Assignment 5

воскресенье, 19 ноября 2017 г. 13:33

#5.1 Sequence of observations (energies): $x_1^T = x_1, x_2, \dots, x_T$

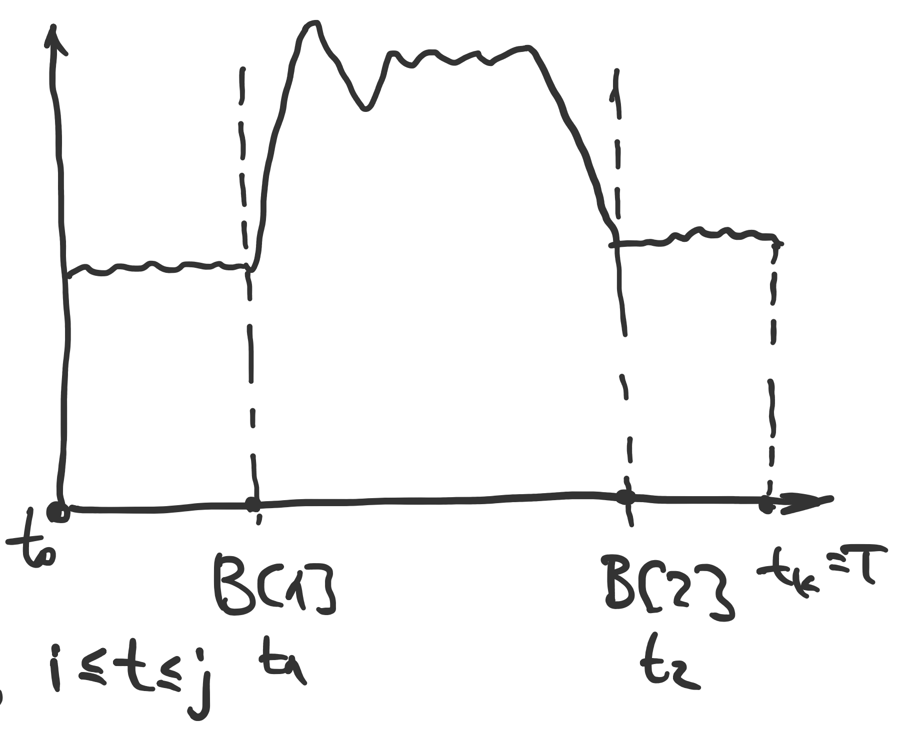
Boundaries: $0 \equiv t_0 < t_1 < t_2 < \dots < t_K = T$ $\quad x_t \in \mathbb{R}$

$K=3 \Rightarrow$ silence - speech - silence

$0 = t_0 < t_1 < t_2 < t_3 = T$

silence | sp. | silence

$B[0] = 0$, $B[1]$, $B[2]$, $B[3] = T$



Local cost $h(x_t; m_{ij}) = \begin{cases} (x_t - m_{ij})^2, & i \le t \le j \\ 0, & \text{otherwise} \end{cases}$

m_{ij} – empirical mean of $x_i^j = x_i, \dots, x_j$

$\Rightarrow h(x_t; m_{ij})$ defines the cost of energy x_t to be in the segment x_i^j

a). global cost: $\sum_{k=1}^{K} \sum_{i=t_{k-1}+1}^{t_k} h(x_i; m_{t_{k-1}+1, t_k})$

Minimisation problem: $D(K,t) = \min\limits_{t_1^K : t_K = t} \left\{ \sum_{k=1}^{K} \sum_{i=t_{k-1}+1}^{t_k} h(x_i; m_{t_{k-1}+1, t_k}) \right\}$

$$D(k,t) = \min_{\substack{t \\ t_1^k : t_k = t}} \left\{ \sum_{k=1}^{K} \sum_{i=t_{k-1}+1}^{t_k} h(x_i; m_{t_{k-1}+1, t_k}) \right\} =$$

$$= \min_{\substack{t \\ t_1^k : t_k = t}} \left\{ \sum_{k=1}^{K-1} \sum_{i=t_{k-1}+1}^{t_k} h(x_i; m_{t_{k-1}+1, t_k}) + \sum_{i=t_{k-1}+1}^{t_k} h(x_i; m_{t_{k-1}+1, t_k}) \right\} =$$

$$= \min_{\substack{t' \\ t_1^{k-1} : t_{k-1} = t'}} \left\{ \sum_{k=1}^{t'} \sum_{i=t_{k-1}+1}^{t_k} h(x_i; m_{t_{k-1}+1, t_k}) + \sum_{i=t'+1}^{t} h(x_i; m_{t'+1, t}) \right\} =$$

$$= \min_{\substack{t, t' \\ t_1 : t' < t}} \left\{ D(k-1, t') + \sum_{i=t'+1}^{t} h(x_i; m_{t'+1, t}) \right\}$$

c). $$\sum_{i=t'+1}^{t} (x_i - m_{t'+1,t})^2 = \sum_{i=t'+1}^{t} (x_i^2 - 2x_i \cdot m_{t'+1,t} + m_{t'+1,t}^2) =$$

$$= \sum_{i=t'+1}^{t} x_i^2 - 2 m_{t'+1,t} \cdot \sum_{i=t'+1}^{t} x_i + (t-t') \cdot m_{t'+1,t}^2 =$$

$$= \left\{ \begin{array}{l} \sum_{i=t'+1}^{t} x_i = \sum_{i=1}^{t} x_i - \sum_{i=1}^{t'} x_i \,;\ \sum_{i=1}^{t} x_i = sum(x_t) \Rightarrow \sum_{i=t'+1}^{t} x_i = sum(x_t) - sum(x_{t'}) \\ \sum_{i=t'+1}^{t} x_i^2 = \sum_{i=1}^{t} x_i^2 - \sum_{i=1}^{t'} x_i^2 = sum(x_t^2) - sum(x_{t'}^2) \\ m_{t'+1,t} = \dfrac{sum(x_t) - sum(x_{t'})}{(t-t')} \end{array} \right\} =$$

$$= sum(x_t^2) - sum(x_{t'}^2) - 2 \cdot \frac{sum(x_t) - sum(x_{t'})}{(t-t')} \cdot (sum(x_t) - sum(x_{t'})) +$$

$$+ (t-t') \cdot \frac{(sum(x_t) - sum(x_{t'}))^2}{(t-t')^2} =$$

$$= sum(x_t^2) - sum(x_{t'}^2) - \frac{(sum(x_t) - sum(x_{t'}))^2}{(t-t')}$$

Sums (partial) of elements and their squares can be computed in advance.