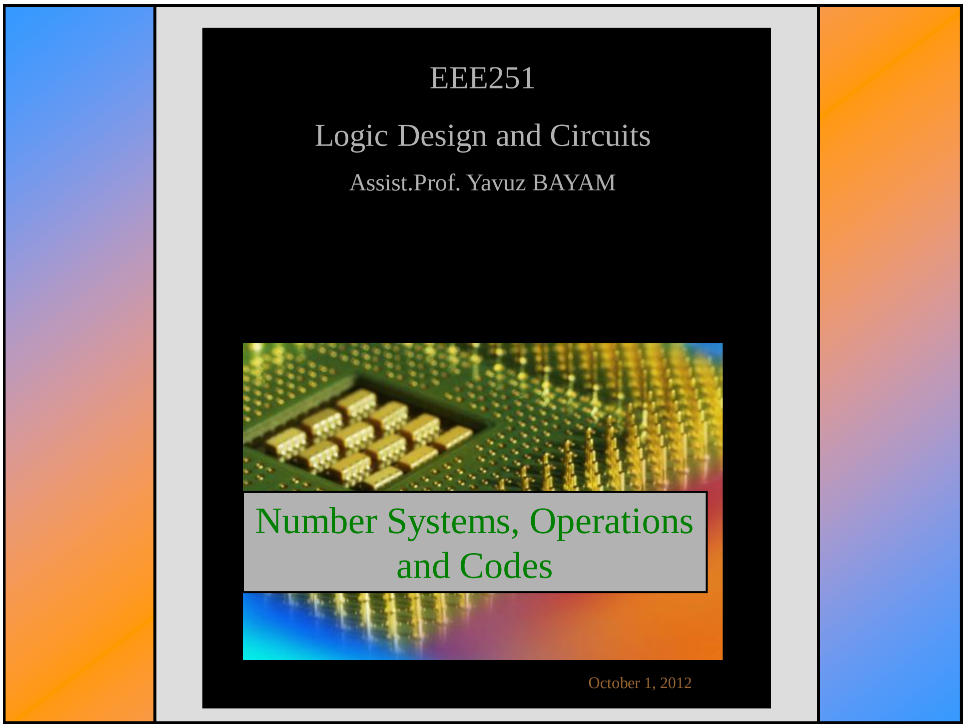


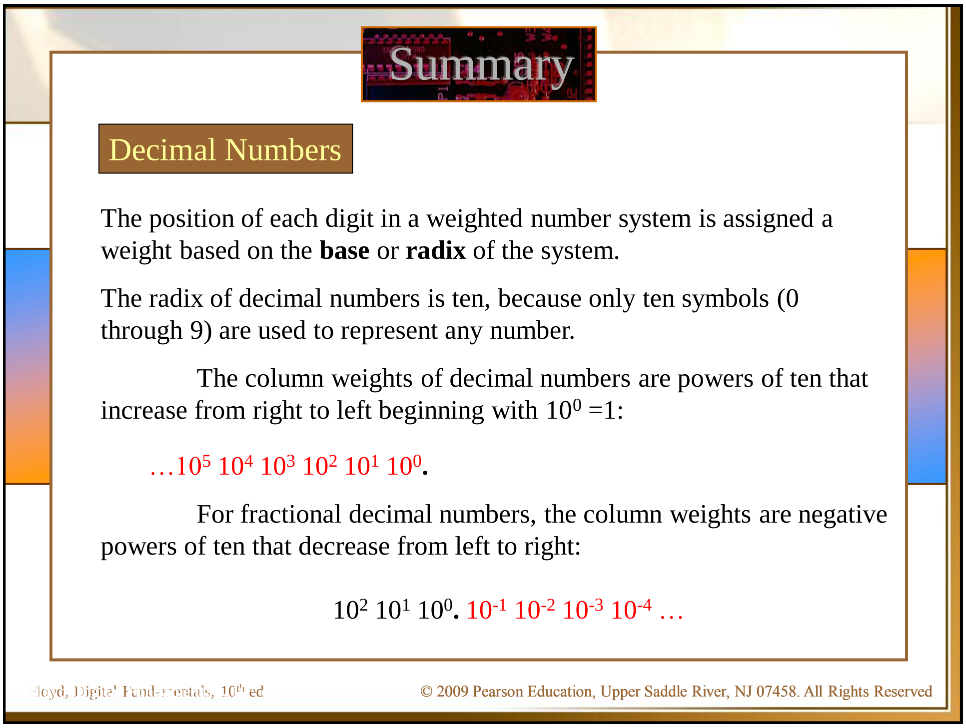
EEE251

Logic Design and Circuits

Assist.Prof. Yavuz BAYAM

Number Systems, Operations and Codes

October 1, 2012



Summary

Decimal Numbers

The position of each digit in a weighted number system is assigned a weight based on the **base** or **radix** of the system.

The radix of decimal numbers is ten, because only ten symbols (0 through 9) are used to represent any number.

The column weights of decimal numbers are powers of ten that increase from right to left beginning with $10^0 = 1$:

$\ldots 10^5\ 10^4\ 10^3\ 10^2\ 10^1\ 10^0.$

For fractional decimal numbers, the column weights are negative powers of ten that decrease from left to right:

$10^2\ 10^1\ 10^0.\ 10^{-1}\ 10^{-2}\ 10^{-3}\ 10^{-4} \ldots$

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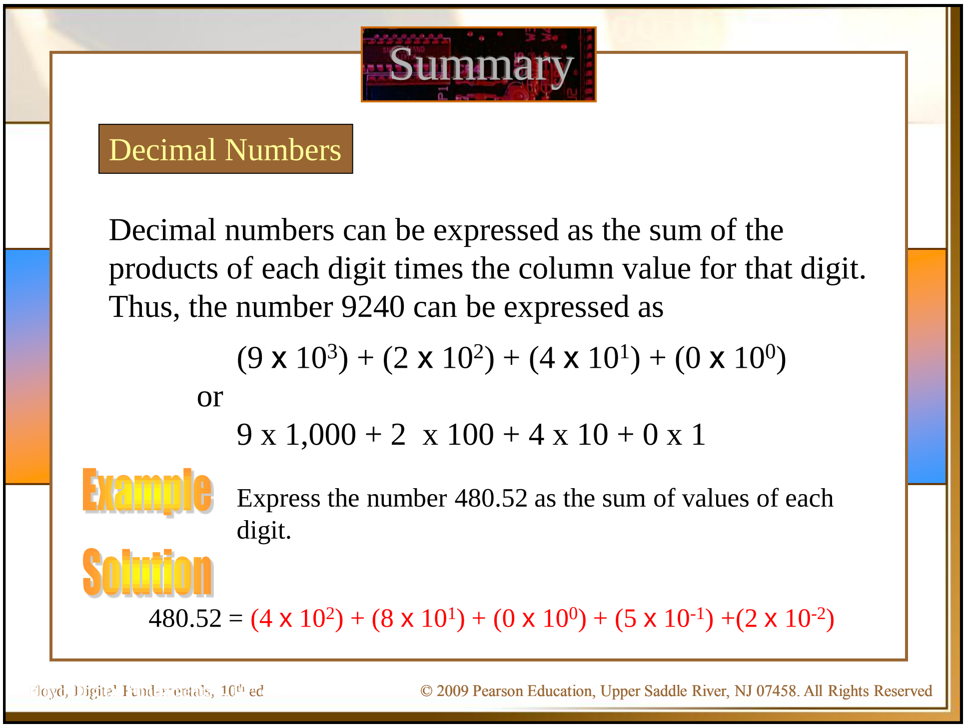

Summary

Decimal Numbers

Decimal numbers can be expressed as the sum of the products of each digit times the column value for that digit. Thus, the number 9240 can be expressed as

$$(9 \times 10^3) + (2 \times 10^2) + (4 \times 10^1) + (0 \times 10^0)$$

or

$$9 \times 1{,}000 + 2 \times 100 + 4 \times 10 + 0 \times 1$$

Example Express the number 480.52 as the sum of values of each digit.

Solution

$$480.52 = (4 \times 10^2) + (8 \times 10^1) + (0 \times 10^0) + (5 \times 10^{-1}) +(2 \times 10^{-2})$$

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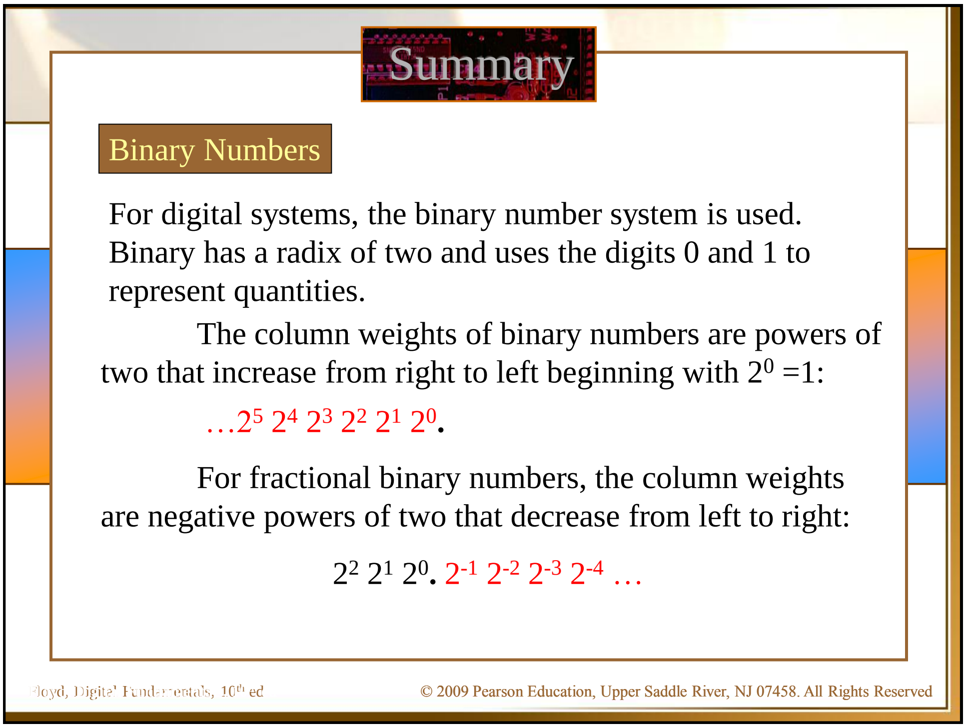

Summary

Binary Numbers

For digital systems, the binary number system is used. Binary has a radix of two and uses the digits 0 and 1 to represent quantities.

The column weights of binary numbers are powers of two that increase from right to left beginning with 2^0 =1:

$$\ldots 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0.$$

For fractional binary numbers, the column weights are negative powers of two that decrease from left to right:

$$2^2\ 2^1\ 2^0.\ 2^{-1}\ 2^{-2}\ 2^{-3}\ 2^{-4} \ldots$$

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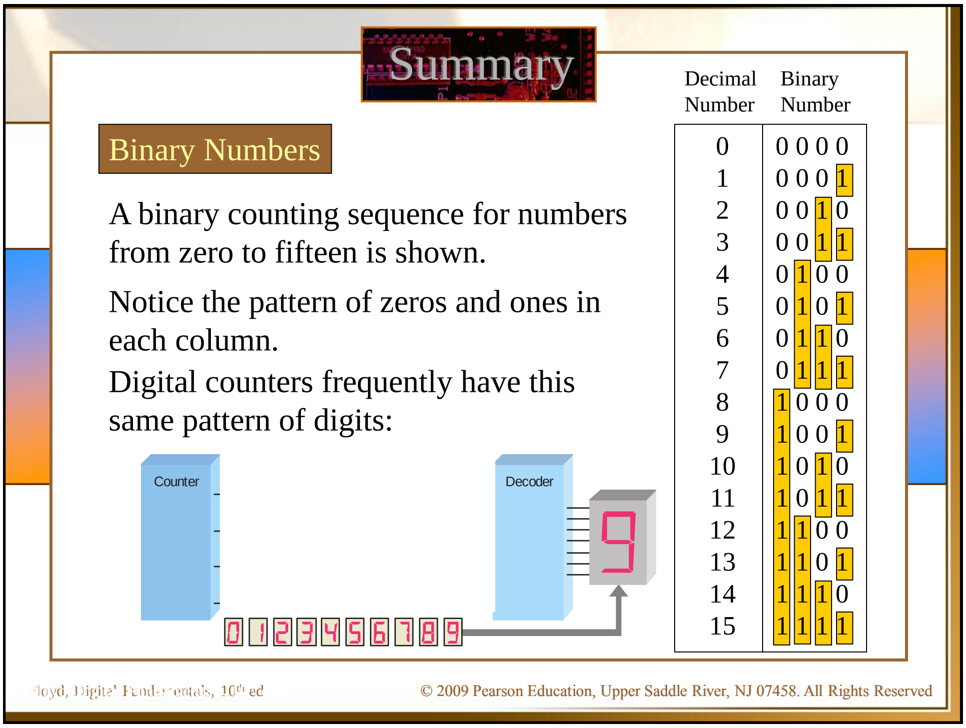

Summary

Binary Numbers

A binary counting sequence for numbers from zero to fifteen is shown.

Notice the pattern of zeros and ones in each column.

Digital counters frequently have this same pattern of digits:

Decimal Number	Binary Number
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
10	1 0 1 0
11	1 0 1 1
12	1 1 0 0
13	1 1 0 1
14	1 1 1 0
15	1 1 1 1

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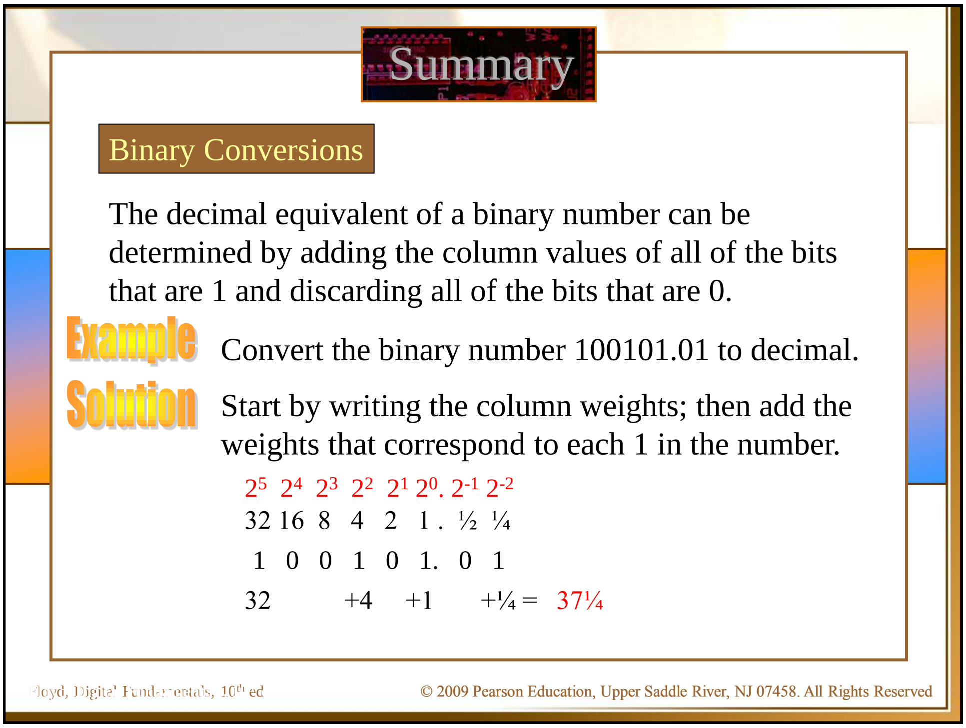

Summary

Binary Conversions

The decimal equivalent of a binary number can be determined by adding the column values of all of the bits that are 1 and discarding all of the bits that are 0.

Example Convert the binary number 100101.01 to decimal.

Solution Start by writing the column weights; then add the weights that correspond to each 1 in the number.

2^5 2^4 2^3 2^2 2^1 2^0. 2^{-1} 2^{-2}
32 16 8 4 2 1 . ½ ¼
1 0 0 1 0 1. 0 1
32 +4 +1 +¼ = 37¼

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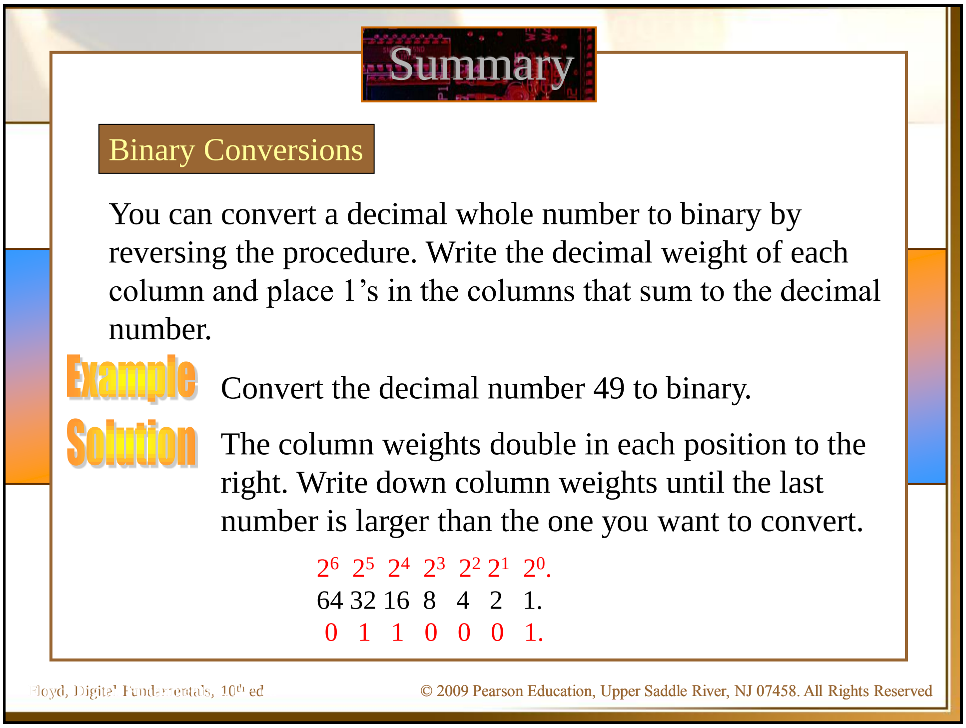

Summary

Binary Conversions

You can convert a decimal whole number to binary by reversing the procedure. Write the decimal weight of each column and place 1's in the columns that sum to the decimal number.

Example Convert the decimal number 49 to binary.

Solution The column weights double in each position to the right. Write down column weights until the last number is larger than the one you want to convert.

2^6 2^5 2^4 2^3 2^2 2^1 2^0.
64 32 16 8 4 2 1.
0 1 1 0 0 0 1.

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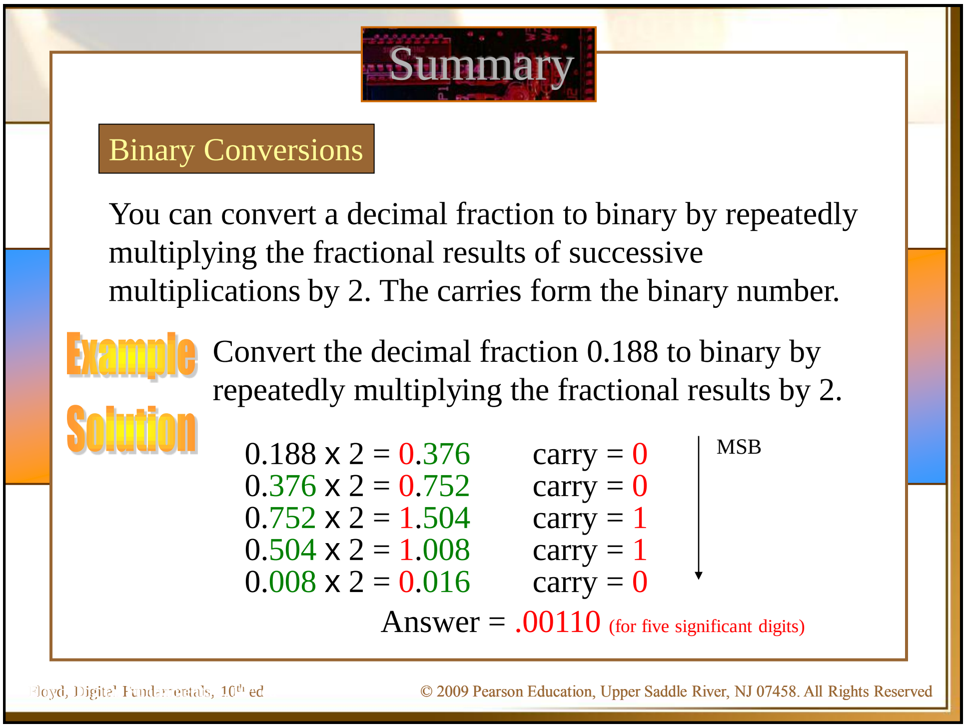

Summary

Binary Conversions

You can convert a decimal fraction to binary by repeatedly multiplying the fractional results of successive multiplications by 2. The carries form the binary number.

Example Convert the decimal fraction 0.188 to binary by repeatedly multiplying the fractional results by 2.

Solution

0.188 x 2 = 0.376 carry = 0 MSB
0.376 x 2 = 0.752 carry = 0
0.752 x 2 = 1.504 carry = 1
0.504 x 2 = 1.008 carry = 1
0.008 x 2 = 0.016 carry = 0

Answer = .00110 (for five significant digits)

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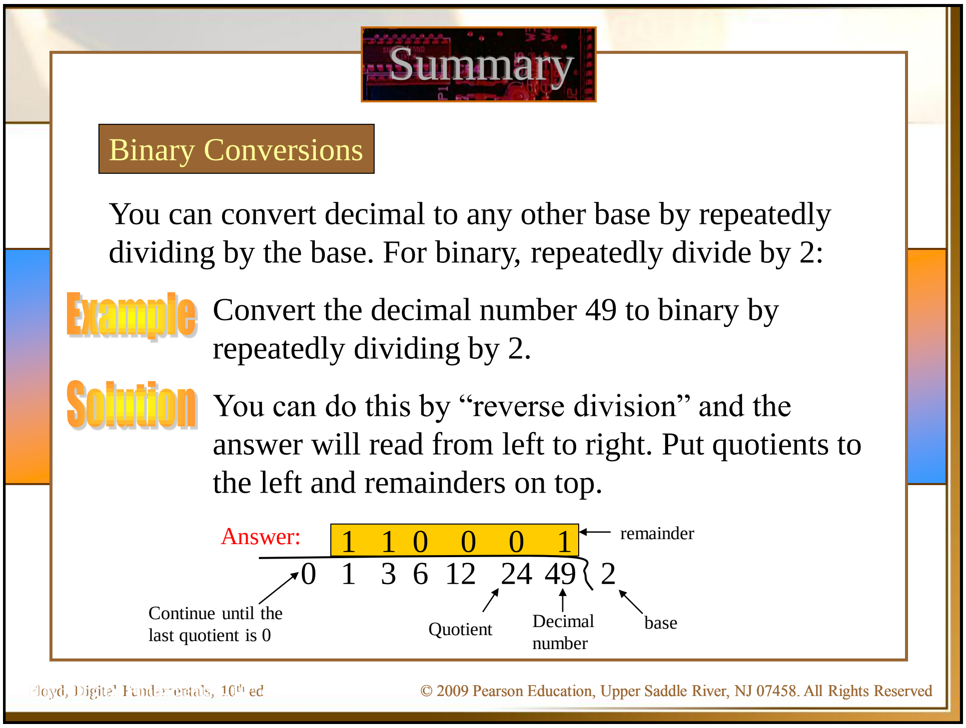

Summary

Binary Conversions

You can convert decimal to any other base by repeatedly dividing by the base. For binary, repeatedly divide by 2:

Example Convert the decimal number 49 to binary by repeatedly dividing by 2.

Solution You can do this by "reverse division" and the answer will read from left to right. Put quotients to the left and remainders on top.

Answer: 1 1 0 0 0 1 ← remainder

0 1 3 6 12 24 49 ⟌ 2

Continue until the last quotient is 0

Quotient

Decimal number

base

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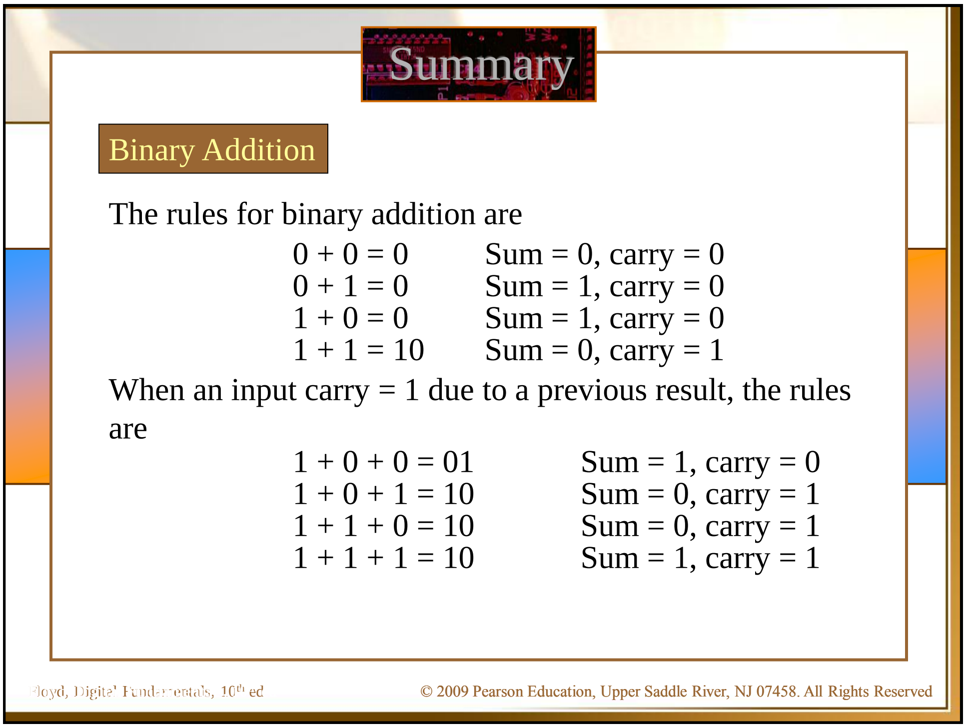

Summary

Binary Addition

The rules for binary addition are

0 + 0 = 0 Sum = 0, carry = 0
0 + 1 = 0 Sum = 1, carry = 0
1 + 0 = 0 Sum = 1, carry = 0
1 + 1 = 10 Sum = 0, carry = 1

When an input carry = 1 due to a previous result, the rules are

1 + 0 + 0 = 01 Sum = 1, carry = 0
1 + 0 + 1 = 10 Sum = 0, carry = 1
1 + 1 + 0 = 10 Sum = 0, carry = 1
1 + 1 + 1 = 10 Sum = 1, carry = 1

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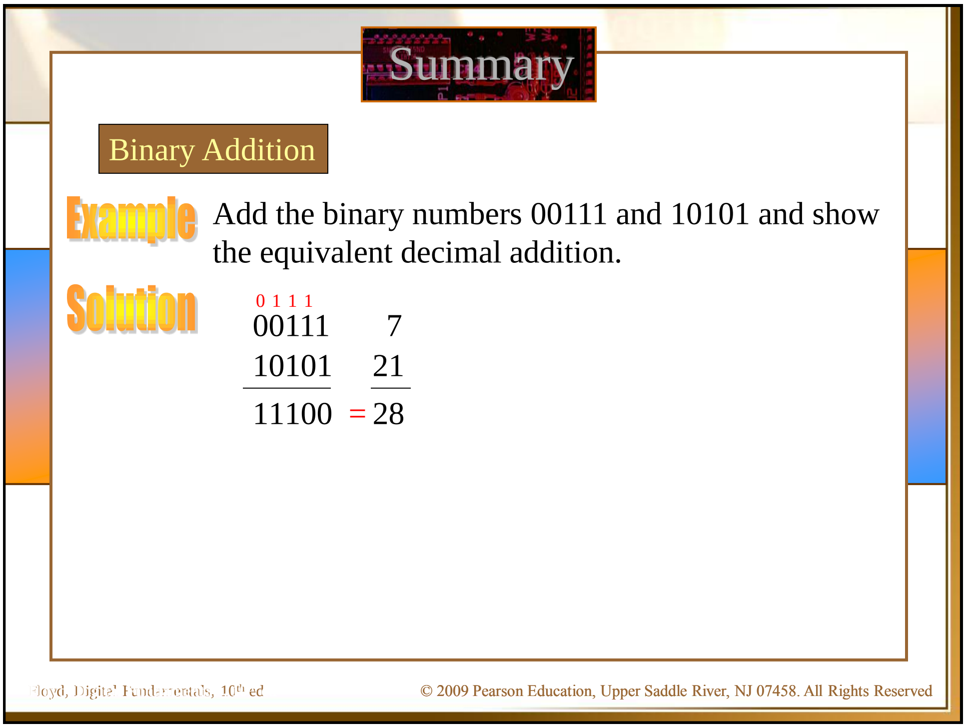

Summary

Binary Addition

Example Add the binary numbers 00111 and 10101 and show the equivalent decimal addition.

Solution

0 1 1 1		
00111		7
10101		21
11100	=	28

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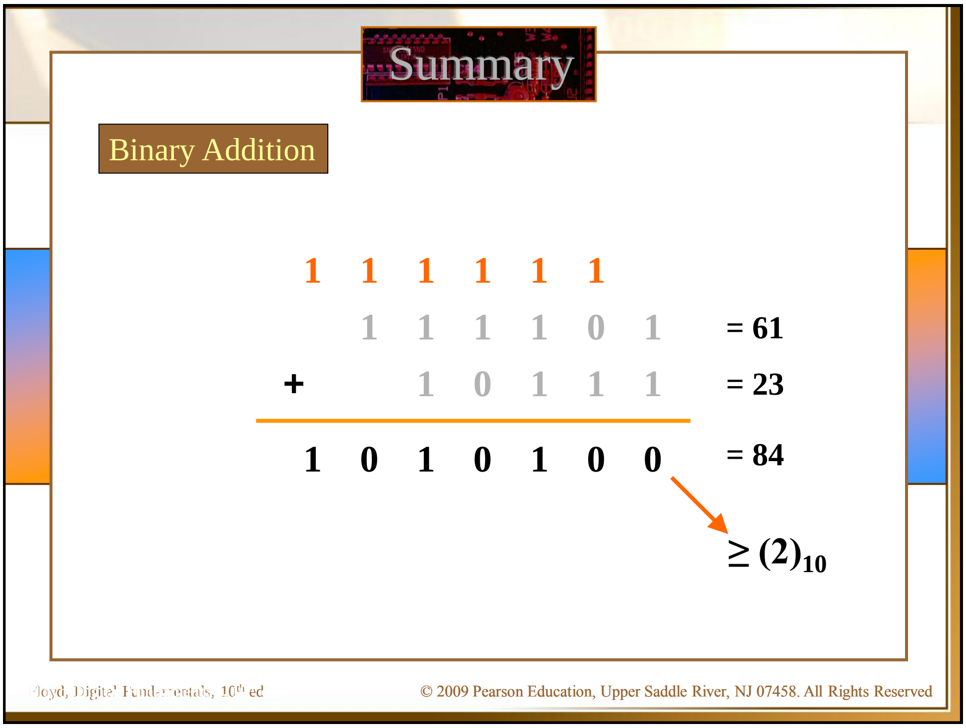

Summary

Binary Addition

	1	1	1	1	1	1		
		1	1	1	1	0	1	= 61
+			1	0	1	1	1	= 23
	1	0	1	0	1	0	0	= 84

$\geq (2)_{10}$

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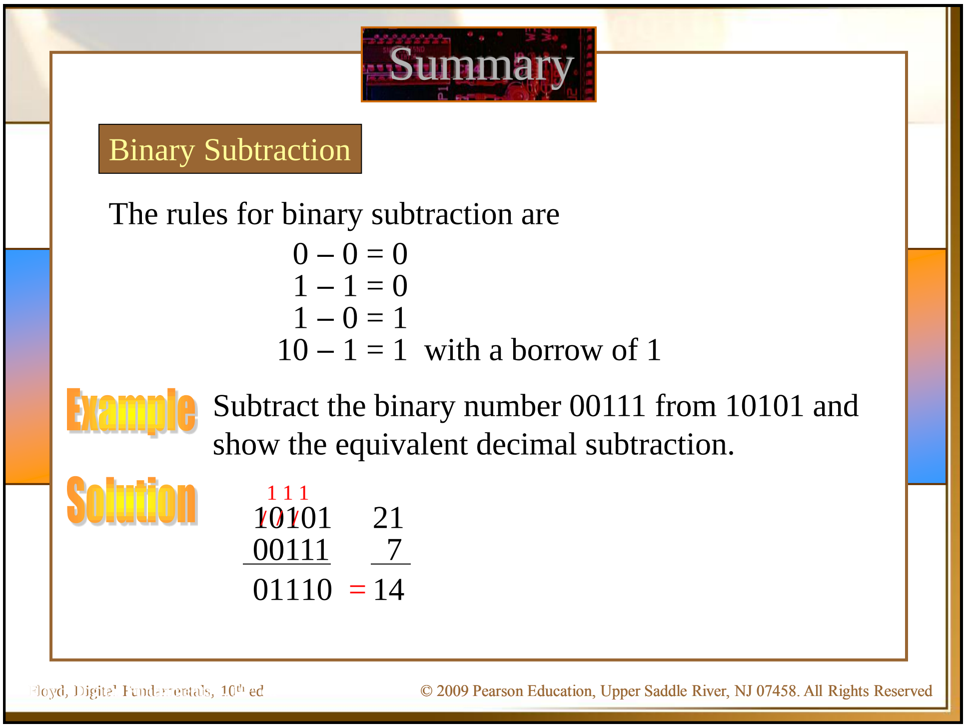

Summary

Binary Subtraction

The rules for binary subtraction are

$0 - 0 = 0$
$1 - 1 = 0$
$1 - 0 = 1$
$10 - 1 = 1$ with a borrow of 1

Example Subtract the binary number 00111 from 10101 and show the equivalent decimal subtraction.

Solution

$$\begin{array}{rcr} {\scriptstyle 1\,1\,1} & & \\ \cancel{1}\cancel{0}\cancel{1}01 & & 21 \\ \underline{00111} & & \underline{7} \\ 01110 & = & 14 \end{array}$$

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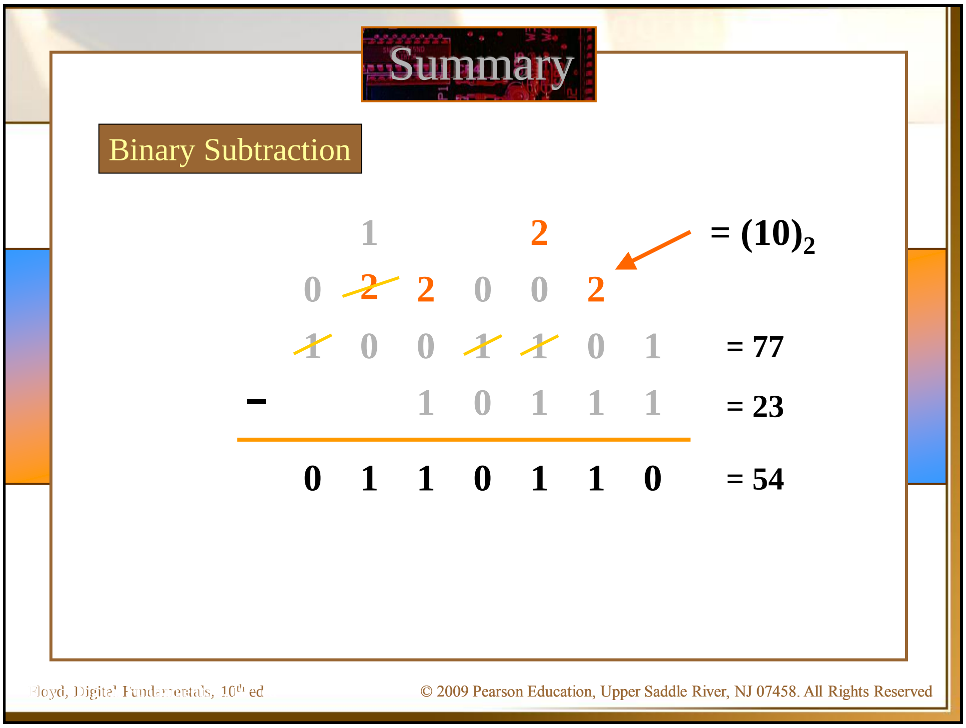

Summary

Binary Subtraction

$$\begin{array}{ccccccccl} & & 1 & & & 2 & & & = (10)_2 \\ & 0 & \cancel{2} & 2 & 0 & 0 & 2 & & \\ & \cancel{1} & 0 & 0 & \cancel{1} & \cancel{1} & 0 & 1 & = 77 \\ - & & & 1 & 0 & 1 & 1 & 1 & = 23 \\ \hline & 0 & 1 & 1 & 0 & 1 & 1 & 0 & = 54 \end{array}$$

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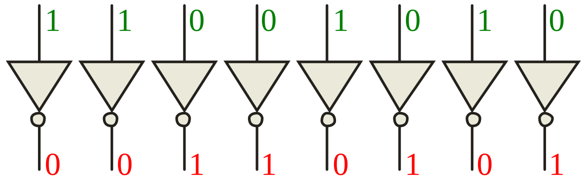
Summary

1's Complement

The 1's complement of a binary number is just the inverse of the digits. To form the 1's complement, change all 0's to 1's and all 1's to 0's.

For example, the 1's complement of 11001010 is
00110101

In digital circuits, the 1's complement is formed by using inverters:



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Summary

1's Complement

- **Example :** Find 1's complement of $(101100)_2$

  $= (010011)_2$

- **Example :** Find 1's complement of $(0.0110)_2$

  $= (0.1001)_2$

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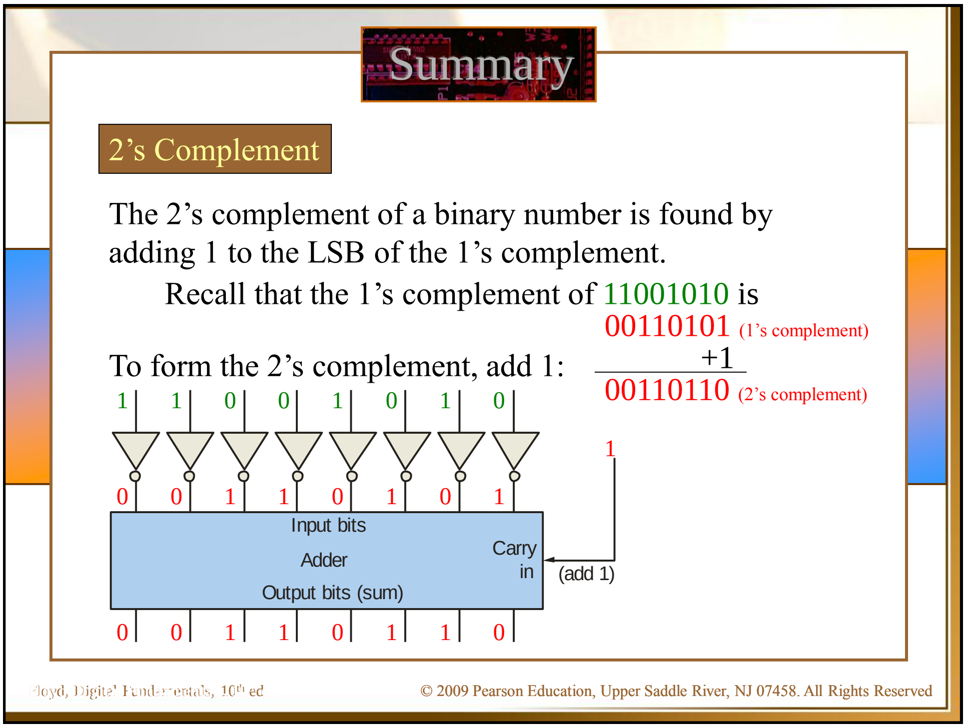

Summary

2's Complement

The 2's complement of a binary number is found by adding 1 to the LSB of the 1's complement.

Recall that the 1's complement of 11001010 is

$$\begin{array}{rl} 00110101 & \text{(1's complement)} \\ +1 & \\ \hline 00110110 & \text{(2's complement)} \end{array}$$

To form the 2's complement, add 1:

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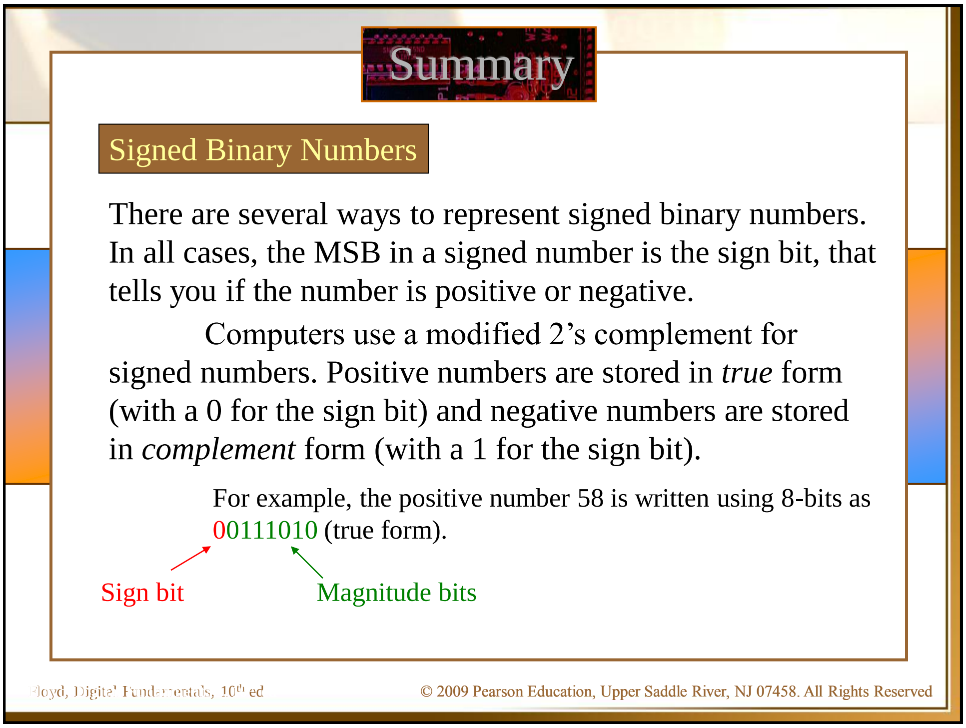

Summary

Signed Binary Numbers

There are several ways to represent signed binary numbers. In all cases, the MSB in a signed number is the sign bit, that tells you if the number is positive or negative.

Computers use a modified 2's complement for signed numbers. Positive numbers are stored in *true* form (with a 0 for the sign bit) and negative numbers are stored in *complement* form (with a 1 for the sign bit).

For example, the positive number 58 is written using 8-bits as 00111010 (true form).

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Summary

Signed Binary Numbers

Negative numbers are written as the 2's complement of the corresponding positive number.

The negative number –58 is written as:

–58 = 11000110 (complement form)

Sign bit Magnitude bits

An easy way to read a signed number that uses this notation is to assign the sign bit a column weight of –128 (for an 8-bit number). Then add the column weights for the 1's.

Example Assuming that the sign bit = –128, show that 11000110 = –58 as a 2's complement signed number:

Solution Column weights: –128 64 32 16 8 4 2 1.

1	1	0	0	0	1	1	0	
–128	+64				+4	+2		= –58

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Summary

Floating Point Numbers

Floating point notation is capable of representing very large or small numbers by using a form of scientific notation. A 32-bit single precision number is illustrated.

S	E (8 bits)	F (23 bits)

Sign bit Biased exponent (+127) Magnitude with MSB dropped

Example Express the speed of light, *c*, in single precision floating point notation. ($c = 0.2998 \times 10^9$)

Solution In binary, $c = 0001\ 0001\ 1101\ 1110\ 1001\ 0101\ 1100\ 0000_2$.

In scientific notation, $c = 1.001\ 1101\ 1110\ 1001\ 0101\ 1100\ 0000 \times 2^{28}$.

S = 0 because the number is positive. $E = 28 + 127 = 155_{10} = 1001\ 1011_2$.

F is the next 23 bits after the first 1 is dropped.

In floating point notation, *c* =

0	10011011	001 1101 1110 1001 0101 1100

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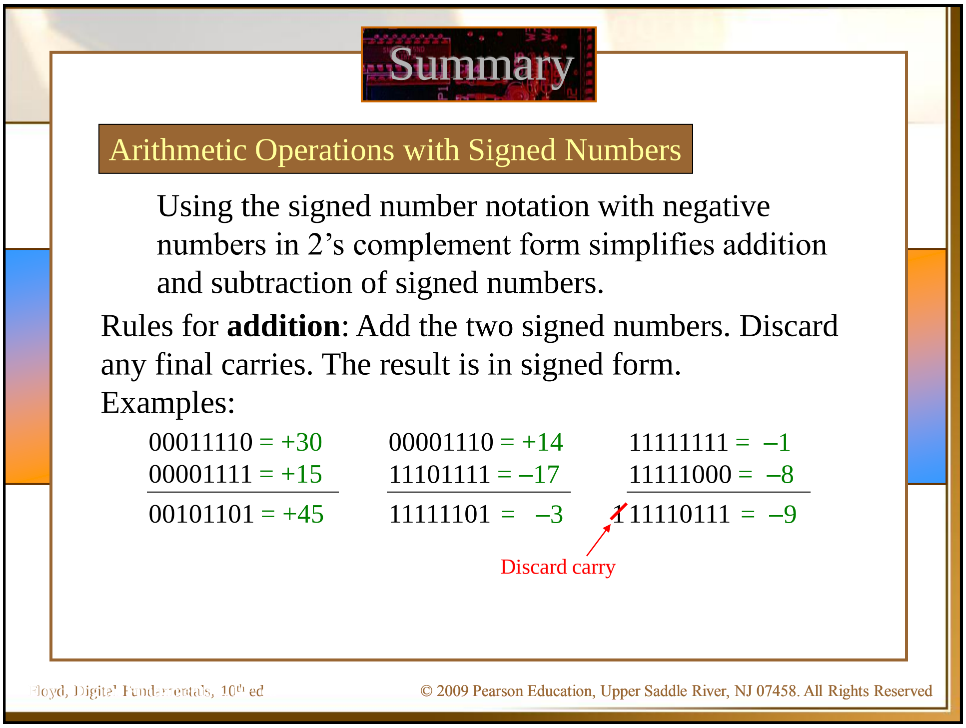

Summary

Arithmetic Operations with Signed Numbers

Using the signed number notation with negative numbers in 2's complement form simplifies addition and subtraction of signed numbers.

Rules for **addition**: Add the two signed numbers. Discard any final carries. The result is in signed form.

Examples:

00011110 = +30	00001110 = +14	11111111 = –1
00001111 = +15	11101111 = –17	11111000 = –8
00101101 = +45	11111101 = –3	~~1~~11110111 = –9

Discard carry

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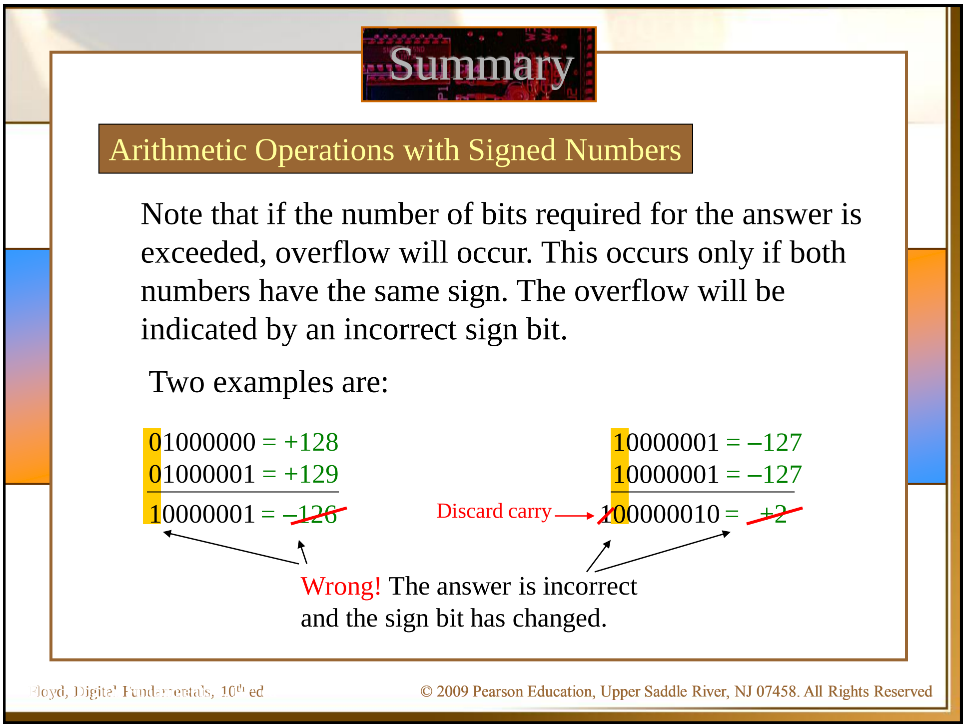

Summary

Arithmetic Operations with Signed Numbers

Note that if the number of bits required for the answer is exceeded, overflow will occur. This occurs only if both numbers have the same sign. The overflow will be indicated by an incorrect sign bit.

Two examples are:

01000000 = +128	10000001 = –127
01000001 = +129	10000001 = –127
10000001 = ~~–126~~	~~1~~00000010 = ~~+2~~

Discard carry

Wrong! The answer is incorrect and the sign bit has changed.

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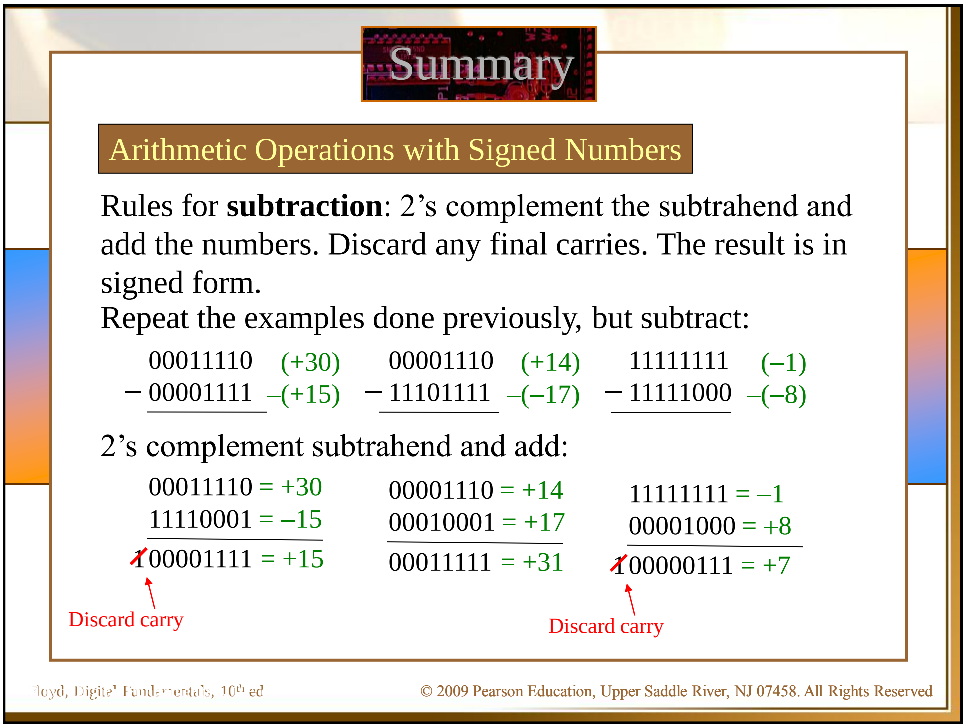

Summary

Arithmetic Operations with Signed Numbers

Rules for **subtraction**: 2's complement the subtrahend and add the numbers. Discard any final carries. The result is in signed form.

Repeat the examples done previously, but subtract:

00011110	(+30)	00001110	(+14)	11111111	(−1)
− 00001111	−(+15)	− 11101111	−(−17)	− 11111000	−(−8)

2's complement subtrahend and add:

00011110 = +30	00001110 = +14	11111111 = −1
11110001 = −15	00010001 = +17	00001000 = +8
$\cancel{1}$00001111 = +15	00011111 = +31	$\cancel{1}$00000111 = +7

Discard carry

Discard carry

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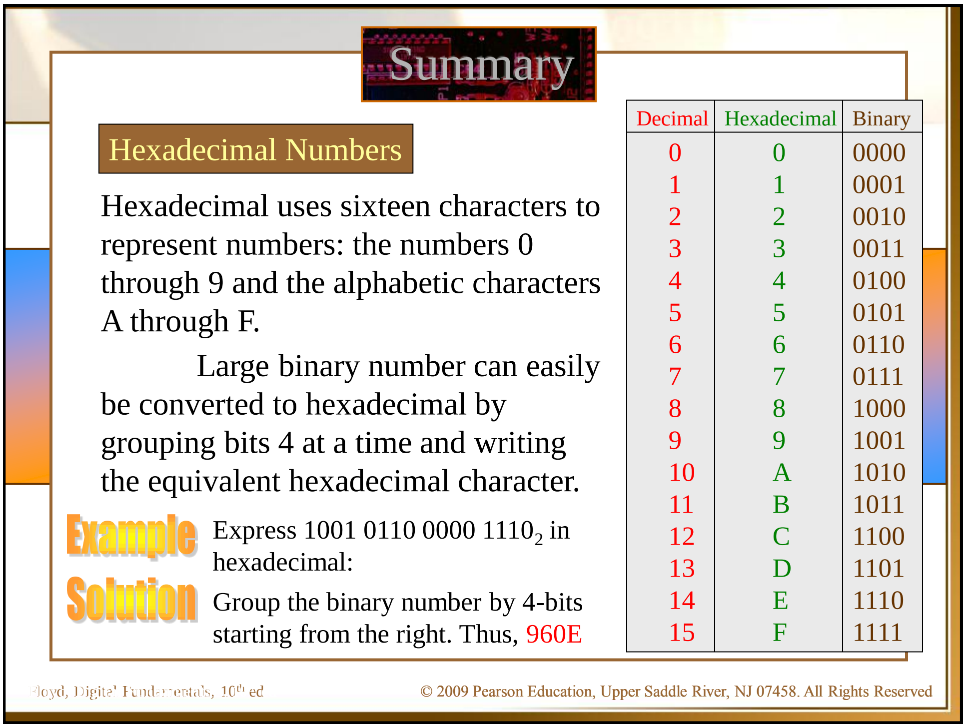

Summary

Hexadecimal Numbers

Hexadecimal uses sixteen characters to represent numbers: the numbers 0 through 9 and the alphabetic characters A through F.

Large binary number can easily be converted to hexadecimal by grouping bits 4 at a time and writing the equivalent hexadecimal character.

Example Express 1001 0110 0000 1110_2 in hexadecimal:

Solution Group the binary number by 4-bits starting from the right. Thus, 960E

Decimal	Hexadecimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

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Summary

Hexadecimal Numbers

Hexadecimal is a weighted number system. The column weights are powers of 16, which increase from right to left.

Column weights { 16^3 16^2 16^1 16^0. / 4096 256 16 1 .

Column weights	16^3	16^2	16^1	16^0
	4096	256	16	1

Example Express $1A2F_{16}$ in decimal.

Solution Start by writing the column weights:

4096 256 16 1
1 A 2 F_{16}

$1(4096) + 10(256) + 2(16) + 15(1) = 6703_{10}$

Decimal	Hexadecimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

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Summary

Octal Numbers

Octal uses eight characters the numbers 0 through 7 to represent numbers. There is no 8 or 9 character in octal.

Binary number can easily be converted to octal by grouping bits 3 at a time and writing the equivalent octal character for each group.

Example Express $1\ 001\ 011\ 000\ 001\ 110_2$ in octal:

Solution Group the binary number by 3-bits starting from the right. Thus, 113016_8

Decimal	Octal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	10	1000
9	11	1001
10	12	1010
11	13	1011
12	14	1100
13	15	1101
14	16	1110
15	17	1111

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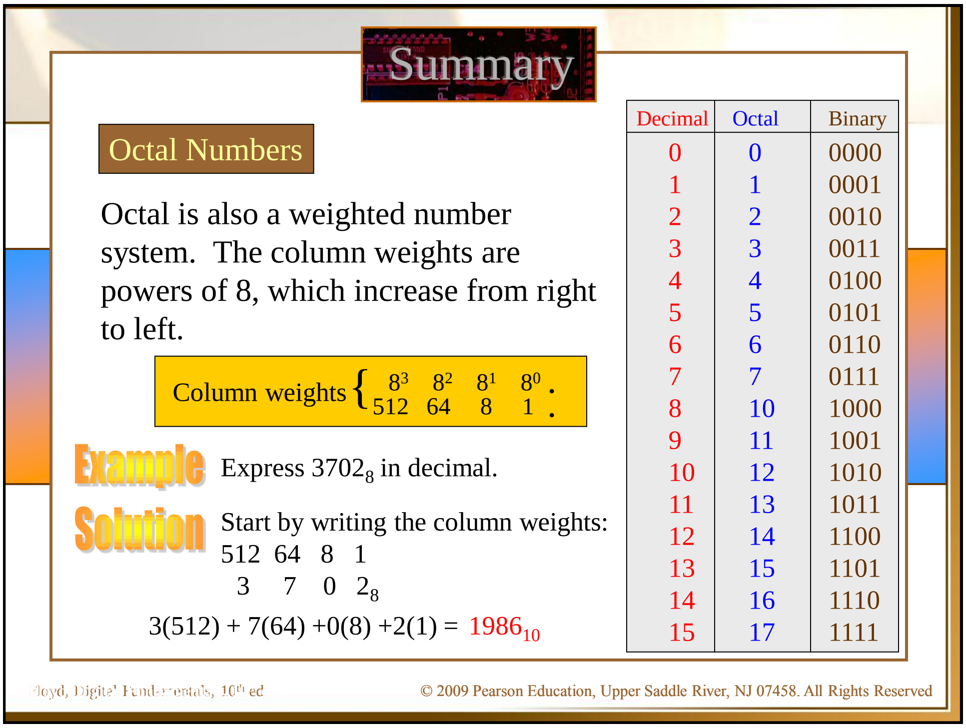

Summary

Octal Numbers

Octal is also a weighted number system. The column weights are powers of 8, which increase from right to left.

Column weights { 8^3 8^2 8^1 8^0 .
512 64 8 1 .

Example Express 3702_8 in decimal.

Solution Start by writing the column weights:
512 64 8 1
3 7 0 2_8

$3(512) + 7(64) + 0(8) + 2(1) = 1986_{10}$

Decimal	Octal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	10	1000
9	11	1001
10	12	1010
11	13	1011
12	14	1100
13	15	1101
14	16	1110
15	17	1111

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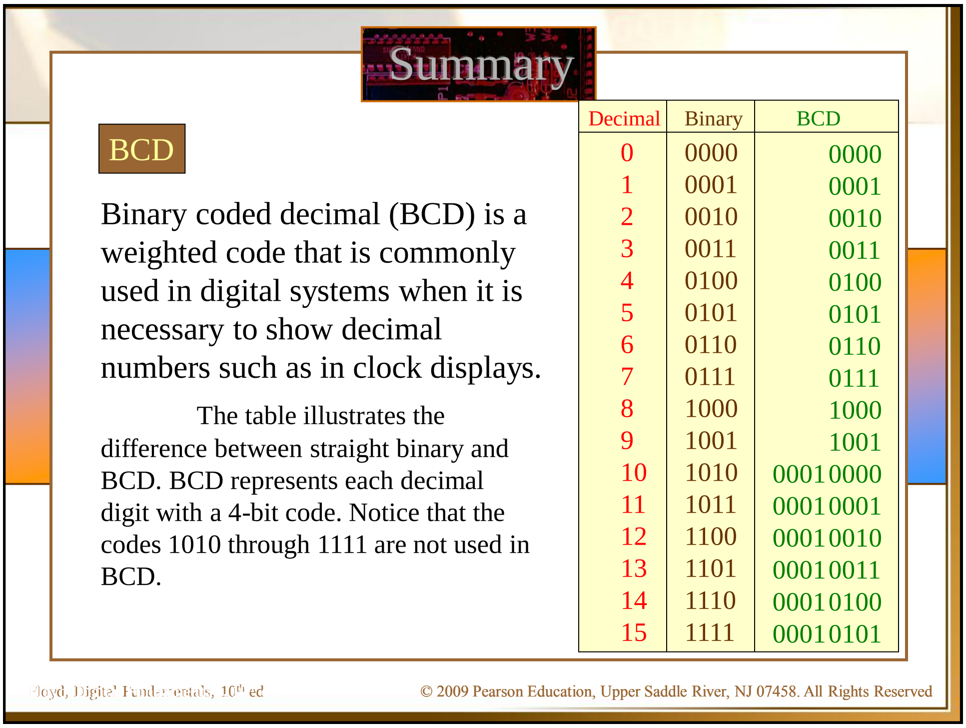

Summary

BCD

Binary coded decimal (BCD) is a weighted code that is commonly used in digital systems when it is necessary to show decimal numbers such as in clock displays.

The table illustrates the difference between straight binary and BCD. BCD represents each decimal digit with a 4-bit code. Notice that the codes 1010 through 1111 are not used in BCD.

Decimal	Binary	BCD
0	0000	0000
1	0001	0001
2	0010	0010
3	0011	0011
4	0100	0100
5	0101	0101
6	0110	0110
7	0111	0111
8	1000	1000
9	1001	1001
10	1010	0001 0000
11	1011	0001 0001
12	1100	0001 0010
13	1101	0001 0011
14	1110	0001 0100
15	1111	0001 0101

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Summary

BCD

You can think of BCD in terms of column weights in groups of four bits. For an 8-bit BCD number, the column weights are: 80 40 20 10 8 4 2 1.

Question: What are the column weights for the BCD number 1000 0011 0101 1001?

Answer:

8000 4000 2000 1000 800 400 200 100 80 40 20 10 8 4 2 1

Note that you could add the column weights where there is a 1 to obtain the decimal number. For this case:

8000 + 200 +100 + 40 + 10 + 8 +1 = 8359_{10}

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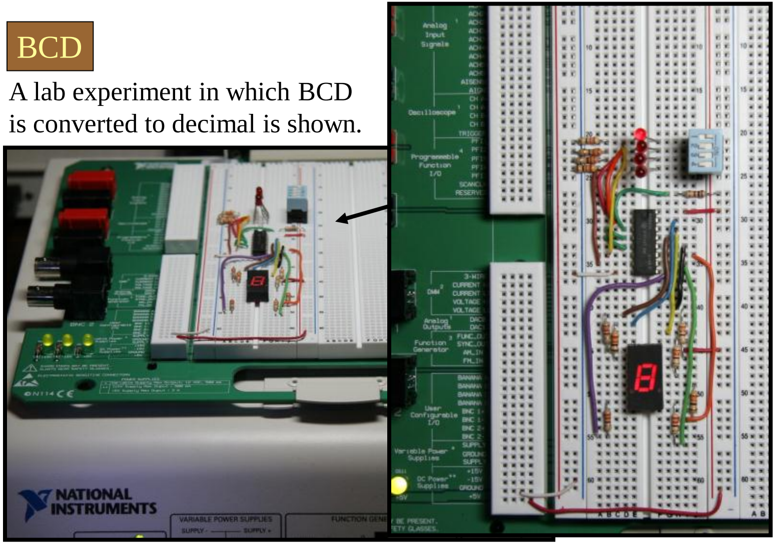
Summary

BCD

A lab experiment in which BCD is converted to decimal is shown.



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Selected Key Terms

Byte	A group of eight bits
Floating-point number	A number representation based on scientific notation in which the number consists of an exponent and a mantissa.
Hexadecimal	A number system with a base of 16.
Octal	A number system with a base of 8.
BCD	Binary coded decimal; a digital code in which each of the decimal digits, 0 through 9, is represented by a group of four bits.

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Selected Key Terms

Alphanumeric	Consisting of numerals, letters, and other characters
ASCII	American Standard Code for Information Interchange; the most widely used alphanumeric code.
Parity	In relation to binary codes, the condition of evenness or oddness in the number of 1s in a code group.
Cyclic redundancy check (CRC)	A type of error detection code.

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Quiz

1. For the binary number 1000, the weight of the column with the 1 is

a. 4

b. 6

[✓ c. 8]

d. 10

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Quiz

2. The 2’s complement of 1000 is

a. 0111

[✓ b. 1000]

c. 1001

d. 1010

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Quiz

3. The fractional binary number 0.11 has a decimal value of

a. ¼
b. ½
[✓ c.] ¾
d. none of the above

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Quiz

4. The hexadecimal number 2C has a decimal equivalent value of

a. 14
[✓ b.] 44
c. 64
d. none of the above

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Quiz

5. Assume that a floating point number is represented in binary. If the sign bit is 1, the

[✓ a.] number is negative

b. number is positive

c. exponent is negative

d. exponent is positive

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Quiz

6. When two positive signed numbers are added, the result may be larger that the size of the original numbers, creating overflow. This condition is indicated by

[✓ a.] a change in the sign bit

b. a carry out of the sign position

c. a zero result

d. smoke

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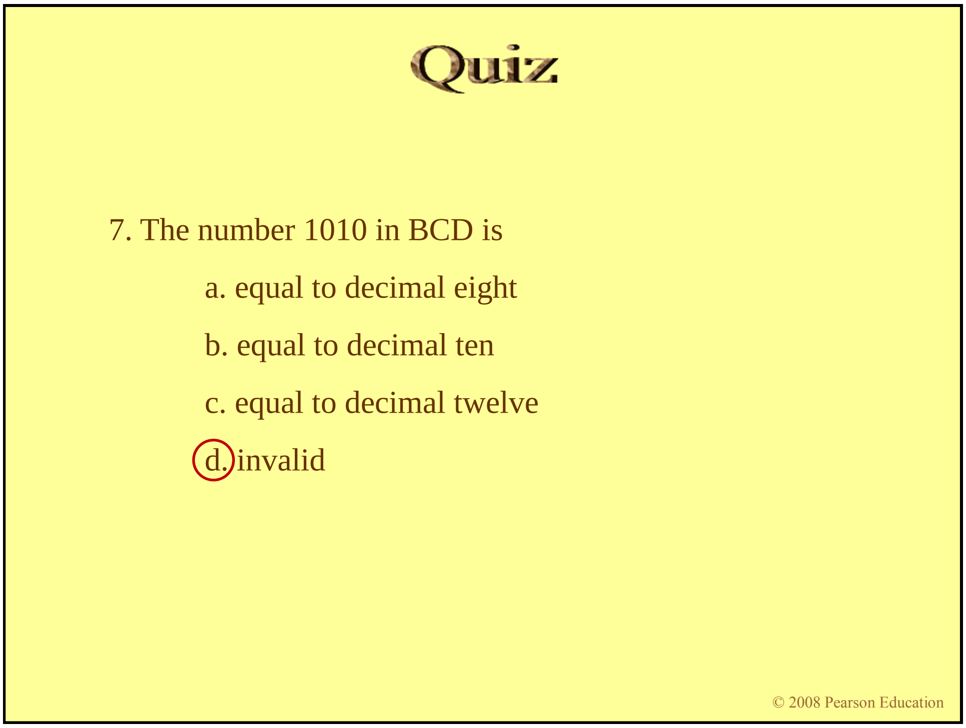

Quiz

7. The number 1010 in BCD is

  a. equal to decimal eight

  b. equal to decimal ten

  c. equal to decimal twelve

  [✓ d.] invalid

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