Class 5 Homework

1. What's the Hamming distance between $\langle 1, 2, 3 \rangle$ and $\langle 4, 2, 6 \rangle$?

2. You recieve the following transmission,

$$1011$$

and the information that if each bit of the message is a component of a vector in $\mathbb{Z}_2^4$, the vector should also be within the vector space $S = \{a\langle 1, 0, 0, 1 \rangle + b\langle 1, 0, 0, 0 \rangle,\ a, b \in \mathbb{Z}_2\}$. Was there an error in the transmission? If so, what's the smallest possible number of bits that could have been flipped, and what was the intended message?

3. Can you define a k=3 error detecting Hamming code? What about a k=3 error *correcting* Hamming code? What's the difference? Which one contains more vectors? What are the maximum and minimum Hamming distances of each vector space?

4. Explain the difference between a k-error correcting and a k-error detecting Hamming code, in general. What should the minimum Hamming distance be between two vectors in a k-error **correcting** space, in terms of k? Why? If you're feeling inspired, come up with some examples.