Hash Table

Dictionary

- **Dictionary:**
  - Dynamic-set data structure for storing items indexed using *keys*.
  - Supports operations Insert, Search, and Delete.
  - Applications:
    - Symbol table of a compiler.
    - Memory-management tables in operating systems.
    - Large-scale distributed systems.
- **Hash Tables:**
  - Effective way of implementing dictionaries.
  - Generalization of ordinary arrays.

Hash Function

- Determines position of key in the array.
- Assume table (array) size is N
- Function $f(x)$ maps any key x to an int between 0 and $N-1$

- For example, assume that N=15, that key x is a non-negative integer between 0 and MAX_INT, and hash function $f(x) = x \% 15$.

Hash Function

Let f(x) = x % 15. Then,

if x =	25	129	35	2501	47	36
f(x)=	10	9	5	11	2	6

Storing the keys in the array is straightforward:

Memory address	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
value	_	_	47	_	_	35	36	_	_	129	25	2501	_	_	_

Thus, delete and find can be done in O(1), and also insert, except…

Collision

What happens when you try to insert: $x = 65$?

$$x = 65$$

$$f(x) = 5$$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
_	_	47	_	_	35	36	_	_	129	25	2501	_	_	_
					65(?)									

This is called a collision.

Handling Collisions

- Separate Chaining
- Open Addressing
  - Linear Probing
  - Quadratic Probing
  - Double Hashing

Separate Chaining

Let each array element be the head of a chain.

```
0    1    2    3    4    5    6    7    8    9   10    11   12   13   14
          ↓              ↓    ↓              ↓    ↓     ↓
         47             65   36            129   25  2501
                         ↓
                        35
```

Where would you store: 29, 16, 14, 99, 127 ?

Separate Chaining

Let each array element be the head of a chain:

Where would you store: 29, 16, 14, 99, 127 ?

```
0   1   2   3   4   5   6   7   8   9   10    11   12  13  14
    ↓   ↓           ↓   ↓   ↓       ↓   ↓     ↓            ↓
   16  47          65  36 127      99  25 2501            14
                    ↓               ↓                      ↓
                   35             129                     29
```

New keys go at the front of the relevant chain.

Separate Chaining: Disadvantages

- Parts of the array might never be used.
- As chains get longer, search time increases to O(n) in the worst case.
- Constructing new chain nodes is relatively expensive (still constant time, but the constant is high).
- Is there a way to use the "unused" space in the array instead of using chains to make more space?

Linear Probing

Let key *x* be stored in element *f(x)=t* of the array

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
		47			35	36			129	25	2501			
					65(?)									

What do you do in case of a collision?

If the hash table is *not full*, attempt to store key in the next array element (in this case $(t+1)\%N$, $(t+2)\%N$, $(t+3)\%N$ ...)

until you find an empty slot.

Linear Probing

Where do you store *65* ?

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
		47			35	36	65		129	25	2501			
					↑	↑	↑							

attempts

Where would you store: 29?

Linear Probing

If the hash table is *not full*, attempt to store key in array elements $(t+1)\%N$, $(t+2)\%N$, ...

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
		47			35	36	65		129	25	2501			29

↑
attempts

Where would you store: 16?

Linear Probing

If the hash table is *not full*, attempt to store key in array elements $(t+1)\%N$, $(t+2)\%N$, ...

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	16	47			35	36	65		129	25	2501			29
	↑													

Where would you store: 14?

Linear Probing

If the hash table is *not full*, attempt to store key in array elements $(t+1)\%N$, $(t+2)\%N$, …

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
14	16	47			35	36	65		129	25	2501			29
↑														↑
														attempts

Where would you store: 99?

Linear Probing

If the hash table is *not full*, attempt to store key in array elements $(t+1)\%N$, $(t+2)\%N$, ...

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
14	16	47			35	36	65		129	25	2501	99		29
									↑	↑	↑	↑		

attempts

Where would you store: 127 ?

Linear Probing

If the hash table is *not full*, attempt to store key in array elements $(t+1)\%N$, $(t+2)\%N$, ...

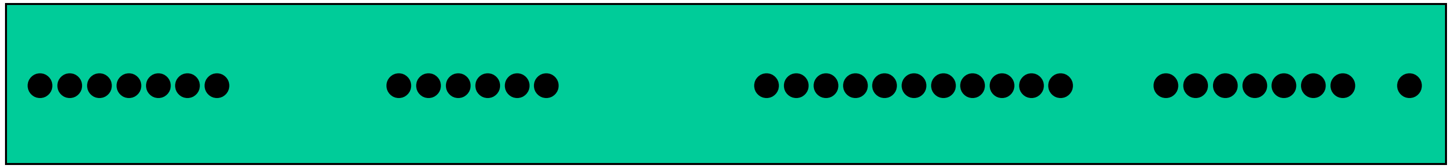
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	16	47			35	36	65	127	129	25	2501	29	99	14
							↑	↑						

attempts

Linear Probing

- Eliminates need for separate data structures (chains), and the cost of constructing nodes.

- Leads to problem of clustering. Elements tend to *cluster* in dense intervals in the array.



- Search efficiency problem remains.
- Deletion becomes trickier….

Deletion problem

- H=KEY MOD 10
- Insert 47, 57, 68, 18, 67

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

0	18
1	67
2	
3	
4	
5	
6	
7	47
8	57
9	68

Deletion problem

- H=KEY MOD 10
- Now say you have deleted 57.
- Now when you will go for 67 deletion, the cursor will look at 7th position. Not match. It will move to next 8th position. No data there. So it will think that 67 is not there. As correct position for 67 should be 8th position.

0	18
1	67
2	
3	
4	
5	
6	
7	47
8	
9	68

Deletion Problem -- SOLUTION

- “Lazy” deletion

- Each cell is in one of 3 possible states:
  - active
  - empty
  - deleted

- For Find or Delete
  - only stop search when EMPTY state detected (not DELETED)

Deletion-Aware Algorithms

- Insert
  - Cell empty or deleted   insert at H, *cell = active*
  - Cell active   H = (H + 1) mod TS

- Find
  - cell empty   NOT found
  - cell deleted   H = (H + 1) mod TS
  - cell active   if key == key in cell -> FOUND
    else H = (H + 1) mod TS

- Delete
  - cell active;   key != key in cell   H = (H + 1) mod TS
  - cell active;   key == key in cell   DELETE; *cell=deleted*
  - cell deleted   H = (H + 1) mod TS
  - cell empty   NOT found

Quadratic Probing

Let key x be stored in element $f(x)=t$ of the array

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
		47			35	36			129	25	2501			
					65(?)									

What do you do in case of a collision?

If the hash table is *not full*, attempt to store key in array elements $(t+1^2)\%N$, $(t+2^2)\%N$, $(t+3^2)\%N$ …

until you find an empty slot.

Quadratic Probing

Where do you store 65 ? *f(65)=t=5 (reason 65 mod 15=5)*

Check for 5, $(5+1^2=6)$, $(5+2^2=9)$, $(5+3^2=14)$

```
0     1     2     3     4     5     6     7     8     9   10     11   12   13   14
           47                35    36               129   25  2501              65
                              ↑     ↑                 ↑                          ↑
                              t  t+1                t+4                        t+9
                              attempts
```

Where would you store: 29?

Quadratic Probing

If the hash table is *not full*, attempt to store key in array elements $(t+1^2)\%N$, $(t+2^2)\%N$ …

Check for 14, $(14+1^2=15,\ 15\%15=0)$

```
 0    1    2    3    4    5    6    7    8    9   10    11   12   13   14
29        47                  35   36                129   25 2501             65
↑                                                                             ↑
t+1                                                                           t
                                                                          attempts
```

Where would you store: 16?

Quadratic Probing

If the hash table is *not full*, attempt to store key in array elements $(t+1^2)\%N$, $(t+2^2)\%N$ ...

Check for 1

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
29	16	47			35	36			129	25	2501			65

↑
t
attempts

Where would you store: 14?

Quadratic Probing

If the hash table is *not full*, attempt to store key in array elements $(t+1^2)\%N$, $(t+2^2)\%N$ …

Check for 14, $(14+1^2=15,\ 15\%15=0)$, $(14+2^2=18,\ 18\%15=3)$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
29	16	47	14		35	36			129	25	2501			65
↑			↑											↑
t+1			t+4											t attempts

Where would you store: 99?

Quadratic Probing

If the hash table is *not full*, attempt to store key in array elements $(t+1^2)\%N$, $(t+2^2)\%N$ …

Check for 9, $(9+1^2=10$, $10\%15=10)$, $(9+2^2=13$, $13\%15=13)$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
29	16	47	14		35	36			129	25	2501		99	65
									↑	↑			↑	
									t	t+1			t+4	

attempts

Where would you store: 127 ?

Quadratic Probing

If the hash table is *not full*, attempt to store key in array elements $(t+1^2)\%N$, $(t+2^2)\%N$ …

Check for 7

Where would you store: 127 ?

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
29	16	47	14		35	36	127		129	25	2501		99	65

↑
t
attempts

Quadratic Probing

- Tends to distribute keys better than linear probing
- Runs the risk of an infinite loop on insertion, unless precautions are taken.
- E.g., consider inserting the key 16 into a table of size 16, with positions 0, 1, 4 and 9 already occupied.
- Therefore, table size should be prime.

Double Hashing

- Use a hash function for the decrement value
  - $Hash(key, i) = H_1(key) - (H_2(key) * i)$
- Now the decrement is a function of the key
  - The slots visited by the hash function will vary even if the initial slot was the same
  - Avoids clustering
- Theoretically interesting, but in practice slower than quadratic probing, because of the need to evaluate a second hash function.

Double Hashing

Let key x be stored in element $f(x)=t$ of the array

Array:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
		47			35	36			129	25	2501			
					65(?)									

What do you do in case of a collision?

Define a second hash function $f_2(x)=d$. Attempt to store key in array elements $(t+d)\%N$, $(t+2d)\%N$, $(t+3d)\%N$ ...

until you find an empty slot.

Double Hashing

- Typical second hash function

$$f_2(x) = R - (x \% R)$$

where R is a prime number, $R < N$ *(max size)*

Double Hashing

Where do you store 65 ? $f(65)=t=5$

Let $f_2(x) = 11 - (x \% 11)$, $f_2(65)=11-(65\%11)$ =11-10= 1, *So, d=1*

Note: Prime number R=11, max size N=15

Attempt to store key in array elements $(t+d)\%N$, $(t+2d)\%N$, $(t+3d)\%N$ …

So try for 5, (5+1)%15=6, (5+2)%15=7

Array:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
		47			35	36	65		129	25	2501			
					↑	↑	↑							
					t	t+1	t+2							

attempts

Double Hashing

If the hash table is *not full*, attempt to store key in array elements $(t+d)\%N$, $(t+2d)\%N$ …

Let $f_2(x) = 11 - (x \% 11)$, $f_2(29)=11-(29\%11)=11-7=4$

$f_2(29)=d=4$

Where would you store: 29?

Check 29%15=14

Array:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
		47			35	36	65		129	25	2501			29
														↑
														t
														attempt

Double Hashing

If the hash table is *not full*, attempt to store key in array elements $(t+d)\%N$, $(t+d)\%N$ …

Let $f_2(x) = 11 - (x \% 11)$ $\quad f_2(16)=d=6$

Where would you store: 16?

Check 16%15=1

Array:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	16	47			35	36	65		129	25	2501			29
	↑													
	t													
	attempt													

Where would you store: 14?

Double Hashing

If the hash table is *not full*, attempt to store key in array elements $(t+d)\%N$, $(t+d)\%N$ …

Let $f_2(x) = 11 - (x \% 11)$ $\quad$ $f_2(14)=d=8$

Check 14%15=14, (14+1*8)%15= 7, (14+2*8)%15= 0

Array:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
14	16	47			35	36	65		129	25	2501			29
↑							↑							↑
t+16							t+8							t

attempts

Where would you store: 99?

Double Hashing

If the hash table is *not full*, attempt to store key in array elements $(t+d)\%N$, $(t+d)\%N$ …

Let $f_2(x) = 11 - (x \% 11)$ $\qquad f_2(99)=d=11$

Check 99%15=9, (99+1*11)%15=5, (99+2*11)%15=1, (99+3*11)%15=12

Array:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
14	16	47			35	36	65		129	25	2501	99		29
	↑				↑				↑			↑		
	t+22				t+11				t			t+33		

attempts

Where would you store: 127 ?

Double Hashing

If the hash table is *not full*, attempt to store key in array elements $(t+d)\%N$, $(t+d)\%N$ …

Let $f_2(x) = 11 - (x \% 11)$ $f_2(127)=d=5$

Check 127%15=7, (127+1*5)%15=12, (127+2*5)%15=2, (127+3*5)%15=7, (127+4*5)%15=12, (127+5*5)%15=2

Array:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
14	16	47			35	36	65		129	25	2501	99		29
		↑					↑					↑		
		t+10					t					t+5		

attempts

Infinite loop!

Performance

Load factor = % of table that's occupied.