Spring 2019 Homework 4

## Regulations:

• Grouping: You are allowed to work in pairs.

• Submission: We provide a latex template for your solutions. Use that template and create a hw4.tar.gz file that includes hw4.tex and all other related files. Tar.gz file should not contain any directories and should create a hw4.pdf file with the following commands, otherwise you will get zero;

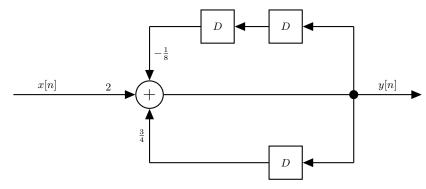
tar xvzf hw4.tar.gz pdflatex hw4.tex

Submit hw4.tar.gz to the COW page of the course.

• **Deadline:** 23:55, 2 June, 2019 (Sunday).

• Late Submission: Not allowed.

1. (30 pts) Consider an LTI system given by the following block diagram:



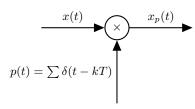
where D is the unit-delay operator.

- (a) (5 pts) Find the difference equation which represents this system.
- (b) (5 pts) Find the frequency response of this system.
- (c) (10 pts) Find the impulse response of this system from its frequency response.
- (d) (10 pts) Find the output y[n] for the input  $x[n] = (\frac{1}{4})^n u[n]$  using the frequency response.
- 2. (10 pts) Suppose that two discrete-time LTI systems with the impulse responses  $h_1[n]$  and  $h_2[n]$  are connected in parallel. We have the following information about this combined system:
  - i. The frequency response of the combined system is;  $H(e^{j\omega}) = \frac{5e^{-j\omega}-12}{e^{-2j\omega}-7e^{-j\omega}+12}$ .
  - ii. The impulse response of the first system is:  $h_1[n] = (\frac{1}{3})^n u[n]$ .

Find  $h_2[n]$ , the impulse response of the second system.

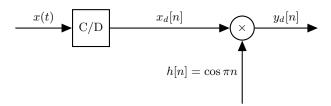
3. (30 pts) Consider the following sampling system where the input is

$$x(t) = \frac{\sin 2\pi t}{\pi t} + \cos 3\pi t$$



- (a) (10 pts) Find and plot the Fourier Transform of x(t).
- (b) (10 pts) What is the Nyquist frequency and period for sampling?
- (c) (10 pts) Find and plot the Fourier Transform of  $x_p(t)$  using the Nyquist rate.

4. (30 pts) Consider the following system with a  $\mathrm{C/D}$  converter:



where the sampling frequency is  $\omega_s = \pi$  and

$$X(j\omega) = \begin{cases} \frac{4}{\pi}\omega, & \text{if } |\omega| \leq \frac{\pi}{4} \\ 0, & \text{otherwise} \end{cases}$$

- (a) (10 pts) Find  $X_d(e^{j\omega})$ .
- (b) (10 pts) Find  $H(e^{j\omega})$ .
- (c) (10 pts) Find  $Y_d(e^{j\omega})$ .