Student Information

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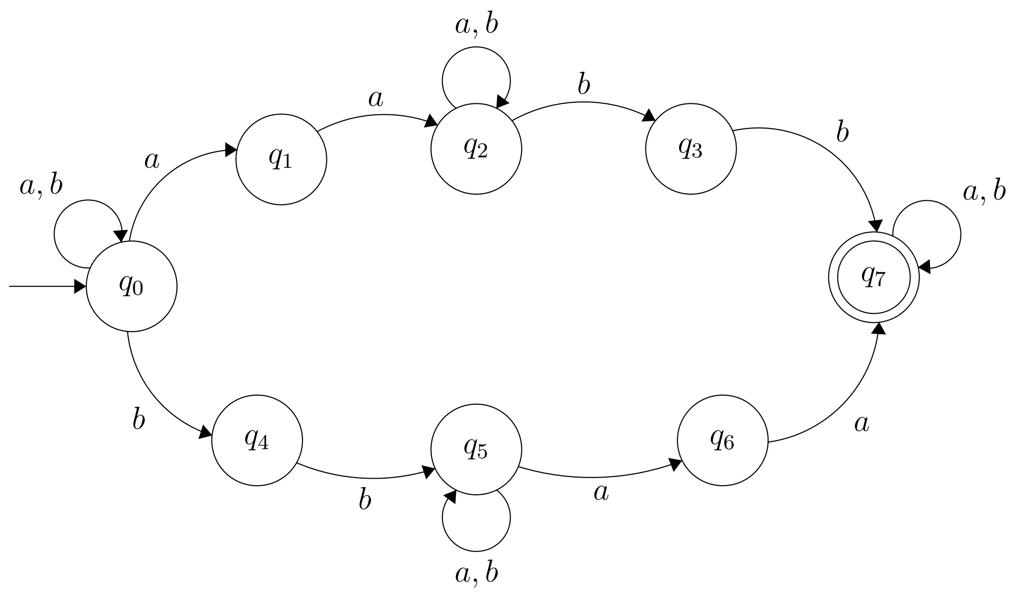
Answer 1

a)

$(a \cup b)^*((aa(a \cup b)^*bb) \cup (bb(a \cup b)^*aa))(a \cup b)^*$

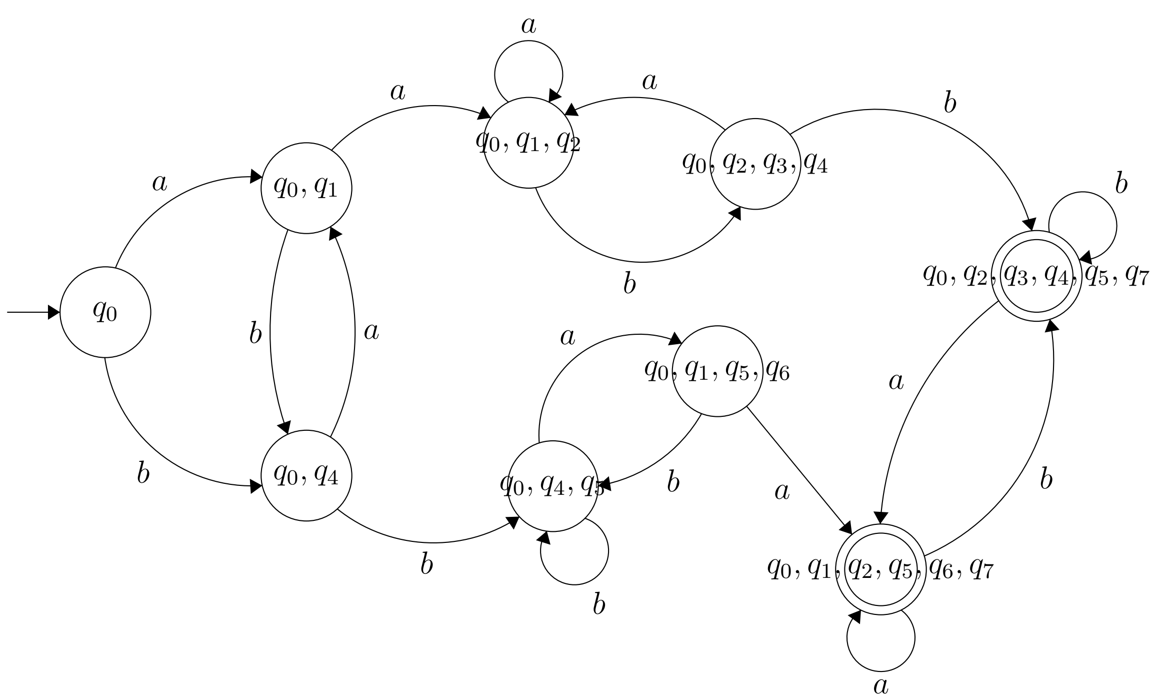
b)

Formally, this machine is $(K, \Sigma, \Delta, s, F)$, where
$K = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$
$\Sigma = \{a, b\}$
$s = q_0$
$F = q_7$ and
$\Delta = \{(q_0, a, q_0), (q_0, b, q_0), (q_0, a, q_1), (q_0, b, q_4), (q_1, a, q_2), (q_2, a, q_2), (q_2, b, q_2), (q_2, b, q_3), (q_3, b, q_7), (q_4, b, q_5), (q_5, a, q_5), (q_5, b, q_5), (q_5, a, q_6), (q_6, a, q_7), (q_7, a, q_7), (q_7, b, q_7)\}$



c)

	a	b
$\delta' > \{q_0\}$	$\{q_0, q_1\}$	$\{q_0, q_4\}$
$\delta'\ \{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_4\}$
$\delta'\ \{q_0, q_4\}$	$\{q_0, q_1\}$	$\{q_0, q_4, q_5\}$
$\delta'\ \{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_2, q_3, q_4\}$
$\delta'\ \{q_0, q_4, q_5\}$	$\{q_0, q_1, q_5, q_6\}$	$\{q_0, q_4, q_5\}$
$\delta'\ \{q_0, q_2, q_3, q_4\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_2, q_3, q_4, q_5, q_7\}$
$\delta'\ \{q_0, q_1, q_5, q_6\}$	$\{q_0, q_1, q_2, q_5, q_6, q_7\}$	$\{q_0, q_4, q_5\}$
$\delta'\ \{q_0, q_2, q_3, q_4, q_5, q_7\}$	$\{q_0, q_1, q_2, q_5, q_6, q_7\}$	$\{q_0, q_2, q_3, q_4, q_5, q_7\}$
$\delta'\ \{q_0, q_1, q_2, q_5, q_6, q_7\}$	$\{q_0, q_1, q_2, q_5, q_6, q_7\}$	$\{q_0, q_2, q_3, q_4, q_5, q_7\}$



d)

$$
\begin{aligned}
(q_0, bbabb) &\vdash w'\ (q_4, babb) \\
&\vdash w'\ (q_5, abb) \\
&\vdash w'\ (q_6, bb) \\
&\vdash w'\ (\emptyset, bb)
\end{aligned}
$$

or

$$
\begin{aligned}
(q_0, bbabb) \quad &\vdash w'\ (q_4, babb) \\
&\vdash w'\ (q_5, abb)
\end{aligned}
$$

$\vdash w^{'}\ (q_5, bb)$
$\vdash w^{'}\ (q_5, b)$
$\vdash w^{'}\ (q_5, \emptyset)$
or
$(q_0, bbabb)\quad \vdash w^{'}\ (q_0, babb)$
$\vdash w^{'}\ (q_0, abb)$
$\vdash w^{'}\ (q_1, bb)$
$\vdash w^{'}\ (\emptyset, bb)$

there is no way that $w^{'}$ accepted by this NFA

Now we look the DFA ;
$(q_0, bbabb)\quad \vdash w^{'}\ (q_0, q_4, babb)$
$\vdash w^{'}\ (q_0, q_4, q_5, abb)$
$\vdash w^{'}\ (q_0, q_1, q_5, bb)$
$\vdash w^{'}\ (q_0, q_4, q_5, b)$
$\vdash w^{'}\ (q_0, q_4, q_5, \emptyset)$

$w^{'}$ is NOT accepted DFA too:
Therefore; $w^{'}$ is NOT accepted this automota.

Answer 2

a)

Pumping Lemma Theorem:
For any regular language L, there exists a pumping length "p", such that any string S ∈ L with $|S| \geq p$, there exists x, y, z ∈ Σ^*, such that x = xyz, and;
(1) $|xy| \leq n$
(2) $|y| \geq 1$ (3) for all $i \geq 0$: xy^iz ∈L

Suppose L_1 were regular and therefore there is a "pumping constant" p for L_1
Choose w = $a^{p+1}b^p$
Look at all decomposition's of w into xyz
a) $|xy| \leq p$ b) $|y| \geq 1$
x=a^α $\alpha \geq 0$
y=a^β $\beta \geq 1$
z=$a^{p+1-\alpha-\beta}b^p$

choose i such that $xy^iz \notin L_1$;
xy^iz=$a^\alpha a^{\beta i} a^{p+1-\alpha-\beta} b^p$
Suppose $a^{p+1+i\beta-\beta}b^p \in L_1$ then;
$p + 1 + i\beta - \beta > p$

$i\beta - \beta + 1 > 0$
choose i=0 and β has to greater and equal to 1: $\beta \geq 1$ then we have;
$-1 + 1 > 0$
$0 > 0$ this is a contradiction!!
Therefore; by pumping lemma theorem L_1 is NOT regular language.

If L_1 was regular, it was possible to create a deterministic finite automata (DFA) for the language. Suppose M_1 = (Q,Σ, δ, s, F) is such a DFA. Then it was possible to design a new DFA $M_1^{'}$ = (Q,Σ, δ, s,Q-F) which rejects a string w if and only if M_1 would accept w. Therefore; L($M_1^{'}$) = Σ^*− L (M_1) = Σ^*− L_1 =$\overline{L_1}$. Since it exists a DFA accepting the language L_1, then L_1 is regular if L_1 is regular. Therefore; Regular languages are closed under complement.

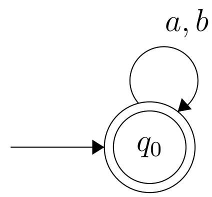
Since regularity is closed under complementation, the considered language can not be regular, because $L_2 = \overline{L_1}$
L_2 and L_1 are complement. If L_2 were regular then L_1 has to be regular.
Thus:
L_2 is NOT regular language.

b)

Suppose L_4 were regular and therefore there is a "pumping constant" p for L_4
Choose w = $a^p b^p$
Look at all decomposition's of w into xyz
a) $|xy| \leq p$ b) $|y| \geq 1$
x=a^α $\alpha \geq 0$
y=a^β $\beta \geq 1$
z=$a^{p-\alpha-\beta} b^p$
choose i such that $xy^i z \notin L_4$;
$xy^i z = a^\alpha a^{i\beta} a^{p-\alpha-\beta} b^p$
$= a^{p+i\beta-\beta} b^p$
Choose i such that $p + i\beta - \beta \neq p$
$i\beta - \beta \neq 0$
$i\beta \neq \beta$ pick i=2 then
There is an integer i such that xyz $\notin L_4$
Thus L_4 is NOT regular language.

Suppose L_5 were regular and therefore there is a "pumping constant" p for L_5
Choose w = $a^{p+1} b^p$
Look at all decomposition's of w into xyz
a) $|xy| \leq p$ b) $|y| \geq 1$
x=a^α $\alpha \geq 0$
y=a^β $\beta \geq 1$
z=$a^{p+1\alpha-\beta} b^p$

choose i such that $xy^iz \notin L_4$;
$xy^iz = a^{\alpha}a^{i\beta}a^{p-\alpha-\beta}b^p$
$=a^{p+i\beta-\beta}b^p$
Choose i such that xyz $\notin L_5$
$p + i\beta - \beta \geq 0$, $\beta \geq 1$ p ≥ 0
Then $p + i\beta - \beta \geq 0$ this expression always consistent;
Thus pumping lemme theorem does not work.
We have to check regularity whether we can construct DFA or not
If we can construct a DFA than language is regular



DFA for L_5 is above
Therefore L_5 is regular language.