Exercise 13 - 1

$$w_{\parallel v} = \text{proj}_v w = \frac{w^T v}{v^T v} v$$

$$w_{\perp v} = w - w_{\parallel v} = w - \frac{w^T v}{v^T v} v$$

$$w_{\perp v} \cdot w_{\parallel v} = \left(w - \frac{w^T v}{v^T v} v\right) \cdot \frac{w^T v}{v^T v} v$$

$$= w \cdot \frac{w^T v}{v^T v} v - \frac{w^T v}{v^T v} v \cdot \frac{w^T v}{v^T v} v$$

$$w_{\parallel v} \cdot w_{\perp v} = \left(\frac{w^T v}{v^T v} v\right)^T \cdot \left(w - \frac{w^T v}{v^T v} v\right)$$

$$= \frac{w^T v}{v^T v} v^T w - \frac{w^T v}{v^T v} \cdot \frac{w^T v}{v^T v} v^T v$$

分配律

scalar

scalar 的分布

$$= \frac{w^T v}{v^T v} v^T w - \frac{w^T v}{v^T v} w^T v$$

交換律相等

$$= 0$$