Introduction

Mathematical Decision-Optimization (DO) Model $M(\mathbf{w})$ (*):

$$
\begin{aligned}
\mathbf{x}^*(\mathbf{w}) \in \arg\min_{\mathbf{x}\in\mathbb{R}^n, \mathbf{y}\in\mathbb{R}^m} \quad & f(\mathbf{x},\mathbf{y},\mathbf{w}) \\
\text{s.t.} \quad & \mathbf{g}(\mathbf{x},\mathbf{y},\mathbf{w}) \leq \mathbf{0} \\
& \mathbf{y} = \mathbf{h}(\mathbf{x},\mathbf{w}) \\
& \mathbf{x} \in \Omega(\mathbf{w})
\end{aligned}
$$

$\mathbf{w} \in \mathbb{R}^k$ – fixed uncontrollable.
$\Omega(\mathbf{w})$ – polytope (possibly unbounded).

(*) Maragno*, D., Wiberg*, H., Bertsimas, D., Birbil, S. I., Hertog, D. d., and Fajemisin, A. (2021). *Mixed-Integer Optimization with Constraint Learning*.

Introduction

Learned DO Model $\widehat{M}(\mathbf{w})$:

$$\begin{aligned} \widehat{\mathbf{x}}^*(\mathbf{w}) \in \arg\min_{\mathbf{x}\in\mathbb{R}^n, \mathbf{y}\in\mathbb{R}^m} \quad & \widehat{f}(\mathbf{x}, \mathbf{y}, \mathbf{w}) && \Longleftarrow \text{learn} \\ \text{s.t.} \quad & \mathbf{g}(\mathbf{x}, \mathbf{y}, \mathbf{w}) \leq \mathbf{0} \\ & \mathbf{y} = \widehat{\mathbf{h}}(\mathbf{x}, \mathbf{w}) && \Longleftarrow \text{learn} \\ & \mathbf{x} \in \Omega(\mathbf{w}) \end{aligned}$$

$\mathbf{w} \in \mathbb{R}^k$ – fixed uncontrollable.
$\Omega(\mathbf{w})$ – polytope (possibly unbounded).

– UN World Food Programme (INFORMS Edelman Award 2021): Food palatability prediction in food basket cost minimization.
– Louisville Metropolitan Sewer District and Tetra Tech (INFORMS Edelman Award 2019 Finalist): Rainfall prediction in wastewater storage maximization.

Quality-Driven Framework

What We **Want**:

$$\widehat{\mathbf{x}}^* = \widehat{\mathbf{x}}^*(\mathbf{w}), \quad \widehat{\mathbf{y}}^* = \mathbf{h}(\widehat{\mathbf{x}}^*, \mathbf{w})$$
$$\mathbf{x}^* = \mathbf{x}^*(\mathbf{w}), \quad \mathbf{y}^* = \mathbf{h}(\mathbf{x}^*, \mathbf{w})$$

(a) close to optimum

$$f(\widehat{\mathbf{x}}^*, \widehat{\mathbf{y}}^*, \mathbf{w}) - f(\mathbf{x}^*, \mathbf{y}^*, \mathbf{w}) \leq \epsilon$$

(b) feasible

$$\mathbf{g}(\widehat{\mathbf{x}}^*, \widehat{\mathbf{y}}^*, \mathbf{w}) \leq \mathbf{0}$$

Quality-Driven Framework

What We **Can**:

known policy: $\mathbf{x}_0 = \mathbf{x}_0(\mathbf{w}), \mathbf{y}_0 = \mathbf{h}(\mathbf{x}_0, \mathbf{w})$

(a) improve upon policy

$$\Pr[f(\mathbf{x}_0, \mathbf{y}_0, \mathbf{w}) - f(\widehat{\mathbf{x}}^*, \widehat{\mathbf{h}}(\widehat{\mathbf{x}}^*, \mathbf{w}), \mathbf{w}) \geq \epsilon] \geq (1 - \delta_1)$$

(b) likely feasible

$$\Pr[\mathbf{g}(\widehat{\mathbf{x}}^*, \widehat{\mathbf{h}}(\widehat{\mathbf{x}}^*, \mathbf{w}), \mathbf{w}) \leq \mathbf{0}] \geq (1 - \delta_2)$$

Quality-Driven Framework

What We **Can**:

known policy: $\mathbf{x}_0 = \mathbf{x}_0(\mathbf{w}), \mathbf{y}_0 = \mathbf{h}(\mathbf{x}_0, \mathbf{w})$

(a) improve upon policy: Probability of Improvement (PoI)

$$\Pr[f(\mathbf{x}_0, \mathbf{y}_0, \mathbf{w}) - f(\widehat{\mathbf{x}}^*, \widehat{\mathbf{h}}(\widehat{\mathbf{x}}^*, \mathbf{w}), \mathbf{w}) \geq \epsilon] \geq (1 - \delta_1)$$

(b) likely feasible: Probability of Constraint Satisfaction (PoCS)

$$\Pr[\mathbf{g}(\widehat{\mathbf{x}}^*, \widehat{\mathbf{h}}(\widehat{\mathbf{x}}^*, \mathbf{w}), \mathbf{w}) \leq \mathbf{0}] \geq (1 - \delta_2)$$

PoI and PoCS

Gaussian Process: f as a "random variable".

$$f|D \sim GP$$

$\Longrightarrow$ Value @ point $(\mathbf{x}, \mathbf{y}, \mathbf{w})$

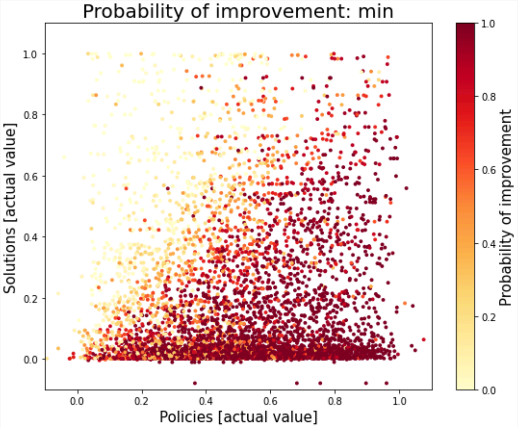
$$f(\mathbf{x}, \mathbf{y}, \mathbf{w}) \sim \mathcal{N}(\mu_{GP}(\mathbf{x}, \mathbf{y}, \mathbf{w}), \sigma^2_{GP}(\mathbf{x}, \mathbf{y}, \mathbf{w}))$$

$\Longrightarrow$ Value @ 2 points $(\mathbf{x}', \mathbf{y}', \mathbf{w}'), (\mathbf{x}'', \mathbf{y}'', \mathbf{w}'')$

$$(f(\mathbf{x}', \mathbf{y}', \mathbf{w}'), f(\mathbf{x}'', \mathbf{y}'', \mathbf{w}'')) \sim \mathcal{N}(\boldsymbol{\mu}_{GP}, \boldsymbol{\Sigma}_{GP})$$

$\Longrightarrow$ PoI and PoCS estimation

PoI and PoCS



PoI of 5593 pairs $(\widehat{\mathbf{x}}^*, \mathbf{x}_0)$ associated with 1718 randomly generated $(\mathbf{f}, \boldsymbol{\Omega}, \mathbf{D}, \mathbf{x}^*)$ in dimensions 2, 3 using **DOFramework**.

FP	2.6%
FN	9.2%

FP: PoI ≥ 0.5, $f(\widehat{\mathbf{x}}^*) \geq f(\mathbf{x}_0)$

FN: PoI ≤ 0.5, $f(\widehat{\mathbf{x}}^*) \leq f(\mathbf{x}_0)$