MIDDLE EAST TECHNICAL UNIVERSITY
DEPARTMENT OF COMPUTER ENGINEERING

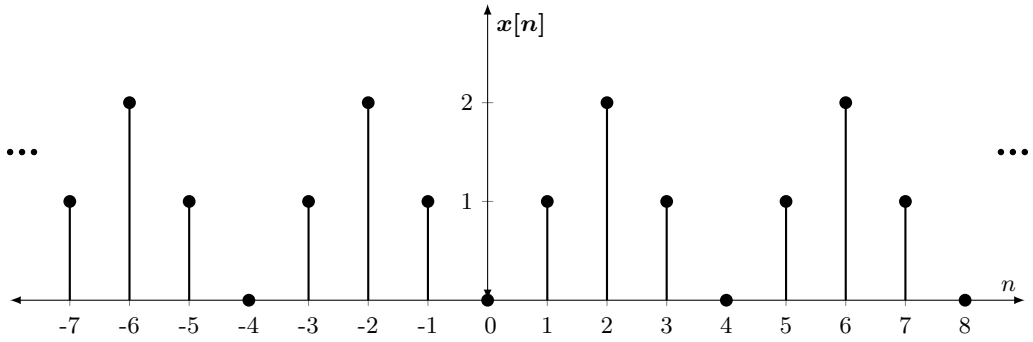
CENG 384

Spring 2019
Homework 3

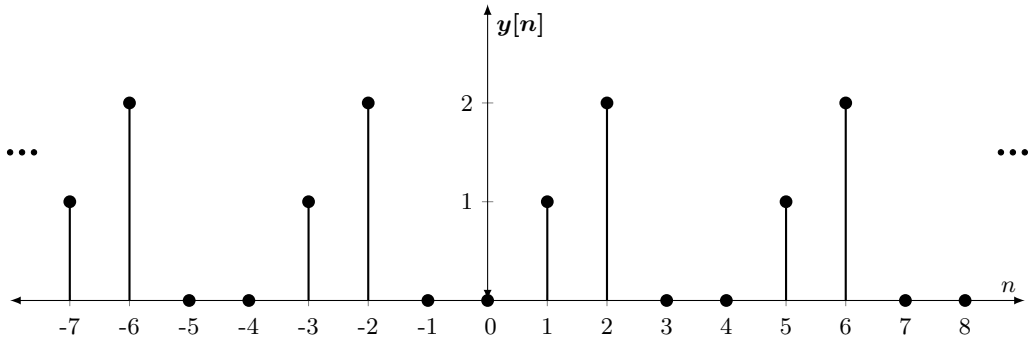
Regulations:

- **Grouping:** You are allowed to work in pairs.
- **Submission:** We provide a latex template for your solutions. Use that template and create a hw3.tar.gz file that includes hw3.tex and all other related files. Tar.gz file should not contain any directories and should create a *hw3.pdf* file with the following commands, otherwise you will get zero;
  tar xvzf hw3.tar.gz
  pdflatex hw3.tex
  Submit hw3.tar.gz to the COW page of the course.
- **Deadline:** 23:55, 14 April, 2019 (Sunday).
- **Late Submission:** Not allowed.

1. (25 pts)
   (a) (10 pts) Find and plot the spectral coefficients of Fourier series for the following discrete time signal, $x[n]$:



   (b) Consider the following discrete time signal, $y[n]$:

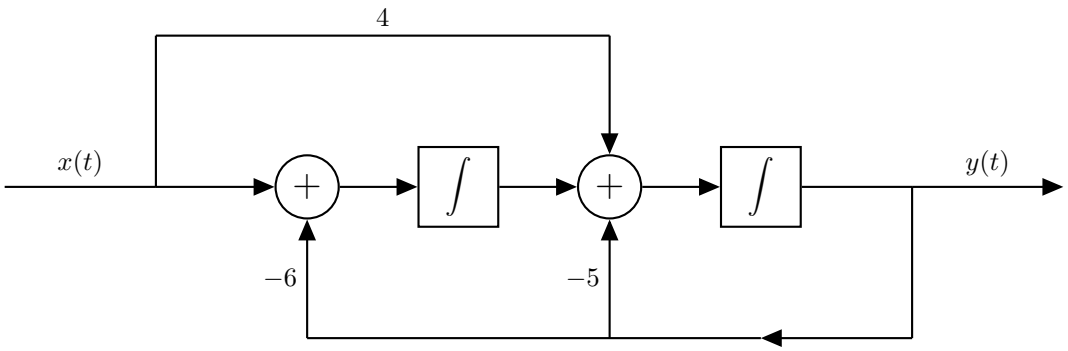


      i. (5 pts) Define $y[n]$ in terms of $x[n]$.
      ii. (10 pts) Find and plot the spectral coefficients of Fourier series for $y[n]$.

2. (20 pts) Determine and plot a discrete-time signal $x[n]$ satisfying the following conditions:
   (a) $x[n]$ is a real and periodic signal with $N = 4$ and has Fourier Series coefficients a_k, which is complex for some k.
   (b) $\sum_{k=-3}^{4} x[k] = 8$.
   (c) $a_{-3} = a_{15}^*$ and $|a_1 - a_{11}| = 1$.
   (d) One of the coefficients is zero.
   (e) $\sum_{k=0}^{3} x[k] \left(e^{-j\pi k/2} + e^{-j\pi 3k/2}\right) = 4$.

3. (20 pts) Consider a periodic signal $x(t)$ which can be represented by the first K Fourier Series coefficients. Determine the impulse response of the system that can yield $x(t)$ when it is contaminated by a noise $r(t)$ (i.e., the input to the system is $x(t) + r(t)$ and the output is $x(t)$), assuming that $r(t)$ is composed of only very high-frequency components (namely, $\mathcal{F}\{r(t)\} = R(j\omega) = 0$ for $|w| \leq K2\pi/T$, where T is the period of $x(t)$).

4. (35 pts) Consider an LTI system given by the following block diagram:



(a) (15 pts) Find the frequency response of this system.

(b) (10 pts) Find the impulse response of this system from its frequency response.

(c) (10 pts) Find the output $y(t)$ for the input $x(t) = \frac{1}{4}e^{-t/4}u(t)$ using the frequency response.