Appendix Table: Derivative Order 3 (Third Derivatives)

RMSE Comparison Across Methods and Noise Levels

1 Performance Table: Derivative Order 3

Method	Noise = 0	Noise = 1e-6	Noise = 1e-3
AAA_Julia	0.53	2.2e05	1.3e09
AAA_lowpres_Julia	0.00	1.8e10	2.6e10
Butterworth_Python	28.9	28.9	28.9
Chebyshev_Python	60.6	8.9e02	1.8e05
FiniteDiff_Python	45.2	1.9e04	2.0e07
GPR_Julia	9.4	9.5	19.9
GP_Matern_1.5_Python	1.6e04	5.9e05	6.9e02
GP_Matern_2.5_Python	23.8	1.4e03	10.1
GP_Matern_Python	1.6e04	5.9e05	6.9e02
GP_RBF_Iso_Python	16.8	16.8	13.8
GP_RBF_Python	16.8	16.8	13.8
JuliaAAAFullOpt_Julia	3.9	3.1e04	4.7e05
JuliaAAALS_Julia	0.02	58.4	2.4e13
JuliaAAASmoothBary_Julia	1.7e04	8.8e03	1.3e09
JuliaAAATwoStage_Julia	0.02	96.4	2.4e13
KalmanGrad_Python	37.8	27.6	34.4
LOESS_Julia	8.1e04	5.2e08	1.1e04
SVR_Python	28.6	28.6	28.6
SavitzkyGolay_Python	23.0	1.8e04	2.2e07
TVDiff_Julia	1.5e02	3.1e03	6.5e03

2 Notes

- RMSE values are shown for third derivative approximation (derivative order 3)
- Green values indicate excellent performance (RMSE < 10)
- Red values indicate severe failure (RMSE > 1000)
- Values represent Root Mean Square Error across test cases
- GPR_Julia shows the most stable performance across noise levels
- Many methods exhibit the "noise cliff" phenomenon - catastrophic failure at higher noise levels
- Scientific notation: $2.2e05$ means 2.2×10^5

3 Key Observations

- **Noise Sensitivity:** Most methods fail catastrophically when noise increases from 1e-6 to 1e-3
- **GPR Stability:** GPR_Julia maintains reasonable performance across all noise levels
- **AAA Methods:** Excellent at zero noise but completely fail with any significant noise
- **Traditional Methods:** Finite differences and polynomial methods show severe noise amplification