CENG 222

Statistical Methods for Computer Engineering

Spring 2021-2022

Homework 1

Due date: 10 04 2022, Sunday, 23:55

Introduction

In this assignment, there are 3 questions related to the 2nd and the 3rd chapter of your text book. While answering the questions, please **show your work** and the steps of your calculations. Give an explanation about what numbers mean in those steps. Otherwise, you may not get any points.

Questions

Q1. (**50 pts.**)

There are three boxes in front of you and each box contains a number of white and black balls as given;

BOX 1: 2 White and 8 Black
BOX 2: 4 White and 11 Black
BOX 3: 3 White and 9 Black

a) If you draw a single ball from each box randomly, what is the probability that at least one of them is white? *(5 pts)*
b) If you draw a single ball from each box randomly, what is the probability that all of them are white? *(5 pts)*
c) You are supposed to draw two balls from the same box and trying to get two white balls. Which box would you choose and why? *(5 pts)*
d) For the question in part c, you are allowed to draw from different boxes. In this case, what would be the order of your draw and why? *(10 pts)*
e) If you draw a single ball from each box randomly, what is the expected value for the number of white balls in total? *(10 pts)*
f) Assume that you are given a single white ball, taken randomly from any one of the boxes. What is the probability that this ball was taken from BOX 1? *(15 pts)*

Q2. (30 pts.)

Frodo and Sam are trying to destroy the one ring before being corrupted by the ring's power.

a) The probability of destroying the ring is 0.9 if Sam is not corrupted and 0.5 if Sam is corrupted. There is 0.1 probability of Sam's corruption. If it is known that the ring is destroyed, what is the probability that Sam is corrupted? *(10 pts)*

b) Frodo is more prone to the ring's power and the probability of Frodo's corruption is 0.25. The probability of destroying the ring drops to 0.2 if Frodo is corrupted and remains at 0.9 if both Frodo and Sam are not corrupted. If both of them are corrupted, there is only 0.05 probability of destroying the ring. If it is known that the ring is destroyed, what is the probability that both Frodo and Sam are corrupted? *(20 pts)*

Note: The corruption of Frodo and Sam are independent events. When solving part b, you can refer to part a for Sam's corruption probabilities.

Q3. (20 pts.)
Assume that A and I denote the number of snowy days in March for Ankara and Istanbul respectively. The joint probability table for the number of snowy days for these two cities are given in Table 1.

a	i	P(A=a, I=i)
1	1	0.18
1	2	0.12
2	1	0.3
2	2	0.2
3	1	0.12
3	2	0.08

Table 1: Joint probability table of random variables A and I.

a) What is the probability that there are four snowy days in total? *(5 pts)*
b) Show whether the snowy days in Ankara and Istanbul are independent or not. *(15 pts)*

Specifications

- You are expected to write your answers in LaTeX format. You can use the given template.

- Please do not skip the calculation steps. Show every step of your work.

- You have a total of 3 late days for this homework. For each day you have submitted late, you will lose 20 points. The homeworks you submit after 3 late days will not be graded.

- Cheating is forbidden. The violators will be punished according to the department regulations.

- Follow the course page on ODTUCLASS for any updates and clarifications. For your questions, you can send an email to "mduymus@ceng.metu.edu.tr".

Submission

Submissions will be done via ODTUCLASS. If you do not have access to ODTUCLASS for some reason, please send an email to assistants about that. You are expected to submit a single PDF file named "hw1.pdf" that contains the compiled version of your latex source as a PDF.