ELEC 7410 Solution: Homework Assignment #1 Aug. 28, 2025

1. (**Modification of Problem 4 in Gubner, Chapter 1.**) With tower as the origin, in Cartesian (x-y) coordinates, source location is (x, y), and it is (r, θ) in polar coordinates with $r = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}(y/x)$ where $-\infty < x, y < \infty$, $0 \le r < \infty$ and $0 \le \theta < 2\pi$.

   (a) $\Omega = \{(x, y) \ : \ x^2 + y^2 \le 10^2\}$ or $\Omega = \{(r, \theta) \ : \ r \le 10\}$.

   (b) $\{(x, y) \ : \ 2^2 \le x^2 + y^2 \le 5^2\}$ or $\{(r, \theta) \ : \ 2 \le r \le 5\}$.

2. (**Problem 7 in Gubner, Chapter 1.**)

   (a) $[1, 4] \cap \Big([0, 2] \cup [3, 5]\Big) = \Big([1, 4] \cap [0, 2]\Big) \cup \Big([1, 4] \cap [3, 5]\Big) = [1, 2] \cup [3, 4]$.

   (b)
$$\begin{aligned}
\Big([0, 1] \cup [2, 3]\Big)^c &= [0, 1]^c \cap [2, 3]^c = \Big((-\infty, 0) \cup (1, \infty)\Big) \cap \Big((-\infty, 2) \cup (3, \infty)\Big) \\
&= \Big((-\infty, 0) \cap \big((-\infty, 2) \cup (3, \infty)\big)\Big) \cup \Big((1, \infty) \cap \big((-\infty, 2) \cup (3, \infty)\big)\Big) \\
&= (-\infty, 0) \cup (1, 2) \cup (3, \infty)\,.
\end{aligned}$$

   (c) $\bigcap_{n=1}^{\infty} \Big(-\frac{1}{n}\,,\ \frac{1}{n}\Big) = \{0\}$.

   (d) $\bigcap_{n=1}^{\infty} \Big[0\,,\ 3 + \frac{1}{2n}\Big) = [0, 3]$.

   (e) $\bigcup_{n=1}^{\infty} \Big[5\,,\ 7 - \frac{1}{3n}\Big] = [5, 7)$.

   (f) $\bigcup_{n=1}^{\infty} \Big[0, n\Big] = [0, \infty)$.

3. $P(A) = 0.4,\ P(B) = 0.8,\ P(A \cup B) = 0.92\,.$

   (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.4 + 0.8 - 0.92 = 0.28\,.$

   (b) We have $B = \Omega \cap B = (A \cup A^c) \cap B = (A \cap B) \cup (A^c \cap B)$. Since $(A \cap B) \cap (A^c \cap B) = \emptyset$, $P(B) = P(A \cap B) + P(A^c \cap B) \Rightarrow 0.8 = 0.28 + P(A^c \cap B)$, therefore, $P(A^c \cap B) = 0.52\,.$

   (c) $P(A^c \cup B) = P(A^c) + P(B) - P(A^c \cap B) = (1 - 0.4) + 0.8 - 0.52 = 0.88\,.$

   (d) We have $A \cap (B \cup A^c) = (A \cap B) \cup (A \cap A^c) = (A \cap B) \cup \emptyset = A \cap B$. Therefore, $P\big(A \cap (B \cup A^c)\big) = P(A \cap B) = 0.28\,.$

4. (**Similar to Problem 54 in Gubner, Chapter 1.**)

   (a) $P(MM) = \frac{130}{70+130} = 0.65.$
   $P(HT) = \frac{70}{70+130} = 0.35 = 1 - P(MM).$

   (b)
$$\begin{aligned}
P(D) = &\underbrace{P(D|MM)}_{0.04}\underbrace{P(MM)}_{0.65} + \underbrace{P(D|HT)}_{0.08}\underbrace{P(HT)}_{0.35} \\
= &0.026 + 0.028 = 0.054
\end{aligned}$$

(c)

$$P(MM|D) = \frac{P(D|MM)P(MM)}{P(D)} = \frac{0.04 \times 0.65}{0.054} = \frac{13}{27} = 0.4815\,.$$

5. (**Similar to Problem 64 in Gubner, Chapter 1**.)

 (a)

$$\begin{aligned} &P\big(\{\text{Exactly one functions }\}\big) \\ &\quad = P\big(\{\text{Airbag 1 fails, Airbag 2 functions }\} \cup \{\text{Airbag 1 functions, Airbag 2 fails}\}\big) \\ &\quad = P\big(\{\text{Airbag 1 fails, Airbag 2 functions }\}\big) + P\big(\{\text{Airbag 1 functions, Airbag 2 fails}\}\big) \\ &\quad = p(1-p) + (1-p)p = 2p(1-p) \end{aligned}$$

 (b)

$$\begin{aligned} P\big(\{\text{None functions }\}\big) &= P\big(\{\text{Airbag 1 fails, Airbag 2 fails}\}\big) \\ &= P\big(\{\text{Airbag 1 fails}\}\big)P\big(\{\text{Airbag 2 fails}\}\big) = p^2 \end{aligned}$$

 (c) $P\big(\{\text{At least one functions }\}\big) = 1 - P\big(\{\text{None functions }\}\big) = 1 - p^2\,.$

6. (**Similar to Problem 67 in Gubner, Chapter 1**.) Let A and B denote the events that Anne and Betty, respectively, catch some fish. Then A^c and B^c denote the events that Anne and Betty, respectively, catch no fish. We are given $P(A^c) = P(B^c) = p$ and therefore, $P(A) = P(B) = 1 - p$. Moreover, $P(A \cap B) = P(A)P(B)$ and $P(A^c \cap B^c) = P(A^c)P(B^c)$ since Anne and Betty catch fish independently.

 (a) Find the probability that none of them catches any fish.

$$P\big(\{\text{None of them catches any fish }\}\big) = P(A^c \cap B^c) = P(A^c)P(B^c) = p^2$$

 (b) Find the probability that at least one of them catches some fish (one or more).

$$\begin{aligned} &P\big(\{\text{at least one of them catches some fish }\}\big) \\ &\quad = P\big(A \cup B\big) = PA) + P(B) - P(A \cap B) = (1-p) + (1-p) - (1-p)^2 = 1 - p^2 \\ &\quad = 1 - P\big(\{\text{None of them catches any fish }\}\big) \end{aligned}$$

 (c) Find the conditional probability that Anne catches no fish given that at least one of them catches no fish.

$$\begin{aligned} P\big(A^c \,|\, A^c \cup B^c\big) &= \frac{P\big(A^c \cap (A^c \cup B^c)\big)}{P\big(A^c \cup B^c\big)} \\ &= \frac{P\big(A^c\big)}{1 - P\big(A \cap B\big)} \quad \text{since } A^c \subset (A^c \cup B^c) \\ &= \frac{p}{1-(1-p)^2} = \frac{p}{p(2-p)} = \frac{1}{2-p}\,. \end{aligned}$$