

ELEC 7410 Homework Assignment #1 Aug. 21, 2025

(6 problems, Due: Aug. 28, 2025)

Any late homework submission will incur 15% penalty per day, with any fraction of a day counted as full day.

1. (10 points) (**Sample Spaces, outcomes, and events**) Modification of Problem 4 in Gubner, Chapter 1.

A cell-phone tower has a circular coverage area of radius 10 km. We observe the source locations of calls received by the tower.

- (a) Suggest a sample space to model all possible source locations of calls that the tower can receive. (Express location in both Cartesian and polar coordinates with tower location as the origin.)
- (b) Using your sample space from part (a) (both versions), what is the event that the source location of a call is between 2 and 5 km from the tower?

2. (10 points) (**Review of set notation**) Problem 7 in Gubner, Chapter 1.

Let Ω denote the set of real numbers, $\Omega = \mathbb{R} = (-\infty, \infty)$.

- (a) Use the distributive law to simplify $[1, 4] \cap ([0, 2] \cup [3, 5])$.

- (b) Use the De Morgan's law to simplify $([0, 1] \cup [2, 3])^c$.

- (c) Simplify $\bigcap_{n=1}^{\infty} \left(-\frac{1}{n}, \frac{1}{n}\right)$.

- (d) Simplify $\bigcap_{n=1}^{\infty} \left[0, 3 + \frac{1}{2n}\right)$.

- (e) Simplify $\bigcup_{n=1}^{\infty} \left[5, 7 - \frac{1}{3n}\right]$.

- (f) Simplify $\bigcup_{n=1}^{\infty} [0, n]$.

3. (10 points) (**Axioms and Properties of Probability**) Let A and B be two events (sets) such that

$$P(A) = 0.4, \quad P(B) = 0.8, \quad P(A \cup B) = 0.92$$

- (a) Find $P(A \cap B)$.
- (b) Find $P(A^c \cap B)$.
- (c) Find $P(A^c \cup B)$.
- (d) Find $P(A \cap (B \cup A^c))$.

4. (10 points) (**Conditional Probability**) Similar to Problem 54 in Gubner, Chapter 1.

The university buys workstations from two different suppliers, Mini Micros (MM) and Highest Technology (HT). On delivery, 4% of MM's workstations are defective, while 8% of HT's workstations are defective. The university buys 130 MM workstations and 70 HT workstations for its computer lab. Suppose you walk into the computer lab and randomly sit down at a workstation.

- (a) What is the probability that your workstation is from MM? From HT?
 - (b) What is the probability that your workstation is defective?
 - (c) Given that your workstation is defective, what is the (conditional) probability that it came from Mini Micros?
5. (10 points) (**Independence**) Similar to Problem 64 in Gubner, Chapter 1.
A new car is equipped with dual airbags. Suppose that they fail independently with probability p .
- (a) What is the probability that **exactly** one airbag functions properly?
 - (b) What is the probability that **neither** airbag functions properly?
 - (c) What is the probability that **at least** one airbag functions properly?
6. (10 points) (**Independence**) Similar to Problem 67 in Gubner, Chapter 1.
Anne and Betty go fishing. Assume they catch fish independently and that each has probability p ($0 < p < 1$) of catching no fish.
- (a) Find the probability that none of them catches any fish.
 - (b) Find the probability that at least one of them catches some fish (one or more).
 - (c) Find the conditional probability that Anne catches no fish given that at least one of them catches no fish.