

Scientific Computing assignment 5

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Problem 5 The following code is used for computing the modified Cholesky factorization of a symmetric H:

```
#!/usr/bin/env python3
# -*- coding: utf-8 -*-
"""
Created on Thu Dec 13 14:07:08 2018

@author: alfred_mac
"""

import numpy as np
import matplotlib.pyplot as plt
import numpy.linalg as la

def mCh(H):
    N = len(H)
    L = np.zeros([N,N])
    l1 = np.zeros([N,1])
    l2 = np.zeros([N,1])

    # Loop running over each element of L across every column in every row
    for i in range(N):
        if (i==0):
            L[i,i] = np.sqrt(abs(H[i,i]))
        else:
            L[i,i] = np.sqrt(abs(H[i,i]-np.dot((L[i].T),L[i])))

        for j in range(N-i-1):
            if (i==0):
                L[j+i+1,i] = H[j+i+1,i]/L[i,i]
            else:
                L[j+i+1,i] = (H[j+i+1,i] - np.dot((L[j+i+1].T),L[i]))/L[i,i]
```

```

eig_H = la.eig(H)
flag = 0

# Loop verifying if the matrix is positive definite by checking if
each eigen value is positive
for i in range(N):
    if (eig_H[0][i]<=0):
        flag = 1

P = True
if (flag==1):
    P = False

return (L,P)

# Below are few input matrices
#A = np.array([[4,12,-16],[12,37,-43],[-16,-43,98]])
#A = np.array([[2,-1,0],[-1,2,-1],[0,-1,2]])
#A = np.array([[2,-1,1,-2],[-1,2,-1,1],[1,-1,2,1],[-2,1,1,-1]])
#A = np.array([[2,-1,0,-2,5],[-1,2,-1,1,-3],
[0,-1,2,1,4],[-2,1,1,0,1],[-1,3,2,1,-5]])
#A = np.eye(11)
#A = np.random.random([7,7])
A = np.random.random([15,15])

(LA,PA) = mCh(A)

f = open("Cholesky_factorization_output.txt","a+")

f.write("A is:\n")
Am = np.matrix(A)
for line in Am:
    np.savetxt(f,line,fmt='%2f')

f.write("Modified Cholesky factorization of A is:\n")
LAm = np.matrix(LA)
for line in LAm:
    np.savetxt(f,line,fmt='%2f')

f.write("A is positive definite: ")
f.write(str(PA))
f.write("\n")
f.close()

```

The following shows the different inputs for which the program was tested

and their corresponding outputs:

A is:
4.00 12.00 -16.00
12.00 37.00 -43.00
-16.00 -43.00 98.00
Modified Cholesky factorization of A is:
2.00 0.00 0.00
6.00 1.00 0.00
-8.00 5.00 3.00
A is positive definite: True

A is:
2.00 -1.00 0.00
-1.00 2.00 -1.00
0.00 -1.00 2.00
Modified Cholesky factorization of A is:
1.41 0.00 0.00
-0.71 1.22 0.00
0.00 -0.82 1.15
A is positive definite: True

A is:
2.00 -1.00 1.00 -2.00
-1.00 2.00 -1.00 1.00
1.00 -1.00 2.00 1.00
-2.00 1.00 1.00 -1.00
Modified Cholesky factorization of A is:
1.41 0.00 0.00 0.00
-0.71 1.22 0.00 0.00
0.71 -0.41 1.15 0.00
-1.41 0.00 1.73 2.45
A is positive definite: False

A is:
2.00 -1.00 0.00 -2.00 5.00
-1.00 2.00 -1.00 1.00 -3.00
0.00 -1.00 2.00 1.00 4.00
-2.00 1.00 1.00 0.00 1.00
-1.00 3.00 2.00 1.00 -5.00
Modified Cholesky factorization of A is:
1.41 0.00 0.00 0.00 0.00
-0.71 1.22 0.00 0.00 0.00
0.00 -0.82 1.15 0.00 0.00
-1.41 0.00 0.87 1.66 0.00
-0.71 2.04 3.18 -1.66 4.74
A is positive definite: False

A is:

1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

Modified Cholesky factorization of A is:

1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00

A is positive definite: True

A is:

0.83	0.76	0.89	0.64	0.97	0.84	0.65
0.18	0.37	0.26	0.82	0.91	0.59	0.04
0.09	0.97	0.63	0.26	0.38	0.89	0.37
0.57	0.56	0.48	0.65	0.40	0.44	0.56
0.51	0.64	0.93	0.32	0.53	0.97	0.51
0.64	0.11	0.96	0.19	0.68	0.32	0.46
0.16	0.61	0.23	0.92	0.11	0.55	0.79

Modified Cholesky factorization of A is:

0.91	0.00	0.00	0.00	0.00	0.00	0.00
0.20	0.57	0.00	0.00	0.00	0.00	0.00
0.10	1.65	1.45	0.00	0.00	0.00	0.00
0.62	0.77	-0.59	0.82	0.00	0.00	0.00
0.56	0.92	-0.44	-1.21	1.51	0.00	0.00
0.70	-0.04	0.66	0.21	0.57	0.99	0.00
0.18	1.00	-0.99	-0.66	-1.42	2.09	2.83

A is positive definite: False

A is:

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0.60 0.51 0.75 0.38 0.23 0.08 0.21 0.57 0.36 0.71 0.87 0.48 0.45 0.88 0.50
0.45 0.14 0.29 0.52 0.21 0.11 0.07 0.32 0.55 0.63 0.52 0.11 0.97 0.07 0.84
0.38 0.72 0.65 0.09 0.70 1.00 0.70 0.37 0.46 0.78 0.28 0.01 0.02 0.99 0.75
0.18 0.75 0.33 0.66 0.82 0.67 0.60 0.35 0.04 0.33 0.07 0.96 0.74 0.91 0.41
0.70 0.55 0.67 0.35 0.97 0.51 0.54 0.27 0.52 0.66 0.95 0.86 0.89 0.51 0.50
0.78 0.81 0.58 0.45 0.73 0.29 0.43 0.41 0.51 0.71 0.19 0.83 0.27 0.76 0.14
0.69 0.00 0.70 1.00 0.67 0.26 0.83 0.55 0.09 0.14 0.53 0.21 0.05 0.42 0.96
0.65 0.88 0.65 0.47 0.76 0.76 0.93 0.79 1.00 0.02 0.47 0.91 0.52 0.91 0.55
0.79 0.81 0.05 0.53 0.89 0.40 0.83 0.25 0.03 0.87 0.91 0.36 0.41 0.48 0.05
0.29 0.63 0.39 0.36 0.87 0.84 0.54 0.89 0.44 0.52 0.12 0.17 0.65 0.28 0.00
0.80 0.94 0.63 0.58 0.73 0.87 0.89 0.40 0.98 0.24 0.43 0.38 0.39 0.02 0.99
0.33 0.46 0.18 0.18 0.10 0.02 0.12 0.73 0.93 0.87 0.56 0.94 0.50 0.56 0.59
0.52 0.66 0.31 0.60 0.00 0.22 0.61 0.43 0.92 0.59 0.25 0.47 0.18 0.03 0.67
0.40 0.84 0.92 0.80 0.17 0.45 0.16 0.23 0.73 0.62 0.31 0.42 0.12 0.76 0.64
0.02 0.64 0.32 0.56 0.04 0.88 0.73 0.13 0.45 0.17 0.51 0.21 0.78 0.70 0.80
Modified Cholesky factorization of A is:
0.77 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.59 0.46 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.49 0.94 0.69 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.23 1.34 -1.53 1.88 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.91 0.05 0.26 0.25 0.11 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
1.01 0.46 -0.51 -0.63 0.77 1.49 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.90 -1.14 1.94 2.81 -11.75 7.91 14.62 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.84 0.84 -0.81 -1.11 4.01 -3.16 5.33 7.55 0.00 0.00 0.00 0.00 0.00 0.00 0.00
1.03 0.45 -1.28 -1.21 5.19 -4.23 6.89 -9.84 13.91 0.00 0.00 0.00 0.00 0.00 0.00
0.38 0.88 -0.90 -1.22 9.24 -5.61 10.90 -15.25 -21.55 30.58 0.00 0.00 0.00 0.00 0.00
1.03 0.74 -0.84 -1.03 2.20 -2.22 3.34 -4.84 -6.77 -9.54 13.58 0.00 0.00 0.00 0.00
0.43 0.47 -0.68 -0.84 0.71 -1.39 1.59 -2.28 -3.21 -4.48 -6.42 9.07 0.00 0.00 0.00
0.68 0.57 -0.82 -0.84 -1.87 -0.16 -1.10 1.41 2.05 3.01 4.01 5.57 8.31 0.00 0.00
0.52 1.17 -0.64 -0.99 0.60 -1.37 1.57 -2.38 -3.27 -4.56 -6.59 -9.38 12.38 18.13 0.00
0.03 1.37 -1.43 -1.84 7.06 -4.81 8.98 -12.67 -17.82 -25.24 -35.57 -50.29 68.90 -
98.78
139.91
A is positive definite: False

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