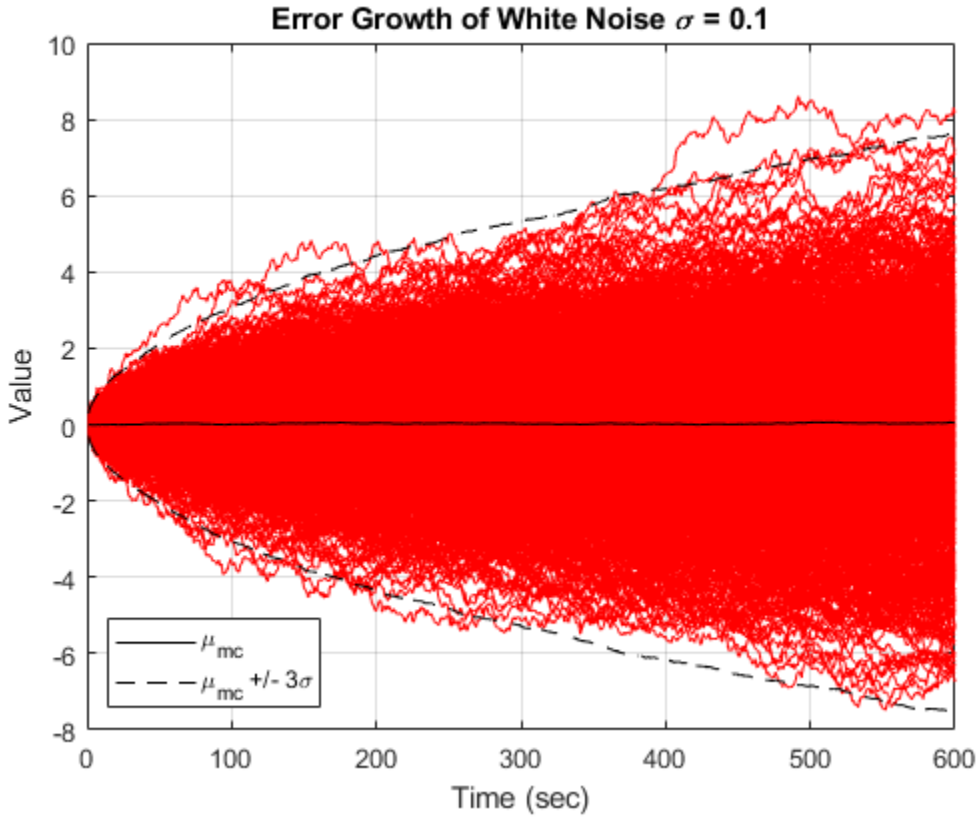
Table of Contents

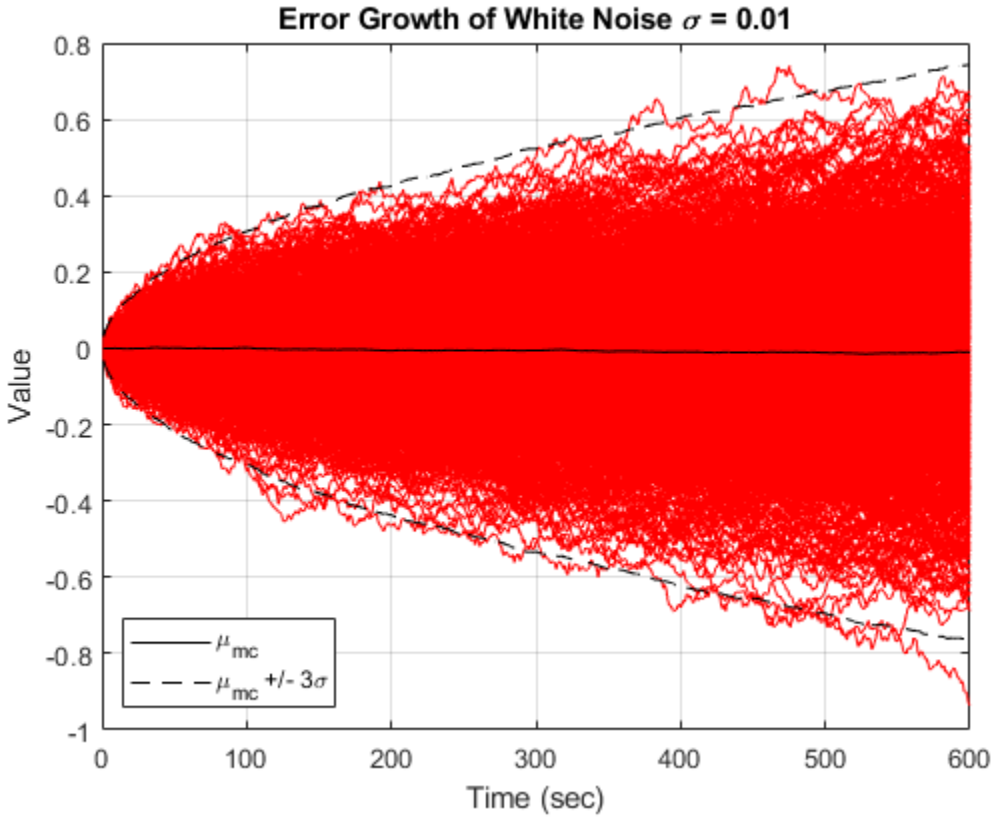

```
clear;clc;close all
```

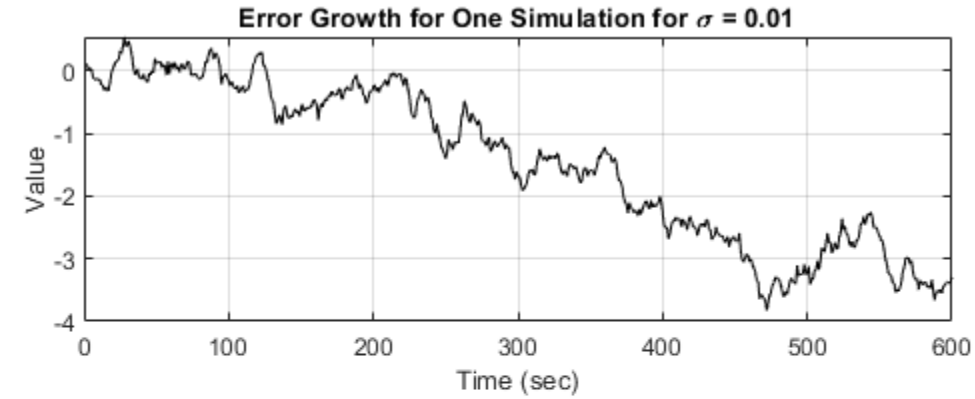
Question 2 - Part A

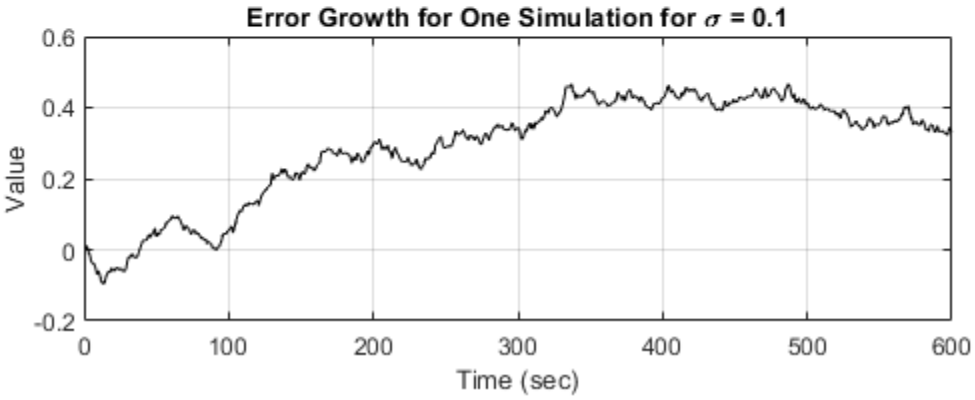
```
sims = 1000;
time = 600;

sigma_1 = 0.1;
sigma_2 = 0.01;

for i = 1 : sims
    x1(i,:) = sigma_1*randn(1);
    x2(i,:) = sigma_2*randn(1);
    for t = 1:time-1
        x1(i,t+1) = sigma_1*randn(1) + x1(i,t);
        x2(i,t+1) = sigma_2*randn(1) + x2(i,t);
    end
end

MC_mean1 = mean(x1);
MC_var1 = var(x1);

MC_mean2 = mean(x2);
MC_var2 = var(x2);

plot(1:time,x1,'r')
hold on
grid on
mu = plot(1:time,MC_mean1,'k');
sig = plot(1:time,MC_mean1+3*sqrt(MC_var1),'k--');
plot(1:time,MC_mean1-3*sqrt(MC_var1),'k--')
title("Error Growth of White Noise \sigma = 0.1")
legend([mu,sig], "\mu_{mc}", "\mu_{mc} +/- 3\sigma","Location","southwest")
xlabel("Time (sec)")
ylabel("Value")

figure
plot(1:time,x2,'r')
hold on
grid on
mu = plot(1:time,MC_mean2,'k');
sigma = plot(1:time,MC_mean2+3*sqrt(MC_var2),'k--');
plot(1:time,MC_mean2-3*sqrt(MC_var2),'k--')
```

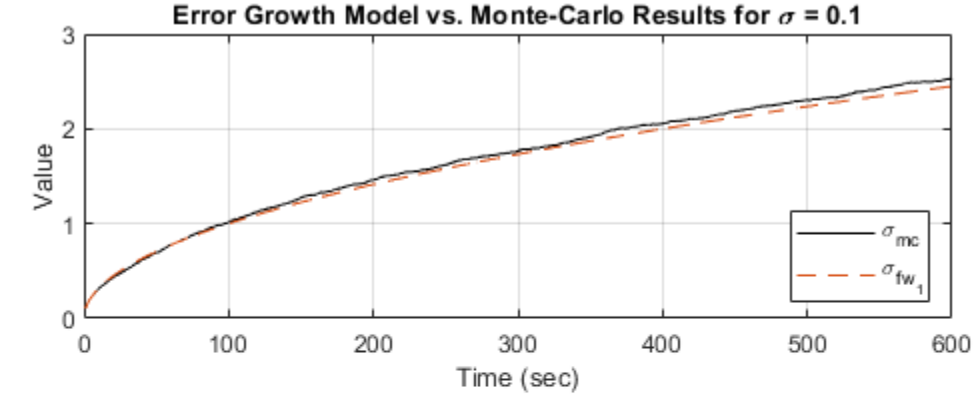
```
title("Error Growth of White Noise \sigma = 0.01")
legend([mu,sig], "\mu_{mc}", "\mu_{mc} +/- 3\sigma","Location","southwest")
xlabel("Time (sec)")
ylabel("Value")

figure
subplot(2,1,1)
plot(1:600,x1(1,:),"k")
grid on
title("Error Growth for One Simulation for \sigma = 0.01")
xlabel("Time (sec)")
ylabel("Value")
subplot(2,1,2)
plot(1:600,x2(1,:),"k")
grid on
title("Error Growth for One Simulation for \sigma = 0.1")
xlabel("Time (sec)")
ylabel("Value")


sigma_f1 = sigma_1*1*sqrt([1:600]);
sigma_f2 = sigma_2*1*sqrt([1:600]);

figure
subplot(2,1,1)
plot(1:600,sqrt(MC_var1),"k")
hold on
grid on
plot(1:600,sigma_f1,"--")
title("Error Growth Model vs. Monte-Carlo Results for \sigma = 0.1")
legend(["\sigma_{mc}" "\sigma_f_{w_1}"],"Location","southeast")
xlabel("Time (sec)")
ylabel("Value")
subplot(2,1,2)
plot(1:600,sqrt(MC_var2),"k")
hold on
grid on
plot(1:600,sigma_f2,"--")
title("Error Growth Model vs. Monte-Carlo Results for \sigma = 0.01")
legend(["\sigma_{mc}" "\sigma_f_{w_2}"],"Location","southeast")
xlabel("Time (sec)")
ylabel("Value")
```

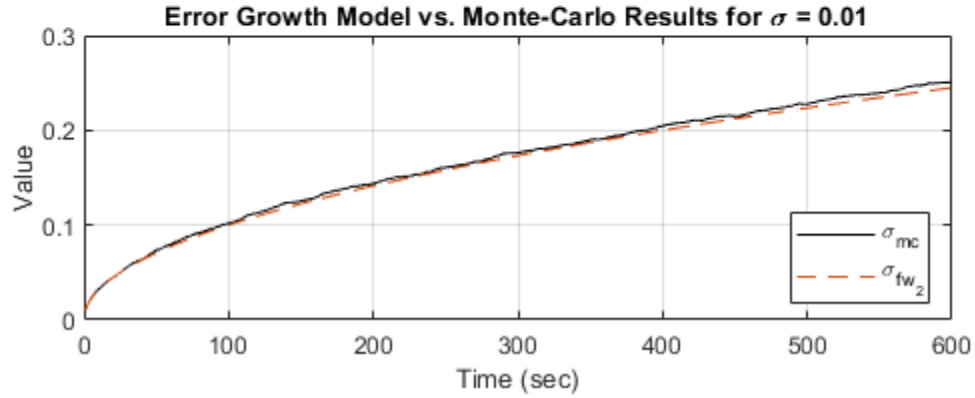

Error Growth of White Noise σ = 0.1
Value
Time (sec)
μ_{mc}
μ_{mc} +/- 3σ


Error Growth of White Noise σ = 0.01
Value
Time (sec)
μ_{mc}
μ_{mc} +/- 3σ


Error Growth for One Simulation for σ = 0.01
Value
Time (sec)

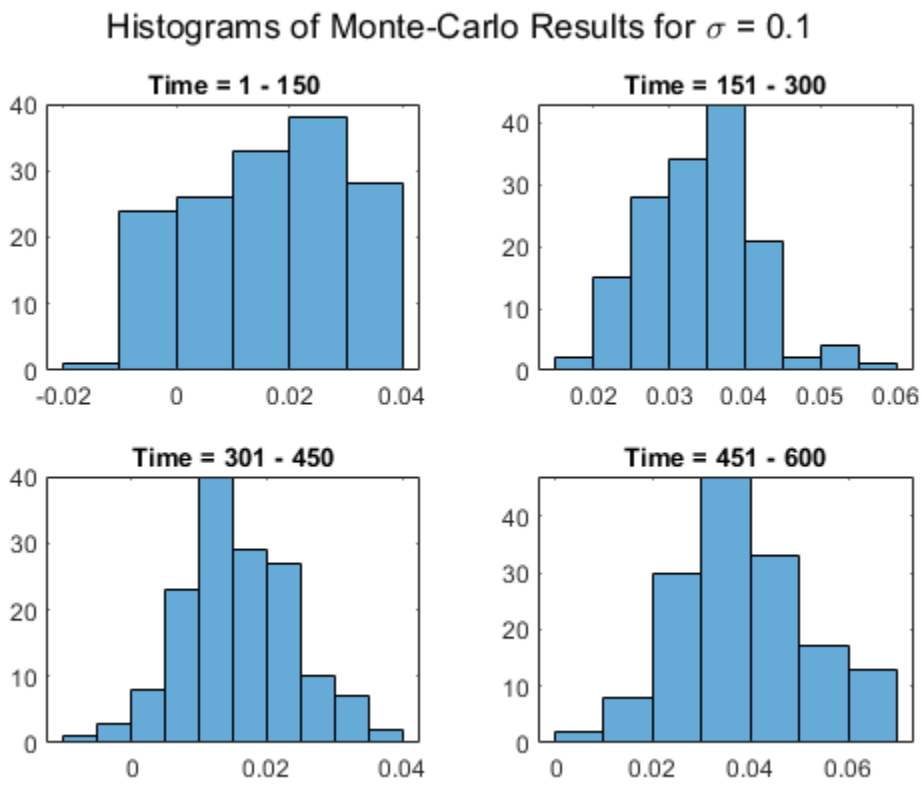

Error Growth for One Simulation for σ = 0.1
Value
Time (sec)

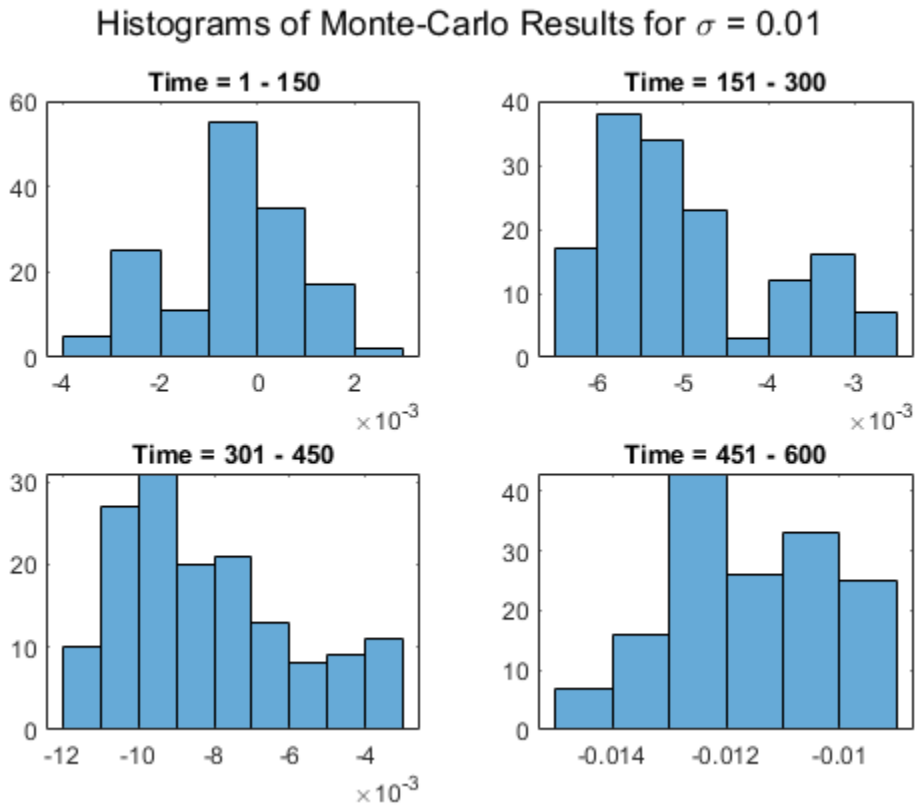

Error Growth Model vs. Monte-Carlo Results for σ = 0.1
Value
Time (sec)
σ_{mc}
σ_{fw_1}


Error Growth Model vs. Monte-Carlo Results for σ = 0.01
Value
Time (sec)
σ_{mc}
σ_{fw_2}

Question 2 - Part B

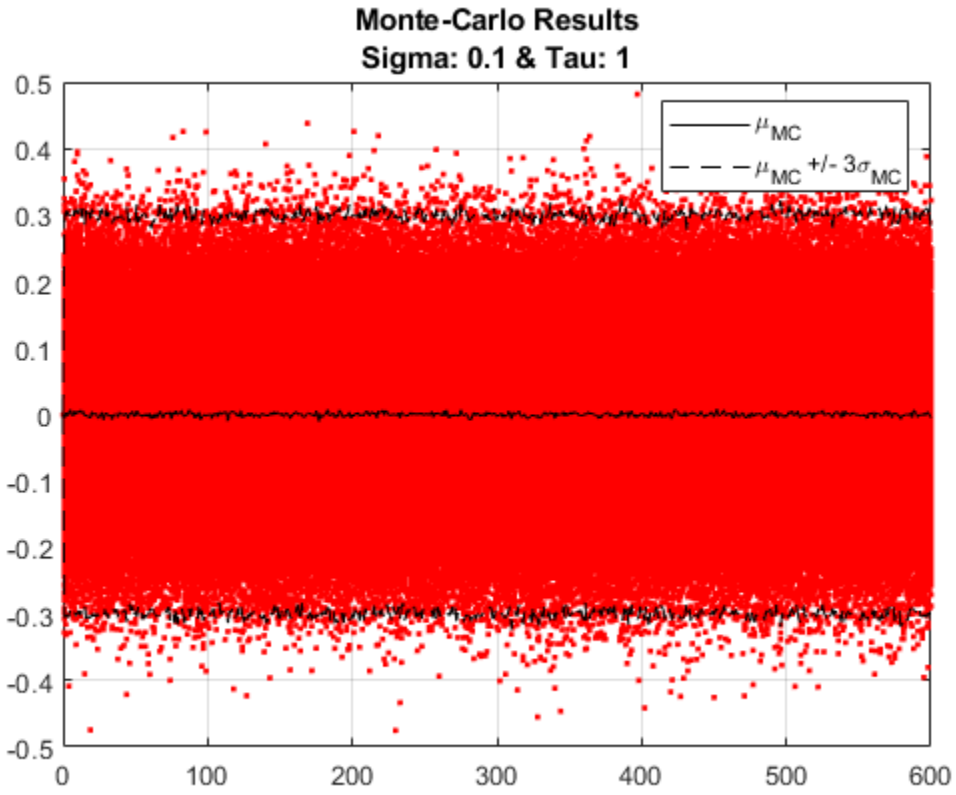
```
figure
sgtitle("Histograms of Monte-Carlo Results for \sigma = 0.1")
subplot(2,2,1)
histogram(MC_mean1(1:150))
title("Time = 1 - 150")
subplot(2,2,2)
histogram(MC_mean1(151:300))
title("Time = 151 - 300")
subplot(2,2,3)
histogram(MC_mean1(301:450))
title("Time = 301 - 450")
subplot(2,2,4)
histogram(MC_mean1(451:600))
title("Time = 451 - 600")

figure
sgtitle("Histograms of Monte-Carlo Results for \sigma = 0.01")
subplot(2,2,1)
histogram(MC_mean2(1:150))
title("Time = 1 - 150")
subplot(2,2,2)
histogram(MC_mean2(151:300))
title("Time = 151 - 300")
subplot(2,2,3)
histogram(MC_mean2(301:450))
title("Time = 301 - 450")
subplot(2,2,4)
histogram(MC_mean2(451:600))
title("Time = 451 - 600")
```

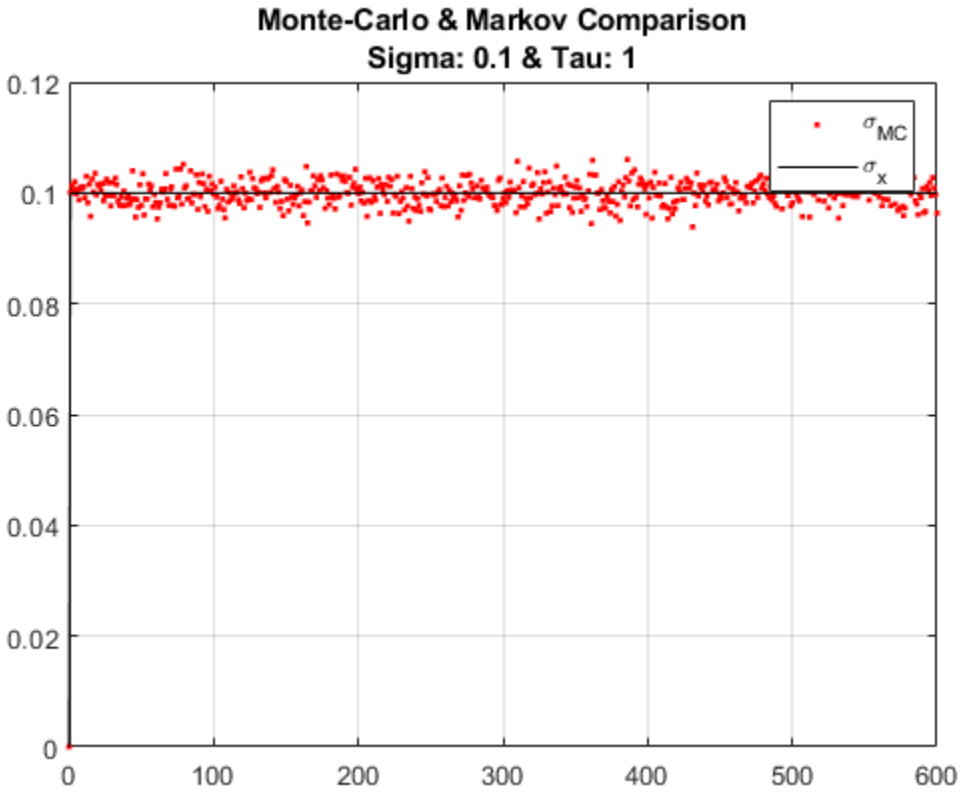

Histograms of Monte-Carlo Results for σ = 0.1
Time = 1 - 150
Time = 151 - 300
Time = 301 - 450
Time = 451 - 600

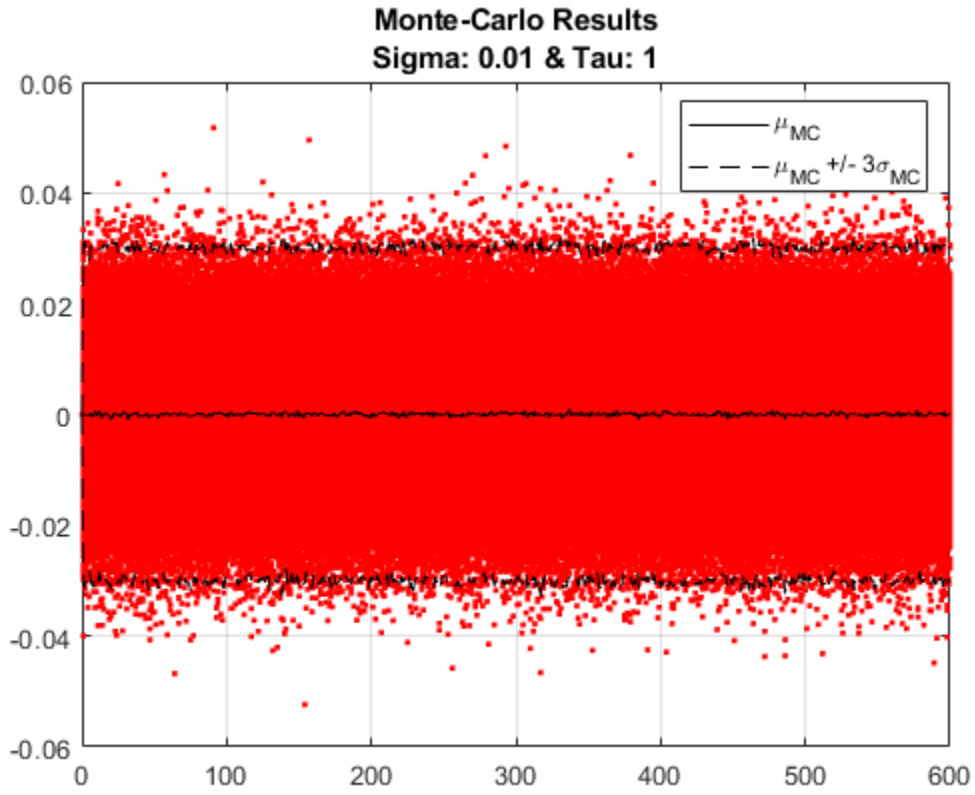

Histograms of Monte-Carlo Results for σ = 0.01
Time = 1 - 150
Time = 151 - 300
Time = 301 - 450
Time = 451 - 600

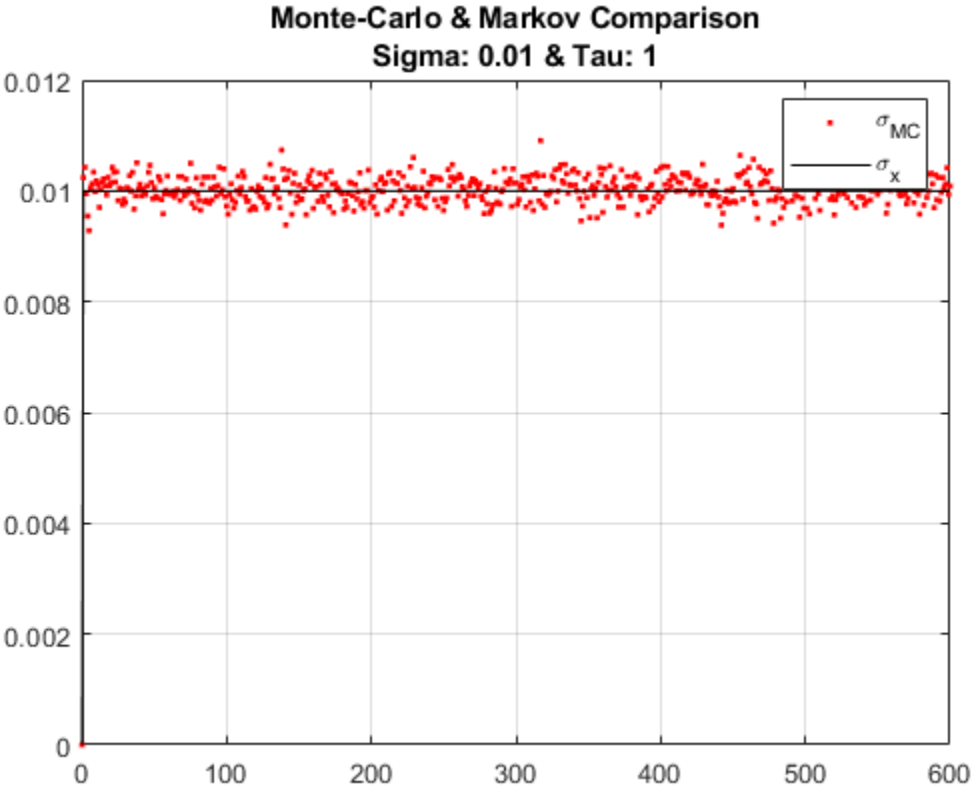
Question 2 - Part C

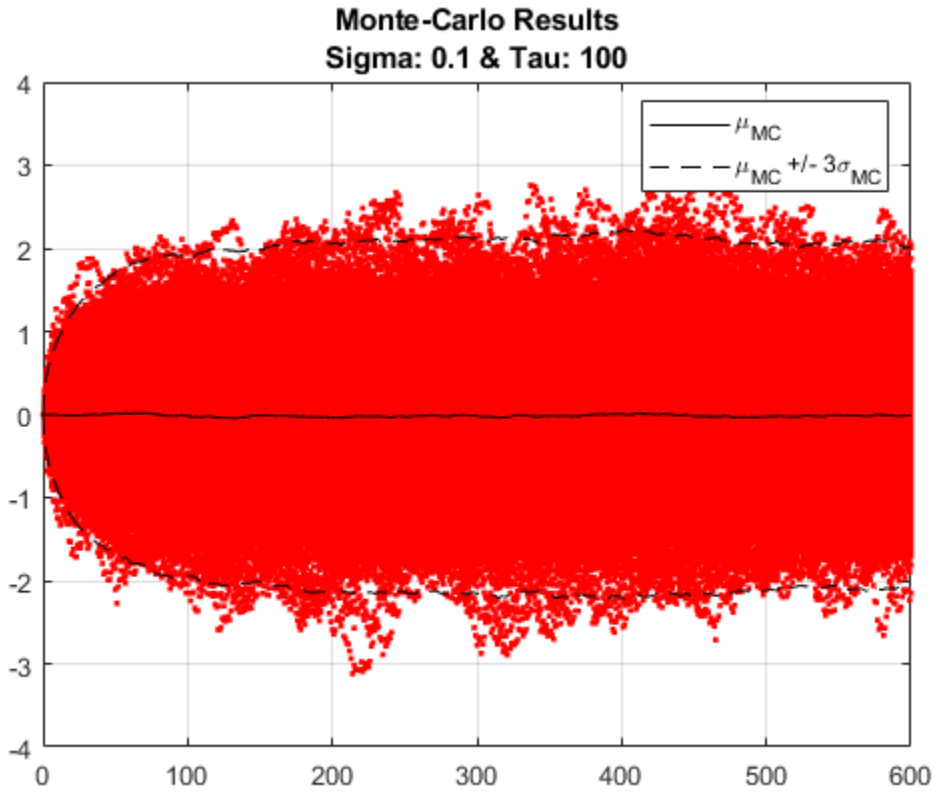
```
tau = [1 100];
sigma = [0.1 0.01];
sims = 1000;
time = 600;
dt = 1;

for i = 1 : length(tau)
    for j = 1 : length(sigma)
        for idx = 1 : sims

            x(1,idx) = 0;

            for t = 1 : time/dt

                x_dot(t,idx) = -(1/tau(i))*x(t,idx) + sigma(j)*randn(1);
                x(t+1,idx)   = x(t,idx) + x_dot(t,idx)*dt;

            end
        end

        X(i,j) = {x};
        MC_mean(i,j) = {mean(x')};
        MC_std(i,j)  = {std(x')};

        figure
        plot(0:dt:time,x,'.r')
        hold on
        grid on
        MU  = plot(0:dt:time,mean(x'),"k");
        SIG = plot(0:dt:time,mean(x') + 3*std(x'),"k--");
        plot(0:dt:time,mean(x') - 3*std(x'),"k--")
        title({"Monte-Carlo Results","Sigma: " + string(sigma(j)) + " & Tau:
" + string(tau(i))})
        legend([MU,SIG],["\mu_{MC}","\mu_{MC} +/- 3\sigma_{MC}"])

        A = (1 - dt/tau(i));
        sig_x = sigma(j)*dt*sqrt((A.^(2*(0:dt:time))-1)/(A^2-1));

        figure
        plot(0:dt:time,std(x'),'r.')
        hold on
        grid on
        plot(0:dt:time,sig_x,'k')
        title({"Monte-Carlo & Markov Comparison","Sigma: " +
string(sigma(j)) + " & Tau: " + string(tau(i))})
        legend(["\sigma_{MC}","\sigma_{x}"])
    end
end
```

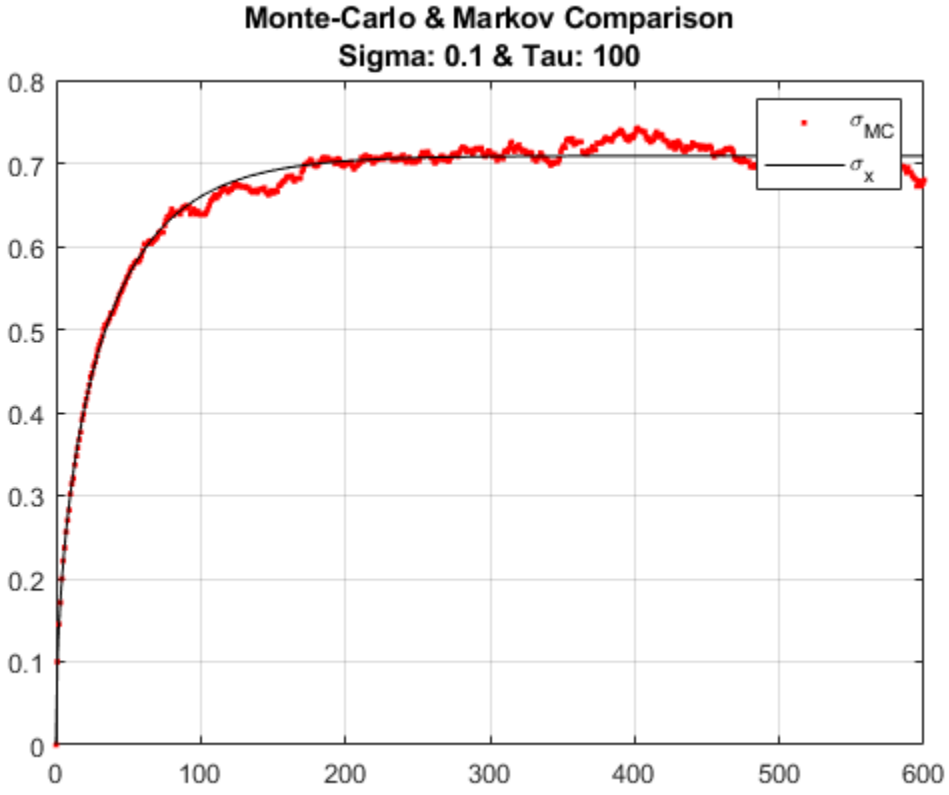

Monte-Carlo Results
Sigma: 0.1 & Tau: 1
μ_{MC}
μ_{MC} +/- $3\sigma_{MC}$
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
100
200
300
400
500
600

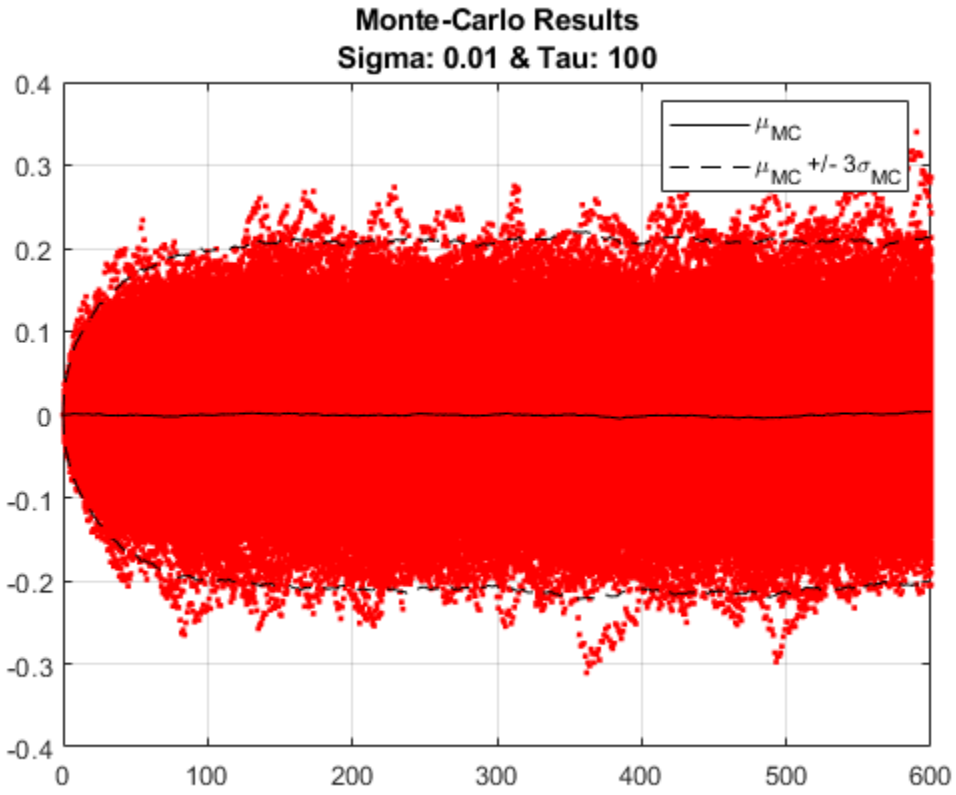

Monte-Carlo & Markov Comparison
Sigma: 0.1 & Tau: 1
σ_{MC}
σ_x
0.12
0.1
0.08
0.06
0.04
0.02
0
0
100
200
300
400
500
600

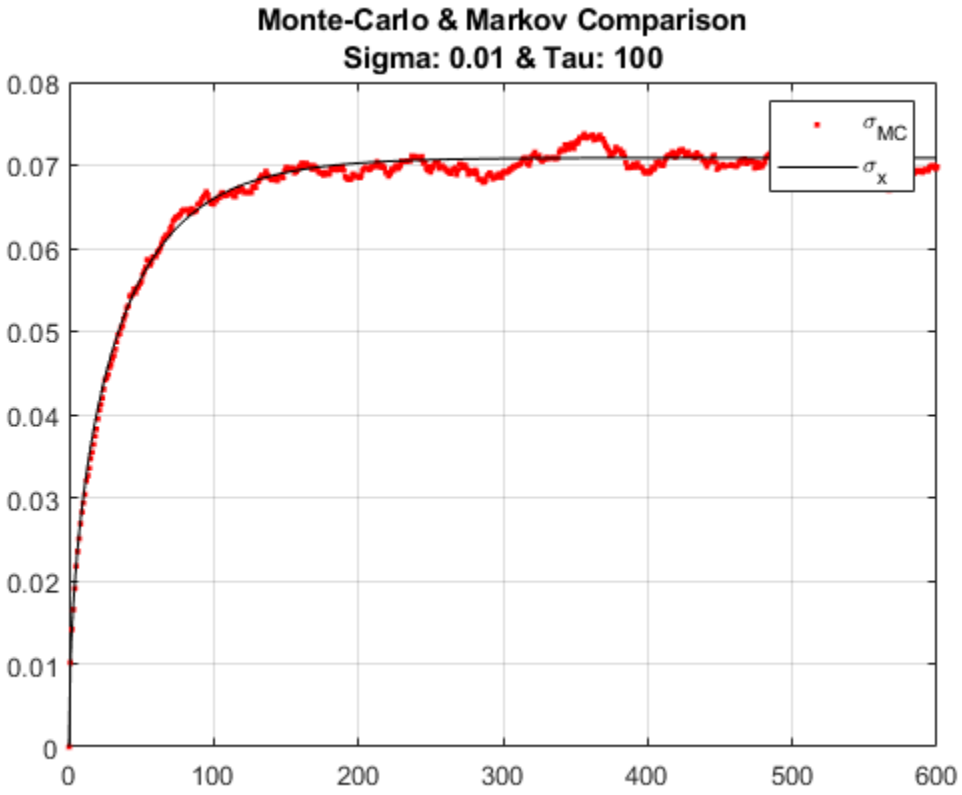

Monte-Carlo Results
Sigma: 0.01 & Tau: 1
μ_{MC}
μ_{MC} +/- $3\sigma_{MC}$
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
0
100
200
300
400
500
600


Monte-Carlo & Markov Comparison
Sigma: 0.01 & Tau: 1
σ_{MC}
σ_x
0.012
0.01
0.008
0.006
0.004
0.002
0
0
100
200
300
400
500
600


Monte-Carlo Results
Sigma: 0.1 & Tau: 100
μ_{MC}
μ_{MC} +/- $3\sigma_{MC}$
4
3
2
1
0
-1
-2
-3
-4
0
100
200
300
400
500
600


Monte-Carlo & Markov Comparison
Sigma: 0.1 & Tau: 100
σ_{MC}
σ_x
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
100
200
300
400
500
600


Monte-Carlo Results
Sigma: 0.01 & Tau: 100
μ_{MC}
μ_{MC} +/- $3\sigma_{MC}$
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
100
200
300
400
500
600


Monte-Carlo & Markov Comparison
Sigma: 0.01 & Tau: 100
σ_{MC}
σ_x
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
100
200
300
400
500
600

Published with MATLAB® R2024b