Regressions with logarithmic data

APEC 3002

BY NATALIA ORDAZ REYNOSO

Exponential Multiplicative Models

- Sometimes we use exponential multiplicative models, such as:
  - $Q = AP^{\alpha}\ e^{\varepsilon}$
- Because that model is not linear in parameters, we cannot run a linear regression directly
- Luckily, logarithms provide us with a way to deal with this. We can transform our model taking logarithms of the model:
- $\ln(Q) = \ln(AP^{\alpha}\ e^{\varepsilon})$, and after applying the properties of logarithms
- It ends up being: $\ln(Q) = \ln(A) + \alpha \ln(p) + \varepsilon$
- Which we can estimate linearly!
- We just need to run a regression on the logarithms of our data

Exponential Multiplicative Models and elasticities

- So in that case, we would run a regression of ln(P) on ln(Q) and get estimates for ln(A) and a
- The estimate for the coefficient of price, a, has an interesting interpretation now: it measures how ln(Q) changes when ln(P) changes
- In the next slide I show that this is equal to the elasticity of Q to P
- If Q is quantity demanded and P is price, then a is an estimate of the price elasticity of demand

Log-log coefficients and elasticities

- We know that a measures how ln(Q) changes when ln(P) changes, in other words:
  - $a = \dfrac{\partial \ln(Q)}{\partial \ln(P)}$
- We also know that $\mathrm{d}\ln(\mathrm{x}) = \dfrac{dx}{x}$
- Which means a=$\dfrac{dQ/Q}{dP/P} = \dfrac{dQ}{dP}\dfrac{P}{Q}$ which is the formula of the elasticity

More generally…

- This means that if you run a regression using logarithmically-transformed variables, you can interpret that variable’s coefficient as the corresponding elasticity
- If your regression’s dependent variable is Quantity demanded, and some of your independent variables are price, and income like this:
  - $\ln(Q) = \alpha + \beta_1 \ln(P) + \beta_2 \ln(Y) + \varepsilon$
- Then you can interpret β_1 as the price elasticity of demand and β_2 as the income elasticity of demand