Spring 2019 Homework 2

Regulations:

• Grouping: You are allowed to work in pairs.

• Submission: We provide a latex template for your solutions. Use that template and create a hw2.tar.gz file that includes hw2.tex and all other related files. Tar.gz file should not contain any directories and should create a hw2.pdf file with the following commands, otherwise you will get zero;

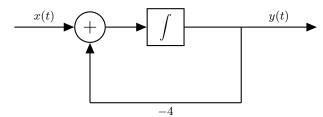
tar xvzf hw2.tar.gz pdflatex hw2.tex

Submit hw2.tar.gz to the COW page of the course.

• **Deadline:** 23:55, 15 March, 2019 (Friday).

• Late Submission: Not allowed.

1. (20 pts) Consider an LTI system given by the following block diagram:



- (a) (5 pts) Find the differential equation which represents this system.
- (b) (15 pts) Find the output y(t), when the input $x(t) = (e^{-t} + e^{-2t})u(t)$. Assume that the system is initially at rest.
- 2. (20 pts) Evaluate the following convolutions.
 - (a) (10 pts) Given $x[n] = \delta[n-1] 3\delta[n-2] + \delta[n-3]$ and $h[n] = \delta[n+1] + 2\delta[n] 3\delta[n-1]$, compute and draw y[n] = x[n] * h[n].
 - (b) (10 pts) Given x(t) = u(t) + u(t-1) and $h(t) = e^{-2t} \cos(t) u(t)$, calculate $y(t) = \frac{dx(t)}{dt} * h(t)$.
- 3. (20 pts) Evaluate the following convolutions.
 - (a) (10 pts) Given $h(t) = e^{-3t}u(t)$ and $x(t) = e^{-t}u(t)$, find y(t) = x(t) * h(t).
 - (b) (10 pts) Given $h(t) = e^t u(t)$ and x(t) = u(t-1) u(t-2), find y(t) = x(t) * h(t).
- 4. (20 pts) Solve the following homogeneous difference equations with the specified initial conditions.
 - (a) $(10 \text{ pts}) \ y[n] 15y[n-1] + 26y[n-2] = 0, y[0] = 10 \text{ and } y[1] = 42.$
 - (b) $(10 \text{ pts}) \ y[n] 3y[n-1] + y[n-2] = 0, y[0] = 1 \text{ and } y[1] = 2.$
- 5. (20 pts) Consider a continuous LTI system represented by the following differential equation, which is initially at rest:

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

- (a) (12 pts) Find the impulse response of this system.
- (b) (8 pts) Using the impulse response you found in (a), analyze if this system is
 - i. causal,
 - ii. memoryless,
 - iii. stable,
 - iv. invertible.