Mechanistic modeling of collective vehicular dynamics reproduces empirical features of urban traffic

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***Abstract*—Despite the recent surge of interest in studying various aspects of traffic dynamics, congestion behavior in urban road networks (which is characterized by extremely high vehicular densities and a large number of intersections often controlled by signals) is not yet well-understood. In this article, we have presented a mechanistic model for reproducing large-scale collective features of congestion behavior from the microscopic dynamics of individual vehicles interacting with each other. By employing a detailed Newtonian description of the time-evolution of velocity and acceleration of each vehicle, and coupling it with a kinetic Monte Carlo simulation approach introduced by us earlier, we have presented an efficient modeling platform for investigating macroscopic patterns of urban congestion. In contrast to discrete cellular automata models of traffic flow, our continuous-time, continuous-space modeling of traffic flow in the presence of stochastic fluctuations approaches closer to reality. We have obtained fundamental diagram of simulated urban traffic in the presence of a signal. The flow appears to be relatively constant over a broad range of traffic densities indicating that the traffic signal plays an important role in smoothing out variations in traffic flow that may result from fluctuations in the vehicle movements. We have also reproduced the empirically observed power-law scaling in the distributions of congestion times. Our results underline the important role played by the heterogeneity in the characteristics of individual vehicles in producing long periods of congestion in urban settings.**

I. Introduction

As a result of advances in computer hardware and simulation techniques, it is now possible to model large-scale social dynamics involving interactions between a large number of autonomous agents. The aim of such models is to understand macroscopic phenomena that is observed at the level of the system or the collective as an emergent phenomena, arising self-organizedly from the micro-dynamics of individual constituents [1]. One of the most striking examples of such phenomena is the transition from free flowing traffic to a jammed state and vice versa when the density of vehicles on a road is altered [2], [3]. Although the dynamics of vehicular traffic has been studied for decades using fluid dynamical models, the introduction of discrete cellular automata models by Nagel and Schreckenberg to study highway traffic [4] has resulted in a surge of interest in describing complex patterns of collective vehicular movement using simple microscopic models. While this has resulted in a deeper understanding of freeway traffic dynamics, the situation in an urban setting, marked by relatively high vehicular densities and the presence of signals that coordinate movement of cross-flowing traffic traveling along several directions, is not yet as well explicated.

Studying urban traffic congestion is extremely significant both in terms of scientific and economic perspective, as a better understanding of how jams occur and the duration for which they persist can help in building more efficient transport infrastructures in cities. Also, as first reported in Ref. [5], the congestion times, i.e., the duration for which a vehicle moves with a speed less than a specified value, follows a heavy-tailed distribution typically described by a power law. This is in stark contrast to an exponential nature for the distribution which is expected in general for a memory-less random process, with the random nature arising from the existence of a large number of independent components. Thus, the existence of non-trivial macroscopic phenomena arising from urban traffic dynamics makes it of great scientific importance also.

By studying the factors that affect traffic congestion, one would be able to deduce aspects which are crucial for the control of urban traffic and for reducing congestion. We have previously proposed a microscopic model of vehicular dynamics that uses a kinetic Monte-Carlo simulation approach [6] that can reproduce the power-law characteristics of the distribution of waiting times. However, there was a phenomenological element in the model specifically in the way heterogeneity is introduced in the response of vehicles to the space available in front of them on a road (the headway distance). As such heterogeneity is crucial for generating the heavy-tailed nature of the congestion time distribution, we have in this article proposed a new mechanistic model that uses a Newtonian description of the movements of each vehicle to provide a more complete description. It is able to represent reality more accurately and allows us to identify which aspects of vehicular heterogeneity plays an important role in governing the exponents of the power-law tails that characterize the nature of the distributions of congestion times. In particular, apart from reproducing such macroscopic features, the model also produces micro-dynamics of individual vehicles that are a closer approximation to reality, e.g., the speed and acceleration time-profiles of actual taxis traveling in major Indian metropolises (Fig. 1).

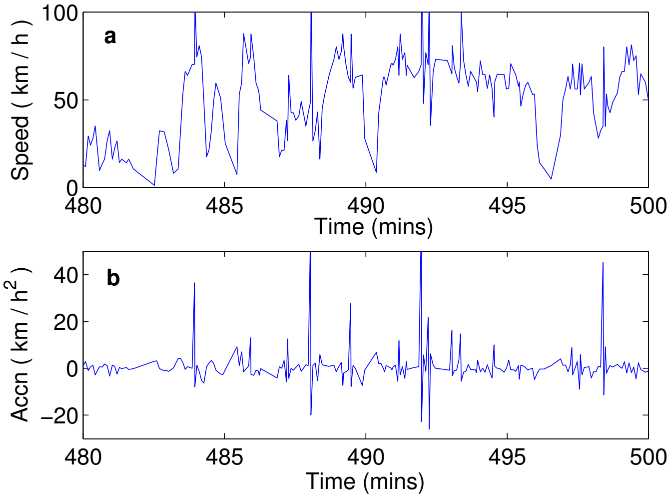

Fig. 1. Time evolution of (a) speed and (b) acceleration of a particular vehicle in New Delhi recorded from GPS trace data during the period 1:30 pm - 1:50 pm (IST) on Jan 10, 2013 obtained from *Traffline* [7].

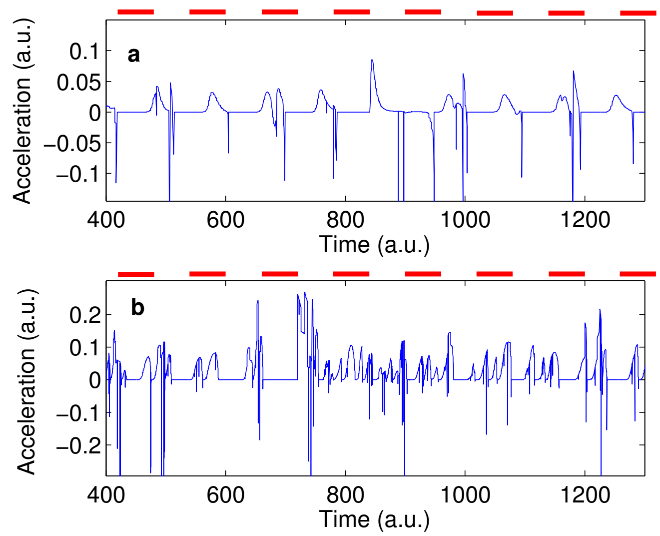

Fig. 2. Time-evolution of acceleration for an individual vehicle from a simulation of the model described in the text with $N = 100$ vehicles. Panel (a) shows the homogeneous situation where all vehicles are identical while in panel (b) the heterogeneous situation where the vehicles have distinct characteristics (randomly chosen from distributions with specified means) is shown. For (a) all the cars have $\gamma = 0.2$, $\beta = 0.2$ and $\tau = 0.3$. For (b) the mean values of the parameters (averaged over all vehicles in the simulation) are $\langle\gamma\rangle = 0.02$, $\langle\beta\rangle = 0.2$ and $\langle\tau\rangle = 0.03$. In both cases the noise ξ is chosen from a uniform distribution over $[0.0.6]$. The periods during which the signal is red is indicated with color red horizontal bars shown along the top of the figure.

II. Model

The kinetic Monte Carlo model that had been introduced earlier in Ref. [6] was majorly to explain the power-law characteristics of the waiting times observed in Ref. [5] and the fundamental relation between mobility, flow and density (which is a general relation). But, in that model, the velocities are abruptly changed every time step (depending on the head-way distance) which is not the case in real life as the empirical data suggests in Fig. 1. Instead, the control mechanisms are in fact acceleration based, namely - accelerator or brake. So, in this paper we intend to improvise the velocity-based model to incorporate the acceleration-based controls and effects. Moreover, the earlier model was phenomenological, whereas this one is mechanistic.

Similar to the model in Ref. [6], here too we assume a single lane with fixed number of vehicles moving only in one direction. To avoid problems with maintaining constant the traffic density by adjusting the input and output rates, circular road is considered. Also, those vehicles are externally controlled by a signal which is placed at one end of the road. The signal controls cross traffic at an intersection (signal controlled characterizes urban traffic). Now, instead of just updating the velocities of each vehicle at every instant, we update acceleration as follows:

$$\begin{aligned}
\frac{dx_i}{dt} &= v_i \\
\frac{dv_i}{dt} &= -(\gamma_i + \xi(x,t))v_i + \beta_i\left(\frac{d_i - v_i^{rel}\tau_i}{d_i+1}\right)
\end{aligned} \tag{1}$$

where,

γ_i : drag rate due to friction of the road surface for car i.

ξ : space and time dependent perturbation (>0) in the drag rate that affects just a random car for each time step, representing possibly a pothole or applying of brake, etc.

β_i : maximum acceleration possible (by pressing the accelerator).

τ_i : reaction time of the car-driver interface of car i.

d_i : headway distance available in front of car i to move without colliding. $d_i = |x_{i+1} - x_i|$, where x_{i+1} and x_i are positions of cars $i+1$ and i.

v_i^{rel} : relative velocity of the car in front with respect to the car under consideration (i). $v_i^{rel} = v_{i+1} - v_i$.

Maximum velocity could then be given by, $v_i^{max} = \beta_i/\gamma_i$ (for a single car in an otherwise empty road $\frac{dv_i}{dt} = -\gamma_i v_i + \beta_i$)

In most of the figures, we either choose identical values of γ, β and τ or random values for each car at the start of each simulation.Without loss of generality, we pick those three parameter values for each car from a gamma distribution, so that we could analyze from a range of distributions. By averaging over several such realizations the probability distribution of the waiting time is calculated. This would therefore provide an insight as to which (inherent) characteristic of the car-driver-road system is crucial for the power-law behavior of the waiting time. It has already been shown that the power-law could emerge just from simple dynamics, without even considering any network structure nor multi-lanes, in Ref. [6]. The influence of external characteristics like the signal cycle, traffic density and duty ratio (of the signals), over the power-law behavior will be published elsewhere.

The first term in the acceleration Eqn. (1) represents the friction due to the road surface and it is proportional to the instantaneous velocity. Similarly, the second term represents

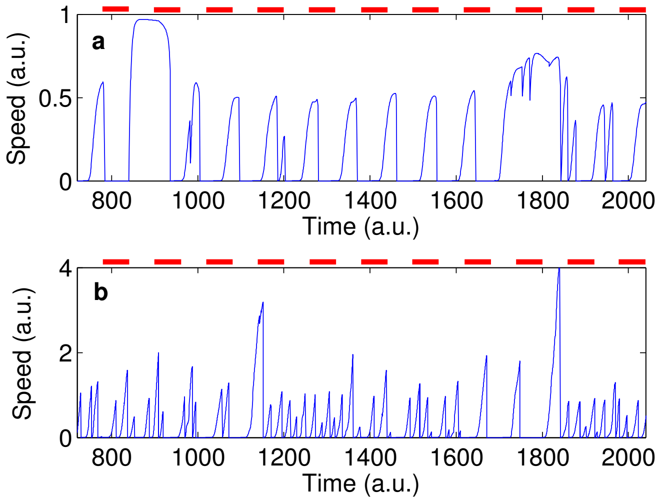

Fig. 3. Time evolution of velocity for an individual vehicle from a simulation of the model described in the text with $N = 100$ vehicles. Panel (a) shows the homogeneous situation where all vehicles are identical while in panel (b) the heterogeneous situation where the vehicles have distinct characteristics (randomly chosen from distributions with specified means) is shown. For (a) all the cars have $\gamma = 0.2$, $\beta = 0.2$ and $\tau = 0.3$. For (b) the mean values of the parameters (averaged over all vehicles in the simulation) are $\langle\gamma\rangle = 0.02$, $\langle\beta\rangle = 0.2$ and $\langle\tau\rangle = 0.03$. In both cases the noise ξ is chosen from a uniform distribution over $[0.0.6]$. The periods during which the signal is red is indicated with color red horizontal bars shown along the top of the figure.

the acceleration given by the driver for every time step. It is directly proportional to the headway distance available for the vehicle in front. We've used an hyperbolic function just to ensure that there is a saturation to maximum value of acceleration. Since there would be a lag in terms of the car-driver system to respond, the actual headway distance $(d - v^{rel}\tau)$ would be lower than the physical headway distance, d. Since $d \gg v_{rel}\tau$ usually,

$$\frac{d_i - v_i^{rel}\tau_i}{d_i - v_i^{rel}\tau_i + 1} \approx \frac{d_i - v_i^{rel}\tau_i}{d_i + 1}$$

Based on those equations (1) for all cars, the time step dt for progressing time is calculated based on two conditions: i) None of the cars collide with the cars in front while the vehicles progress with constant acceleration during dt. ii) None of the cars' velocity goes below zero. The largest such dt value is chosen for updating. At time n (using Newton's 2nd law we arrive at),

$$dt_n = \min_i \left(\frac{-v_{i,n-1} + \sqrt{v_{i,n-1}^2 + 2a_{i,n-1}d_{i,n-1}}}{a_{i,n-1}} \right) \quad (2)$$

where $v_{i,n-1}$ is velocity, $a_{i,n-1}$ is acceleration and $d_{i,n-1}$ is headway distance at time n for the car i. dt_n is now the time step at time n. This adaptive step-size allows us to mimic continuous-time, continuous-space dynamics more accurately. Note that the determinant (or the expression inside the square root) always remains non-negative as it is just the square of the final velocity (or the velocity just after the time step).

III. Results

We have considered the vehicular density (i.e., the fraction of road surface occupied by cars) to be 0.5 in all cases reported here. Also, the signal cycle is about 60 time units for each red and green light. Given that, we observe from Fig. 2(a) that when the cars and drivers are identical, the acceleration is a periodic function that synchronizes with the signal cycle. On the other hand, in Fig. 2(b) where the agents (car-driver systems) are assumed to be non-identical, the acceleration does not behave periodically, but rather in an erratic fashion. In fact, the maximum magnitude in both positive and negative directions has increased, which could possibly mean that the agents are applying stronger braking/acceleration because of the unpredictability.

Such contrasting pattern in the acceleration gets smoothened out in the velocity variation as time progresses, Fig. 3. Even though the velocity variation in Fig. 3 (b) is more smooth than the one with the stochastic noise in Ref. [6], it still has spikes and appears to be more unpredictable compared to Fig. 3 (a). Again, because of higher maximum magnitudes for acceleration, we observe higher maximum values of speed also in Fig. 3(b), even though overall average velocity of the system remains almost the same in both cases.

By comparing the Figs. 2(b) and 3(b) with Figs. 1(b) and 1(a) respectively, we could quite conclude that our model's prediction about the evolution of velocity or acceleration over time agrees well with the empirical data.

In Fig. 4, we plot the space-time trajectory of each car as it moves along the single-lane road towards the signal (marked on the right end). We notice from the plot on the top of Fig. 4 that the cars start behaving in a periodic and deterministic fashion when the agents are assumed to be identical, whereas there arises a lot of random perturbations in the trajectory of the cars when the agents are assumed to be non-identical to each other (bottom of Fig. 4). So, when the agents are non-identical, lot of internal perturbations start arising, even if there is no effect of a signal. In the simulation, it has been made sure that the cars never collide at any instant, so, we see that the trajectories never intersect nor touch each other.

The fundamental diagram in Fig. 5 shows how mobility $\langle v\rangle$ and flow J varies as a function of density ρ.

Density here is defined as the number of cars per unit car length of the road, $\rho = \frac{P}{L}$, where P is the number of cars and L is the road length.

Mobility is given by,

$$\langle v\rangle = \frac{\text{Total distance travelled by all cars during simulation}}{(\text{Total duration of simulation})P}$$

and flow is $J = \langle v\rangle\rho$. The dotted red line shows how the transition from free flow to jammed state occurs in an highway (meaning in the absence of signals). From the blue line plot, we observe that, unlike in an highway, the transition is not abrupt, but rather the flow in the traffic flow with intersections (meaning with traffic signals) is actually similar for a wide range of density values, causing a flattened plateau, Fig. 5(b).

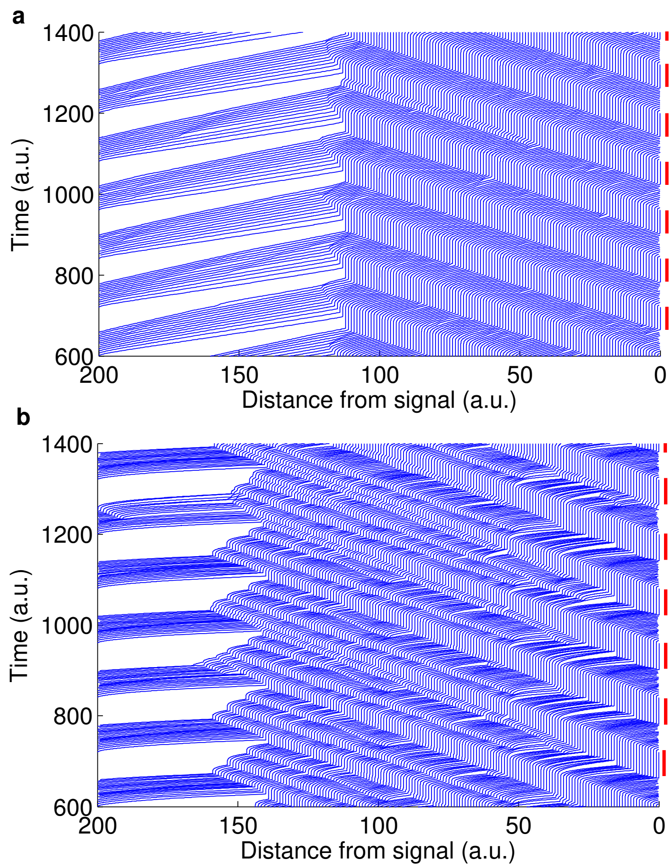

Fig. 4. Spatio-temporal evolution of simulated traffic showing trajectories of individual vehicles with a signal located at the right end that prevents vehicular movement at periodic intervals comparing the homogeneous situation (a) where all cars are identical with the heterogeneous case (b) where the cars have distinct (randomly chosen) characteristics. The vehicular density is (i.e., the fraction of road surface occupied by cars) is 0.5 in all cases. The ordinate indicates time so that when a vehicle slows down the line becomes more vertical. The red vertical bars at the right margin of the center and bottom panels indicate when cars are not allowed to move past the signal (the duration of the signal cycle is 120 a.u., with the light being green and red for equal amounts of time).

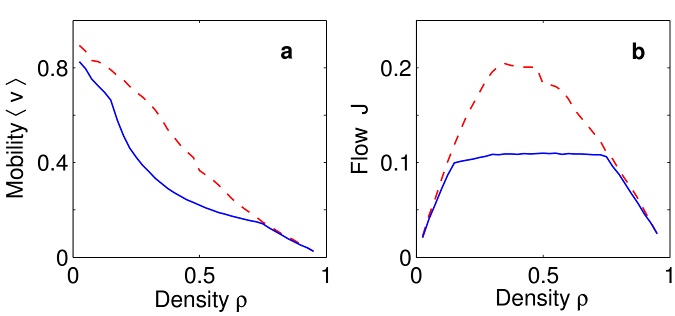

Fig. 5. The macroscopic fundamental diagrams representing the traffic dynamics generated by the model presented here showing the dependence of (a) mobility, i.e., average speed, and (b) flow, i.e., the average number of moving vehicle per unit time, on the vehicular density. The situation in the presence of a traffic signal with a signal cycle of 120 time units (continuous curves) is compared with the diagram obtained in the absence of any intersection or signal (broken curves).

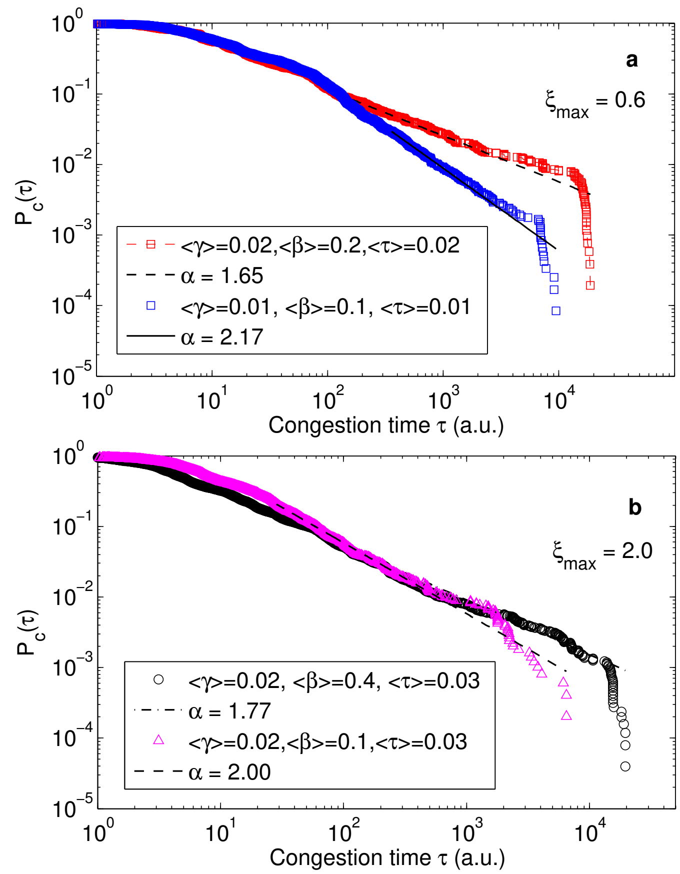

Fig. 6. The complementary cumulative probability distribution of the congestion time τ, i.e., the duration for which a vehicle moves with a speed less than a specified value (threshold speed = 0.1 a.u.), shown for the model in the heterogeneous situation where all vehicles have distinct characteristics. The mean values of the parameters are for the different cases shown are indicated in the legend. The vehicles are subject to random noise ξ in their deceleration which is generated from a uniform distribution between $[0, \xi_{max}]$. The two panels compare situations with (a) low and (b) high levels of noise, ξ_{max}. The heavy-tailed nature of the distributions have been fit to power-law forms, viz., $P_c(\tau) \sim \tau^{(1-\alpha)}$ and the exponent values α for each of the curves are estimated using maximum likelihood technique.

The mobility becomes more concave as a function of density, Fig. 5(a).

We find that using our acceleration-based model, the congestion times follow a power-law distribution,

$$P_c(\tau) \sim \tau^{(1-\alpha)} \tag{3}$$

with varying exponent values α (where α is a non-negative real number), as shown in Fig. 6 and Fig. 7. When the agents are assumed to be identical, we've noticed from previous figures that the system very closely resembles a deterministic one. So, we would only be considering the non-identical case here. As the values of ξ, $\langle\gamma\rangle$, $\langle\beta\rangle$ and $\langle\tau\rangle$ are varied, the power-law distribution of the congestion times have exponents very close to $1.6 - 2.2$. But, when we assume stochastic fluctuations in the parameter values, meaning heterogeneity of the parameters γ, β and τ, we then get a range of power-law exponents, from $1.8 - 3.2$ for different choice of

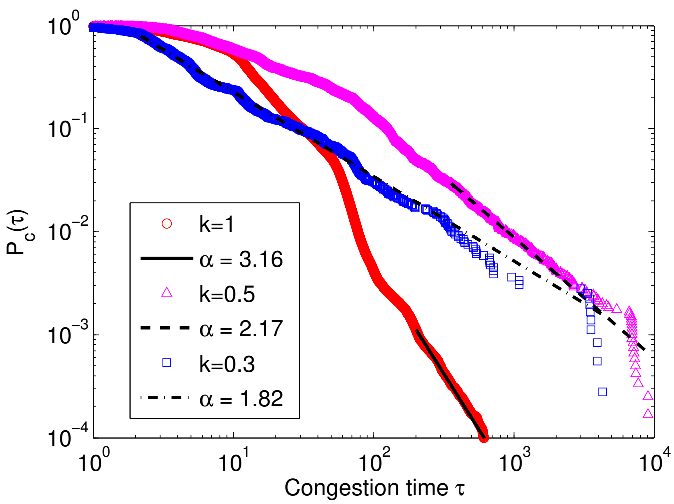

Fig. 7. The complementary cumulative distribution of the congestion time τ for the model in the heterogeneous situation where all vehicles have distinct characteristics. The different curves correspond to different types of heterogeneity for the distribution of parameter values, obtained by varying the shape parameter k of a gamma distribution. In all other simulations shown here k has been chosen to be 0.5.

shape parameter of the gamma distribution (especially for sub-exponential distributions), as shown in Fig. 7.

The exponent values of the power-law are obtained using the maximum-Likelihood method described in Clauset *et. al.* [8]. This provides a best power-law fit to the distribution that is obtained from the simulation.

IV. Conclusion

In this report, we've presented a novel approach towards modelling the traffic congestion using a microscopic Monte-Carlo model. Following Newton's laws, with some fluctuations in terms of the non-identical nature of the car-driver system, we have shown that one could obtain waiting times following a power-law distribution. This therefore does not introduce any major fluctuations directly into the dynamics, as was done in Ref. [6]. Moreover, the range of exponent values, α's, agrees very well with the range of exponent values obtained from empirical data of various Indian urban metropolises, namely Bangalore, Bombay and New Delhi [6],[5].

Acknowledgments

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