

CENG 280

Fundamental Structures for Computation

Spring 2021-2022

Homework 2

Regulations

1. The homework is **due by May 14th 2022, 23:59. Late submission is not allowed.**
2. This homework consists of two parts: Graded Questions and Self-Study Questions. As the name implies, Self-Study Questions will not be graded (but you are recommended to solve them).
Therefore, **please only submit answers to graded questions.**
3. **Submissions must be made via ODTUClass.** Do not send your homework via e-mail, or do not bring any hardcopy.
4. You can use any typesetting tool (LaTeX, Word, etc.) or handwriting while writing the homework. However, **you must upload your solutions as a pdf file.** Other formats will not be considered for grading.
5. Name pdf files you will submit as **<yourstudentid>_hw2** (e.g. **2345678_hw2.pdf**). Write your name and student id number at the top of the page. In case you violate the naming convention, you will receive a penalty of 5 points (over 100).
6. Send an e-mail to **garipler@metu.edu.tr** if you need to get in contact.
7. **This is an individual homework, which means you have to answer the questions on your own.** Any contrary case will be considered as cheating and university regulations about cheating will be applied.

Graded Questions

Question 1

- Write a context-free grammar for the language
 $L_1 = \{w \mid w \in \{a,b\}^* \wedge w \text{ has twice as many } b\text{'s as } a\text{'s}\}$
- Write a context-free grammar for the language
 $L_2 = \{a^n b^m \mid m, n \in \mathbb{N} \wedge m \leq n \leq 2m\}$
- (*) Formally define and draw a PDA that accepts L_1 .
- (**) Write a context-free grammar for the language $L_3 = L_1 \cup L_2$.

* Use the construction given at the proof of the Lemma 3.4.1 in your textbook.

**Use the construction given at the proof of the Theorem 3.5.1 in your textbook.

Question 2

Given $G_1 = \{V, \Sigma, R, S\}$ where $V = \{0,1,S,A\}$, $\Sigma = \{0,1\}$, and $R = \{S \rightarrow AS \mid e, A \rightarrow A1 \mid 0A1 \mid 01\}$

- Show that G_1 is ambiguous.
- Give an unambiguous grammar for $L(G_1)$. (i.e. disambiguate the given grammar.)
- Give the leftmost derivation of the string 00111 from the grammar you have constructed at part-b and draw the corresponding parse tree.

Self-Study Questions

Exercise 1

For the string *abbbaabb*, show an accepting computation of the PDA you have drawn at Question 1-c.

Exercise 2

Use the strong version of the pumping theorem to prove that the language $L = \{a^{n!} \mid n \geq 0\}$ is not context-free.

Exercise 3

Show that $L = \{\omega \in \{a, b, c\}^* \mid \omega \text{ has equal numbers of } a's, b's, \text{ and } c's\}$ is not context-free.

(Hint: Use the fact that intersection of a context-free language with a regular language is context-free.)