CENG 384 - Signals and Systems for Computer Engineers
Spring 2018-2019
Written Assignment 3

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1. (a) Period N = 4 and $w_0 = \dfrac{\pi}{2}$ in this equation

$a_k = \dfrac{1}{N} \sum_{n=<N>}$ x(n)e^{-jw_0kn}

$a_k = \dfrac{1}{4}(0 + e^{-j\frac{\pi}{2}k} + 2e^{-j\frac{\pi}{2}k2} + e^{-j\frac{\pi}{2}k3}) = \dfrac{1}{4}((\text{cosk}\frac{\pi}{2}$ - jsink$\frac{\pi}{2}$) + 2(coskπ - jsinkπ) + (cosk$\frac{3\pi}{2}$ - jsink$\frac{3\pi}{2}$))

$a_0 = \dfrac{1}{4}$((1-0) + 2(1-0) + (1-0)) = 1

$a_1 = \dfrac{1}{4}$((0-j) + 2(-1-0) + (0+j)) = -$\dfrac{1}{2}$

$a_2 = \dfrac{1}{2}$((-1-0) + 2(1-0) + (-1-0)) = 0

$a_3 = \dfrac{1}{4}$((0+j) + 2(-1-0) + (0-j)) = -$\dfrac{1}{2}$

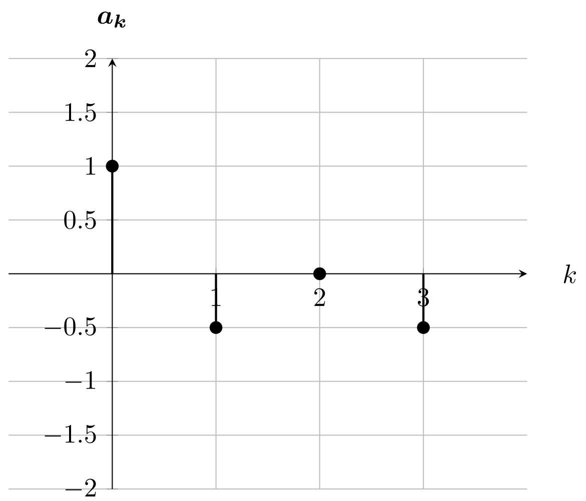

Figure 1: k vs. a_k of x(n)

(b) i)

y(n) = x(n) - $\sum_{k=-\infty}^{\infty} \delta$(n + 1 - 4k)

ii)

Period N = 4 and $w_0 = \dfrac{\pi}{2}$ in this equation

$a_k = \dfrac{1}{N} \sum_{n=<N>}$ x(n)e^{-jw_0kn}

$a_k = \dfrac{1}{4}(0 + e^{-j\frac{\pi}{2}k} + 2e^{-j\frac{\pi}{2}k2} + 0) = \dfrac{1}{4}$((cosk$\frac{\pi}{2}$ - jsink$\frac{\pi}{2}$) + 2(coskπ - jsinkπ))

$a_0 = \dfrac{1}{4}$((1-0) + 2(1-0)) = $\dfrac{3}{4}$

$a_1 = \dfrac{1}{4}$((0-j) + 2(-1-0)) = $\dfrac{-j-2}{4}$, $\quad |a_1| = \sqrt{(-\frac{1}{4})^2 + (-\frac{2}{4})^2} = 0.56$

$a_2 = \dfrac{1}{2}$((-1-0) + 2(1-0)) = $\dfrac{1}{4}$

$a_3 = \dfrac{1}{4}$((0+j) + 2(-1-0)) = $\dfrac{j-2}{4}$, $\quad |a_3| = \sqrt{(\frac{1}{4})^2 + (-\frac{2}{4})^2} = 0.56$

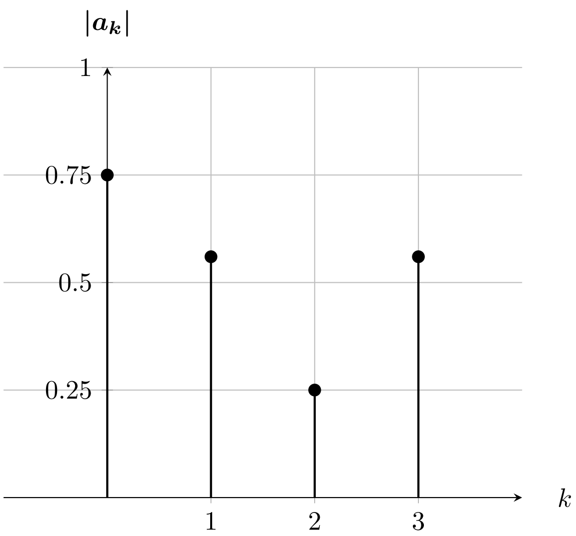

Figure 2: k vs. $|a_k|$ of y(n)

2. a) N = 4 and $w_0 = \dfrac{\pi}{2}$

   b) We know that x(n) is periodic. So, $\sum_{k=0}^{3}$ x(k) = 4

   c) $a_{-3} = a_1$ and also $a_{15} = a_{11} = a_3$. Then, $|a_1$ - $a_3| = 1$ and $a_1 = a_3^*$

   d) One of the coefficients is zero

   e) $\frac{1}{4}\sum_{k=0}^{3}$ x(k)$(e^{-jk\frac{\pi}{2}} + e^{-jk\frac{3\pi}{2}})$ = 1. So, $a_1 + a_3 = 1$

   $a_0 = \frac{1}{4}$(x(0) + x(1) + x(2) + x(3)) = 1

   a_1 = a + jb and a_3 = a - jb. So, (a + jb) + (a - jb) = 1, 2a = 1. Then, a = $\frac{1}{2}$ and b = $\frac{1}{2}$

   $a_1 = \dfrac{1+j}{2}$ and $a_3 = \dfrac{1-j}{2}$ and $a_2 = 0$

   $$x(0) = 1 + \frac{1+j}{2} + 0 + \frac{1-j}{2} = 2$$
   $$x(1) = 1 + \frac{-1+j}{2} + 0 + \frac{-1-j}{2} = 0$$
   $$x(2) = 1 + \frac{-1-j}{2} + 0 + \frac{-1+j}{2} = 0$$
   $$x(3) = 1 + \frac{1-j}{2} + 0 + \frac{1+j}{2} = 2$$

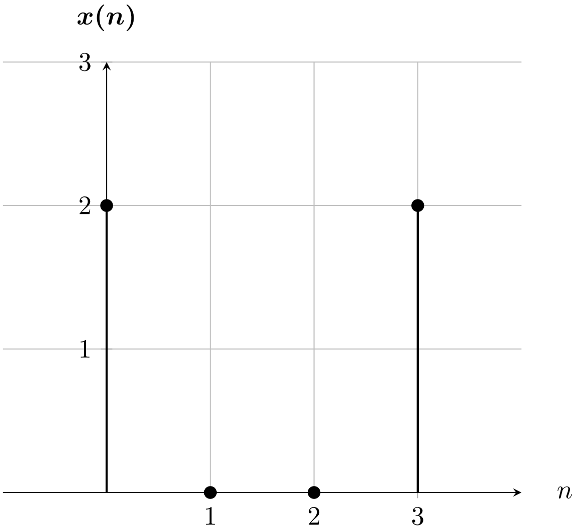

Figure 3: n vs. $x(n)$

3. $w_0 = K\dfrac{2\pi}{T}$

   $-w_0 < w < w_0$ between this integral Y(jw) = X(jw)

   x(t) + r(t)→ | h(t) | → y(t) = x(t)

   x(t) = e^{jwt} → | h(t) | → y(t) = H(jw)e^{jwt}

   As we said on the above Y(jw) = X(jw). So, the frequency response must be 1 (H(jw) = 1)

   $$h(t) = \frac{1}{2\pi}\int_{-w_0}^{w_0} H(jw)e^{jwt}dw = \frac{1}{2\pi}\frac{(e^{jwt_0} - e^{-jwt_0})}{jt} = \frac{1}{t\pi}\frac{(e^{jwt_0} - e^{-jwt_0})}{2j}$$

   h(t)= $\dfrac{1}{t\pi}sin(w_0t)$

4. (a) $y''(t) + 5y'(t) + 6y(t) = 4x'(t) + x(t)$

x(t) = $e^{jwt} \rightarrow |h(t)| \rightarrow y(t) = H(jw)e^{jwt}$

$x'(t) = (jw)e^{jwt}$

$y'(t) = (jw)e^{jwt}H(jw)$

$y''(t) = (jw)^2e^{jwt}H(jw)$

$y''(t) + 5y'(t) + 6y(t) = 4x'(t) + x(t)$

$(jw)^2e^{jwt}H(jw) + 5(jw)e^{jwt}H(jw) + 6H(jw)e^{jwt} = 4(jw)e^{jwt} + e^{jwt}$

$H(jw)((jw)^2 + 5(jw) + 6) = 4(jw) + 1$

$$H(jw) = \frac{4(jw)+1}{((jw)^2 + 5(jw) + 6)} = \frac{4(jw)+1}{(jw+3)(jw+2)}$$

(b) $H(jw) = \dfrac{4(jw)+1}{(jw+3)(jw+2)} = \dfrac{A}{(jw+3)} + \dfrac{B}{(jw+2)}$ A(jw+2) + B(jw+3) = 4(jw)+1

A = 11 and B = -7 So, H(jw) = $\dfrac{11}{(jw+3)}$ - $\dfrac{7}{(jw+2)}$

Inverse Fourier Transform of $H(jw)$ = ((11e^{-3t}) - (7e^{-2t}))u(t)

(c) x(jw) = $\int_{-\infty}^{\infty} x(t) * e^{-jwt}dt$

$$= \frac{1}{4}\int_0^\infty e^{-(\frac{t}{4})}e^{-jwt}dt = \frac{1}{4}\int_0^\infty e^{-t(jw+\frac{1}{4})}dt = \frac{1}{4}\frac{e^{-t(jw+\frac{1}{4})}}{-(jw+\frac{1}{4})}0to\infty$$

X(jw) = $\dfrac{1}{4(jw)+1}$

$$H(jw) = \frac{4(jw)+1}{(jw+2)(jw+3)}$$

Y(jw) = X(jw).H(jw) = $\dfrac{1}{(jw+2)(jw+3)}$

As a result, inverse Fourier Transform of Y(jw) is equal to y(t);

y(t) = (e^{-2t} - e^{-3t})u(t)