CENG 384 - Signals and Systems for Computer Engineers
Spring 2018-2019
Written Assignment 1

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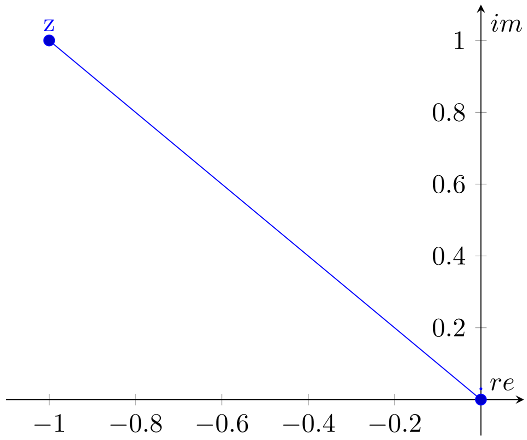
1. (a) 1.
z = x + yj
3z + 4 = 2j - $\overline{z}$
3x +3yj + 4 = 2j -x + yj
4x + 2yj = -4 + 2j
x = -1, y = 1
z = -1 + j
$\overline{z}$ = -1 - j
$|z|^2$ = z.$\overline{z}$ = (-1 + j).(-1 - j) = 2
$\theta = tan^{-1}\frac{b}{a} = tan^{-1}(-1)$ = -45 or 180-45 because of real part is positive and imaginary part negative θ = 135
r = $\sqrt{a_2 + b_2} = \sqrt{2}$
a = $\sqrt{2}$sin(135) = 1 and b = $\sqrt{2}$cos(135) = -1

2.



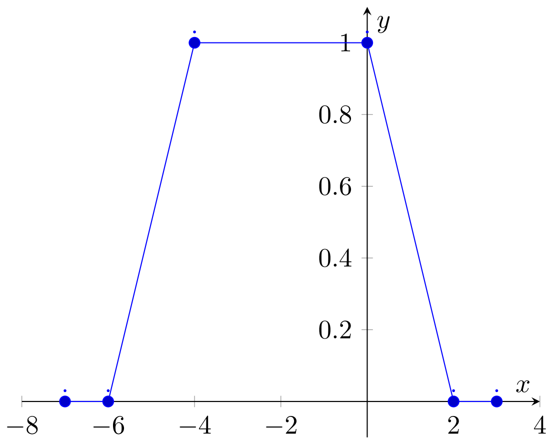
(b) $z^3 = r^3 * e^{3j\theta}$ = 4^3(cos(3 θ) + j*sin(3 θ))
r = 4
cos(3 θ) = 0
j.sin(3 θ) = j
$\theta = \frac{\pi}{6}$
z = 4*(cos($\frac{\pi}{6}$) + j*sin($\frac{\pi}{6}$)) = 4*$e^{j*\frac{\pi}{6}}$

(c) $\frac{(1-j)(1+j\sqrt{3})}{1+j} = \frac{(1-j)(1-j)(1+j\sqrt{3})}{(1+j)(1-j)} = \frac{(-2j)(1+j\sqrt{3})}{2} = -j + \sqrt{3}$

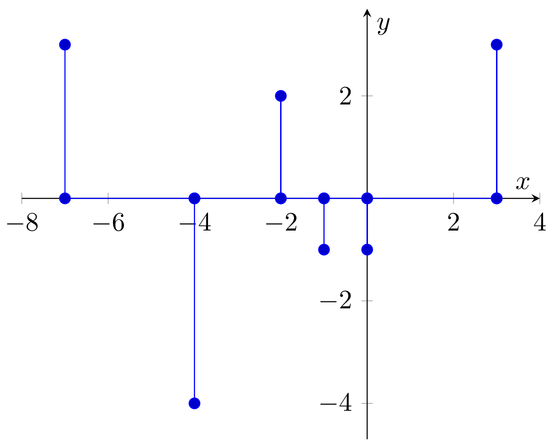
r = $\sqrt{3+1}$ = 2 and $\theta = tan^{-1}\frac{-1}{\sqrt{3}}$

(d) z = -j(cos($*\frac{\pi}{2}$) + jsin($*\frac{\pi}{2}$)) = -j*cos($*\frac{\pi}{2}$) + sin($*\frac{\pi}{2}$)
cos($*\frac{\pi}{2}$) = sin(0)
sin($*\frac{\pi}{2}$) = cos(0)
z = cos(0) - j*sin(0)
|z| = r = 1 So, z = $e^{j\frac{\pi}{2}}$

2. .



3. (a) -



(b) $3.\delta$(n+7) - $4.\delta$(n+4) + $2.\delta$(n+2) - $1.\delta$(n+1) - $1.\delta$(n) + $3.\delta$(n-3)

4. (a) The fundamental period of cos($\frac{13\pi*n}{10}$) is $T_1 = \frac{10*2*\pi}{13*\pi} = \frac{20}{13}$.
The fundamental period of sin($\frac{7\pi*n}{3}$) is $T_2 = \frac{3*2*\pi}{7*\pi} = \frac{6}{7}$.
The fundamental period of x$[n] = \frac{ekok(20,6)}{ebob(13,7)} = 60$

or

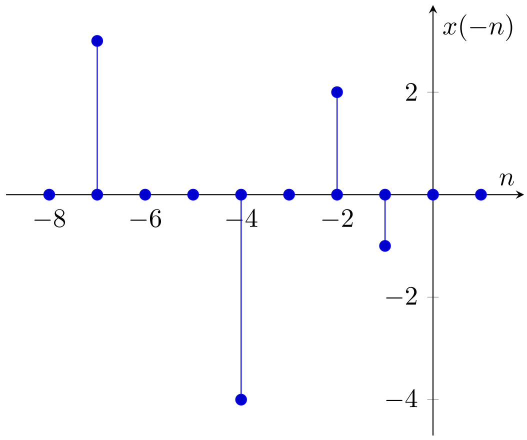
y(n) = cos($\frac{39\pi*n}{30}$) + sin($\frac{70\pi*n}{30}$) so the fundamental frequency is $\frac{\pi}{30}$ than T = $\frac{2*\pi}{\frac{\pi}{30}} = 60$.

(b) The fundamental frequency is $w_0 = 5$
T = $\frac{2*\pi}{w0}$*k if x(n) is periodic, k and T should be an integer. Then, it is not periodic.

(c) The fundamental period of x(t) = $\frac{2*\pi}{3\pi} = \frac{2}{3}$

(d) -je^{j5t} = -j(cos(5t) +jsin(5t)) = -jcos(5t) + sin(5t)
= cos(5t + $\frac{\pi}{2}$) + jsin(5t + $\frac{\pi}{2}$)
$= e^{j(5t+\frac{\pi}{2})}$
the fundamental period of x$(t) = \frac{2\pi}{5}$

5. In this figure x(n) $\neq$ x(-n) and x(n) $\neq$ -x(-n). So, the signal is neither even nor odd.



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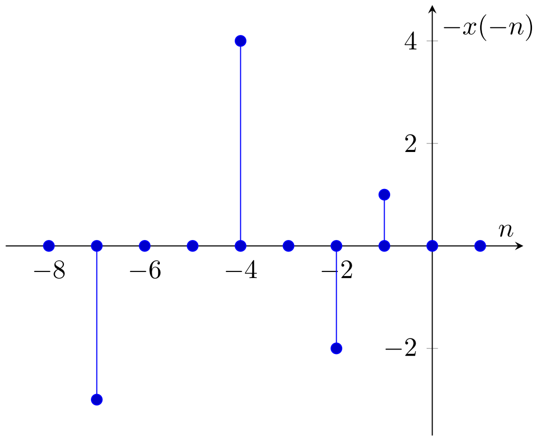

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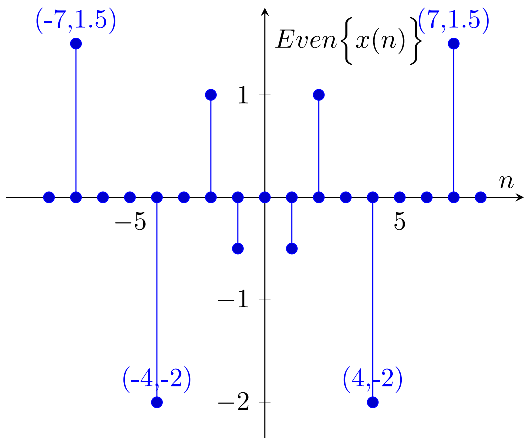
Any signal can be represented by its even and odd parts.

x(n) = Odd$\Big\{$x(n)$\Big\}$ + Even$\Big\{$x(n)$\Big\}$

Even$\Big\{$x(n)$\Big\}$ = $\frac{1}{2}$[x(n) + x(-n)] Odd$\Big\{$x(n)$\Big\}$ = $\frac{1}{2}$[x(n) - x(-n)]

Odd$\Big\{$x(n)$\Big\}$ + Even$\Big\{$x(n)$\Big\}$ = $\frac{1}{2}$[x(n) + x(-n) + x(n) - x(-n)] = $\frac{2x(n)}{2}$ = x(n)



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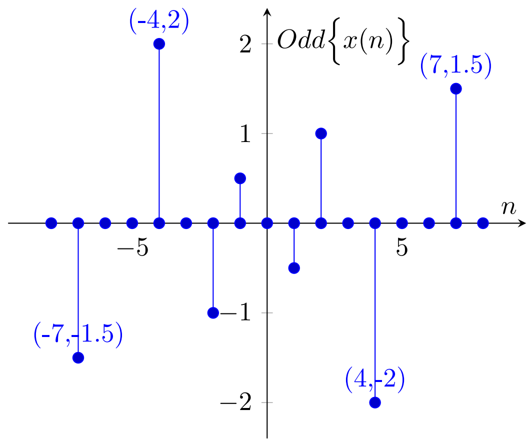

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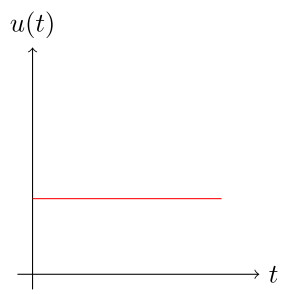
. Even$\Big\{$x(n)$\Big\}$ = $\frac{1}{2}$[x(n) + x(-n)] Odd$\Big\{$x(n)$\Big\}$ = $\frac{1}{2}$[x(n) - x(-n)]

As a result, addition of these plots will give us x(n) in Figure 2.

6. (a) y(t) = x(2t-3)

$\Rightarrow$ This system has memory, because output signal y(t) depends on the present and past value of the input signal x(t).

$\Rightarrow$ A system is stable for an output signal [y(t)] which is bounded for any bounded input signal [x(t)]. For a constant A > 0, |x(t)| < A for all t. For a constant B > 0, |y(t)| < B for all t. Let x(t) = u(t) [u(t) is a bounded signal]. If y(t) is bounded then system is stable.



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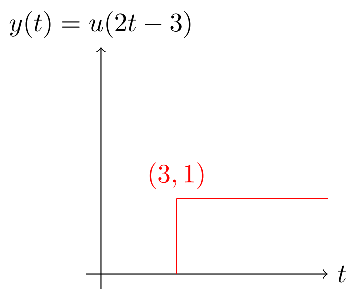

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Shifting and scaling the time doesn't effect the value of signal u(t). We just shift the signal 3 times right. Hence, this system is stable.

$\Rightarrow$ This system depends on past value of input signal x(t). Also, there is no dependence of future value of x(t). As a result, the system is causal.

$\Rightarrow$ Let x_1(t) $\rightarrow$ System $\rightarrow$ y_1(t) x_2(t) $\rightarrow$ System $\rightarrow$ y_2(t)

. y_1(t) = x_1(2t-3) y_2(t) = x_2(2t-3)

y_1(t) + y_2(t) = x_1(2t-3) + x_2(2t-3)

ax_1(t) + bx_2(t) = ax_1(2t-3) + bx_2(2t-3)

So, ay_1(t) + by_2(t) = ax_1(t) + bx_2(t)

Finally, the system is linear.

$\Rightarrow$ It has an one-to-one mapping. So, the system is invertible. y^{-1} = x $(\frac{t+3}{2})$

$\Rightarrow$ Let x_1(t) $\rightarrow$ System $\rightarrow$ y_1(t) x_2(t) $\rightarrow$ System $\rightarrow$ y_2(t)

. x_2(t) = x_1(t-t_0)

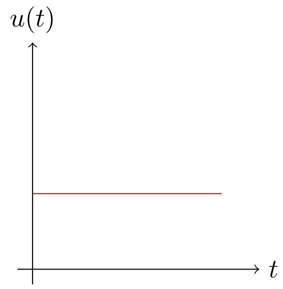
. y_1(t) = x_1(2t-3) y_2(t) = x_1(2t-2t_0-3)

y_1(t-t_0) = x_1(2t-2t_0-3). Then, y_2(t) = y_1(t-t_0). Hence, the system is time-invariant.

(b) y(t) = tx(t)

$\Rightarrow$ The system is memoryless because output signal y(t) doesn't depend on the past value of input signal x(t). For y(t) = h(x(t- t_o)) t_o = 0 for all t

$\Rightarrow$ Let x(t) = u(t). If y(t) is bounded then the system is stable.



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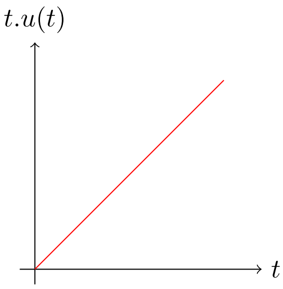

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Input signal x(t) is bounded; but output signal y(t) is not bounded. Hence, the system is not stable.

⇒ The system is causal because it is independent of future value of input signal x(t).
y(t) = h(x(t- t_o)) t_o = 0 for all t

⇒ Let x_1(t) → System → y_1(t) $\qquad$ x_2(t) → System → y_2(t)

. $\quad$ y_1(t) = $t.x_1$(t) $\qquad$ y_2(t) = $t.x_2$(t)

y_1(t) + y_2(t) = t.[x_1(t) + x_2(t)]

a.x_1(t) + b.x_2(t) = a.tx_1(t) + b.tx_2(t)

So, a.y_1(t) + b.y_2(t) = a.x_1(t) + b.x_2(t). Finally, the system is linear.

⇒ It has an one-to-one mapping. So, the system is invertible. $\qquad$ y^{-1} = x(t).$\frac{1}{t}$

⇒ Let x_1(t) → System → y_1(t) $\qquad$ x_2(t) → System → y_2(t)

. $\qquad$ x_2(t) = x_1(t-t_0)

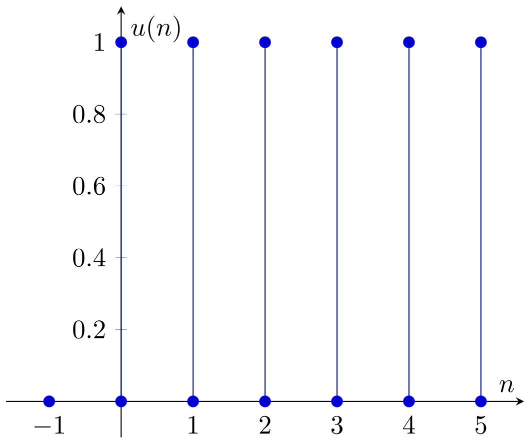
. $\quad$ y_1(t) = t.x_1(t) $\qquad$ y_2(t) = t.x_1(t-t_0)

y_1(t-t_0) = (t-t_0)x_1((t-t_0)). Then, y_2(t) = y_1(t-t_0). Hence, the system is time-varying.

(c) y(n) = x(2n-3)

⇒ This system has memory, because output signal y(n) depends on the present and past value of the input signal x(n).

⇒ Let x(n) = u(n). If y(n) is bounded then the system is stable.



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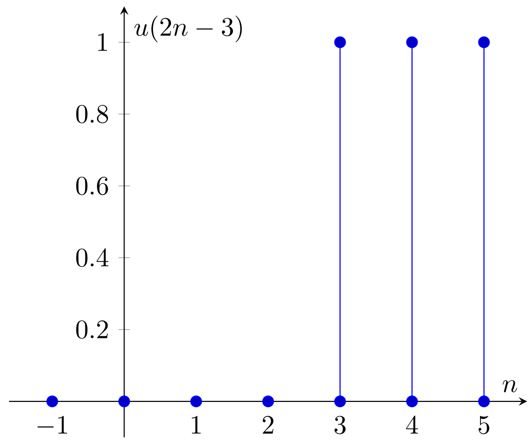

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Sifting and scaling doesn't effect the value of signal u(n). We just shifted the signal 3 times right. Then the system is stable.

$\Rightarrow$ This system depends on future value of input signal x(n). For n = 5 y[5] = x[7]. Hence, the system is not causal.

$\Rightarrow$ Let x_1(n) $\rightarrow$ System $\rightarrow$ y_1(n) x_2(n) $\rightarrow$ System $\rightarrow$ y_2(n)

. y_1(n) = x_1(2n-3) y_2(n) = x_2(2n-3)

y_1(n) + y_2(n) = x_1(2n-3) + x_2(2n-3)

a.x_1(n) + b.x_2(n) = ax_1(2n-3) + bx_2(2n-3)

So, a.y_1(n) + b.y_2(n) = a.x_1(n) + b.x_2(n). Finally, the system is linear.

$\Rightarrow$ y^{-1} = x(.$\frac{n+3}{2}$) is not invertible, because in some values of x(n) the output signal y(n) takes values which is not integer.

$\Rightarrow$ Let x_1(n) $\rightarrow$ System $\rightarrow$ y_1(n) x_2(n) $\rightarrow$ System $\rightarrow$ y_2(n)

. x_2(n) = x_1(n-n_0)

. y_1(n) = x_1(2n-3) y_2(n) = x_1(2n-2n_0-3)

y_1(n-n_0) = x_1(2n-2n_0-3). Then, y_2(n) = y_1(n-n_0). Hence, the system is time-varying.

(d) y(n) = $\sum_{k=1}^{\infty}$x(n-k)

$\Rightarrow$ This system has memory, because output signal y(n) depends on the present and past value of the input signal x(n). For y = 0, y[0] = x[-1] + x[-2] ... 0 > -1,-2...

$\Rightarrow$ For any bounded input x(n) it has unbounded output y(n) so it is not stable.

$\Rightarrow$ This system depends on past value of input signal x(n). There is no dependence on future value of x(n). For any n $\rightarrow$ n-k < n in k from zero to infinity. Hence, the system is causal.

$\Rightarrow$ Let x_1(n) $\rightarrow$ System $\rightarrow$ y_1(n) x_2(n) $\rightarrow$ System $\rightarrow$ y_2(n)

. y_1(n) = $\sum_{k=1}^{\infty} x_1$(n-k) y_2(n) = $\sum_{k=1}^{\infty} x_2$(n-k)

y_1(n) + y_2(n) = $\sum_{k=1}^{\infty} x_1$(n-k) + $\sum_{k=1}^{\infty} x_2$(n-k)

a.x_1(n) + b.x_2(n) = a$\sum_{k=1}^{\infty} x_1$(n-k) + b$\sum_{k=1}^{\infty} x_2$(n-k)

So, a.y_1(n) + b.y_2(n) = a.x_1(n) + b.x_2(n). Finally, the system is linear.

$\Rightarrow$ x[n] = y[n+1] - y[n] so it is invertible.

$\Rightarrow$ Let x_1(n) $\rightarrow$ System $\rightarrow$ y_1(n) x_2(n) $\rightarrow$ System $\rightarrow$ y_2(n)

. x_2(n) = x_1(n-n_0)

. y_1(n) = $\sum_{k=1}^{\infty} x_1$(n-k) y_2(n) = $\sum_{k=1}^{\infty} x_1$(n-n_0-k)

y_1(n-n_0) = $\sum_{k=1}^{\infty} x_1$(n-n_0-k). Then, y_2(n) = y_1(n-n_0). Hence, the system is time-invariant.