Middle East Technical University | Department of Computer Science

CENG 280

Formal Languages and Abstract Machines

Spring 2021-2022

Homework 1

Regulations

1. The homework is **due by April 12th 2022, 23:59. Late submission is not allowed.**
2. This homework consists of two parts: Graded Questions and Self-Study Questions. As the name implies, Self-Study Questions will not be graded (but you are recommended to solve them). Therefore, **please only submit answers to graded questions.**
3. **Submissions must be made via ODTUClass**. Do not send your homework via e-mail, or do not bring any hardcopy.
4. You can **use any typesetting tool** (LaTex, Word, etc.) while writing the homework. However, **you must upload your solutions as a single vectorized (searchable) pdf file**. **Other formats and handwritten answers will not be considered for grading.**
5. Name pdf files you will submit as **<yourstudentid>_hw1 (e.g. 2132807_hw1.pdf)**. In case you violate the naming convention, you will receive a penalty of 5 points (over 100).
6. Send an e-mail to **garipler@metu.edu.tr** if you need to get in contact.
7. **This is an individual homework, which means you have to answer the questions on your own.** Any contrary case will be considered as cheating and university regulations about cheating will be applied.

<u>Graded Questions</u>

Question 1

Given $L_0 = \{\omega \in \{a, b\}^* \mid \omega \; contains \; at \; least \; 1 \; \boldsymbol{aa} \; substring \; and \; at \; least \; 1 \; \boldsymbol{bb} \; substring\}$

a. Write a regular expression that generates the language.
b. Formally define and draw an NFA that recognizes the language.
c. Using subset construction algorithm*, construct an equivalent DFA for the NFA you have constructed at (b). Clearly show each step of (the application of) the subset construction algorithm. Draw the equivalent DFA you have constructed.
d. Employing yields in one step relation** ($\vdash$) between configurations, trace the string $\omega' =$ “bbabb” on the given NFA and on the equivalent DFA you have constructed. For both automata, show whether ω' is accepted by that automaton or not.

Question 2

Using the pumping lemma*** for regular languages **and/or** closure properties

a. Given $L_2 = \overline{L_1}$ and $L_1 = \{\, a^m b^n \mid m > n \; and \; m, n \in \mathbb{N}\}$ prove whether L_2 is regular or not.
b. Given $L_4 = \{\, a^n b^n \mid n \in \mathbb{N}^+\}$, $L_5 = \{\, a^m b^n \mid m, n \in \mathbb{N}\}$ and $L_6 = b^* a (ab^* a)^*$ prove whether $L_4 \cup L_5 \cup L_6$ is regular or not.

*Check Theorem 2.2.1 and Example 2.2.4 in your textbook.

**Check Example 2.1.1 in your textbook for DFA and Example 2.2.1 for NFA.

***Check Theorem 2.4.1 in your textbook

Self-Study Questions

Exercise 1

Give formal proofs to state whether the following sets are finite, countably infinite or uncountable. State any mapping explicitly and give clear references to known theorems if used.

a. The set $D = \{L^* \mid L\ is\ a\ finite\ language\ over\ the\ unary\ alphabet\ \Sigma = \{a\}\ \ and\ L^* is\ not\ regular.\}$

b. The set of all languages over the binary alphabet $\Sigma = \{0, 1\}$ (This is, $2^{\{0,1\}^*}$).

c. The set of all regular languages over the binary alphabet $\Sigma = \{0, 1\}$.

Exercise 2

For each of the finite automata given below, construct a regular expression by eliminating states one-by-one.

(While doing the conversion, at each step you will derive a smaller generalized finite automaton equivalent to the given FA. Studying example 2.3.2 in your textbook may be helpful.)

a. $A_1 = \{K_1, \Sigma_1, \Delta_1, s_1, F_1\}\ \ where\ \ K_1 = \{q_0, q_1, q_2\}\ ,\ \ \Sigma_1 = \{\mathrm{a}, \mathrm{b}\}\ ,\ \ \mathrm{s}_1 = \mathrm{q}_0\ ,\ \ F_1 = \{q_0, q_2\}\ ,$
$\Delta_1 = \{(q_0, a, q_1), (q_0, b, q_1), (q_1, a, q_1), (q_1, b, q_2), (q_2, a, q_0), (q_2, b, q_1)\}$

b. $A_1 = \{K_2, \Sigma_2, \Delta_2, s_2, F_2\}\ \ where\ \ K_2 = \{q_0, q_1, q_2, q_3, q_4\}\ ,\ \ \Sigma_1 = \{\mathrm{a}, \mathrm{b}\}\ ,\ \ \mathrm{s}_2 = \mathrm{q}_0\ ,\ \ F_2 = \{q_3\}\ ,$

$$
\begin{aligned}
\Delta_2 = \{\ \ & (q_0, b, q_1), (q_0, b, q_3), (q_0, \epsilon, q_2), \\
& (q_1, a, q_1), \\
& (q_2, a, q_2), (q_2, a, q_4), (q_2, b, q_2) \\
& (q_3, a, q_1), (q_3, \epsilon, q_4) \\
& (q_4, a, q_2), (q_4, a, q_3), (q_4, b, q_4) \\
\}\ \ &
\end{aligned}
$$