Dynamics of Urban Traffic Congestion: Acceleration-based Kinetic Monte-Carlo model

Author
The Institute of Mathematical Sciences,
CIT Campus, Taramani, Chennai 600113, India
Email:

***Abstract*—Abstract abstract abstract.**

I. Introduction

As a result of the advancement in computing architecture and numerical techniques, large-scale social dynamics are nowadays modelled in a simpler manner using just the interactions between autonomous agents. Through such approach, the macroscopic changes can be observed at the level of the entire system as a result of the micro-dynamics of the individual constituents [1]. For example, the transition of the traffic state from free flowing to jammed state and vice versa [2], [3].

Although vehicle traffic has been studied for decades using fluid dynamical models, after Nagel and Schreckenberg introduced a much simpler cellular automata model of highway traffic flow [4], there was an increased interest towards describing complicated patterns of traffic using elegant microscopic models. Yet, there has been very little work done regarding urban traffic.

Studying urban traffic turns out to have more significance both in terms of academical and economical point of view, as the study of traffic jams and their lifetimes might be helpful in building more efficient transport infrastructure in a city. Also, as mentioned earlier in [5], the congestion times (or the waiting times of vehicles in the jam) follows a power-law (as opposed to an exponential), thereby making the study of congestion even more interesting.

By studying the critical factors that affects traffic congestion times, one would be able to deduce what aspects of traffic could be used to control and perhaps to reduce congestion time in traffic. Earlier [6], our team has provided with a kinetic Monte-Carlo model that explains how power-law characteristic of the waiting time emerges when there is some heterogeneity in the way cars respond to headway distances. Here, we've used acceleration-based updating scheme to resemble reality more closely. More importantly, in the velocity-based model, the stochastic fluctuations that were used did not have a clear mapping to reality. So, in this acceleration-based model, we're able to reproduce similar results by just considering the fact that different characteristics for each car and each driver, meaning each car and driver are non-identical to each other.

From fig. 1, we observe that the velocity varies rather in a relatively continuous fashion, whereas, the acceleration varies quite abruptly over time. This aspect is not captured by any of the earlier models.

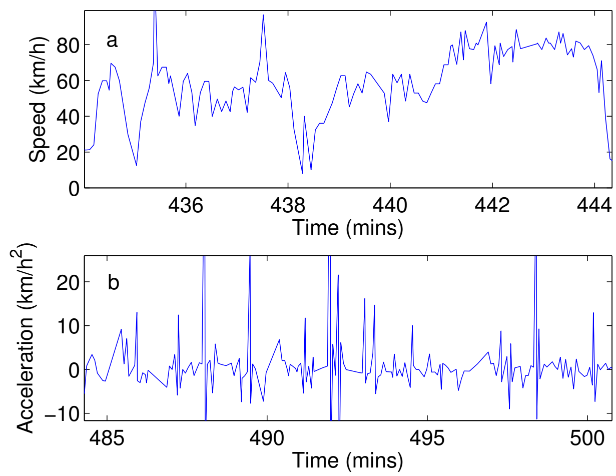

Fig. 1. a) Variation of velocity over time on a day, from a taxi in Delhi using GPS data obtained by [7] during Jan, 2013. b) Similarly, the variation of acceleration over time is also plotted

II. Model

The kinetic Monte Carlo model that had been introduced earlier in [6] was majorly to explain the power-law characteristics of the waiting times observed in [5]. In that model, the velocities are abruptly changed every time step (depending on the headway distance). Although it was possible to reproduce the power law characteristics of waiting times and the fundamental relation between mobility, flow and density, in real life, the control mechanisms are in fact acceleration based, namely - accelerator or brake. So, in this paper we intend to improvise the velocity-based model to incorporate the acceleration-based controls and effects. This would in turn smoothen out the velocity vs time plot as expected from the empirical data.

Similar to the model in [6], here too we assume a single lane with fixed number of (identical) vehicles moving only in one direction. In order to avoid boundary issues, a circular road is considered. Also, those vehicles are externally controlled by a signal which is placed at one end. This signal basically represents the cross-over traffic. Now, instead of just updating the velocities of each vehicle at every instant, we update both velocity and acceleration as follows:

$$
\begin{aligned}
\frac{dx}{dt} &= v \\
\frac{dv}{dt} &= -(\alpha + \xi(x,t))v + \beta\left(\frac{d + v_{rel}\tau}{d+1}\right)
\end{aligned} \tag{1}
$$

where,

α : drag rate due to friction of the road surface.

ξ : space and time dependent positive glitch in the drag rate that affects just a random car for each time step, representing possibly a pothole or applying of brake, etc.

β : maximum acceleration possible (using accelerator). Maximum velocity $v_{max} = \beta/\alpha$ (for single car in empty road $\frac{dv}{dt} = -\alpha v + \beta$)

τ : reaction time of the car-driver interface.

d : headway distance available in front of every car to move without colliding.

v_{rel} : relative velocity of the car in front with respect to the car under consideration.

The values of α, β and τ are chosen randomly for each car throughout the simulation. For a generalized choice of distribution, we pick those three parameter values for each car from a gamma distribution, so that we could choose from a range of distributions. Several such realizations are collectively used for the probability distribution of the waiting time. This would therefore provide an insight as to which (inherent) characteristic of the car-driver-road system is crucial for the power-law behaviour of the waiting time. It has already been shown that the power-law could emerge just from simple dynamics, without even considering any network structure nor multi-lanes, in [6]. The influence of external characteristic like the signal cycle, traffic density and duty ratio (of the signals), over the power-law behaviour is already being studied in [9].

The first term in the acceleration eqn. 1 represents the friction due to the road surface and it is proportional to the instantaneous velocity. Similarly, the second term represents the acceleration given by the driver for every time step. It is directly proportional to the headway distance available for the vehicle in front. We've used an hyperbolic function just to make sure an upper bound for the acceleration. Since there would be a lag in terms of the car-driver system to respond, the actual headway distance would be lower than the physical headway distance. So, the second term is corrected for that. Since $d \gg v_{rel}\tau$ usually, we've corrected only for the numerator.

Based on those equations 1, the time step dt for progressing time is calculated based on two conditions: i) None of the cars collide with the cars in front while the vehicles progress with constant acceleration during dt. ii) None of the cars' velocity goes below zero (which is not a bad assumption based on reality). The largest such dt value is chosen for updating. Basically at time n (using Newton's 2nd law we arrive at),

$$
dt_n = \min_i \left(\frac{-v_{i,n-1} + \sqrt{v_{i,n-1}^2 + 2a_{i,n-1}d_{i,n-1}}}{a_{i,n-1}} \right) \tag{2}
$$

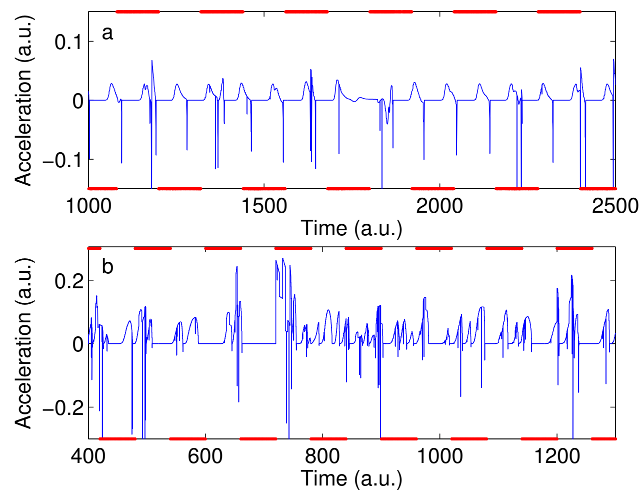

Fig. 2. Variation of acceleration over time from the simulation. The traffic signal is marked with red. a) shows the variation of the instantaneous acceleration for a single car when we assume all identical cars with identical parameter values. b) similarly shows the variation of instantaneous acceleration when the parameter values for each car is different from each other.

where v is velocity, a is acceleration, d is headway distance, n is the time instant and i represents vehicle. dt_n is now the time step at time n. This adaptive step-size allows us to mimic continuous-time, continuous-space dynamics more accurately. *Note*: The determinant (or the expression inside the square root) always remains non-negative as it is just the square of the final velocity (or the velocity just after the time step).

III. Results

The vehicular density (i.e., the fraction of road surface occupied by cars) is 0.5 in all cases. Also, the signal cycle is about 60 time units for each red and green light. Given that, we observe from fig. 2a) that when the cars and drivers are identical, the acceleration kind of synchronizes with the signal cycle and behaves quite periodically. Whereas, in fig. 2b) when the agents (car-driver systems) are assumed to be non-identical, the acceleration doesn't behave periodically, but rather in an erratic fashion. In fact, the maximum magnitude in both positive and negative directions has increased, which could possibly mean that the agents are quite behaving in an unpredictable and "aggressive" manner.

Such contrasting pattern in the acceleration gets smoothened out in the velocity variation as time progresses, fig 3. Even though the velocity variation in fig. 3b) is more smooth than the one with the stochastic noise in [6], it still has spikes and appears to be more unpredictable compared to fig. 3a). Again, because of higher maximum magnitudes for acceleration, we observe higher maximum values of speed also in fig. 3b), even though overall average velocity of the system remains almost the same in both cases.

By comparing the figures 2b) and 3b) with figures 1b) and 1a) respectively, we could quite conclude that our model's

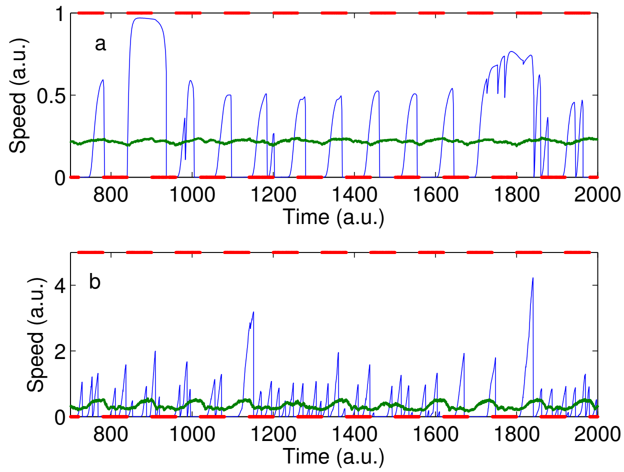

Fig. 3. Variation of velocity over time in the simulation. Blue line shows the instantaneous velocity of a single car, whereas the dotted green line represents the instantaneous average velocity across the system. The swapping of the traffic signal is marked by red where marking over '1' represents the red signal and '0' represents green signal. a) shows velocity variation over time when the cars are assumed to be 'identical' to each other, in the sense that the car-specific parameters are all same for each one of them. b) shows the velocity variation when the cars are assigned with different parameter values, namely α, β and τ.

prediction about the evolution of velocity or acceleration over time agrees well with the empirical data.

In fig. 4, we plot the space-time trajectory of each car as it moves along the single-lane road towards the signal (marked on the right end). We notice from the plot on the top of fig. 4 that the cars start behaving in a periodic and deterministic fashion when the agents are assumed to be identical, whereas there arises a lot of random perturbations in the trajectory of the cars when the agents are assumed to be non-identical to each other (bottom of fig. 4). So, when the agents are non-identical, lot of internal perturbations start arising, even if there is no effect of a signal. In the simulation, it has been made sure that the cars never collide at any instant, so, we see that the trajectories never intersect nor touch each other. Such constraint is ensured by picking the time step in such a way that the cars move with constant acceleration over the duration of that time step without colliding nor decreasing it's velocity below zero, meaning we assume that cars rather stop instead of moving in the reverse direction.

The fundamental diagram in fig. 5 shows how mobility and flow varies as a function of density. Density here is defined as the number of cars per per unit car length of the road. The dotted red line shows how the transition from free flow to jammed state occurs in an highway. Unlike in highway, from the blue line plot, we observe that the transition is not abrupt, but rather the flow in the traffic flow with intersections (meaning traffic signals) is actually similar to a wide range of density values, causing a flattened plateau, fig. 5b). The mobility actually becomes more concave as a function of density, fig. 5a).

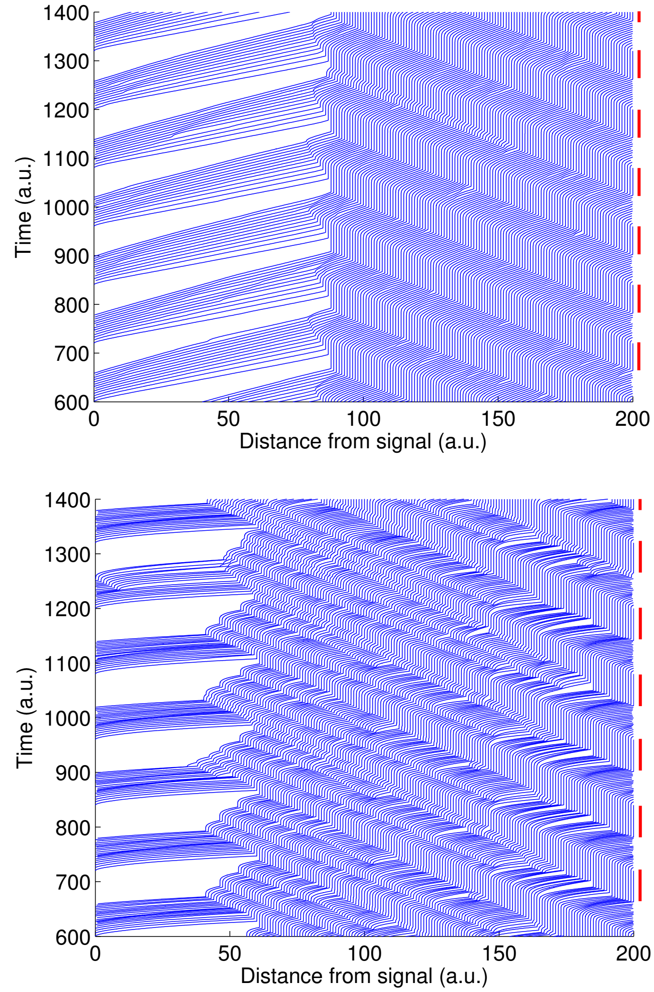

Fig. 4. Both the plots show the space-time evolution of all cars. Each line represents the space-time trajectory of a single car. Traffic signals are marked with red on the right side for reference. When we consider identical car-driver system (top), the space-time evolution of the cars kind of synchronize with each other and resembles very close to that of the deterministic case in [6], whereas we could observe (bottom) that there are various other perturbations causing an increase in the length of the jam when the car-driver systems are considered to be non-identical. Interestingly, unlike in the identical case (top), the non-identical nature of individuals (bottom) results in the reduction of the effect of any perturbation as it propagates backwards.

Now, we find that using our acceleration-based model, the congestion times follow a power-law distribution,

$$P(T \leq \tau) \sim \tau^{-m} \tag{3}$$

with varying exponent values m (where m is a non-negative real number), as shown in fig. 6 and fig. 7. When the agents are assumed to be identical, we've noticed from previous figures that the system very closely resembles a deterministic one. So, we would only be considering the non-identical case here. As the values of ξ, $\langle\alpha\rangle$, $\langle\beta\rangle$ and $\langle\tau\rangle$ are varied, the power-law distribution of the congestion times have exponents very close to $1.7-2.2$. But, when we assume stochastic fluctuations in

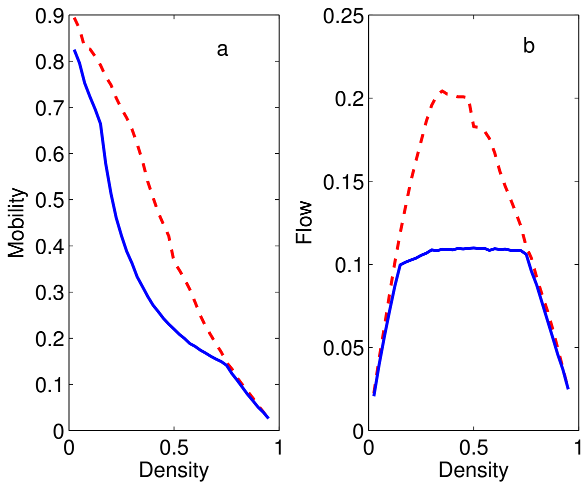

Fig. 5. a) Mobility vs density. b) Flow vs density. Blue line is for the case when there is signal and red dotted line is when there is no signal, meaning if it was just an highway

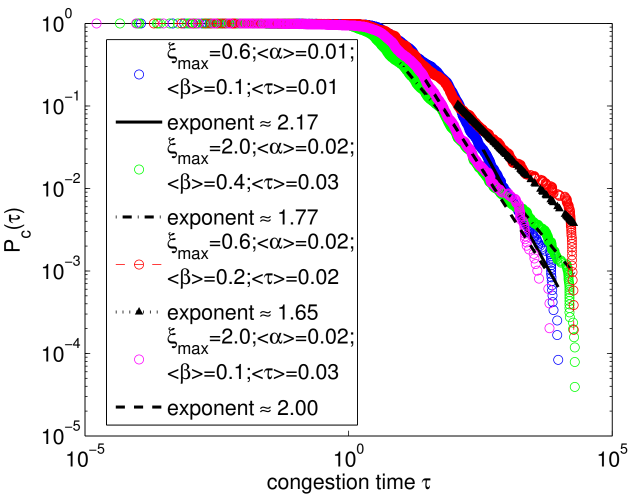

Fig. 6. Waiting time distribution when we consider different mean values for different acceleration parameters.

the parameter values, meaning heterogeneity of the parameters α, β and τ, we then get a range of power-law exponents, from $1.8-3.2$ for different choice of shape parameter of the gamma distribution (especially for sub-exponential distributions), as shown in fig. 7.

The exponent values of the power-law is actually obtained using the Maximum-Likelihood method given by Clauset., et. al. in [8]. This provides a best power-law fit to the distribution that is obtained from the simulation.

IV. Conclusion

In this report, we've presented a novel approach towards modelling the traffic congestion using a microscopic Monte-Carlo model. Following Newton's laws, with some fluctuations in terms of the non-identical nature of the car-driver system,

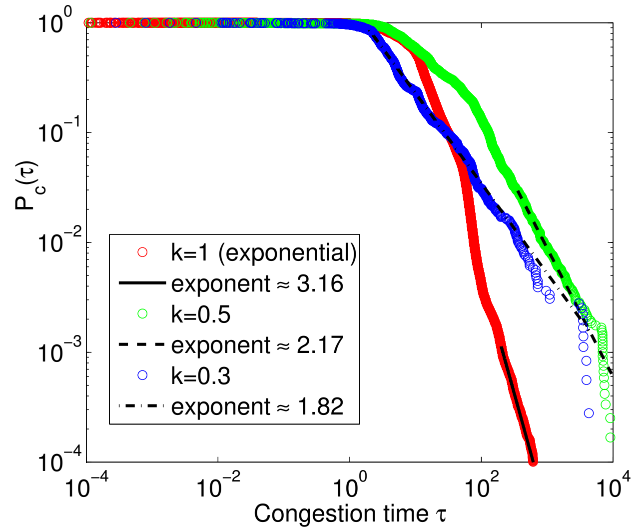

Fig. 7. Waiting time distribution for different type of heterogeneity for the acceleration parameters. Basically, the shape parameter of the gamma distribution is varied to represent a wide range of distributions.

we have shown that one could obtain waiting times following a power-law distribution. This therefore does not introduce any fluctuation directly into the dynamics, as as done in [6]. Moreover, the range of exponent values, m's, agrees very well with the range of exponent values obatined from empirical data of various Indian urban cities, namely Bangalore, Bombay and Delhi [6],[5].

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