Aluno: Victor Rafael Ordozgoite
Disciplina: Projeto e Análise de Algoritmos
Professor: Luis Cuevas Rodríguez

Algoritmos recursivos

a) $T(n) = 2T\left(\frac{n}{2}\right) + n$

1ª iteração: $\boxed{2T\left(\frac{n}{2}\right) + n}$

2ª iteração: $2\left[2T\left(\frac{n}{4}\right) + \frac{n}{2}\right] + n$

$\boxed{4T\left(\frac{n}{4}\right) + 2n}$

3ª iteração: $4\left[2.T\left(\frac{n}{8}\right) + \frac{n}{4}\right] + 2n$

$\boxed{8T\left(\frac{n}{8}\right) + 3n}$

4ª iteração: $8\left[2.T\left(\frac{n}{16}\right) + \frac{n}{8}\right] + 3n$

$\boxed{16T\left(\frac{n}{16}\right) + 4n}$

i-ésima iteração: $\boxed{2^i T\left(\frac{n}{2^i}\right) + in}$

Conhecendo $T(1)$, precisamos que:

$\frac{n}{2^i} = 1 \Rightarrow 2^i = n$

$\log_2 2^i = \log_2 n \Rightarrow i \log_2 2 = \log_2 n \therefore \boxed{i = \log_2 n}$

Portanto faremos $\log_2 n$ iterações:

$T(n) = 2^{\log_2 n}\, T(1) + \log_2 n \cdot n \Rightarrow T(n) = n + n \log_2 n \therefore \boxed{O(n \log_2 n)}$

1ª iteração:

b) $\boxed{T(n) = 8T(\frac{n}{2}) + n^2}$

2ª iteração: $8\left[8.T(\frac{n}{4}) + \frac{n^2}{4}\right] + n^2$

$$\boxed{64.T(\frac{n}{4}) + 3n^2}$$

3ª iteração: $64\left[8.T(\frac{n}{8}) + \frac{n^2}{16}\right] + 3n^2$

$$\boxed{512T(\frac{n}{8}) + 7n^2}$$

4ª iteração: $512\left[8T(\frac{n}{16}) + \frac{n^2}{64}\right] + 7n^2$

$$\boxed{4096T(\frac{n}{16}) + 15n^2}$$

i-ésima iteração: $8^i T(\frac{n}{2^i}) + (2^i - 1)n^2$

Conhecendo T(1), precisamos que:

$$\frac{n}{2^i} = 1 \Rightarrow 2^i = n \rightsquigarrow \log_2 2^i = \log_2 n$$

$$i \cancel{\log_2 2} = \log_2 n \Rightarrow \boxed{i = \log_2 n}$$

Portanto, faremos $\log_2 n$ iterações

$$T(n) = 8^{\log_2 n} \cancel{T(1)} + (2^{\log_2 n} - 1)n^2 = 2^{3\log_2 n} + (n-1)n^2$$

$$T(n) = n^3 + n^3 - n^2 \Rightarrow T(n) = 2n^3 - n^2$$

$\therefore$ $\boxed{O(n^3)}$

c) $T(n) = 7T(\frac{n}{2}) + n^2$

Pelo teorema mestre:

$\log_2 7 = 2,8 > K = 2$, $P = 0$

$\therefore \boxed{\Theta(n^{\lg 7})}$

d) $T(n) = 2T(\frac{n}{4}) + 1$

$\log_4 2 = \frac{1}{2} > K = 0 \quad P = 0$

$\Theta(n^{1/2})$

e) $T(n) = 2T(\frac{n}{4}) + \sqrt{n}$

$\log_4 2 = \frac{1}{2} = K = \frac{1}{2} \quad P = 0$

$\Theta(n^k \log^{p+1} n) = \boxed{\Theta(n^{\frac{1}{2}} \log n)}$

f) $T(n) = 2T(\frac{n}{4}) + n$

$\log_4 2 = \frac{1}{2} < K = 1 \quad P = 0$

$\Theta(n^k \lg^p n) = \boxed{\Theta(n)}$

g) $T(n) = 2T\left(\frac{n}{4}\right) + n^2$

$\log_4 2 = \frac{1}{2} < K = 2 \quad P = 0$

$\Theta(n^K \log^P n) = \boxed{\Theta(n^2)}$

h) $T(n) = 2T\left(\frac{n}{2}\right) + n^4$

$\log_2 2 = 1 < K = 4 \quad P = 0$

$\Theta(n^K \log^P n) = \boxed{\Theta(n^4)}$

2) a) $T(n) = \begin{cases} \Theta(1), \text{ se } n \leq 1 \\ T(n-1) + \Theta(1), \text{ se } n > 1 \end{cases}$

$\cancel{T(1)} = \Theta(1)$

$\cancel{T(2)} = \cancel{T(1)} + \Theta(1)$

$\cancel{T(3)} = \cancel{T(2)} + \Theta(1)$

$\vdots$

$+ \; T(n) = T(\cancel{n-1}) + \Theta(1)$

$T(n) = n \cdot \Theta(1) \quad \therefore \quad \boxed{\Theta(n)}$

b) $T(n) = \begin{cases} \Theta(1), & \text{se } n \leq 1 \\ 2T(n-1) + \Theta(1), & \text{se } n > 1 \end{cases}$

Iteração 2: $2[2T(n-2) + \Theta(1)] + \Theta(1)$

$$4T(n-2) + 3\Theta(1)$$

iteração 3: $4[2T(n-3) + \Theta(1)] + 3\Theta(1)$

$$8T(n-3) + 7\Theta(1)$$

iteração 4: $8[2T(n-4) + \Theta(1)] + 7\Theta(1)$

$$16T(n-4) + 15\Theta(1)$$

i-ésima iteração: $2^i T(n-i) + (2^i - 1)\Theta(1)$

Conhecendo $T(1)$, precisamos que:

$n - i = 1 \Rightarrow \underline{i = n-1}$

Portanto faremos $n-1$ iterações

$$2^{n-1} \cdot \overset{\Theta(1)}{\cancel{T(1)}} + (2^{n-1} - 1)\Theta(1)$$

$$2^{n-1} \cdot \Theta(1) + 2^{n-1} \cdot \Theta(1) - \Theta(1)$$

$$2 \cdot 2^{n-1} \cdot \Theta(1) - \Theta(1)$$

$$2^{n-1+1} \cdot \Theta(1) - \Theta(1)$$

$$2^n \cdot \Theta(1) - \Theta(1) \therefore \boxed{\Theta(2^n)}$$

c) $T(n) = \begin{cases} \Theta(1) & , \text{se } n \leq 1 \\ T(n-1) + \Theta(1), & \text{se } n > 1 \end{cases}$

$\cancel{T(1)} = \Theta(1)$
$\cancel{T(2)} = \cancel{T(1)} + \Theta(1)$
$\cancel{T(3)} = \cancel{T(2)} + \Theta(1)$
$\vdots$
$+ T(n) = \cancel{T(n-1)} + \Theta(1)$

$T(n) = n \cdot \Theta(1) \therefore \boxed{\Theta(n)}$

d) $T(n) = \begin{cases} \Theta(1) & , \text{se } n \leq 1 \\ T(\frac{n}{2}) + \Theta(1), & \text{se } n > 1 \end{cases}$

$\log_2 1 = 0 \qquad K = 0 \qquad P = 0$

$\Theta(n^k \log^{P+1} n) = \boxed{\Theta(\log n)}$

e) $T(n) = \begin{cases} \Theta(1), & \text{se } n \leq 1 \\ T(n-1) + \Theta(n) + \Theta(1), & \text{se } n > 1 \end{cases}$

$\cancel{T(1)} = \Theta(1)$
$\cancel{T(2)} = \cancel{T(1)} + \Theta(n) + \Theta(1)$
$\cancel{T(3)} = \cancel{T(2)} + \Theta(n) + \Theta(1)$
$\vdots$
$+ T(n) = \cancel{T(n-1)} + \Theta(n) + \Theta(1)$

$T(n) = (n-1) \cdot \Theta(n) + n \cdot \Theta(1) \therefore \boxed{\Theta(n^2)}$