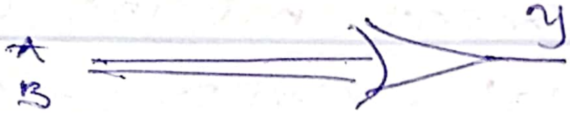
COS 203 28/09/2023

Logical Expression.

Logical gates

Types of Logical gates.

① OR



$y = A+B$

A	B	
1	0	1
0	1	1
1	1	1
0	0	0

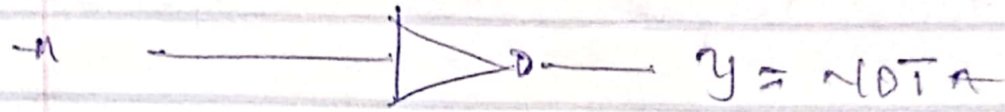
② ~~[illegible]~~

② AND

$y = A \cdot B$

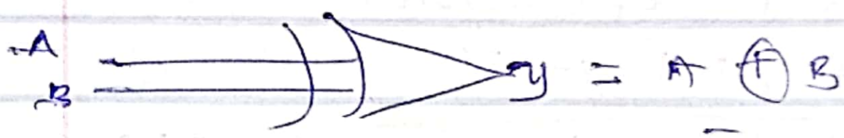
A	B	
0	0	0
1	0	0
0	1	0

(5) NOT / complement.



A	nA
1	0
0	1

(6) Exclusive OR (XOR)



$A \cdot \bar{B} + \bar{A} \cdot B$

compliment means opposite.

A logical expression is a statement that can either be true or false

For example, a<b? is a logical expression. It can either be true or false depending on what the values of a and b are given,

Logical expressions forms basis of computing. Boolean expression is the digital logic used to analyze gates and switching circuits such as AND, OR and NOT gate function.

Boolean functions are implemented by using logic gates. logic gates are used to carry out logical ~~expression~~ operation on single or multiple binary input to give one binary output.

In simple terms, logic gates are the electronic circuit in a digital system.

Types of basic logic gates

There are several basic logic gates used in performing operations in digital system, the common ones are AND, OR, NOT and XOR gates.

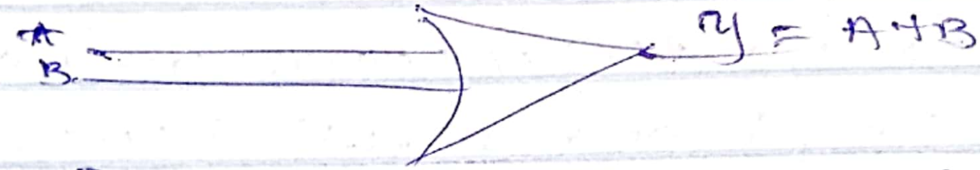
Additionally, these gates can also be found in a combination of one or two. Therefore we get other gates such as NAND, NOR,

XNOR, ~~NOR~~, XNAND.

OR gate

In an OR gate, the output of an OR gate attains state one if one or more inputs attains state one.

A 2 input OR gate.



The boolean expression of OR gate is $y = A + B$,

Truth table

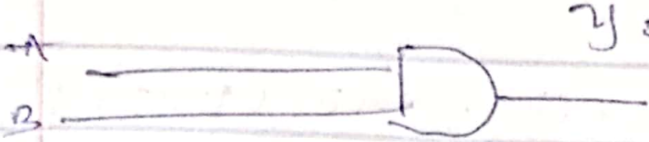
A	B	y
1	1	1
1	0	1
0	1	1
0	0	0

The truth table of a simple 2 input OR table is given as ←

(2) AND gate

In the AND gate, the output of an AND gate attains state 1 if and only if all input are in state one

2 input AND gate



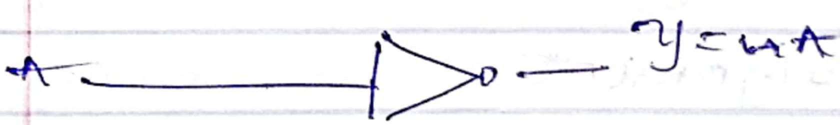
Boolean expression

$Y = A \cdot B$

A	B	Y
1	0	0
1	1	1
0	1	0
0	0	0

(5) NOT gate

In a NOT gate, the output of a NOT gate attains state 1 if and only if the input does not attain state 1.



Boolean $Y = \neg A$ or $\bar{A}$ or A'

A	$\neg A$
1	0
0	1

(4) Exclusive OR (XOR) gate

In an XOR gate, the output of a 2 input XOR gate attains state 1 if one has only input and attains state 1.

A	B	Y
0	1	1
1	1	0
0	0	0
1	0	1

If A or B as an input of 1 alone it is true (1) but when both are true or false it is 0 (zero)

Boolean Express

$y = A \oplus B$ or $A \cdot \bar{B} + \bar{A} \cdot B$

Assignment

Draw the diagram, write the boolean expression and truth tables of the following gates.

- NOR
- NAND
- XNOR
- XNAND
- XOR

Boolean function representation
Sum of product and product of sum.
- A boolean expression is an expression which consists of variable, constants (0 = false, 1 = true) and logical operators which result in a true or false.

A boolean function is an algebraic form of boolean expression.

A boolean function of n variables is represented by $f(x_1, x_2, x_3, \ldots x_n)$.

By using boolean laws and theory we can simplify the boolean functions of a digital circuits.

⇒ Different ways of representing a boolean function are:-

1. Sum of product (SOP) form
2. product of sums (POS) form
3. canonical form.

There are 2 types of canonical forms:-

- Sum of min terms or canonical (SOP)
- product of max terms or canonical (pos).

Boolean expression can be standardized by using these 2 standard forms
- SOP form
- POS form.

Standardization of boolean equations makes the implementation, evolution and simplification easier and more systematic.

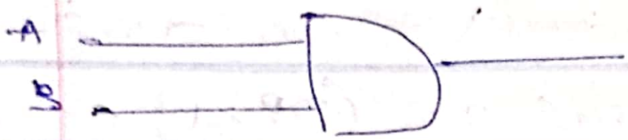
Sum of product form (SOP) :- is a method of simplifying the boolean expression of logic gates. In this boolean function (SOP) form, boolean function representation, the variables are operated by (AND) product to form a product term and all these product terms are (ORed) (sum or added) together to get the final function.

A sum of product form can be formed by adding or summing 2 or more product terms with a boolean addition operation"

i.e the product terms are defined by using the AND operation and the sum term is defined by using OR operation.

In boolean algebra the multiplication of 2 integers is equivalence to the binary AND operation there by producing a product ~~term~~ term when 2 or more input variables are (ANDed) in other words in boolean algebra, the AND function is the equivalence of multiplication and so it output state represent the product of its input.

AND gate (product)



$Q = A \cdot B$

2 inputs AND gate.

Thus the boolean ~~operation~~ equation for a 2 input AND gate is given as

$Q = A \cdot B$

Q = A and B represent AND gate

For a product term, these input variables

* Annulment law:- a term ANDed with zero is equal to 0

$(A \cdot 0 = 0)$.

* Idempotent law:- $(A \cdot A = A)$

$(B \cdot B = B)$

* Compliment law: $A \cdot \bar{A} = 0$

$B \cdot \bar{B} = 0$

* Commutative law:- $A \cdot 1 = 1 \cdot A$

Example

Simplify the boolean Equation:

$F = AB + BC + \bar{B}C$

Solution.

$F = AB + BC + \bar{B}C$

$f = AB + C(B + \bar{B})$

$f = AB +$

A AND B. For a product term these input variables can be either "true" or "false" "1" or "0" or be of a complemented form, so A.B, A'.B, or A.B are all classed as product terms.

The product (AND) Term

In Boolean Algebra, "product" means the ANDing of the terms with the variables in a product term having one instance in its true form or in its complemented form so that the resulting product cannot be simplified further. These are known as minterms. So how can the operation of this "product" function be shown in Boolean Algebra?

A product term can have one or two independent variables such as A and B, or it can have one or two fixed constants, again 0 and 1. These variables and constants can be used in a variety of different combinations and produce a product result as shown in the following lists:

Boolean Algebra Product Terms

Variables	Constants
$A \cdot 0 = 0$	$0 \cdot 0 = 0$
$A \cdot 1 = A$	$0 \cdot 1 = 0$
$A \cdot A = A$	$1 \cdot 0 = 0$
$A \cdot \bar{A} = 0$	$1 \cdot 1 = 1$

Note that a Boolean "variable" can have one of two values, either "1" or "0" and can change its value. For example, A = 0, or A = 1 whereas a Boolean

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"Constant" which can also be in the form of a "1" or "0", is a fixed value and therefore cannot change.

Then from the above, any given Boolean product can be simplified to a single constant or variable a a brief description of the various Boolean Laws given below where "A" represents a variable inp

Annulment Law – A term ANDed with 0 is always equal to 0 $(A \cdot 0 = 0)$

Identity Law – A term ANDed with 1 is always equal to the term $(A \cdot 1 = A)$

Idempotent Law – A term ANDed with itself is always equal to the term $(A \cdot A = A)$

Complement Law – A term ANDed with its complem. is always equal to 0 $(A \cdot \bar{A} = 0)$

Commutative Law – The order in which two terms are ANDed is the same $(A \cdot 1 = 1 \cdot A)$

Examples

$AB + ABC + CDE$

$(AB)' + ABC + C\bar{D}\bar{E}$

SOP form can be obtained by

(1) Writing an AND term for each input combination, which produces HIGH output (2) Writing the input variable

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If the value is 1, and write the Complement of the variables if its value is 0. (3) OR the AND terms to obtain the output function.

Ex: Boolean expression for majority function

$F = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$

Truth Table

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$AB + AC + BC$

Checking

By Idempotence law, ~~we~~ states know that

$([ABC + ABC)] + ABC) = (ABC + ABC) = ABC$

Now the function $F = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$

$= \bar{A}BC + A\bar{B}C + AB\bar{C} + ([ABC + ABC)] + ABC)$

$= (\bar{A}BC + ABC) + (ABC + A\bar{B}C) + (ABC + AB\bar{C})$

$= (\bar{A} + A)BC + A(B + \bar{B})C + AB(C + \bar{C})$

$= BC + AC + AB$

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(2) Simplify the Boolean function

$F = AB + BC + \bar{B}C$

Solution $F = AB + BC + \bar{B}C$

$= AB + C(B + \bar{B})$

$= AB + C$

(3) Simplify the Boolean function $F = A + A'B$

Solution $F = A + A'B$

$= (A + A)(A + B)$

$= A + B$

Exercise

Simplify the following Boolean function and use Truth table or Boolean Law to check your answer

1. $F = \bar{A}\bar{B}C + \bar{A}BC + AB'$
2. $F = AB + (AC)' + AB'C(AB + C)$