\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$.

One can easily prove there are no $\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma},\boldsymbol{\delta}$ such that:

$$(\boldsymbol{\alpha}|0\rangle+\boldsymbol{\beta}|1\rangle)\otimes(\boldsymbol{\gamma}|0\rangle+\boldsymbol{\beta}|1\rangle)=\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$$

So, the resulting state is not the product of two quantum states, this is an entangled state.

With maths :

Stage 1 (H on q0) :

$$(H\otimes I)|00\rangle =$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ 1 & 0 & -1 & 0\\ 0 & 1 & 0 & -1\end{pmatrix}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$$

Stage 2: CNOT(0,1)

$$\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix} \times \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$$