CENG 384 - Signals and Systems for Computer Engineers
Spring 2018-2019
Written Assignment 2

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1. (a) $\frac{dy(t)}{dt}$ = x(t) - 4y(t)

   (b) y(t) = y_p(t) + y_h(t)

   $\frac{dy(t)}{dt}$ + 4y(t) = 0

   $\lambda - 4 = 0$

   $\lambda = 4$

   y_h(t) = Ae^{-4t}

   x(t) = $e^{-t} + e^{-2t}$

   for $e^{-t} \rightarrow y_{p1} = H(-1)e^{-t}$

   for $e^{-2t} \rightarrow y_{p2} = H(-2)e^{-2t}$

   for x(t) $\rightarrow y_p = H(-2)e^{-2t} + H(-1)e^{-t}$

   H(-1) = $\frac{1\lambda^0}{1*\lambda^0 - 4*\lambda^1}$ = $\frac{1}{5}$

   H(-2) = $\frac{1\lambda^0}{1*\lambda^0 - 4*\lambda^1}$ = $\frac{1}{9}$

   $y_p = H(-2)e^{-2t} + H(-1)e^{-t} = \frac{1}{9}e^{-2t} + \frac{1}{5}e^{-t}$

   y(x) = $y_p + y_h = \frac{1}{9}e^{-2t} + \frac{1}{5}e^{-t} + Ae^{-4t}$

   The system is initially at rest. So, y(0) = 0 and $y^{'}$ = 0 for t < 0

   y(0) = $\frac{1}{9}e^0 + \frac{1}{5}e^0 + Ae^0$

   A = $\frac{-14}{45}$

   y(x) = $\frac{1}{9}e^{-2t} + \frac{1}{5}e^{-t} + \frac{-14}{45}e^{-4t}$

2. (a) x[n]*$\delta[n-n_0]$ = x[n-n_0]

By distribution rule x[n]*h[n] = x[n]*($\delta[n+1]+2\delta[n]-3\delta[n-2]$)

$= x[n]*\delta[n+1]+2(x[n]*\delta[n])-3(x[n]*\delta[n-2])$

= x[n+1] + 2x[n] - 3x[n-2]

x[n] = $\delta[n-1]-3\delta[n-2]+\delta[n-3]$

y[n] = $\delta[n]-3\delta[n-1]+\delta[n-2]$ + 2*($\delta[n-1]-3\delta[n-2]+\delta[n-3]$) -3($\delta[n-3]-3\delta[n-4]+\delta[n-5]$)

So, y[n] = $\delta[n]-\delta[n-1]-8\delta[n-2]$ + $11\delta[n-3]$ - $3\delta[n-4]$

(b) .

$\frac{du(t)}{dt}=\delta(t)$

$\frac{dx(t)}{dt}=\frac{du(t)}{dt}+\frac{du(t-1)}{dt}=\delta(t)+\delta(t-1)$

$\frac{dx(t)}{dt}$*h(t) = ($\delta(t)+\delta(t-1)$)*h(t)

$=\delta(t)*h(t)+\delta(t-1)*h(t)$

= h(t) + *h(t-1) = $(e^t+e^{(t-1)})$u(t)

3. (a) .

y(t) = $\int_0^t e^{-z}e^{-3(t-z)}dz$

$=\int_0^t e^{-3t}e^{2z}dz$

$=e^{-3t}*(e^{2t}-1)$

y(t) = $(e^{-2t}$ – (e^{-3t}) * u(t).

(b) x(t)*h(t) = $\int_{-\infty}^{\infty}x(z)h(t-z)dz$

$=\int_0^{\infty}e^t(u(t-z-1)-u(t-z-2))dz$

The system is nonzero in (t-2) < z < (t-1).

For t <= 1, y(t) = 0. -For 1 < t <= 2;

y(t) = $\int_0^{t-1}e^z dz=e^{t-1}$ - 1.

-For t > 2;

y(t) = $\int_{t-2}^{t-1}e^z dz=e^{t-1}$ - e^{t-2}.

4. (a) y^2 - 15y + 26y = 0

(y-2)*(y-13) = 0

y_1 = 13 , y_2 = 2

y[n] = c_1*13^n + c_2*2^n

for y[0] = 10 = c_1*13^0 + c_2*2^0

10 = c_1 + c_2

for y[1] = 42 = c_1*13^1 + c_2*2^1

$c_1 = 2$

$c_2 = 8$

y[n] = $2 * 13^n$ + $8 * 2^n$

(b) y^2 - 3y + 1 = 0

y1 = -$\frac{b+\sqrt{b^2-4ac}}{2a}$ = $\frac{3+\sqrt{5}}{2}$, y2 = -$\frac{b-\sqrt{b^2-4ac}}{2a}$ = $\frac{3-\sqrt{5}}{2}$

y[n] = $c_1 * (\frac{3+\sqrt{5}}{2})^n$ + $c_2 * (\frac{3-\sqrt{5}}{2})^n$

for y[0] = 1 = c_1 + c_2

for y[1] = 2 = $c_1 * (\frac{3+\sqrt{5}}{2})$ + $c_2 * (\frac{3-\sqrt{5}}{2})$

c_1 = $\frac{\sqrt{5}+1}{2\sqrt{5}}$

c_2 = $\frac{\sqrt{5}-1}{2\sqrt{5}}$

y[n] = $\frac{\sqrt{5}+1}{2\sqrt{5}} * (\frac{3+\sqrt{5}}{2})^n$ + $\frac{\sqrt{5}-1}{2\sqrt{5}} * (\frac{3-\sqrt{5}}{2})^n$

5. (a) y(t) = h(t) and x(t) = $\lambda(t)$

h^2 + 6h +8 = 0

(h+4)(h+2) = 0

h = -4,-2

$h_h(t) = k_1 e^{-4t}$ + $k_2 e^{-2t}$

$h^{''}(t) + 6h^{'}(t) + h(t) = 2\lambda(t)$

Take both sizes integral

$\int_{-0}^{+0} h^{''}(t)dt + 6\int_{-0}^{+0} h^{'}(t)dt + \int_{-0}^{+0} h(t)dt = \int_{-0}^{+0} 2\lambda(t)$dt

0 + 6(h(+0) - h(-0)) = 2

h(+0) = $\frac{1}{3}$ The system is initially at rest

h(+0) = $k_1 e^0$ + $k_2 e^0$

$k_1 + k_2 = \frac{1}{3}$

for t = -1

h(-1) = $k_1 e^4$ + $k_2 e^2$ = 0

$k_1 e^4 = -k_2 e^2$

$k_2 = -k_1 e^2$

$(1-e^2)k_1 = \frac{1}{3}$

$k_1 = \frac{1}{3(1-e^2)}$

$k_2 = \frac{-e^2}{3(1-e^2)}$

h(t) = $\frac{1}{3(1-e^2)}e^{-4t} - \frac{e^2}{3(1-e^2)}e^{-2t}$

(b) -The system is causal because it is initially at rest.

-The system is has memory, because it is related to the past value of the output signal y(t).

-The system is not stable, because the output signal y(t) is not a bounded signal.

-The system is non-invertible, because it has many-to-one mapping.