Exercise 13-5

Orthonormal matrix 定義

$$\langle e_i, e_j \rangle = \begin{cases} 1 & \text{when } i=j \\ 0 & \text{when } i \neq j \end{cases}$$

(a)

$$A = \begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix} \quad e_1 = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} \quad e_2 = \begin{bmatrix} -\sqrt{3} \\ 1 \end{bmatrix}$$

$e_1 \cdot e_2 = 0 \quad \|e_1\| = \|e_2\| = 2,$

not a orthonormal matrix

(b) $A = \frac{1}{5} \begin{bmatrix} 3 & 4 & 0 \\ -4 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$$e_1 = \frac{1}{5}\begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix} \quad e_2 = \frac{1}{5}\begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix} \quad e_3 = \frac{1}{5}\begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$$

$e_1 \cdot e_2 = e_2 \cdot e_3 = e_1 \cdot e_3 = 0$

$\|e_1\| = \|e_2\| = \|e_3\| = 1$

是 orthonormal matrix

(c) $A = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -\sqrt{2}/6 & \sqrt{2}/6 & 2\sqrt{2}/3 \\ 2/3 & -2/3 & 2/3 \end{bmatrix}$

$$e_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{\sqrt{2}}{6} \\ \frac{2}{3} \end{bmatrix} \quad e_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{\sqrt{2}}{6} \\ -\frac{2}{3} \end{bmatrix} \quad e_3 = \begin{bmatrix} 0 \\ \frac{2\sqrt{2}}{3} \\ \frac{2}{3} \end{bmatrix}$$