CENG 384 - Signals and Systems for Computer Engineers
Spring 2018-2019
Written Assignment 4

OREN, Zeki
e2264612@ceng.metu.edu.tr

KOSEN, Emrah
e1942317@ceng.metu.edu.tr

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1. (a) y[n] = 2x[n] - $\frac{1}{8}$y[n-2] + $\frac{3}{4}$y[n-1]

   $\frac{1}{8}$y[n-2] - $\frac{3}{4}$y[n-1] + y[n] = 2x[n]

   (b) x[n] = e^{jwn}

   y[n] = $H(jw).e^{jwn}$

   $\frac{1}{8}$ $H(jw).e^{jw(n-2)}$ - $\frac{3}{4}H(jw).e^{jw(n-1)}$ + $H(jw).e^{jwn}$ = $2e^{jwn}$

   $\frac{1}{8}$ $H(jw).e^{-2jw}$ - $\frac{3}{4}H(jw).e^{-jw}$ + $H(jw) = 2$

   H(jw).($\frac{1}{8}e^{-2jw} - \frac{1}{8}e^{-jw} + 1$) = 2

   H(jw) = $\dfrac{2}{\frac{1}{8}e^{-2jw} - \frac{3}{4}e^{-jw} + 1}$

   (c) H(jw) = $\dfrac{2}{\frac{1}{8}e^{-2jw} - \frac{3}{4}e^{-jw} + 1}$

   H(jw) = $\dfrac{-2}{1 - \frac{1}{4}e^{-jw}} + \dfrac{4}{1 - \frac{1}{2}e^{-jw}}$

   h[n] = $-2(\frac{1}{4})^n u[n] + 4(\frac{1}{2})^n u(n)$

(d) y[n] = x[n]*h[n]

$$X(e^{jw}) = \frac{1}{1-\frac{1}{4}e^{-jw}}$$

$$Y(e^{jw}) = X(e^{jw}).H(e^{jw})$$

$$= \frac{1}{1-\frac{1}{4}e^{-jw}}.(\frac{-2}{1-\frac{1}{4}e^{-jw}} + \frac{4}{1-\frac{1}{2}e^{-jw}})$$

$$-2.(\frac{1}{1-\frac{1}{4}e^{-jw}})^2 + \frac{4}{(1-\frac{1}{4}e^{-jw}).(1-\frac{1}{2}e^{-jw})}$$

$$-2.(\frac{1}{1-\frac{1}{4}e^{-jw}})^2 + \frac{32}{(4-e^{-jw}).(2-e^{-jw})}$$

$$\frac{32}{(4-e^{-jw}).(2-e^{-jw})} = \frac{A}{(4-e^{-jw})} + \frac{B}{(2-e^{-jw})} = \frac{-16}{(4-e^{-jw})} + \frac{16}{(2-e^{-jw})}$$

$$Y(e^{jw}) = -2.(\frac{1}{1-\frac{1}{4}e^{-jw}})^2 + -4(\frac{1}{1-\frac{1}{4}e^{-jw}}) + 8(\frac{1}{1-\frac{1}{2}e^{-jw}})$$

y[n] = -2.(n + 1).$(\frac{1}{4})^n$u[n] - 4$(\frac{1}{4})^n$u[n] + 8$(\frac{1}{2})^n$u[n]

2. $H_1(jw) = \frac{1}{1-\frac{1}{3}e^{-jw}} + H_2(jw)$

$$= \frac{5e^{-jw}-12}{e^{-2jw}-7e^{-jw}+12}$$

$$= \frac{5e^{-jw}-12}{(e^{-jw}-4).(e^{-jw}-3)}$$

$$= \frac{A}{(e^{-jw}-4)} + \frac{B}{(e^{-jw}-3)}$$

$$= \frac{8}{(e^{-jw}-4)} - \frac{3}{(e^{-jw}-3)}$$

$$= \frac{-2}{(1-\frac{1}{4}e^{-jw})} + \frac{1}{(1-\frac{1}{3}e^{-jw})}$$

h[n] = -2$(\frac{1}{4})^n$u[n] + $(\frac{1}{3})^n$u[n]

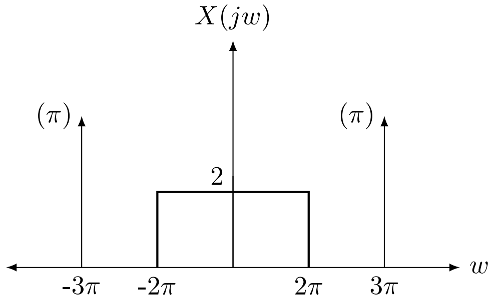
h[n] = $h_1[n] + h_2[n]$ = -2$(\frac{1}{4})^n$u[n] + $(\frac{1}{3})^n$u[n]

h[n] = $h_1[n] + h_2[n]$ = -2$(\frac{1}{4})^n$u[n] + $h_2[n]$

$h_2[n]$ = -2$(\frac{1}{4})^n$u[n]

3. (a) x(t) $= \dfrac{sin(2\pi t)}{\pi t} + cos(3\pi t)$

   X(jw) = $\pi(\delta$(w-3π) + δ(w+3 π)) + 2rect(w)



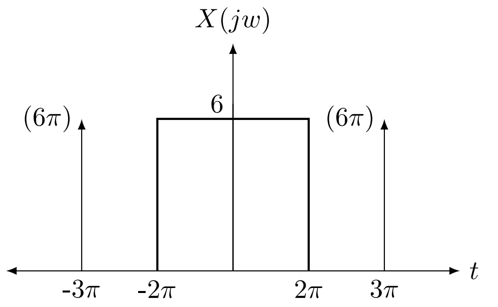
(b) $w_m = 3\pi$

   $w_s = 2w_m = 6\pi$

   T $= \dfrac{2\pi}{6\pi} = \dfrac{1}{3}$

(c) $X_p(jw) = \dfrac{1}{T}\sum_{k=-\infty}^{\infty} X(j(w - kw_s))$

   $= 3\sum_{k=-\infty}^{\infty}$ [$\pi(\delta(w - k6\pi - 3\pi) + \delta(w - k6\pi + 3\pi))$ + 2rect(w-k6π)]



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4. (a) $X_p(e^{jw}) = \dfrac{1}{T}\sum_{k=-\infty}^{\infty} X(j(w - \pi k))$

   X_d(n) = X_p(j$\dfrac{w}{T}$)

   $X_d(e^{jw}) = \dfrac{1}{2}\sum_{k=-\infty}^{\infty} \dfrac{2w}{\pi} - \pi k$ |w| $\leq \dfrac{\pi}{2}$

(b) $H(e^{jw}) = \pi(\delta(w + \pi) + \delta(w - \pi))$

(c) $Y_d(e^{jw}) = X_d(e^{jw}) * H(e^{jw})$

$$Y_d(e^{jw}) = \frac{1}{2}\sum_{k=-\infty}^{\infty}[\frac{2w}{\pi} - \pi k] * [\pi(\delta(w+\pi) + \delta(w-\pi))]$$

$$Y_d(e^{jw}) = \frac{\pi}{2}\sum_{k=-\infty}^{\infty}\frac{2(w+\pi)}{\pi} + \frac{2(w-\pi)}{\pi} - 2\pi k$$