Student Information

Name : Satılmış Örencik

ID : 2396273

Answer 1

a)

P(at least one of them is white) = 1 - P(None of them is white)
=1-[P(pick black from box 1) * P(pick black from box 2) * P(pick black from box 3)]

P(at least one of them is white) = 1- $(\frac{8}{10} * \frac{11}{15} * \frac{9}{12})$

P(at least one of them is white) = 1- $\frac{264}{600}$
P(at least one of them is white)=0.56

b)

P(All of them are white) = P(pick white from box 1) * P(pick white from box 2) * P(pick white from box 3)
P(All of them are white) = $\frac{2}{10} * \frac{4}{15} * \frac{3}{12}$
P(All of them are white) = $\frac{2}{15}$
P(All of them are white) = 0.013333

c)

We have to compare probability of picking two white ball from each box
P(Picking two white ball from box 1) = $\frac{\binom{2}{2}}{\binom{10}{2}}$
P(Picking two white ball from box 2) = $\frac{\binom{4}{2}}{\binom{15}{2}}$
P(Picking two white ball from box 3) = $\frac{\binom{3}{2}}{\binom{12}{2}}$

P(Picking two white ball from box 1) = $\frac{1}{45}$
P(Picking two white ball from box 2) = $\frac{6}{105}$
P(Picking two white ball from box 3) = $\frac{3}{66}$

P(Picking two white ball from box 1) = 0.02222
P(Picking two white ball from box 2) = 0.05714
P(Picking two white ball from box 3) = 0.04545
Since the probability of picking two white ball from box 2 is biggest we have to choose it.

d)

We have to compare 4 different probabilities picking box 1 and box 2 , picking box 1 and box 3 , picking box 2 and box 3 and lastly picking 2 balls from box 2. We have already eliminated above 2 other options picking 2 balls from box1 or box3.

$P($ Picking one white from box1 and one white from box2 $)= \frac{\binom{2}{1}}{\binom{10}{1}} * \frac{\binom{4}{1}}{\binom{15}{1}}$

$P($ Picking one white from box1 and one white from box3 $)= \frac{\binom{2}{1}}{\binom{10}{1}} * \frac{\binom{3}{1}}{\binom{12}{1}}$

$P($ Picking one white from box2 and one white from box3 $)= \frac{\binom{4}{1}}{\binom{15}{1}} * \frac{\binom{3}{1}}{\binom{12}{1}}$
$P($ Picking two white ball from box 2 $) = 0.05714$ is already calculated above
$P($ Picking one white from box1 and one white from box2 $)= 0.05333$
$P($ Picking one white from box1 and one white from box3 $)=0.05$
$P($ Picking one white from box2 and one white from box3 $)= 0.06666$
Probability of picking one white from box 2 and one white from box 3 is the biggest.
Hence; we have to choose them.

e)

	0	1	2	3	4
1	$\frac{28}{45}$	$\frac{16}{45}$	$\frac{1}{45}$	0	0
2	$\frac{330}{1365}$	$\frac{660}{1365}$	$\frac{330}{1365}$	$\frac{44}{1365}$	$\frac{1}{1365}$
3	$\frac{84}{220}$	$\frac{108}{220}$	$\frac{27}{220}$	$\frac{1}{220}$	0

Horizontal axis display number of number of white ball which is taken
Vertical axis display the boxes

$E_1 = 0 + 1 * \frac{16}{45} + 2 * \frac{1}{45} = \frac{18}{45}$
$E_2 = 0 + 1 * \frac{660}{1365} + 2 * \frac{330}{1365} + 3 * \frac{44}{1365} + 4 * \frac{1}{1365} = \frac{16}{15}$
$E_3 = 0 + 1 * \frac{108}{220} + 2 * \frac{27}{220} + 3 * \frac{1}{220} + 0 = \frac{3}{4}$
Number of white balls in total = $E_1 + E_2 + E_3$
Number of white balls in total = $\frac{18}{45} + \frac{16}{15} + \frac{3}{4}$
Number of white balls in total = 2.216667

f)

Bayes Rule−− >> $P(\mathrm{A} \mid \mathrm{B}) = \frac{P(B|A) \cdot P(A)}{P(B)}$
$P(A)$: Choosing Box1
$P(B)$: Picking White Ball
$P(B) = P(C) + P(D) + P(E)$
where;
$P(C)$: Picking White Ball from Box1

$P(C')$: Choosing Box1
$P(D)$: Picking White Ball from Box2
$P(D')$: Choosing Box2
$P(E)$: Picking White Ball from Box3
$P(E')$: Choosing Box3

$P(\text{A} \mid \text{B}) = \frac{P(B|A) \cdot P(A)}{P(C) \cdot P(C') + P(D) \cdot P(D') + P(E) \cdot P(E')}$

$P(\text{A} \mid \text{B}) = \frac{\frac{2}{10} \cdot \frac{1}{3}}{\frac{2}{10} \cdot \frac{1}{3} + \frac{4}{15} \cdot \frac{1}{3} + \frac{3}{12} \cdot \frac{1}{3}}$

$P(\text{A} \mid \text{B}) = 0.27906$

Answer 2

a)

Bayes Rule $-->>$ $P(\text{A} \mid \text{B}) = \frac{P(B|A) \cdot (A)}{P(B)}$

$= P(\text{A} \mid \text{B}) = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|\widetilde{A}) \cdot P(\widetilde{A})}$
$P(\text{Sam Corrupted} \mid \text{Ring Destroyed}) =$

$= \frac{P(\text{Ring Destroyed} \mid \text{Sam Corrupted}) * P(\text{Sam Corrupted})}{P(\text{Ring Destroyed} \mid \text{Sam Corrupted}) * P(\text{Sam Corrupted}) + P(\text{Ring Destroyed} \mid \text{Sam Not Corrupted}) * P(\text{Sam NOT Corrupted})}$

$P(\text{Sam Corrupted} \mid \text{Ring Destroyed}) = \frac{(0.5) \cdot (0.1)}{(0.5) \cdot (0.1) + (0.9) \cdot (0.9)}$

$P(\text{Sam Corrupted} \mid \text{Ring Destroyed}) = 0.05813$

b)

From Bayes rule above we have ;
Bayes Rule$--> P(\text{A} \mid \text{B}) = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|\widetilde{A}) \cdot P(\widetilde{A})}$

$P(A) = P(\text{Both Are Corrupted})$
$P(B) = P(\text{Ring Destroyed})$
for $P(B|\widetilde{A}) \cdot P(\widetilde{A})$ we have to check 3 option;
$P(B|\widetilde{A}) \cdot P(\widetilde{A}) = P(B|\widetilde{A}) \cdot P(\widetilde{A}) + [$
$P(B|\text{Frodo Corrupted and Sam Not Corrupted}) \cdot P(\text{Frodo Corrupted}) \cdot P(\text{Sam NOT Corrupted}) +$
$P(B|\text{Sam Corrupted and Frodo NOT Corrupted}) \cdot P(\text{Sam Corrupted}) \cdot P(\text{Frodo NOT Corrupted})]$

$P(\text{A} \mid \text{B}) = \frac{(0.05) \cdot (0.25) \cdot (0.1)}{(0.05) \cdot (0.25) \cdot (0.1) + (0.9) \cdot (0.9) \cdot (0.75) + (0.2) \cdot (0.25) \cdot (0.9) + (0.5) \cdot (0.1) \cdot (0.75)}$

$P(\text{A} \mid \text{B}) = 0.00180$

Answer 3

a)

$P_{(A,I)}(a,i)$	1	2	$P_A(a)$
1	0.18	0.12	0.3
2	0.3	0.2	0.5
3	0.12	0.08	0.2
$P_I(i)$	0.6	0.4	

$E(4) = P(a = 2, i = 2) + P(a = 3, i = 1)$
$E(4) = 0.2 + 0.12$
$E(4) = 0.32$

b)

for Independence $P(x = a, y = b) = P(a) \cdot P(b)$ for every value
$P(a = 1, i = 1) = P(a = 1) \cdot P(i = 1)$, $0.18 = 0.3 \cdot 0.6$
$P(a = 1, i = 2) = P(a = 1) \cdot P(i = 2)$, $0.12 = 0.3 \cdot 0.4$
$P(a = 2, i = 1) = P(a = 2) \cdot P(i = 1)$, $0.3 = 0.5 \cdot 0.6$
$P(a = 2, i = 2) = P(a = 2) \cdot P(i = 2)$, $0.2 = 0.5 \cdot 0.4$
$P(a = 3, i = 1) = P(a = 3) \cdot P(i = 1)$, $0.12 = 0.2 \cdot 0.6$
$P(a = 3, i = 2) = P(a = 3) \cdot P(i = 2)$, $0.08 = 0.2 \cdot 0.4$
It is correct for every variable
Therefore; snowy days in Ankara and İstanbul are independent.