ASYMPTOTIC NOTATION

EXAMPLES

Ques1 → Find upper bound for :

$$f(n) = 3n + 8$$

Solution: For finding the upper bound,

we have
$$3n + 8 \le c_1 n$$

Or
$$3 + \frac{8}{n} \le c_1 \quad \text{[Dividing by } n\text{]}$$

Keeping $c_1 = 4$, Lets find the value of n_0 :

n	eq: $3 + \frac{8}{n}$	Value	Condition
1	$3 + \frac{8}{1} = 11$	$11 \le 4$	X
2	$3 + \frac{8}{2} = 3 + 4 = 7$	$7 \le 4$	X
3	$3 + \frac{8}{3} = 3 + 2.6 = 5.6$	$5.6 \le 4$	X
4	$3 + \frac{8}{4} = 3 + 2 = 5$	$5 \le 4$	X
5	$3 + \frac{8}{5} = 3 + 1.6 = 4.6$	$4.6 \le 4$	X
6	$3 + \frac{8}{6} = 3 + 1.3 = 4.3$	$4.3 \le 4$	X
7	$3 + \frac{8}{7} = 3 + 1.1 = 4.4$	$4.4 \le 4$	X
n_0 → 8	$3 + \frac{8}{8} = 3 + 1 = 4$	$4 \le 4$	✓
9	$3 + \frac{8}{9} = 3 + 0.8 = 3.8$	$3.8 \le 4$	✓

SO ON...

$$\therefore \quad \boxed{3n + 8 = O(n) \text{ with } c_1 = 4 \text{ and } n_0 = 8}$$

Ques 2: Find Upper Bound for $100n + 5$.

Solution 1: $100n + 5 \le cn$

Let c be 101, so the eq^n becomes:

$$100n + 5 \le 101n$$

Dividing by n, we have:

$$100 + \frac{5}{n} \le 101$$

On further solving we get:

for all $n \ge 5$, ie. $n_0 = 5$ and $c = 101$ a sol^n exists

$$\therefore \quad \boxed{100n + 5 = O(n) \text{ with } c = 101 \text{ and } n_0 = 5}$$

Solution 2: $100n + 5 \le cn$

Let c be 105, so eq^n becomes

$$100n + 5 \le 105n$$

Dividing by n, we get: $100 + \frac{5}{n} \le 105$.

On further solving we get:

for all $n > 1$, $n_0 = 1$ and $c = 105$, a sol^n exists.

$$\therefore \quad \boxed{100n + 5 = O(n) \text{ with } c = 105 \text{ and } n_0 = 1}$$

Ques 3: Find lower bound for $f(n) = 5n^2$.

Solution: $\exists c, n_0$ such that: $0 \le cn^2 \le 5n^2$

$\Rightarrow \quad cn^2 \le 5n^2$

Dividing by n^2, we have

$\Rightarrow \quad c \le 5$ and $n_0 = 1$.

$$\boxed{\therefore\ 5n^2 = \Omega(n^2) \text{ with } c = 5 \text{ and } n_0 = 1}$$

Ques 4: Prove $f(n) = 100n + 5 \neq \Omega(n^2)$.

Solution: $\exists c, n_0$ such that: $0 \le cn^2 \le 100n + 5$

$100n + 5 \le 100n + 5n \ (\forall n \ge 1) = 105n$

$\Rightarrow \quad cn^2 \le 105n$

$\Rightarrow$ Dividing by n^2, we get

$\Rightarrow \quad \boxed{c \le \frac{105}{n}}$ OR $\boxed{n \le \frac{105}{c}}$

X X

*** CONTRADICTION*:** n cannot be smaller than a constant. So, we can say that

$f(n) = 100n + 5 \neq \Omega(n^2)$.

Proved.

Ques 5: Find Θ bound for $f(n) = \frac{n^2}{2} - \frac{n}{2}$

Solution: $\exists\, c_1, c_2, n_0$, such that

$$0 \le c_1 n^2 \le \frac{n^2}{2} - \frac{n}{2} \le c_2 n^2$$

$$\Rightarrow \quad c_1 n^2 \le \frac{n^2}{2} - \frac{n}{2} \le c_2 n^2$$

Left Hand Inequality

$$c_1 n^2 \le \frac{n^2}{2} - \frac{n}{2}$$

Dividing by n^2, we have:

$$\boxed{c_1 \le \frac{1}{2} - \frac{1}{2 \cdot n}}$$

Now:

n	eq^n	Value
1	$\frac{1}{2} - \frac{1}{2 \cdot 1}$	$\frac{1}{2} - \frac{1}{2} = 0$ [ZERO]
2	$\frac{1}{2} - \frac{1}{2 \cdot 2}$	$\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ [NON-ZERO +ve]
3	$\frac{1}{2} - \frac{1}{2 \cdot 3}$	$\frac{1}{2} - \frac{1}{6} = \frac{1}{3}$
⋮	⋮	⋮ So ON

$\therefore \frac{n^2}{2} - \frac{n}{2} = \Omega(n^2)$ for

$c_1 = \frac{1}{4}$ and $n_0 = 2$

Right Hand Inequality

$$\frac{n^2}{2} - \frac{n}{2} \le c_2 n^2$$

Dividing by n^2, we have:

$$\boxed{\frac{1}{2} - \frac{1}{2n} \le c_2}$$

Now: [Let $c_2 = \frac{1}{2}$]

n	eq^n	Value
1	$\frac{1}{2} - \frac{1}{2}$	$0 \le \frac{1}{2}$ ✓
2	$\frac{1}{2} - \frac{1}{4} = \frac{2-1}{4} = \frac{1}{4}$	$\frac{1}{4} \le \frac{1}{2}$ ✓
3	$\frac{1}{2} - \frac{1}{2 \cdot 3} = \frac{3-1}{6} = \frac{2}{6}$	$\frac{1}{3} \le \frac{1}{2}$ ✓
		⋮ So ON

$\therefore \frac{n^2}{2} - \frac{n}{2} = O(n^2)$ for

$c_2 = \frac{1}{2}$ and $n_0 = 1$

$$\boxed{\therefore \frac{n^2}{2} - \frac{n}{2} = \Theta(n^2) \text{ for } c_1 = \frac{1}{4},\ c_2 = \frac{1}{2} \text{ and } n_0 = 2}$$