ELEC 7410 Homework Assignment #5 Oct. 14, 2025

(6 problems, Due: Oct. 21, 2025)

Any late homework submission will incur 15% penalty per day, with any fraction of a day counted as full day.

1. (10 points) (**Transformations of Random Vectors**) Problem 21, Chapter 8, Gubner.

   21. Let X and Y have joint density $f_{XY}(x,y)$. Let $U := X+Y$ and $V := X-Y$. Find $f_{UV}(u,v)$.

2. (10 points) (**Transformations of Random Vectors**) Problem 23, Chapter 8, Gubner.

   23. Let X and Y be independent Laplace(λ) random variables. Put $U := X$ and $V := Y/X$. Find $f_{UV}(u,v)$ and $f_V(v)$. Compare with Problem 33(c) in Chapter 7.

3. (10 points) (**Linear Estimation of Random Vectors**) Problem 28, Chapter 8, Gubner.

   28. Let X and W be independent $N(0,1)$ random variables, and put $Y := X^3 + W$. Find A and b that minimize $\mathsf{E}[|X-\widehat{X}|^2]$, where $\widehat{X} := AY + b$.

4. (10 points) (**Linear Estimation of Random Vectors**) Problem 29, Chapter 8, Gubner.

   29. Let $X \sim N(0,1)$ and $W \sim$ Laplace(λ) be independent, and put $Y := X+W$. Find the linear MMSE estimator of X based on Y.

5. (10 points) (**Definition of Multivariate Gaussian**) Problem 3, Chapter 9, Gubner.

   3. Let $X \sim N(0,1)$ and put $Y := 3X$.

      (a) Show that X and Y are jointly Gaussian.

      (b) Find their covariance matrix, $\mathsf{cov}([X,Y]')$.

      (c) Show that they are not jointly continuous. *Hint:* Show that the conditional cdf of Y given $X = x$ is a unit-step function, and hence, the conditional density is an impulse.

6. (10 points) (**Characteristic Function**) Problem 11, Chapter 9, Gubner.

   11. Let X be a random vector with joint characteristic function $\varphi_X(\nu) = e^{j\nu' m - \nu' C \nu/2}$. For any coefficients a_i, put $Y := \sum_{i=1}^n a_i X_i$. Show that $\varphi_Y(\eta) = \mathsf{E}[e^{j\eta Y}]$ has the form of the characteristic function of a scalar Gaussian random variable.