

# CENG222

## Statistical Methods for Computer Engineering

Spring 2021-2022

### Homework 2

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Due: April 24th, 2022, Sunday 23:59

#### Question 1 (30 points)

Let a joint density function of two continuous variables  $X \in [-1, 1]$  and  $Y \in [-1, 1]$  be given as

$$f_{(X,Y)}(x,y) = \begin{cases} \frac{1}{\pi} & \text{if } x^2 + y^2 \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

- a) ( 5 pts) Are  $X$  and  $Y$  independent?
- b) (10 pts) What are the marginal pdfs for  $X$  and  $Y$ ?
- c) ( 5 pts) What is the expected value of  $X$ ?
- d) (10 pts) What is the variance of  $X$ ?

#### Question 2 (30 points)

In a test, subject  $A$  and subject  $B$  are put in different rooms. In front of each of them is a button. Each subject is asked to push the button after a simultaneously played beep sound. The elapsed time until the subjects push the button,  $T_A$  and  $T_B$ , are independent and uniformly distributed on  $[0, 100]$  seconds.

- a) ( 5 pts) What are the joint density function  $f(t_A, t_B)$  and the joint cdf  $F(t_A, t_B)$ ?
- b) ( 5 pts) What is the probability that subject  $A$  pushes the button in the first 10 seconds and subject  $B$  in the last 10 seconds?
- c) (10 pts) What is the probability that subject  $A$  pushes the button at most 20 seconds after subject  $B$ ?
- d) (10 pts) Subjects fail the test if their elapsed time differ by more than 30 seconds. What is the probability that they pass the test?

(Hint: You may find it helpful to think in terms of the ratio of the regions corresponding to the asked events to the  $[0, 100] \times [0, 100]$  square)

#### Question 3 (20 points)

- a) (10 pts) Let  $X_n = \text{Exponential}(\lambda_n)$  for  $1 \leq n \leq N$  be independent random variables and  $T = \min(X_1, X_2, \dots, X_N)$ . What is the cdf of  $T$ ?
- b) (10 pts) We are given 10 different computers  $C_1, C_2, \dots, C_{10}$  the lifetimes of which are exponential random variables with means  $\frac{10}{n}$  years ( $1 \leq n \leq 10$ ), respectively. Each computer's lifetime is independent from the others. What is the expected time before one of the computers fails?

#### Question 4 (20 points)

Assume there are a total of 30,000 registered students in METU and 74% of the students are enrolled in an undergraduate program, 16% work towards their Master's degrees, and 10% are pursuing a doctoral degree. 100 students from METU participate in a poll. Estimate the following using the central limit theorem.

- a) (10 pts) The probability that at least 70% of participants are undergraduate students.
- b) (10 pts) The probability that at most 5% of participants are pursuing a doctoral degree.

## Regulations

1. Make sure that your solutions are cleanly written and well organized.
2. Do not cheat.
3. Late submission is allowed up to 3 days with a penalty of 20 points for each day.
4. You will submit a single PDF through odtuclass.
5. Follow odtuclass for possible updates and discussions.