1) $X \sim \text{POISSON}(\lambda)$, GIVEN $X = n$, $Y \sim \text{BERNOULLI}(1/(n+1))$, FIND $P(X=n \mid Y=1)$

$$P(X=n \mid Y=1) = \frac{P(Y=1 \mid X=n)\,P(X=n)}{P(Y=1)}$$

$$= \frac{\frac{1}{n+1}\left(\frac{\lambda^n}{n!}e^{-\lambda}\right)}{\sum_{n=0}^{\infty} P(Y=1 \mid X=n)P(X=n)}$$

$$= \frac{\frac{1}{n+1}\left(\frac{\lambda^n}{n!}e^{-\lambda}\right)}{\sum_{n=0}^{\infty} \frac{1}{n+1}\left(\frac{\lambda^n}{n!}e^{-\lambda}\right)}$$

$$= \frac{\frac{1}{n+1}\left(\frac{\lambda^n}{n!}e^{-\lambda}\right)}{e^{-\lambda}\sum_{n=0}^{\infty} \frac{\lambda^n}{n!(n+1)}}$$

$$= \frac{\frac{1}{n+1}\left(\frac{\lambda^n}{n!}e^{-\lambda}\right)}{e^{-\lambda}\sum_{n=0}^{\infty} \frac{\lambda^n}{(n+1)!}}$$

$$= \frac{\frac{1}{n+1}\left(\frac{\lambda^n}{n!}e^{-\lambda}\right)}{e^{-\lambda}\sum_{Y=1}^{\infty} \frac{\lambda^{Y-1}}{Y!}}$$

$$= \frac{\frac{1}{n+1}\left(\frac{\lambda^n}{n!}e^{-\lambda}\right)}{\frac{e^{-\lambda}}{\lambda}\sum_{Y=1}^{\infty} \frac{\lambda^Y}{Y!}}$$

= TAYLOR SERIES EXPANSION W/OUT x_0 (e^x)

$$\frac{\frac{1}{n+1}\left(\frac{\lambda^n}{n!}e^{-\lambda}\right)}{e^{-\lambda}\left(\frac{e^{\lambda}-1}{\lambda}\right)}$$

SIMPLIFY

$$P(X=n \mid Y=1) = \frac{\lambda^{n+1}e^{-\lambda}}{(n+1)!\,(1-e^{-\lambda})}$$

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2) A) FIND PROB. THAT Y FAILS IN 2 MONTHS

$Y \sim \exp(\mu)$

$$P(Y \leq 2) = \int_0^2 \mu e^{-\mu x} dx$$

$$= \left[-e^{-\mu x}\right]_0^2 = \left[-e^{-2\mu} - (-1)\right]$$

$$\boxed{P(Y \leq 2) = 1 - e^{-2\mu}}$$

B) FIND PROB THAT BOTH X & Y FAIL WITHIN A YEAR

$$P(X \leq 12 \cap Y \leq 12) = P(X \leq 12)P(Y \leq 12) \quad X \sim \exp(\lambda)$$

INDEPENDENT

$P(X \leq 12 \cap Y \leq 12)$ = FOLLOWING FORM SOLVED IN A)

$$(1 - e^{-12\lambda})(1 - e^{-12\mu})$$

C) FIND PROB. EITHER X OR Y FAIL WITHIN A YEAR

$$P(X \leq 12 \cup Y \leq 12) = 1 - P(X > 12 \cap Y > 12)$$

$$= 1 - P(X > 12)P(Y > 12)$$

$$= 1 - e^{-12\lambda} e^{-12\mu}$$

$$\boxed{= 1 - e^{-12(\lambda - \mu)}}$$

$$P(X > 12) = \int_{12^-}^{\infty} \lambda e^{-\lambda x} dx$$

$$= \left[-e^{-\lambda x}\right]_{12^-}^{\infty}$$

$$= 0 - (-e^{-12\lambda})$$

$$= e^{-12\lambda}$$

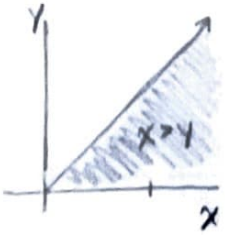
D) FIND PROB IF X FAILS AFTER Y. GIVEN $1/\mu = 48$ & $1/\lambda = 24$

SINCE INDEPENDENT $f_{xy}(x,y) = f_x(x) f_y(y)$

$$= \lambda e^{-\lambda x} \mu e^{-\mu y}$$

$$= \lambda\mu e^{-\lambda x} e^{-\mu y}$$

$$P(X>Y) = \int_{Y=0}^{\infty} \int_{X=Y}^{\infty} \lambda\mu e^{-\lambda x} e^{-\mu y} \, dx\, dy$$



$$= \int_{Y=0}^{\infty} \mu e^{-(\lambda-\mu)y} \, dy$$

$$= \frac{\mu}{-(\lambda+\mu)} e^{-(\lambda+\mu)y} \Big]_{Y=0}^{\infty}$$

$$= \frac{\mu}{(\lambda+\mu)}$$

$$= \frac{(1/48)}{(1/24 + 1/48)}$$

$$\boxed{P(X>Y) = 1/3}$$

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3) FIND PROB AT LEAST ONE FLIGHT A DAY MAKES MONEY

$$X \sim \exp(\lambda) \quad E[X] = 1/\lambda = 20 \Big\} \; X \sim \exp(1/20)$$

$$P\left(\bigcup_{i=1}^{5}(X_i > 25)\right) = 1 - P\left(\bigcap_{i=1}^{5}(X_i \le 25)\right)$$

$$= \text{INDEPENDENT}\quad 1 - \prod_{i=1}^{5} P(X_i \le 25)$$

$$= 1 - (1 - e^{-5/4})^5$$

$$\boxed{P\left(\bigcup_{i=1}^{5}(X_i > 25)\right) = 0.815}$$

$$P(X \le 25) = \int_0^{25} 1/20\, e^{-x/20}\, dx$$

$$= \left[-e^{-x/20}\right]_0^{25}$$

$$= \left(-e^{-25/20} - (-1)\right)$$

$$= 1 - e^{-5/4}$$

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4) SHOW MOMENT GENERATING FUNCTION OF Y IS IDENTICAL TO $X \sim EXP(1)$

$$M_X = \int_{-\infty}^{\infty} e^{sx} e^{-x}\, dx = \int_0^{\infty} e^{(s-1)x} dx = \left[\frac{e^{(s-1)x}}{s-1} \right]_0^{\infty}$$

$$M_X = \frac{1}{1-s}$$

$X \sim UNIFORM(0,1) \quad Y = \ln(1/x) = -\ln(x)$

$$M_Y = \int_{-\infty}^{\infty} e^{sy} f_x(x)\, dx = \int_{-\infty}^{\infty} e^{-s\ln x} f_x(x)\, dx$$

$$= f_x(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & \text{ELSE} \end{cases}$$

$$\int_0^1 x^{-s} \cdot 1\, dx$$

$$= \frac{x^{(-s+1)}}{(-s+1)} \Big]_0^1$$

$$= \frac{1 - 0}{1-s}$$

$$M_Y = \frac{1}{1-s}$$

$$\boxed{M_Y = M_X \Rightarrow Y \sim X \sim exp(1)}$$

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5) A) $X \sim EXP(1)$, COMPUTE MARKOV INEQUALITY, CHEBYSHEV BOUND, CHERNOFF BOUND ON $P(X \geq a)$. FIND $P(X \geq a)$

MARKOV INEQUALITY

$$E\{X\} = 1/\lambda = 1$$

$$P(X \geq a) \leq \frac{E\{X\}}{a} = \frac{1}{a} \Rightarrow \boxed{P(X \geq a) \leq \frac{1}{a}}$$

CHEBYSHEV INEQUALITY

$$E\{X^2\} = \frac{n!}{\lambda^n} = 2$$

$$P(X \geq a) \leq P(|X - E\{X\}| \geq a) = \frac{E\{X^2\}}{a}$$

$$\boxed{P(X \geq a) \leq \frac{2}{a^2}}$$

CHERNOFF BOUND

$$P(X \geq a) \leq \min_{s \geq 0} \left[e^{-as} \overbrace{E\{e^{sx}\}} \right]$$

M.G.F SOLVED IN QUESTION #4

$$\leq \min_{s \geq 0} \left[e^{-as} \left(\frac{1}{1-s} \right) \right]$$

$y(s)$

$$\frac{dy(s)}{ds} = -ae^{-as}\left(\frac{1}{1-s}\right) + e^{-as}\left(\frac{1}{(1-s)^2}\right) = 0$$

$$\frac{-a}{1-s} + \frac{1}{(1-s)^2} = 0$$

$$\frac{-a + as + 1}{(1-s)^2} = 0$$

$$s = \frac{a-1}{a} = 1 - \frac{1}{a}$$

$$P(X \geq a) \leq y\left(\frac{a-1}{a}\right) = e^{-a+1}\left(\frac{1}{1/a}\right)$$

$$\boxed{P(X \geq a) = ae^{-a+1}}$$

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P(X≥a) TRUE

$$P(X \geq a) = \int_a^{\infty} e^{-x} dx = \left[-e^{-x} \right]_a^{\infty}$$

$$= 0 - (-e^{-a})$$

$$\boxed{P(X \geq a) = e^{-a}}$$

WHERE DOES MARKOV < CHEBYSHEV

$$\frac{1}{a} = \frac{2}{a^2}$$

$$a = 2$$

$$\boxed{0 < a \leq 2}$$

B) MATLAB PLOTS SHOWN ON NEXT PAGE

TYPE	RANGE OF a: $0 \leq a \leq 6$	$6 \leq a \leq 20$
MARKOV	(0, 2)	N/A
CHEBYSHEV	(2, ~5.34)	N/A
CHERNOFF	(~5.34, 6]	[6.0, 20] *SMALLEST*

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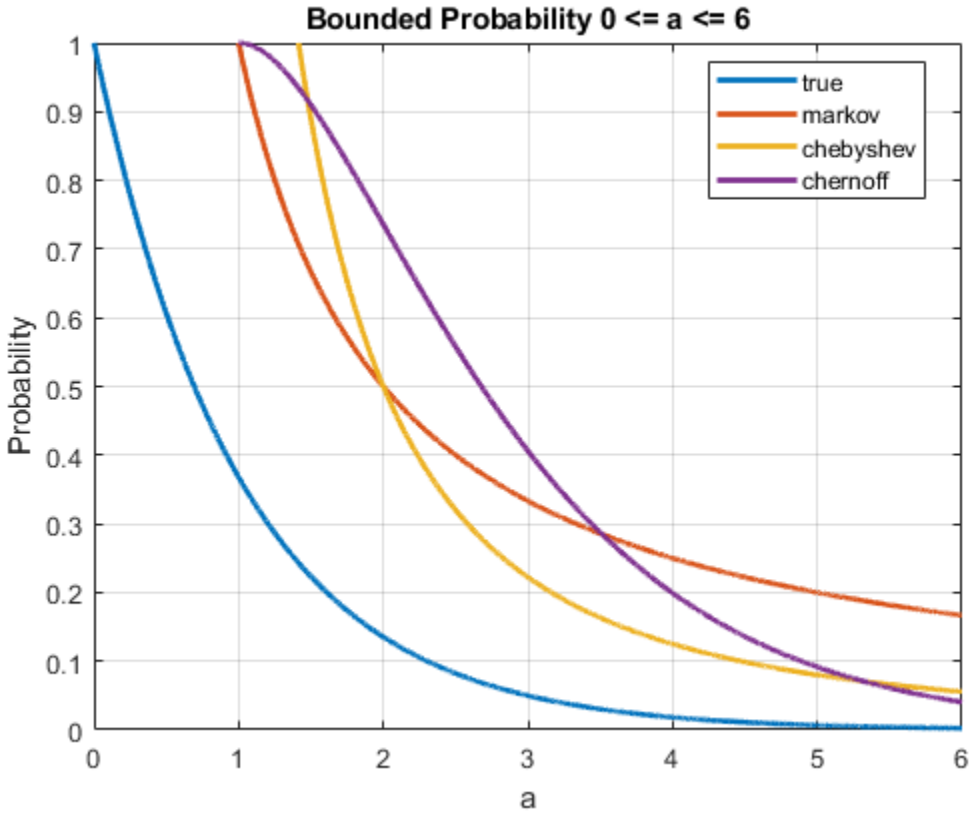
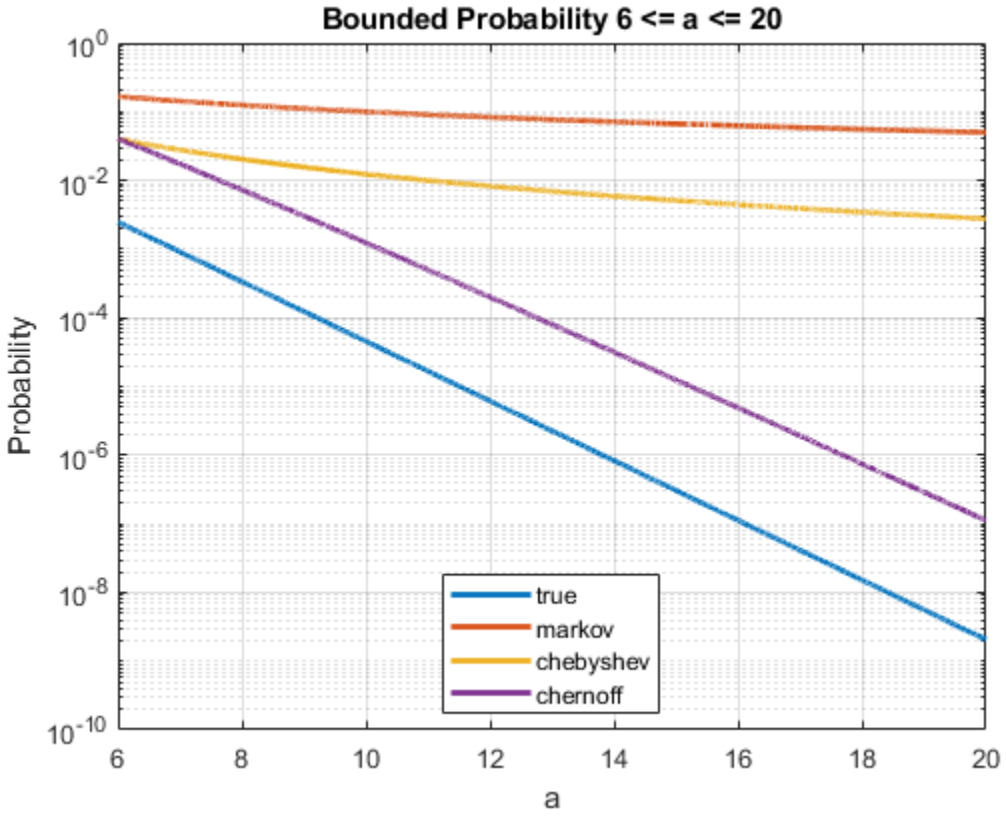
given

0 <= a <= 6



6 <= a <= 20



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(6) IF X & Y ARE INDEPENDENT ~ EXP(λ), FIND $E[\max(X,Y)]$

CDF. $Z = \max(X,Y)$

$F_Z(z) = P(\max(X,Y) \le z) = P(X \le z \cap Y \le z)$

$= \text{INDEPENDENT}$

$P(X \le z)\, P(X \le z)$

$= (1 - e^{-\lambda z})(1 - e^{-\lambda z})$

$F_z(z) = (1 - e^{-\lambda z})^2$

$\frac{dF_z(z)}{dz} = f_z(z) = 2(-\lambda)(-e^{\lambda z})(1 - e^{-\lambda z})$

$f_z(z) = 2\lambda e^{-\lambda z}(1 - e^{-\lambda z}) \quad z \ge 0$

$E[Z] = \int_0^\infty z f_z(z)\, dz = \int_0^\infty z(2\lambda e^{-\lambda z}(1 - e^{-\lambda z}))\, dz$

$= 2\lambda \left[\int_0^\infty z e^{-\lambda z} dz - \int_0^\infty z e^{-2\lambda z} dz \right]$

* SOLVED W/ WOLFRAM ALPHA *

$= 2\lambda \left[1/\lambda^2 - 1/4\lambda^2 \right]$

$= 2\lambda \left[3/4\lambda^2 \right]$

$$\boxed{E[\max(X,Y)] = \frac{3}{2}\lambda}$$