

Student Information

Name : Satılmış ÖRENCİK

ID : 2396273

Answer 1

a) sample size $N=10$ and $\sigma=3$

Sample Mean $\bar{X} = \frac{X_1 + \dots + X_n}{N}$

Therefore; $\bar{X} = 16.96$

$(1 - \alpha) = 0.90$ for 90% confidence level $\alpha = 0.10$ also $\frac{\alpha}{2} = 0.05$

$q_{0.05} = -z_{0.05}$ and $q_{0.95} = z_{0.05}$

From table A-4 in textbook we find $q_{0.95} = 1.645$ substitutes these values to the formula below and we have ;

$$\begin{aligned}\bar{X} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{N}} &= 16.96 \pm 1.645 \cdot \frac{3}{\sqrt{10}} \\ &= 16.96 \pm 1.56 \text{ or } [15.40, 18.52]\end{aligned}$$

$(1 - \alpha) = 0.99$ confidence level $\alpha = 0.01$ also $\frac{\alpha}{2} = 0.005$

$q_{0.005} = -z_{0.005}$ and $q_{0.995} = z_{0.005}$

From table A-4 in textbook we find $q_{0.995} = 2.5758 \cong 2.58$ substitutes these values to the formula below and we have ;

$$\begin{aligned}\bar{X} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{N}} &= 16.96 \pm 2.58 \cdot \frac{3}{\sqrt{10}} \\ &= 16.96 \pm 2.45 \text{ or } [14.51, 19.41]\end{aligned}$$

b) We have given $\Delta=1.55$, $(1-\alpha) = 0.98$ and $\alpha = 0.02$, standard deviation given $\sigma = 3$

We need to solve $n \geq \left(\frac{z_{0.01} \cdot \sigma}{\Delta}\right)^2$

$$= \left(\frac{2.33 \cdot 3}{1.55}\right)^2 = 20.34$$

This is the minimum sample size to assure Δ and we are only allowed to round it up,

Hence; we need a sample of at least 21 observations.

Answer 2

a) No, standard deviation and the mean alone ARE NOT satisfactory as the statistics for the restaurant. Besides, from these two values of mean and standard deviation, In addition, we have to need another parameter called as standard deviation to measure and the spread of this data. Hence, standard deviation is NECESSARY for the computation.

b) The restaurant has a lower rating was given 7.5 : Null Hypothesis $H_0 : \mu < 7.5$

Alternate Hypothesis $H_A : \mu \geq 7.5$

Given sample size $N = 256$, mean $\bar{X} = 7.4$ and Standard deviation $\sigma = 0.8$

$$z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{N}}}$$

$$z = \frac{7.5 - 7.4}{\frac{0.8}{\sqrt{256}}}$$

$$z = \frac{0.1}{0.05}$$

$$z = 2$$

Critical value for 5% level of significance $\alpha = 0.05$ and $\alpha/2 = 0.025$

From table A-4 in textbook we find $q_{0.975} = z_{0.025} = 1.96$

Since z value is GREATER than critical value ($1.96 < 2$), we reject NULL Hypothesis.

Therefore, we can detect that the rating significant ≥ 7.5

Hence, restaurant A is involved in my list of candidate restaurants to order food.

c) Given that Standard deviation has changed to $\sigma = 1$:

$$\text{Then } z = \frac{7.5 - 7.4}{\frac{1}{\sqrt{256}}}$$

$$z = \frac{0.1}{0.0625}$$

$$z = 1.6$$

Since z value is NOT greater than critical value ($1.96 > 1.6$) calculated above, we can observe that FAIL to reject the NULL Hypothesis.

For this reason, I DO NOT include this candidate list of my restaurants.

d) Given that the value (7.6) is greater than mean of the population(7.5), we DO NOT bother about the standard deviation as well.

Hence, we DO NOT NEED TO resort a statistical test.

Answer 3

a) We can observe that below calculations are given:

Computer A : Sample size $N_1 = 20$, Mean $\bar{X}_1 = 211$ and standard deviation $s_1 = 5.2$

Computer B : Sample size $N_2 = 32$, Mean $\bar{X}_2 = 133$ and standard deviation $s_2 = 22.8$

Null Hypothesis $H_0 : \mu_a - \mu_b \geq 90$ Alternative Hypothesis $H_A : \mu_a - \mu_b < 90$

Now start part a: Population variances are the same Therefore, we have to use: Pooled sample variance:

$$s_p^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2 + \sum_{i=1}^m (Y_i - \bar{Y})^2}{(n+m-2)}$$

$$= \frac{(n_1-1) \cdot s_1^2 + (n_2-1) \cdot s_2^2}{(n_1+n_2-2)}$$

$$= \frac{(20-1) \cdot 5.2^2 + (32-1) \cdot (22.8)^2}{20+32-2} = 332.58$$

$$\text{Test statistic equals } t = \frac{\bar{X} - \bar{Y} - D}{\sqrt{s_p^2 \cdot (\frac{1}{n} + \frac{1}{m})}} = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{s_p^2 \cdot (\frac{1}{N_1} + \frac{1}{N_2})}}$$

$$= \frac{(211-133-90)}{\sqrt{332.58 \cdot (\frac{1}{20} + \frac{1}{32})}} = -2.30855 \cong -2.31$$

degrees of freedom in short $df = n+m-2 = N_1 + N_2 - 2 = 20 + 32 - 2 = 50$

p-value = T.DIST(-2.31, 50, 1) = 0.0125

confidence level of significance 1% -- $>> 0.01 = \alpha$

as p-value $> \alpha$, DO NOT reject NULL Hypothesis

Hence:

There is NOT enough proof to conclude that the computer B supports better improvement at 0.01

b) For different standard deviations we have this formula:

$$t = \frac{\bar{X} - \bar{Y} - D}{\sqrt{\frac{s_x^2}{n} + \frac{s_y^2}{m}}} = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}}}$$

$$= \frac{(211-133-90)}{\sqrt{\frac{(5.2)^2}{20} + \frac{(22.8)^2}{32}}} = -2.86$$

for degrees of freedom we have Satterthwaite approximation formula:

$$df = \frac{(\frac{(s_x)^2}{n} + \frac{(s_y)^2}{m})^2}{\frac{(s_x)^4}{n^2 \cdot (n-1)} + \frac{(s_y)^4}{m^2 \cdot (m-1)}} = \frac{(\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2})^2}{\frac{(s_1)^4}{n_1^2 \cdot (n_1-1)} + \frac{(s_2)^4}{n_2^2 \cdot (n_2-1)}}$$

$$df = 35.97 \cong 35$$

$$\text{p-value} = \text{T.DIST}(-2.86, 35, 1) = 0.0035$$

as p-value $< \alpha$ (0.01 that is calculated above), REJECT NULL Hypothesis

Hence;

There is enough evidence to conclude that the computer B supports BETTER improvement at 0.01