Ex. 1

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28.4

1. a)

   b) The algorithm:

   - Find min cost cycle cover - denoted by $C = (c_1, \dots, c_k)$. For every $i \in [k]$, define $e_i = (u_i, v_i)$ as an edge in c_i.
   - $G \leftarrow \{(u_k, v_1)\}$
   - for $i = 1$ to $k - 1$ do:
     - $G \leftarrow G \cup (c_i \setminus \{e_i\} \cup \{u_i, v_{i+1}\})$
   - $G \leftarrow G \cup (c_i \setminus \{e_i\} \cup \{u_i, v_{i+1}\})$

   Proof. We will show :

   I G is Hamiltonian cycle.

   II cost G is at most $\frac{4}{3}OPT$.

   I We will show the edges in G admit Hamiltonian cycle. We start by v_1 and go throug edges of cycle c_1 until the node u_1 than take the edge u_1, v_2 and continue in this fashion until reaching node u_k, then taking the edge $\{(u_k, v_1)\}$ and we done,

   II $cost(C) \leq OPT$ because the optimal solution is feasible solution for the cycle cover problem. As each cycle is at least of size of 3 we have that $k \leq \frac{|V|}{3}$. G replace k edges of size at least 1 with k edges of size at most 2, then:

   $$G \leq cost(C) + k \leq cost(C) + \frac{|V|}{3} \leq OPT + \frac{|V|}{3} \leq \frac{4}{3}OPT$$

   And the last inequality is due to the fact that the optimal solution visits $|V|$ edges of weight one at least.

   $\square$

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3.

4.