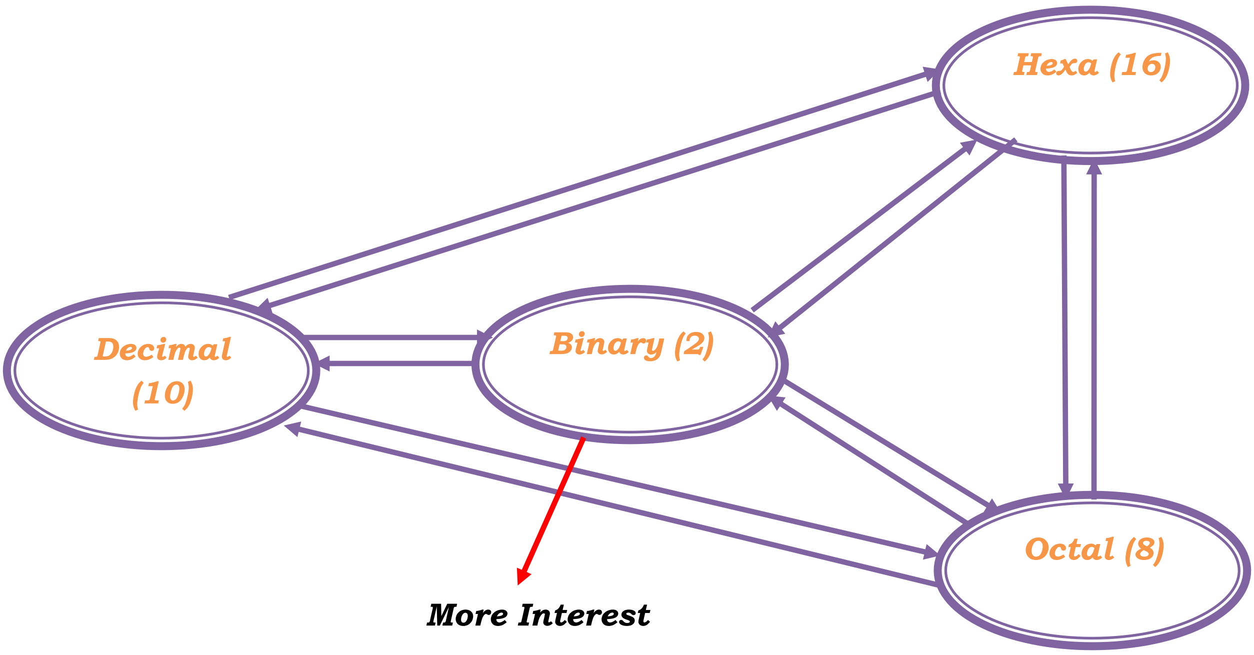
Digital System Design

Lecture 4

Digital Number Systems II

3. Convert a number from one number system to another

Conversion between number bases:



Way we need conversion?

- ✓ We need decimal system for ***real world*** (for presentation and input): for example: we use 10-based numbering system for input and output in digital calculator.
- ✓ We need binary system inside calculator for ***calculation***.

a) *Binary to decimal conversions:*

- ✓ ***Rule***: any binary number can be converted to its decimal equivalent simply by ***summing together the weights of the various positions in the binary number which contains a 1***.

Example 1: Convert 11011_2 to its decimal equivalent

1	1	0	1	1	
↓ +	↓ +	↓ +	↓	↓	$= 16+8+2+1= 27_{10}$
2^4	$+2^3$	0	2^1	2^0	

Example 2: Convert 10110101_2 to decimal equivalent

$2^7 + 0+ 2^5+ 2^4+ 0+ 2^2+ 0+ 2^0 = 181_{10}$

b) *Decimal to binary conversions:*

- ✓ There are two ways to convert a decimal number to its equivalent binary representation

1. ***The reverse of the binary-to-decimal conversion process*** *(optional). The decimal number is simply expressed as a sum of powers of 2 and then 1_2 and 0_2 are written in the appropriate bit positions.*

Example 1:-Convert 45_{10} to binary number

$45_{10} = 32+ 8+ 4+ 1= 2^5+ 0+ 2^3+ 2^2+ 0+ 2^0 =101101_{(2)}$

Example 2:-Convert 76_{10} to binary number

$76_{10}= 64+ 8+ 4 = 2^6+ 2^3+ 2^2 = 1001100_2$

2. ***Repeated division***: *Repeating division the decimal number by* **2** *and writing down the remainder after each division until a quotient of* **0** *is obtained*.

Note:
The binary result is obtained by writing the first remainder as the LSB and the last remainder as the MSB.

General Rule 1: Conversion from decimal to other base

1. Divide decimal number by the base (2, 8, 16,…).
2. The remainder is the lowest-order digit.
3. Repeat first two steps unit no divisor remains.

General Rule 2: Decimal fraction conversion to anther base

1. Multiply decimal number by the base (2, 8,…).
2. The integer is the highest-order digit.
3. Repeat first two steps until fraction becomes zero.

Example 1 Convert 25_{10} to binary number

$\frac{25}{2}$ = 12 + remainder of 1 (LSB)

$\frac{12}{2}$ = 6 + remainder of 0

$\frac{6}{2}$ = 3 + remainder of 0

$\frac{3}{2}$ = 1 + remainder of 1

$\frac{1}{2}$ = 0 + remainder of 1 (MSB)

$25_{10} = 1\ 1\ 0\ 0\ 1_2$

Example 2 Convert 13_{10} to binary number

Division by 2		Quotient integer	remainder
$\frac{13}{2}$	=	6	1 (a_0)
$\frac{6}{2}$		3	0 (a_1)
$\frac{3}{2}$		1	1 (a_2)
$\frac{1}{2}$		0	1 (a_3)
Answer		$(13)_{10} = (a_3\ a_2\ a_1\ a_0) = (1101)_2$	

Example 3: Convert 0.625_{10} to binary number

Multiply by 2	Integer		Fraction	coefficient
0.625*2 =	1	+	0.25	$a_1 = 1$
0.250*2 =	0	+	0.50	$a_2 = 0$
0.500*2 =	1	+	0(stop)	$a_3 = 1$
Answer	$(0.625)_{10} = (0.a_1\ a_2\ a_3)_2 = (0.101)_2$			

Correct order

c) *Octal-to-decimal*

- ✓ To convert, we need to ***multiply each octal digit by its positional weight***.

Example 1

$372_{(8)} = (3*8^2) + (7*8^1) + (2*8^0) = (3*64) + 56+2 = 250_{10}$

Example 2

$24.6_8 = (2*8^1) + (4*8^0) + (6*8^{-1}) = 20.75_{10}$

d) *Decimal to octal*

- ✓ ***Repeated division by 8.***

Example 1: Convert 266_{10} to octal number.

$\frac{266}{8} = 33$ + remainder of 2 (LSD)

$\frac{33}{8} = 4$ + remainder of 1

$\frac{4}{8} = 0$ + remainder of 4

$266_{10} = 4\ 1\ 2_{(8)}$

Example 2: Convert 0.35_{10} to octal number.

Multiply by 8	Integer		Fraction	coefficient
0.35*8=	2	+	0.80	$a_1 = 2$
0.8*8 =	6	+	0.40	$a_2 = 6$
0.4*8 =	3	+	0.20	$a_3 = 3$
0.2*8 =	1	+	0.60	$a_4 = 1$
0.6*8 =	4	+	0.80	$a_5 = 4$
Answer	$(0.35)_{10} = (0.a_1\, a_2\, a_3\, a_4\, a_5)_2 = (0.26314)_8$			

Repeated "stop"

Correct order

e) Hexa-to-decimal

Example 1:-Convert $356_{(16)}$ to decimal:

$356_{(16)}$ =

$(3*16^2) + (5*16^1) + (6*16^0) = 3*256 + 80 + 6 = 854_{(10)}$

Example 2:-Convert $2AF_{(16)}$ to decimal:

$2AF_{(16)}$ =

$(2*16^2) + (10*16^1) + (15*16^0) = 512+160+15 = 687_{(10)}$

f) Decimal-to-hexa:(using repeated division by 16)

Example 1: Convert 423_{10} to hex number.

$\frac{423}{16} = 26$ + remainder of 7 (LSD)

$\frac{26}{16} = 1$ + remainder of 10

$\frac{1}{16} = 0$ + remainder of 1

$423_{10} = 1\ A\ 7_{(16)}$

g) *Hexa-to-binary:*

- ✓ Each hexa digit is converted to its ***four-bit binary equivalent***:

Example 1: Convert $9F2_{(16)}$ to its binary equivalent

9	F	2
↓	↓	↓
1001	1111	0010

$9F2_{(16)} = 100111110010_{(2)}$

Example 2: Convert $BA6_{(16)}$ to binary equivalent

$BA6_{(16)} = (\underline{1011}\ \ \underline{1010}\ \ \underline{0110})_2$

h) *Binary-to-hexa*

- ✓ ***The binary numbers are grouped into groups of four bits*** and each group is converted to its equivalent hexa digit.
- ✓ ***Zeros*** are added as needed to complete a four-bit group.

Example 1: Convert $1110100110_{(2)}$ to hexa equivalent

Solution:

Added zeros →	<u>0011</u>	<u>1010</u>	<u>0110</u>
	↓	↓	↓
	3	A	6

$1110100110_{(2)} = 3A6_{(16)}$

Example 2: Convert 101011111_2 to hexa equivalent

Solution:

$\underline{1}\ \underline{0101}\ \underline{1111}_2 = 15F_{(16)}$

i) *Octal to binary conversion:*

- ✓ Conversion each octal digit to its ***three bit binary equivalent***.

Conversion Table								
Octal digit	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Binary equivalent	000	001	010	011	100	101	110	111

- ✓ Using this table, we can convert any octal number to binary by individually converting each digit.

Example 1: Convert $472_{(8)}$ to binary number

Solution:

$$\begin{array}{ccc} 4 & 7 & 2 \\ \downarrow & \downarrow & \downarrow \\ 100 & 111 & 010 \end{array}$$

$472_{(8)} = 100111010_{(2)}$

Example 2: Convert $5431_{(8)}$ to binary number

Solution:

$\underline{5431_{(8)}} = \underline{101}\ \ \underline{100}\ \ \underline{011}\ \ \underline{001} = 101100011001_{(2)}$

j) Binary to octal conversion:

- ✓ The bits of the binary number are grouped into group of 3 bits starting at the LSB, then each group is converted to its octal equivalent (see table).

Example 1: Convert $11010110_{(2)}$ to octal equivalent

Solution:

$$\boxed{\text{Added zero}} \rightarrow \begin{array}{ccc} \underline{011} & \underline{010} & \underline{110} \\ \downarrow & \downarrow & \downarrow \\ 3 & 2 & 6 \end{array}$$

$11010110_{(2)} = 326_{(8)}$

Note:
Zero was placed to the left of the MSB to produce groups of 3 bits.

General example:

Convert 177_{10} to its eight-bit binary equivalent by first converting to octal.

Solution:

$$\frac{177}{8} = 22 + \text{remainder of } 1 \text{ (LSD)}$$

$$\frac{22}{8} = 2 + \text{remainder of } 6$$

$$\frac{2}{8} = 0 + \text{remainder of } 2$$

$$177_{10} = 2\ 6\ 1_{(8)}$$

- ✓ Thus $177_{10} = 261_{(8)}$, now we can quickly convert this octal number to its binary equivalent **010110001** to get eight bit representation.

  So:

  $177_{10} = 1011000_{(2)}$

***Important Note*: this method of decimal-to-octal-to-binary conversion is often quicker than going directly from decimal to binary, especially for large numbers.**

4. Advantage of octal and hexadecimal systems:

1. Hexa and octal number are used as a "***short hand***" way to represent stings of bits.
2. Error prone to write the binary number, in hex and octal ***less error***.
3. The octal and hexadecimal number systems are both used (***in memory addressing and microprocessor technology***).