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Exercises:

1) Sign extension is necessary, especially when it is a negative number, say,

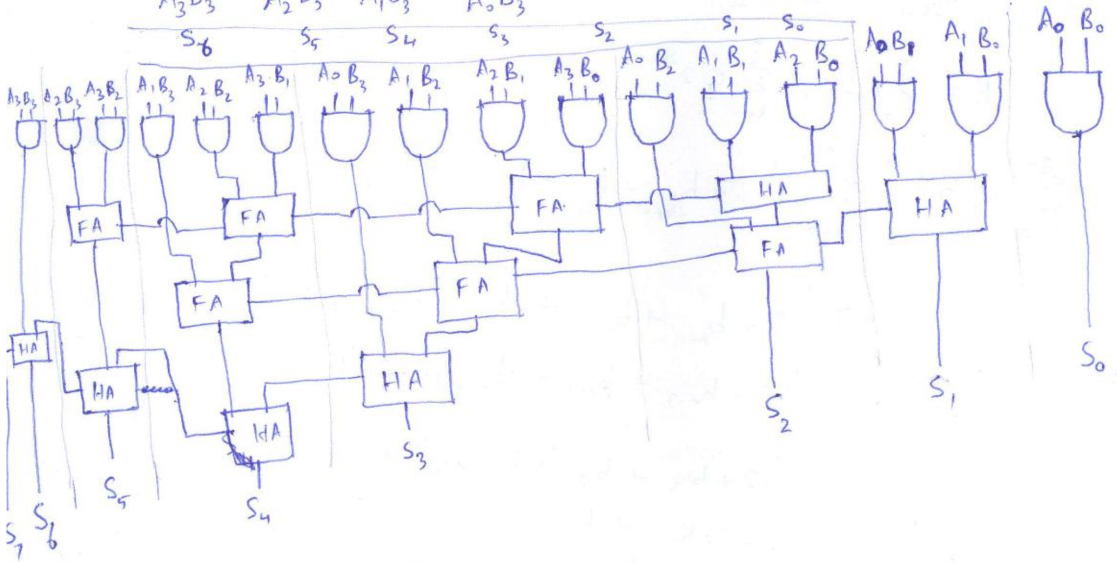
$(3)_{10} = (011)_2$ in 3 bits, but when sign extended to 4 bits, becomes, $(3)_{10} = (0011)_2$.

whereas $(-3)_{10} = (101)_2$ in 3 bits, but when sign extended to 4 bits,

becomes, $(-3)_{10} = (1101)_2$.

2) Example: (for 4 bits).

$$\begin{array}{rrrrrrr}
 & & & A_3 & A_2 & A_1 & A_0 \\
 & & \times & B_3 & B_2 & B_1 & B_0 \\
\hline
 & & & A_3B_0 & A_2B_0 & A_1B_0 & A_0B_0 \\
 & & A_3B_1 & A_2B_1 & A_1B_1 & A_0B_1 & \\
 & A_3B_2 & A_2B_2 & A_1B_2 & A_0B_2 & & \\
A_3B_3 & A_2B_3 & A_1B_3 & A_0B_3 & & &
\end{array}$$



3) It becomes directly the 2's complement expression.

let $B = \sum_{i=0}^{n-1} b_i 2^i = b_{n-1} \times 2^{n-1} + b_{n-2} \times 2^{n-2} + \cdots + b_0$.

if $b_{n-1} = 0 \Rightarrow B = \sum_{i=0}^{n-2} b_i 2^i$

if $b_{n-1} = 1 \Rightarrow -B = b_{n-1} \cdots b_0$ with $b_{n-1} = 0$.

suppose $B = -b_{n-1} \times 2^{n-1} + \sum_{i=0}^{n-2} b_i 2^i$.

then $-B = -(1-b_{n-1}) 2^{n-1} + \sum_{i=0}^{n-2} (1-b_i) 2^i + 1$

$$= -2^{n-1} + b_{n-1} 2^{n-1} + \sum_{i=0}^{n-2} 2^i - \sum_{i=0}^{n-2} b_i 2^i + 1$$

$$= \cancel{-2^{n-1}} + b_{n-1} 2^{n-1} + \frac{\cancel{2^{n-1}} \cancel{-1}}{2-1} - \sum_{i=0}^{n-2} b_i 2^i \cancel{+1}$$

$$= b_{n-1} 2^{n-1} - \sum_{i=0}^{n-2} b_i 2^i$$

$\therefore B + (-B) = 0$.

4) $c_k = -2 \times b_{2k+1} + b_{2k} + b_{2k-1}$ for $k = 0, \ldots, \frac{n}{2} - 1$ with $b_{-1} = 0$.

$$B = \sum_{i=0}^{n-1} b_i 2^i = \sum_{k=0}^{\frac{n}{2}-1} c_k 2^{2k}.$$

$$\sum_{k=0}^{\frac{n}{2}-1} c_k 2^{2k} = \sum_{k=0}^{\frac{n}{2}-1} \left(-2 \times b_{2k+1} + b_{2k} + b_{2k-1}\right) 2^{2k}.$$

$$= (-2 \times b_{n-1} + b_{n-2} + b_{n-3}) \times 2^{n-2} +$$
$$(-2 \times b_{n-3} + b_{n-4} + b_{n-5}) \times 2^{n-4} +$$
$$\vdots$$
$$(-2 \times b_3 + b_2 + b_1) \times 2^2 +$$
$$(-2 \times b_1 + b_0 + b_{-1}).$$