MECH 5970/6970
Homework #3 (Due: 3/19/2025)

1. Show mathematically that the error from the sum ***OR*** difference of two independent random measurements created from $y = 3a \pm 4b$, where *a* and *b* are both zero mean with variance σ_a^2 and σ_b^2 results in:

$$\mu_y = 0 \text{ and } \sigma_y = \sqrt{9\sigma_a^2 + 16\sigma_b^2}$$

   Perform 1000 run monte-carlo simulation to verify your results (compare to HW#1). Plot a histogram of the monte-carlo simulation to verify the output y is Gaussian.

2. This problem is to look at noise models.
   a) Perform a 1000 run monte-carlo simulation (of 10 minutes) to look at the error growth of a random walk (integrated white noise). Use a white noise with 1-sigma value of 0.1 and 0.01 and compare the results. Plot the mean and standard deviation of the monte-carlo simulation along with one run of the simulation (show that the random walk is zero mean with a standard deviation is $\sigma_{\int w} = \sigma_w \Delta t \sqrt{k} = \sigma_w \sqrt{t \times \Delta t}$ (where ***k*** is sample number).
   b) ***Bonus***: Provide a Histrogram of the monte-carlo simulation at a few select time slots
   c) Perform a 1000 monte-carlo simulation to look at the error growth of a 1st order markov process (integrated filtered noise) of the form $\dot{x} = -\frac{1}{\tau}x + w$. Use the same noise characteristics as above and compare the results with a 1 second and 100 second time constant (this results in 4 combinations). Comment on how changing the time constant and changing the standard deviation of the noise effects the error. Show that the 1st order markov process is zero mean with a standard deviation of is $\sigma_x = \sigma_w \Delta t \sqrt{\left(\frac{A^{2t}-1}{A^2-1}\right)}$ where $A = \left(1 - \frac{\Delta t}{\tau}\right)$. Note that for a positive time constant (i.e. stable system) the standard deviation has a steady state value.

3. Determine the expected uncertainty for an L1-L2 ionosphere free pseudorange measurement, L1-L5, and L2-L5 ionosphere free pseudorange. Assuming all measurements have the same accuracy (L1, L2, L5) which will provide the best ionosphere estimate?

4. Show that the differential GPS problem is linear. In other words derive the following expression:

$$\Delta\rho = [uv_x \; uv_y \; uv_z \; 1] \begin{bmatrix} r_x \\ r_y \\ r_z \\ c\delta t_{ab} \end{bmatrix}$$

5. Set up your own 2D planar trilateration problem. Place the SVs at (0,300) (100,400), (700,400), and (800,300). Generate a range measurement for a base station at (400,0) and a user at (401,0).
    a. Solve for the position of the user using 2 SVs and then 4 SVs assuming no clock errors. How does the PDOP change for the two cases?
    b. Solve for the position of the user assuming you need to solve for the user clock bias. What is the PDOP with all 4 satellites.
    c. Calculate a differential solution between the base and user using a single difference model and assuming you must solve for a clock bias between the base station and user. What is the PDOP with all 4 satellites?
    d. Calculate a differential solution between the base and user using a double difference model to remove the clock bias between the base station and user. What is the PDOP with all 4 satellites?
    e. Assuming the range error is zero mean with unit variance, what is the order of accuracy in the above 4 solution methods?

6. (Bonus for Undergrads/Required for Grads). Repeat problem #4 using 4 and 8 SV positions from Lab #2 (4,7,8,9,16,21,27,30). Comment on any difference or similarities with the planar problem in #3.

7. Chapter 2, Problem 1a and 1b for PRN#4. Repeat 1a for PRN #7

8. Using your PRN sequence for PRN 4 and 7, repeat problem #2 from HW#1. Compare the results to the results for your made up sequence.
    a. Plot the histogram on each sequence
    b. Plot the spectral analysis on each sequence
    c. Plot the autocorrelation each sequence with itself (i.e. a sequence delay cross correlation)
    d. Plot the cross autocorrelation between the two sequences

9. Take your C/A code from problem above (i.e. PRN #4 and #7) and multiply it times the L1 Carrier (your C/A code must be in the form -1 and +1). Perform a spectral analysis (magnitude) on the resultant signal. You will need to make sure to "hold" your C/A code bits for the correct length of time (I suggest using a sample rate 10x the L1 carrier frequency – meaning each chip of the C/A code will be used for 10 samples of the sine wave).