Ex. 1

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28.4

1. a) Let $G = (V, E), w$ be our graph and the weight function on the edges accordingly. We will create bipartite graph $G' = (V \times \{0\}, V \times \{1\}, E')$ where,
$$E' = \{((u, 0), (v, 1)) : (u, v) \in E\}$$
With weight function $w' : E' \to R$, s.t. $w'(((u, 0), (v, 1))) = w((u, v))$. We will run weighted prefect matching and receive M. We will build the cycle cover accordingly to M, cycle by cycle. c_0 will be constructed by taking and delete an edge $((u, 0), (v, 1))$ in M and add (u, v) to c_0, go on by take $((v, 0), (w, 1))$ in M, remove it from M and add (v, w) to the cycle until we will reach a node which is matched to $(u, 1)$. By then we will finish one cycle and if there are more edges in M we will construct a new cycle c_1 and so on until there are no more edges to delete in M.
b) The algorithm:
    - Find min cost cycle cover - denoted by $C = (c_1, \ldots, c_k)$. For every $i \in [k]$, define $e_i = (u_i, v_i)$ as an edge in c_i.
    - $G \leftarrow \{(u_k, v_1)\}$
    - for $i = 1$ to $k - 1$ do:
        - $G \leftarrow G \cup (c_i \setminus \{e_i\} \cup \{u_i, v_{i+1}\})$
    - $G \leftarrow G \cup (c_i \setminus \{e_i\} \cup \{u_i, v_{i+1}\})$

Proof. We will show :

I G is Hamiltonian cycle.

II cost G is at most $\frac{4}{3}OPT$.

I We will show the edges in G admit Hamiltonian cycle. We start by v_1 and go throug edges of cycle c_1 until the node u_1 than take the edge u_1, v_2 and continue in this fashion until reaching node u_k, then taking the edge $\{(u_k, v_1)\}$ and we done,

II $cost(C) \leq OPT$ because the optimal solution is feasible solution for the cycle cover problem. As each cycle is at least of size of 3 we have that $k \leq \frac{|V|}{3}$. G replace k edges of size at least 1 with k edges of size at most 2, then:

$$G \leq cost(C) + k \leq cost(C) + \frac{|V|}{3} \leq OPT + \frac{|V|}{3} \leq \frac{4}{3}OPT$$

And the last inequality is due to the fact that the optimal solution visits $|V|$ edges of weight one at least.

$\square$

2. (a) We build MST $T = (R, E')$ on the sub graph which includes only nodes in R. Our algorithm will return T which is also a feasible solution. We will show $c(T)$ is at most $2OPT$. Let $\tilde{T} = (\tilde{V}, \tilde{E})$ be the steiner tree which has $c(\tilde{T}) = OPT$. $c(\tilde{T}) = \sum_{v \in \tilde{V}} c(v) + \sum_{e \in \tilde{E}} c(e)$. In the same way as we showed in class we can have that:

$$2 \cdot \sum_{e \in \tilde{E}} c(e) \geq \sum_{e \in E'} c(e)$$

and since $\sum_{v \in R} = 0$ we conclude:

$$c(T) = \sum_{v \in R} c(v) + \sum_{e \in E'} c(e) = \sum_{e \in \tilde{E}} c(e) \leq 2OPT$$

(b)

(c) We will run the

3.

4.