

Exercises:

Designing an FIR filter in Python.

FIR Filters:

Qn: If a digital low-pass filter is designed for a sampling frequency of $f_s = 1000\text{Hz}$ and cutoff frequency $f_c = 200\text{Hz}$, then if we use the same coefficients on a sampling frequency of $f_s = 4000\text{Hz}$, what cutoff frequency does that correspond to?

Ans: The normalized frequency should be the same for the cutoff frequency,

$$\therefore \frac{200}{1000} = \frac{f_c}{4000} \Rightarrow f_c = 800\text{Hz}.$$

Qn: Why $20 \log$ and not $10 \log$?

$$\text{Ans: Power} \propto V^2 \Rightarrow \frac{\text{output Power}}{\text{input Power}} = \left(\frac{V_{\text{output}}}{V_{\text{input}}}\right)^2 = \left(\frac{V_o}{V_i}\right)^2.$$

$$\Rightarrow 10 \log \left(\frac{\text{output power}}{\text{input power}} \right) = 20 \log \left(\frac{V_o}{V_i} \right).$$

Qn: Why -40 and not $+40$?

Ans: The power attenuated & decreases.

$$\therefore P_o < P_i \text{ & } V_o < V_i \Rightarrow 20 \log \left(\frac{V_o}{V_i} \right) < 0 \\ \Rightarrow \frac{V_o}{V_i} < 1 \Rightarrow 20 \log \left(\frac{V_o}{V_i} \right) < 0.$$

 FIR Design and Simulation with Python:

Frequency Content:

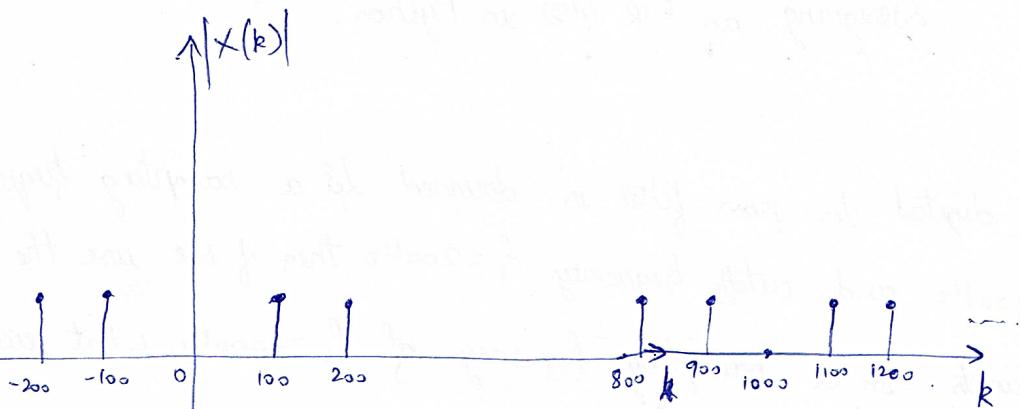
Qn: Why is the peak amplitude the value that you see?

$$\text{Ans: } x[n] = s = \sin \left(2\pi \times \frac{100n}{1000} \right) + \sin \left(2\pi \times \frac{200n}{1000} \right) = e^{j2\pi \times \frac{100n}{1000}} - e^{-j2\pi \times \frac{100n}{1000}} + e^{j2\pi \times \frac{200n}{1000}} - e^{-j2\pi \times \frac{200n}{1000}}$$

$$X(k) = \text{FFT}(s) = \sum_{n=0}^{999} x_n e^{-j2\pi k \frac{n}{N}} = \frac{1}{2i} \left[\delta(k-100) - \delta(k-900) + \delta(k-200) - \delta(k-800) \right]$$

(2)

\therefore in frequency domain, we have,



\therefore we see peaks at $k = 100, 200, 800 \text{ & } 900$,

$$|X(k)| = \frac{1}{2} \text{ at those } k \text{ values.}$$

$$20 \times \log_{10} \left(\frac{1}{2} \right) \approx -6.02.$$

Qn: Why is there a noisy floor?

Ans: In python, the plot of sine had been of an interpolation of points sampled at 0.001 ms. So, though, it resembles a sine wave, it has slightly abrupt slopes which are discontinuous. This rises to higher frequency noise in addition to the desired signal.

Qn: Change the time base to the following: ~~and~~ ~~for~~

$$t = \text{arange}(0, 1.001, 1.0/1000)$$

and repeat the above steps. Why does the FFT look so different.

Ans: In this case, $x[n] = S = \sin\left(2\pi \times \frac{100n}{1000}\right) + \sin\left(2\pi \times \frac{200n}{1000}\right)$.

$$\text{Let } X(k) = \sum_{n=0}^{1000} x_n e^{-j2\pi \frac{kn}{1001}} \quad , \quad k = 0, \dots, 1000.$$

\therefore This imbalance between $\frac{100n}{1000}$ & $\frac{200n}{1000}$ with $\frac{kn}{1001}$ gives rise to such a different FFT.

(3)

Filter coefficients:

Qn: What is the attenuation of the above figure in the stop band?

Ans: ~~about -76.5 dB about -46.37 dB~~, about -37.19 dB.

Qn: What happens if you reduce the number of taps to 5?

Ans: The attenuation of the figure in the stop band increases to about ~~-20 dB~~.

-11.61 dB

Qn: How many taps do you need for your filter for a 30dB stopband attenuation?

Ans: ~~about 15 taps~~ about 14 taps.

Qn: What happens to signals in the range $\omega \in 200-300\text{Hz}$ with the above filter?

Ans: The signals gets attenuated at variable degrees ranging from ~~0dB to -46.37 dB~~
0dB to -37.19 dB

Filtering:

Qn: What does the filtered signal look like?

Ans: Ref figure - "Filtered Signal".

Qn: When input is 8 bits, how many bits required before decimal place for output?

Ans: sine function "S" runs from $-\sqrt{2}$ to $\sqrt{2}$. So, we just need one bit for magnitude & (0 or 1) and one bit for the sign.
Hence we need only 2 bits.

Qn: How many bits in the coefficients of the LPF, for 30dB attenuation.

Ans: We need 8 bits. Because, then LPF becomes 20dB attenuated at stop band.

FIR Coefficients