Dynamics of Urban Traffic Congestion: Acceleration-based Kinetic Monte-Carlo model

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***Abstract*—Abstract abstract abstract.**

I. Introduction

Large-scale social dynamics are nowadays modelled using interacting agents, where macroscopic changes can be observed at the level of the entire system as a result of the micro-dynamics of the individual constituents, with advances in computing infrastructure and numerical techniques [1]. For example, how traffic changes from free flowing to jammed state and vice versa [2], [3].

Although vehicle traffic has been studied for decades using fluid dynamical models, only after Nagel and Schreckenberg's cellular automata model of highway traffic flow [4], there was increased concentration of simplistic computational models to describe various aspects of traffic. Even so, there has been very little work done regarding urban traffic.

Studying urban traffic turns out to have more significance both in terms of academical and economical point of view, as the study of traffic jams and their lifetimes might be helpful in building more efficient transport infrastructure in a city. Also, as mentioned earlier in [5], the congestion times (or the waiting times of vehicles in the jam) follows a power-law, thereby making the study of congestion much more interesting.

II. Model

The kinetic Monte Carlo model that had been introduced earlier in [6] was majorly to explain the power-law characteristics of the waiting times observed in [5]. In that model, the velocities are abruptly changed every time step (depending on the headway distance). Although it was possible to reproduce the power law characteristics of waiting times and the fundamental relation between mobility, flow and density, in real life, the control mechanisms are in fact acceleration based, namely - accelerator or brake. So, in this paper we intend to improvise the velocity-based model to incorporate the acceleration-based controls and effects. This would in turn smoothen out the velocity vs time plot as expected from the empirical data.

Similar to the model in [6], here too we assume a single lane with fixed number of (identical) vehicles moving only in one direction. In order to avoid boundary issues, a circular road is considered. Also, those vehicles are externally controlled by a signal which is placed at one end. This signal basically represents the cross-over traffic. Now, instead of just updating the velocities of each vehicle at every instant, we update both velocity and acceleration as follows:

$$\begin{aligned} \frac{dx}{dt} &= v \\ \frac{dv}{dt} &= -(\alpha + \xi(x,t))v + \beta\left(\frac{d + v_{rel}\tau}{d+1}\right) \end{aligned} \tag{1}$$

where,

α : drag rate due to friction of the road surface.

ξ : space and time dependent positive glitch in the drag rate that affects just a random car for each time step, representing possibly a pothole or applying of brake, etc.

β : maximum acceleration possible (using accelerator). Maximum velocity $v_{max} = \beta/\alpha$ (for single car in empty road $\frac{dv}{dt} = -\alpha v + \beta$)

τ : reaction time of the car-driver interface.

The values of α, β and τ are chosen randomly for each car throughout the simulation. Several such realizations are collectively used for the probability distribution of the waiting time. This would therefore provide an insight as to which (inherent) characteristic of the car-driver-road system is crucial for the power-law behaviour of the waiting time. It has already been shown that the power-law could emerge just from simple dynamics, without even considering any network structure nor multi-lanes, in [6]. The influence of external characteristic like the signal cycle, traffic density and duty ratio (of the signals), over the power-law behaviour is already being studied in [9].

The first term in the acceleration eqn. 1 represents the friction due to the road surface and it is proportional to the instantaneous velocity. Similarly, the second term represents the acceleration given by the driver for every time step. It is directly proportional to the headway distance available for the vehicle in front. We've used an hyperbolic function just to make sure an upper bound for the acceleration. Since there would be a lag in terms of the car-driver system to respond, the actual headway distance would be lower than the physical headway distance. So, the second term is corrected for that. Since $d \gg v_{rel}\tau$ usually, we've corrected only for the numerator.

Based on those equations 1, the time step dt for progressing time is calculated based on two conditions: i) None of the cars collide with the cars in front while the vehicles progress with constant acceleration during dt. ii) None of the cars' velocity goes below zero (which is not a bad assumption based on reality). The largest such dt value is chosen for updating. Basically at time n (using Newton's 2nd law we arrive at),

$$dt_n = \min_i \left(\frac{-v_{i,n-1} + \sqrt{v_{i,n-1}^2 + 2a_{i,n-1}d_{i,n-1}}}{a_{i,n-1}} \right) \quad (2)$$

where v is velocity, a is acceleration, d is headway distance, n is the time instant and i represents vehicle. dt_n is now the time step at time n. This adaptive step-size allows us to mimic continuous-time, continuous-space dynamics more accurately. *Note*: The determinant (or the expression inside the square root) always remains non-negative as it is just the square of the velocity at the end of the time step.

III. Results

The vehicular density (i.e., the fraction of road surface occupied by cars) is 0.3 in all cases. Each line denotes a the space-time trajectory of a single vehicle moving along the road.

IV. Conclusion

Here we have presented a novel kinetic Monte Carlo simulation approach for studying the dynamics of urban traffic congestion.

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