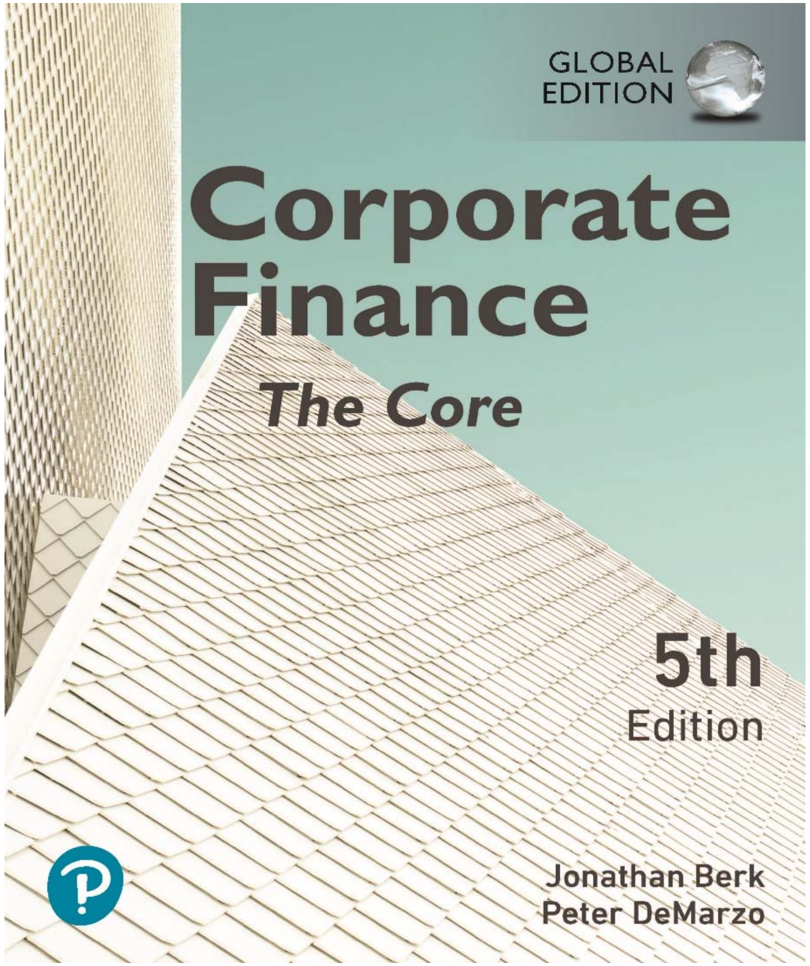
Corporate Finance: The Core

Fifth Edition, Global Edition



Chapter 4

The Time Value of Money

Chapter Outline (1 of 2)

4.1 The Timeline

4.2 The Three Rules of Time Travel

4.3 Valuing a Stream of Cash Flows

4.4 Calculating the Net Present Value

4.5 Perpetuities and Annuities

Chapter Outline (2 of 2)

4.6 Using an Annuity Spreadsheet or Calculator

4.7 Non-Annual Cash Flows

4.8 Solving for the Cash Payments

4.9 The Internal Rate of Return

Learning Objectives (1 of 4)

- Draw a timeline illustrating a given set of cash flows.
- List and define three rules of time travel.
- Calculate the future value of the following:
  - A single sum
  - An uneven stream of cash flows, starting either now or sometime in the future
  - An annuity, starting either now or sometime in the future
  - Several cash flows occurring at regular intervals, which grow at a constant rate each period

Learning Objectives (2 of 4)

- Calculate the present value of the following
  - A single sum
  - An uneven stream of cash flows, starting either now or sometime in the future
  - An infinite stream of identical cash flows
  - An annuity, starting either now or sometime in the future
  - An infinite stream of cash flows that grow at a constant rate each period
  - Several cash flows occurring at regular intervals, which grow at a constant rate each period

Learning Objectives (3 of 4)

- Given four out of the following five inputs for an annuity, compute the fifth: (a) present value, (b) future value, (c) number of periods, (d) periodic interest rate, (e) periodic payment.
- Given three out of the following four inputs for a single sum, compute the fourth: (a) present value, (b) future value, (c) number of periods, (d) periodic interest rate.

Learning Objectives (4 of 4)

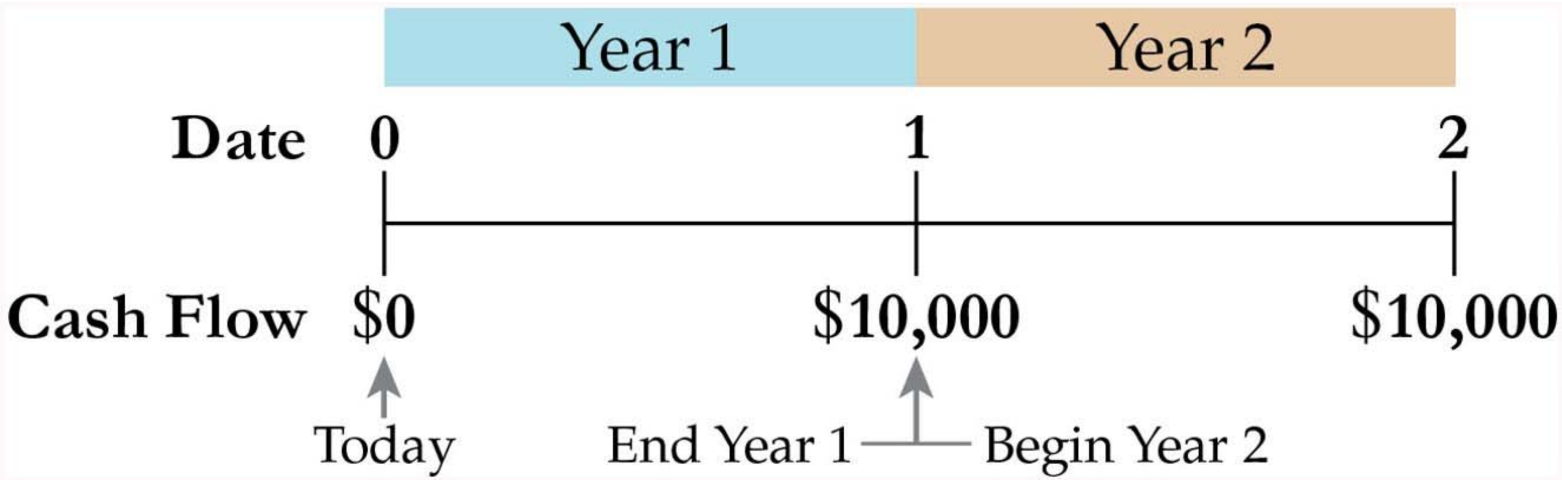
- Given cash flows and present or future value, compute the internal rate of return for a series of cash flows.

4.1 The Timeline (1 of 4)

- A timeline is a linear representation of the timing of potential cash flows.
- Drawing a timeline of the cash flows will help you visualize the financial problem.

4.1 The Timeline (2 of 4)

- Assume that you made a loan to a friend. You will be repaid in two payments, one at the end of each year over the next two years.

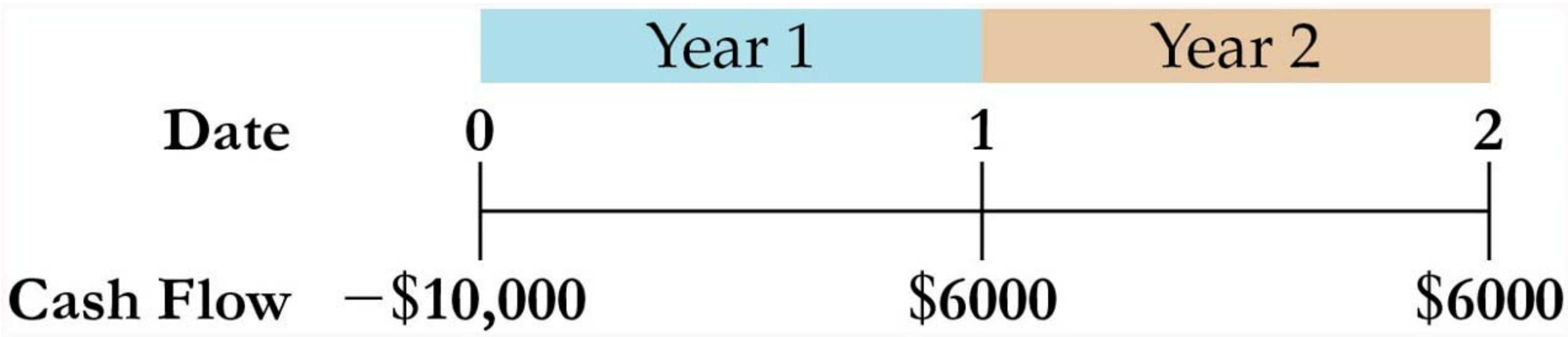


4.1 The Timeline (3 of 4)

- Differentiate between two types of cash flows
  - Inflows are positive cash flows.
  - Outflows are negative cash flows, which are indicated with a – (minus) sign.

4.1 The Timeline (4 of 4)

- Assume that you are lending $10,000 today and that the loan will be repaid in two annual $6,000 payments.



- The first cash flow at date 0 (today) is represented as a negative sum because it is an outflow.
- Timelines can represent cash flows that take place at the end of any time period—a month, a week, a day, etc.

Textbook Example 4.1 (1 of 2)

Constructing a Timeline

- **Problem**
  - Suppose you must pay tuition of $10,000 per year for the next two years. Your tuition payments must be made in equal installments at the start of each semester. What is the timeline of your tuition payments?

Textbook Example 4.1 (2 of 2)

Solution

- Assuming today is the start of the first semester, your first payment occurs at date 0 (today). The remaining payments occur at semester intervals. Using one semester as the period length, we can construct a timeline as follows:

4.2 The Three Rules of Time Travel

- Financial decisions often require combining cash flows or comparing values. Three rules govern these processes.

Table 4.1 The Three Rules of Time Travel

Rule 1	Only values at the same point in time can be compared or combined.	
Rule 2	To move a cash flow forward in time, you must compound it.	Future Value of a Cash Flow $FV_n = C \times (1 + r)^n$
Rule 3	To move a cash flow backward in time, you must discount it.	Present Value of a Cash Flow $PV = C \div (1 + r)^n = \frac{C}{(1 + r)^n}$

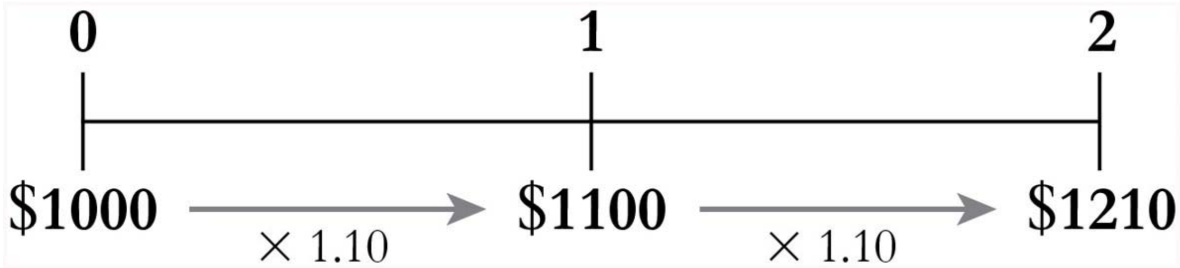
Rule 1: Comparing and Combining Values

- A dollar today and a dollar in one year are not equivalent.
- It is only possible to compare or combine values at the same point in time.
  - Which would you prefer: A gift of \$1,000 today or \$1,210 at a later date?
  - To answer this, you will have to compare the alternatives to decide which is worth more. One factor to consider: How long is "later?"

Rule 2: Moving Cash Flows Forward in Time (1 of 2)

- To move a cash flow forward in time, you must compound it.
  - Suppose you have a choice between receiving $1,000 today or $1,210 in two years. You believe you can earn 10% on the $1,000 today but want to know what the $1,000 will be worth in two years.

Rule 2: Moving Cash Flows Forward in Time (2 of 2)

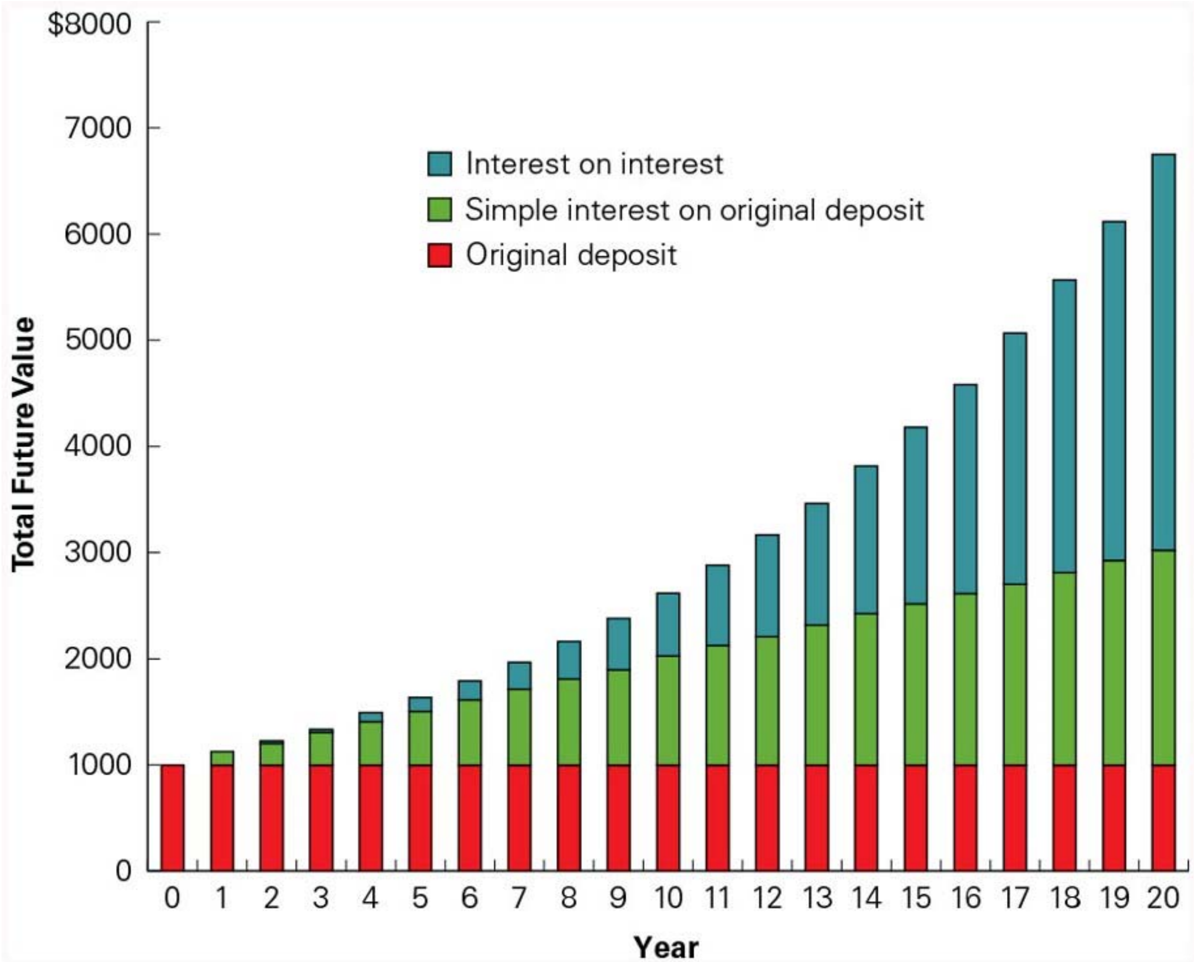


- Future Value of a Cash Flow

Future Value of a Cash Flow

$$FV_n = C \times \underbrace{(1+r) \times (1+r) \times \cdots \times (1+r)}_{n \text{ times}} = C \times (1+r)^n \qquad (4.1)$$

Figure 4.1 The Composition of Interest over Time



Textbook Example 4.2 (1 of 2)

The Power of Compounding

- **Problem**
  - Suppose you invest $1,000 in an account paying 10% interest per year. How much will you have in the account in seven years? In 20 years ? In 75 years?

Textbook Example 4.2 (2 of 2)

Solution

- You can apply Eq. 4.1 to calculate the future value in each case

$$
\begin{array}{lll}
\text{7 years:} & \$1000 \times (1.10)^{7} & = \$1948.72 \\
\text{20 years:} & \$1000 \times (1.10)^{20} & = \$6727.50 \\
\text{75years:} & \$1000 \times (1.10)^{75} & = \$1,271,895.37
\end{array}
$$

  - Note that at 10% interest, your money will nearly double in 7 years. After 20 years, it will increase almost sevenfold. And if you invest for 75 years, you will be a millionaire!

Rule 3: Moving Cash Flows Back in Time

- To move a cash flow backward in time, we must discount it.
- Present Value of a Cash Flow

$$PV = C \div (1+r)^n = \frac{C}{(1+r)^n}$$

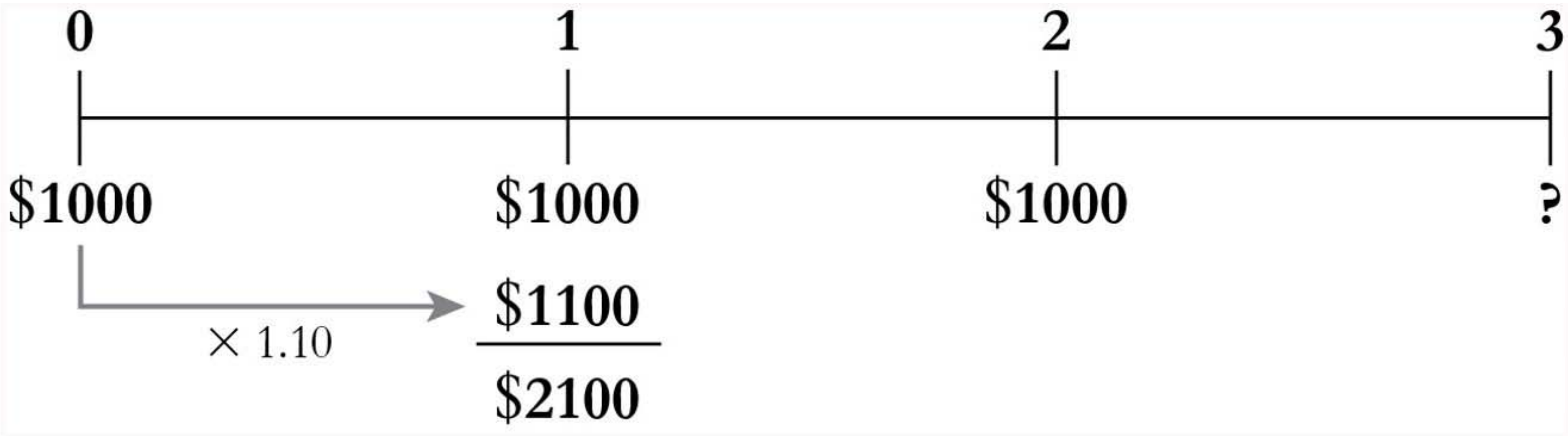
Applying the Rules of Time Travel (1 of 5)

- Recall the first rule: It is only possible to compare or combine values at the same point in time. So far we've only looked at comparing.
  - Suppose we plan to save $1,000 today, and $1,000 at the end of each of the next two years. If we can earn a fixed 10% interest rate on our savings, how much will we have three years from today?

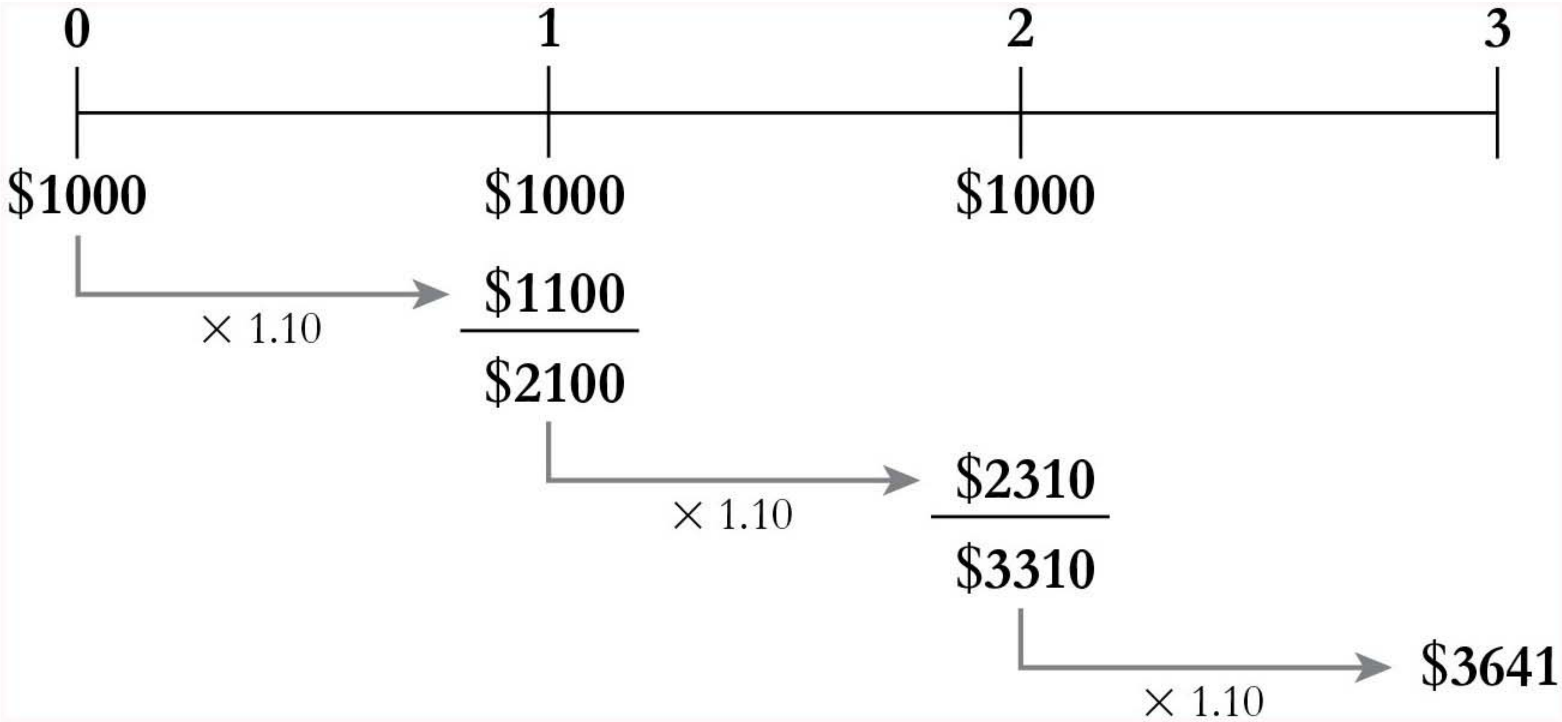
Applying the Rules of Time Travel (2 of 5)

- The time line would look like this:

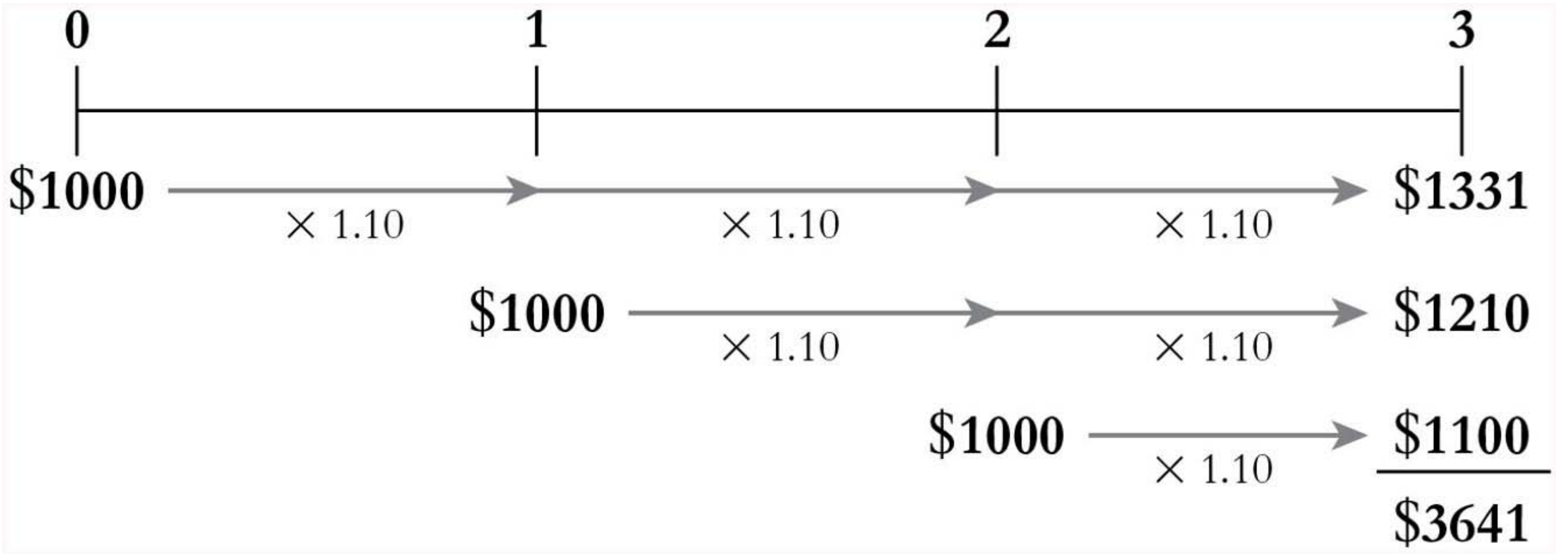
Applying the Rules of Time Travel (3 of 5)



Applying the Rules of Time Travel (4 of 5)



Applying the Rules of Time Travel (5 of 5)



Textbook Example 4.4 (1 of 3)

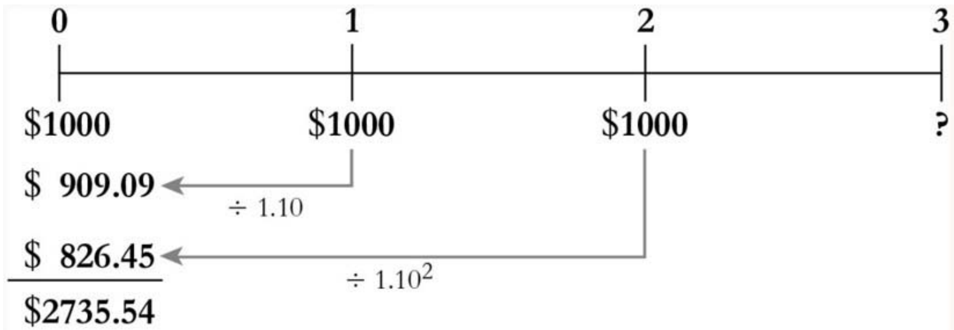
Computing the Future Value

- **Problem**
  - Let's revisit the savings plan we considered earlier: we plan to save $1,000 today and at the end of each of the next two years. At a fixed 10% interest rate, how much will we have in the bank three years from today?

Textbook Example 4.4 (2 of 3)

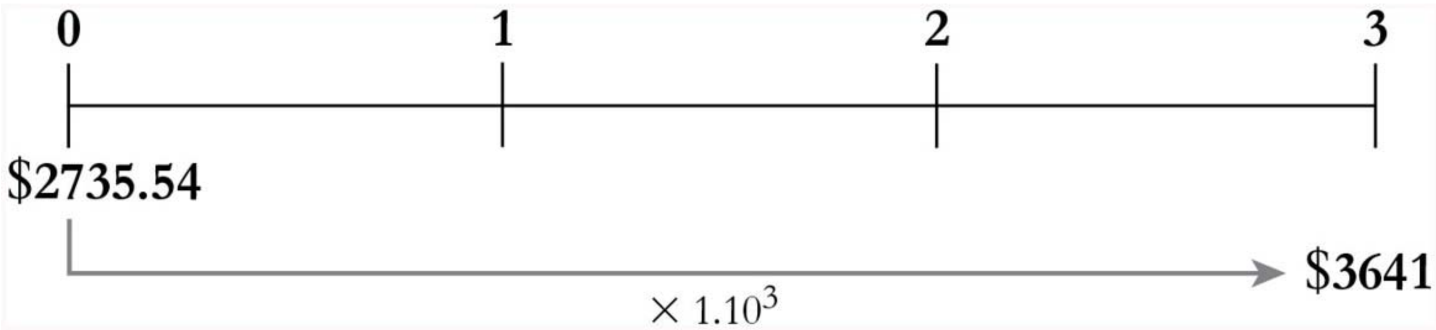
Solution

- Let's solve this problem in a different way than we did earlier. First, compute the present value of the cash flows. There are several ways to perform this calculation. Here we treat each cash flow separately and then combine the present values.



Textbook Example 4.4 (3 of 3)

- Saving $2,735.54 today is equivalent to saving $1,000 per year for three years. Now let's compute its future value in year 3:

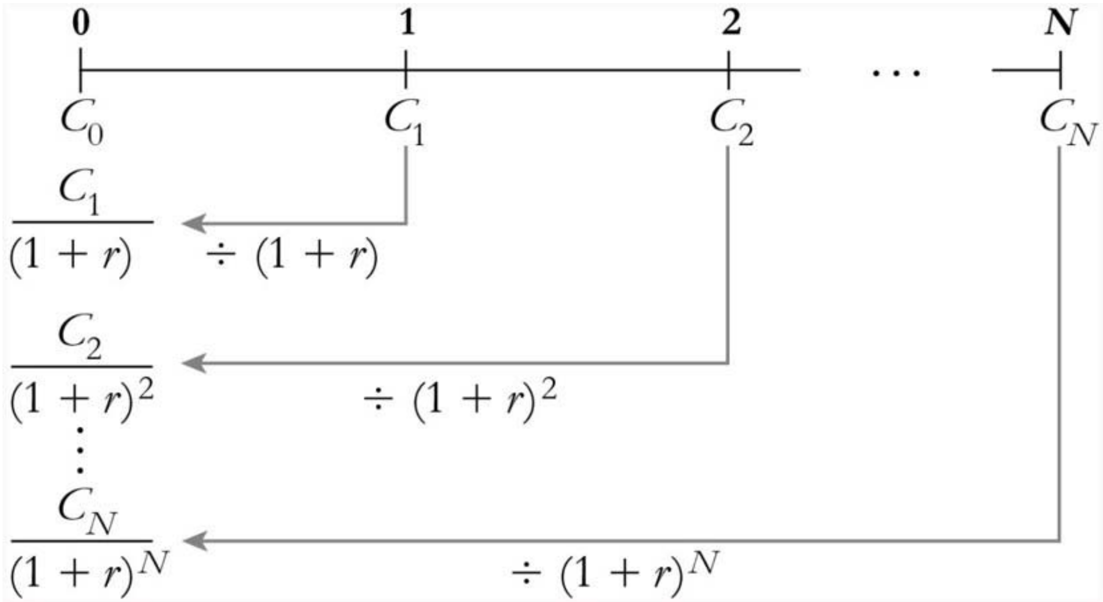


- This answer of $3,641 is precisely the same result we found earlier. As long as we apply the three rules of time travel, we will always get the correct answer.

4.3 Valuing a Stream of Cash Flows (1 of 2)

- Based on the first rule of time travel we can derive a general formula for valuing a stream of cash flows: if we want to find the present value of a stream of cash flows, we simply add up the present values of each.

4.3 Valuing a Stream of Cash Flows (2 of 2)



- Present Value of a Cash Flow Stream

$$PV = \sum_{n=0}^{N} PV(C_n) = \sum_{n=0}^{N} \frac{C_n}{(1+r)^n}$$

Textbook Example 4.5 (1 of 4)

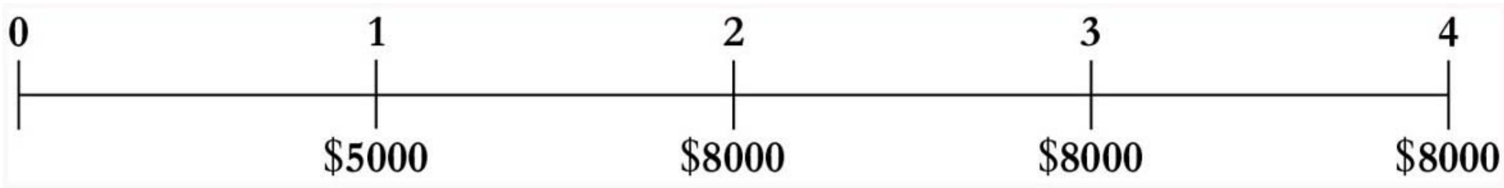
Present Value of a Stream of Cash Flows

- **Problem**
  - You have just graduated and need money to buy a new car. Your rich uncle Henry will lend you the money so long as you agree to pay back within four years, and you offer to pay him the rate of interest that he would otherwise get by putting his money in a savings account. Based on your earnings and living expenses, you think you will be able to pay him $5,000 in one year, and then $8000 each year for the next three years. If uncle Henry would otherwise earn 6% per year on his savings, how much can you borrow from him?

Textbook Example 4.5 (2 of 4)

Solution

- The cash flows you can promise Uncle Henry are as follows:



- How much money should Uncle Henry be willing to give you today in return for your promise of these payments? He should be willing to give you an amount that is equivalent to these payments in present value terms. This is the amount of money that it would take him to produce these same cash flows, which we calculate as follows:

$$PV = \frac{5000}{1.06} + \frac{8000}{1.06^2} + \frac{8000}{1.06^3} + \frac{8000}{1.06^4}$$
$$= 4716.98 + 7119.97 + 6716.95 + 6336.75$$
$$= 24{,}890.65$$

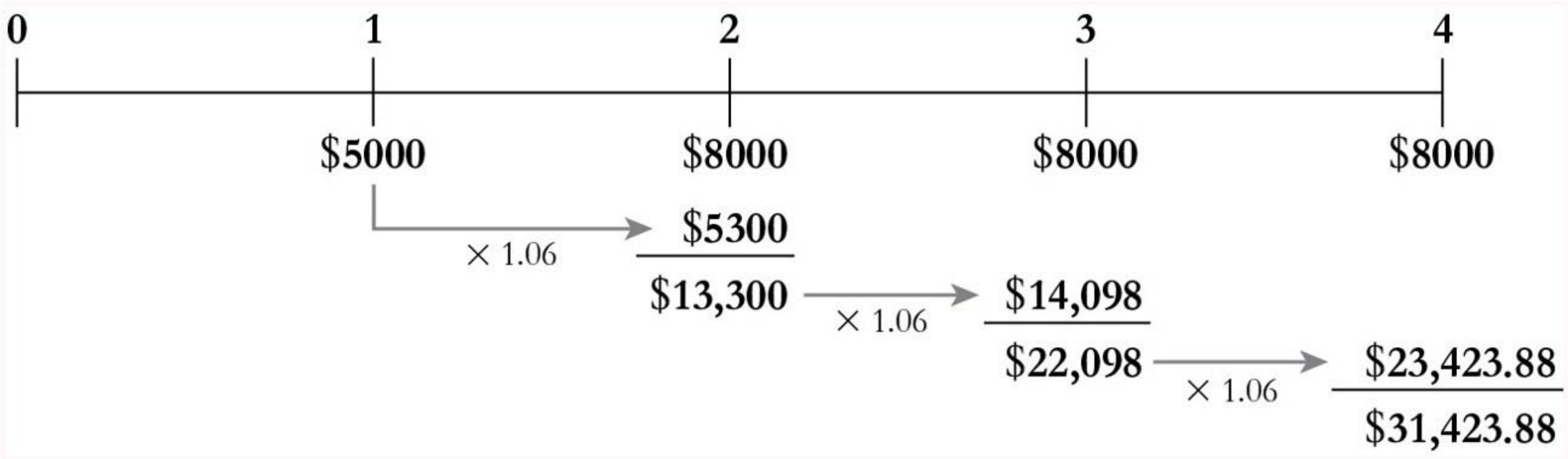
Textbook Example 4.5 (3 of 4)

- Thus, Uncle Henry should be willing to lend you $24,890.65 in exchange for your promised payments. This amount is less than the total you will pay him $(\$5{,}000 + \$8{,}000 + \$8{,}000 + \$8{,}000 = \$29{,}000)$ due to the time value of money.
- Let's verify our answer. If your uncle kept his $24,890.65 in the bank today earning 6% interest, in four years he would have

$$FV = \$24{,}890.65 \times (1.06)^4 = \$31{,}423.87 \text{ in four years}$$

- Now suppose that Uncle Henry gives you the money, and then deposits your payments to him in the bank each year. How much will he have four years from now? We need to compute the future value of the annual deposits. One way to do so is to compute the bank balance each year:

Textbook Example 4.5 (4 of 4)



- We get the same answer both ways (within a penny, which is because of rounding).

Future Value of Cash Flow Stream

- Future Value of a Cash Flow Stream with a Present Value of PV

$$FV_n = PV \times (1 + r)^n$$

4.4 Calculating the Net Present Value

- Calculating the NPV of future cash flows allows us to evaluate an investment decision.
- Net Present Value compares the present value of cash inflows (benefits) to the present value of cash outflows (costs).

Textbook Example 4.6 (1 of 4)

Net Present Value of an Investment Opportunity

- **Problem**
  - You have been offered the following investment opportunity: if you invest \$1,000 today, you will receive \$500 at the end of each of the next three years. If you could otherwise earn 10% per year on your money, should you undertake the investment opportunity?

Textbook Example 4.6 (2 of 4)

Solution

- As always, we start with a timeline. We denote the upfront investment as a negative cash flow (because it is money we need to spend) and the money we receive as a positive cash flow.

- To decide whether we should accept this opportunity, we compute the NPV by computing the present value of the stream:

$$NPV = -1000 + \frac{500}{1.10} + \frac{500}{1.10^2} + \frac{500}{1.10^3} = \$243.43$$

Textbook Example 4.6 (3 of 4)

- Because the NPV is positive, the benefits exceed the costs and we should make the investment. Indeed, the NPV tells us that taking this opportunity is like getting an extra $243.43 that you can spend today. To illustrate, suppose you borrow $1,000 to invest in the opportunity and an extra $243.43 to spend today. How much would you owe on the $1,243.43 loan in three years? At 10% interest, the amount you would owe would be

$$FV = (\$1000 + \$243.43) \times (1.10)^3 = \$1655 \text{ in three years}$$

Textbook Example 4.6 (4 of 4)

- At the same time, the investment opportunity generates cash flows. If you put these cash flows into a bank account, how much will you have saved three years from now? The future value of the savings is

$$FV = (\$500 \times 1.10^2) + (\$500 \times 1.10) + \$500 = \$1655 \text{ in three years}$$

- As you see, you can use your bank savings to repay the loan. Taking the opportunity therefore allows you to spend $243.43 today at no extra cost.

4.5 Perpetuities and Annuities (1 of 2)

- Perpetuities
  - When a constant cash flow will occur at regular intervals forever it is called a perpetuity.

4.5 Perpetuities and Annuities (2 of 2)

- The value of a perpetuity is simply the cash flow divided by the interest rate.
- Present Value of a Perpetuity

$$PV(C \text{ in perpetuity}) = \frac{C}{r}$$

Textbook Example 4.7 (1 of 2)

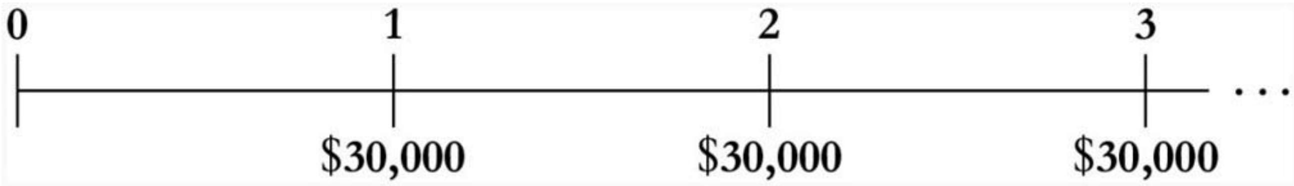
Endowing a Perpetuity

- **Problem**
  - You want to endow an annual MBA graduation party at your alma mater. You want the event to be a memorable one, so you budget $30,000 per year forever for the party. If the university earns 8% per year on its investments, and if the first party is in one year's time, how much will you need to donate to endow the party?

Textbook Example 4.7 (2 of 2)

Solution

- The timeline of the cash flows you want to provide is



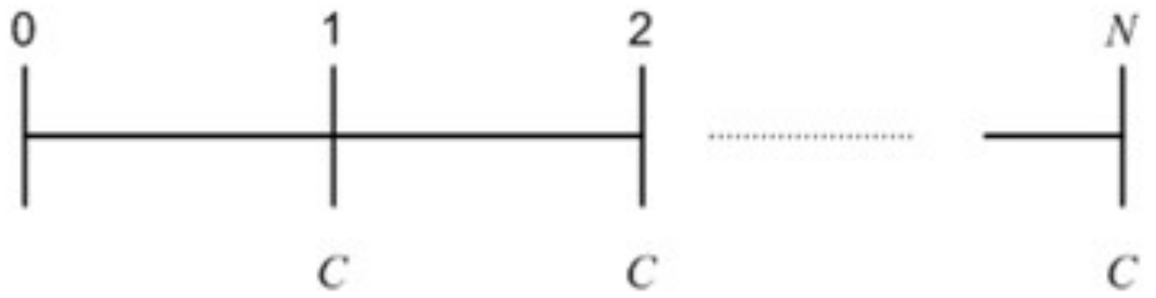
- This is a standard perpetuity of $30,000 per year. The funding you would need to give the university in perpetuity is the present value of this cash flow stream. From the formula,

$$PV = C/r = \frac{\$30{,}000}{0.08} = \$375{,}000 \text{ today}$$

- If you donate $375,000 today, and if the university invests it at 8% per year forever, then the MBA swill have $30,000 every year for their graduation party.

4.5 Perpetuities and Annuities

- Annuities
  – When a constant cash flow will occur at regular intervals for a finite number of *N* periods, it is called an annuity.



  – Present Value of an Annuity

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots + \frac{C}{(1+r)^N} = \sum_{n=1}^{N} \frac{C}{(1+r)^n}$$

Present Value of an Annuity (3 of 3)

- For the general formula

PV(annuity of C for N periods) =

$$\frac{C}{r}\left(1-\frac{1}{(1+r)^{\mathrm{N}}}\right)$$

Textbook Example 4.8 (1 of 3)

Present Value of a Lottery Prize Annuity

- **Problem**
  - You are the lucky winner of the $30 million state lottery. You can take your prize money either as (a) 30 payments of $1 million per year (starting today), or (b) $15 million paid today. If the interest rate is 8%, which option should you take?

Textbook Example 4.8 (2 of 3)

Solution

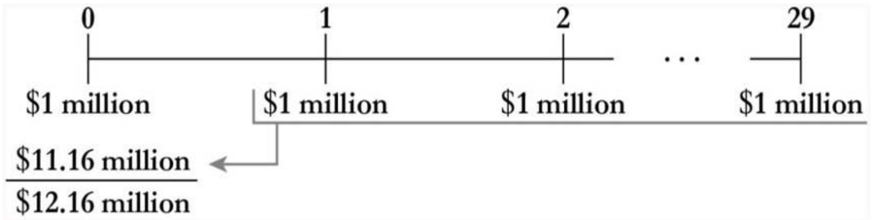
- Option (a) provides $30 million of prize money but paid annually. In this case, the cash flows are an annuity in which the first payment begins immediately, sometimes called an **annuity due**.
- Because the first payment is paid today, the last payment will occur in 29 years (for a total of 30 payments). We can compute the present value of the final 29 payments as a standard annuity of $1 million per year using the annuity formula:

$$PV\left(29 \text{ yr annuity of } \$1 \ \frac{\text{million}}{\text{yr}}\right) = \$1 \text{ million} \times \frac{1}{.08}\left(1 - \frac{1}{1.08^{29}}\right)$$
$$= \$11.16 \text{ million today}$$

- Adding the $1 million we receive upfront, this option has a present value of $12.16 million:

Textbook Example 4.8 (3 of 3)

- Adding the $1 million we receive upfront, this option has a present value of $12.16 million:



- Therefore, the present value of option (a) is only $12.16 million, and so it is more valuable to take option (b) and receive $15 million upfront—even though we receive only half the total cash amount. The difference, of course, is due to the time value of money. To see that (b) really is better, if you have the $15 million today, you can use $1 million immediately and invest the remaining $14 million at an 8% interest rate. This strategy will give you $\$14 \text{ million} \times 8\% = \$1.12 \text{ million per year}$ in perpetuity! Alternatively, you can spend $\$15 \text{ million} - \$11.16 \text{ million} = \$3.84$ million today, and invest the remaining $11.16 million, which will still allow you to withdraw $1 million each year for the next 29 years before your account is depleted.

Future Value of an Annuity

- Future Value of an Annuity

$$
\begin{aligned}
FV\ \text{(annuity)} &= PV \times (1+r)^N \\
&= \frac{C}{r}\left(1 - \frac{1}{(1+r)^N}\right) \times (1+r)^N \\
&= C \times \frac{1}{r}\left((1+r)^N - 1\right)
\end{aligned}
$$

Textbook Example 4.9 (1 of 3)

Retirement Savings Plan Annuity

- **Problem**
  - Ellen is 35 years old, and she has decided it is time to plan seriously for her retirement. At the end of each year until she is 65, she will save $10,000 in a retirement account. If the account earns 10% per year, how much will Ellen have saved at age 65?

Textbook Example 4.9 (2 of 3)

Solution

- As always, we begin with a timeline. In this case, it is helpful to keep track of both the dates and Ellen's age:

- Ellen's savings plan looks like an annuity of $10,000 per year for 30 years. (**Hint**: It is easy to become confused when you just look at age, rather than at both dates and age. A common error is to think there are only $65 - 36 = 29$ payments. Writing down both dates and age avoids this problem.)

Textbook Example 4.9 (3 of 3)

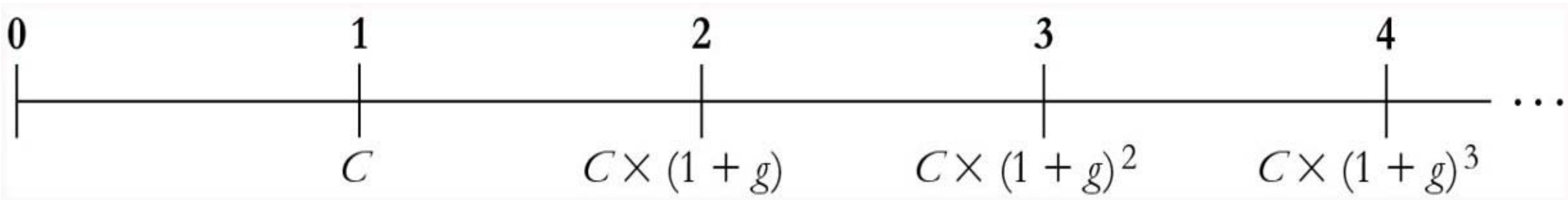
Solution

- To determine the amount Ellen will have in the bank at age 65, we compute the future value of this annuity:

$$
\begin{aligned}
FV &= \$10{,}000 \times \frac{1}{0.10}(1.10^{30} - 1) \\
&= \$10{,}000 \times 164.49 \\
&= \$1.645 \text{ million at age } 65
\end{aligned}
$$

Growing Cash Flows (1 of 2)

- Growing Perpetuity
  - Assume you expect the amount of your perpetual payment to increase at a constant rate, g.
- Present Value of a Growing Perpetuity



$$PV \text{ (growing perpetuity)} = \frac{C}{r-g}$$

Textbook Example 4.10 (1 of 2)

Endowing a Growing Perpetuity

Problem

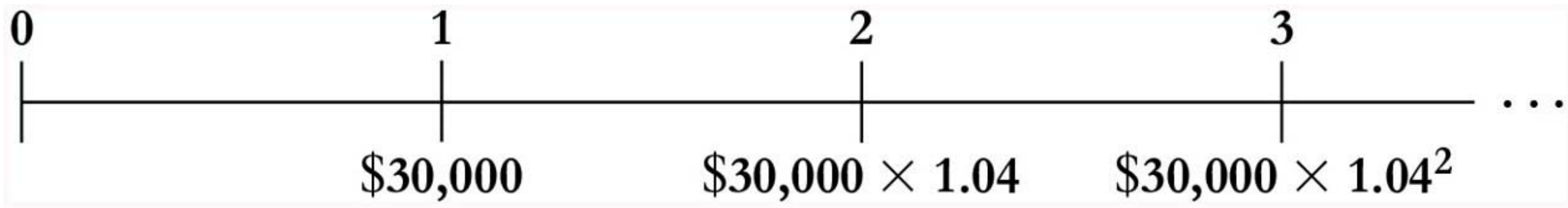
In example 4.7, you planned to donate money to your alma mater to fund an annual $30,000 MBA graduation party. Given an interest rate of 8% per year , the required donation was the present value of

$$PV = \frac{\$30{,}000}{0.08} = \$375{,}000 \text{ today}$$

Before accepting the money, however, the MBA student association has asked that you increase the donation to account for the effect of inflation on the cost of the party in future years. Although $30,000 is adequate for next year's party, the students estimate that the party's cost will rise by 4% per year thereafter. To satisfy their request, how much do you need to donate now?

Textbook Example 4.10 (2 of 2)

Solution



The cost of the party next year is $30,000, and the cost then increases 4% per year forever. From the timeline, we recognize the form of a growing perpetuity. To finance the growing cost, you need to provide the present value today of

$$PV = \frac{\$30{,}000}{0.08 - 0.04} = \$750{,}000 \text{ today}$$

You need to double the size of your gift!

Growing Cash Flows (2 of 2)

- Growing Annuity
  - The present value of a growing annuity with the initial cash flow *c*, growth rate *g*, and interest rate *r* is defined as:
  - Present Value of a Growing Annuity

$$PV = C \times \frac{1}{(r-g)}\left(1-\left(\frac{1+g}{(1+r)}\right)^{N}\right)$$

Textbook Example 4.11 (1 of 3)

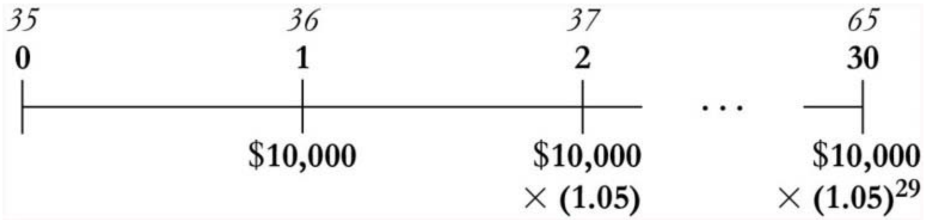
Retirement Savings with a Growing Annuity

- **Problem**
  - In Example 4.9, Ellen considered saving $10,000 per year for her retirement. Although $10,000 is the most she can save in the first year, she expects her salary to increase each year so that she will able to increase her savings by 5% per year. With this plan, if she earns 10% per year on her savings, how much will Ellen have saved at age 65?

Textbook Example 4.11 (2 of 3)

Solution

- Her new savings plan is represented by the following timeline:



- This example involves a 30-year growing annuity, with a growth rate of 5%, and an initial cash flow of $10,000. The present value of this growing annuity is given by

$$PV = \$10{,}000 \times \frac{1}{0.10 - 0.05}\left(1 - \left(\frac{1.05}{1.10}\right)^{30}\right)$$

$$= \$10{,}000 \times 15.0463$$

$$= \$150{,}463 \text{ today}$$

Textbook Example 4.11 (3 of 3)

- Ellen's proposed savings plan is equivalent to having \$150,463 in the bank **today**. To determine the amount she will have at age 65, we need to move this amount forward 30 years:

$$\begin{aligned} FV &= \$150{,}463 \times 1.10^{30} \\ &= \$2.625 \text{ million in 30 years} \end{aligned}$$

- Ellen will have saved \$2.625 million at age 65 using the new savings plan. This sum is almost \$1 million more than she had without the additional annual increases in savings.

4.6 Using an Annuity Spreadsheet or Calculator

- Spreadsheets simplify the calculations of TVM problems
  - $NPER$
  - $RATE$
  - PV
  - PMT
  - FV
- These functions all solve the problem:

$$NPV = PV + PMT \times \frac{1}{RATE}\left(1 - \frac{1}{(1+RATE)^{NPER}}\right) + \frac{FV}{(1+RATE)^{NPER}} = 0$$

Textbook Example 4.12

Computing the Future Value in Excel

- **Problem**
  - Suppose you plan to invest $20,000 in an account paying 8% interest. How much will you have in the account in 15 years?

Textbook Example 4.12 (2 of 4)

Solution

- We represent this problem with the following timeline:

Textbook Example 4.12 (3 of 4)

Solution

- To compute the solution, we enter the four variables we know $(\text{NPER} = 15, \text{RATE} = 8\%, \text{PV} = -20{,}000, \text{PMT} = 0)$ and solve for the one we want to determine (FV) using the Excel function FV(RATE, NPER, PMT, PV). The spreadsheet here calculates a future value of $63,443.

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	15	8.00%	–20,000	0		
Solve for FV					**63,443**	= FV (0.08,15,0, –20000)

Textbook Example 4.12 (4 of 4)

- Note that we entered PV as a negative number (the amount we are putting **into** the bank), and FV is shown as a positive number (the amount we can take **out** of the bank). It is important to use signs correctly to indicate the direction in which the money is flowing when using the spreadsheet functions.
- To check the result, we can solve this problem directly:

$$FV = \$20{,}000 \times 1.08^{15} = \$63{,}443$$

Textbook Example 4.13 (1 of 3)

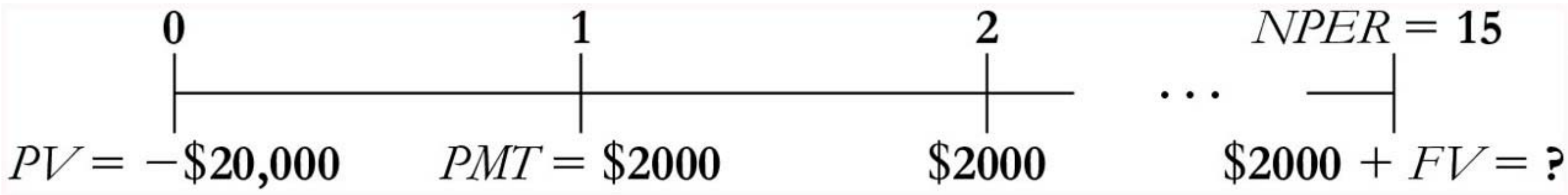
Using the Annuity Spreadsheet

- **Problem**
  - Suppose that you invest $20,000 in an account paying 8% interest. You plan to withdraw $2,000 at the end of each year for 15 years. How much money will be left in the account after 15 years?

Textbook Example 4.13 (2 of 3)

Solution

- Again, we start with the timeline showing our initial deposit and subsequent withdrawals:



Textbook Example 4.13 (3 of 3)

Solution

- Note that *PV* is negative (money **into** the bank), while *PMT* is positive (money **out** of the bank). We solve for the final balance in the account, *FV* , using the annuity spreadsheet:

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	15	8.00%	-20,000	2,000		
Solve for FV					**9139**	= FV(0.08,15,2000, -20000)

4.7 Non-Annual Cash Flows

- The same time value of money concepts apply if the cash flows occur at intervals other than annually.
- The interest and number of periods must be adjusted to reflect the new time period.

Textbook Example 4.14 (1 of 3)

Evaluating an annuity with monthly cash flows

- **Problem**
  - You are about to purchase a new car and have two options to pay for it. You can pay $20,000 in cash immediately, or you can get a loan that requires you to pay $500 each month for the next 48 months (four years). If the monthly interest rate you earn on your cash is 0.5%, which option should you take?

Textbook Example 4.14 (2 of 3)

Solution

- Let's start by writing down the timeline of the loan payments:

- The timeline shows that the loan is a 48-period annuity. Using the annuity formula the present value is

$$PV(48-\text{period annuity of }\$500) = \$500 \times \frac{1}{0.005}\left(1 - \frac{1}{1.005^{48}}\right)$$
$$= \$21{,}290$$

Textbook Example 4.14 (3 of 3)

- Alternatively, we may use the annuity spreadsheet to solve the problem:

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	48	0.50%		500	0	
Solve for PV			**(21,290)**			= PV(0.005,48,500,0)

- Thus, taking the loan is equivalent to paying $21,290 today, which is costlier than paying cash. You should pay cash for the car.

4.8 Solving for the Cash Payments (1 of 2)

- Sometimes we know the present value or future value, but we do not know one of the variables we have previously been given as an input.

4.8 Solving for the Cash Payments (2 of 2)

- For example, when you take out a loan you may know the amount you would like to borrow but may not know the loan payments that will be required to repay it.

Loan or Annuity Payment

$$C = \frac{P}{\frac{1}{r}\left(1 - \frac{1}{(1+r)^N}\right)}$$

Textbook Example 4.15 (1 of 3)

Computing a Loan Payment

- **Problem**
  - Your biotech firm plans to buy a new DNA sequencer for $500,000. the seller requires that you pay 20% of the purchase price as a down payments, but is willing to finance the remainder by offering a 48-month loan with equal monthly payments and an interest rate of 0.5% per month. What is the monthly loan payment?

Textbook Example 4.15 (2 of 3)

Solution

- Given a down payment of $20\% \times \$500{,}000 = \$100{,}000$, your loan amount is \$400,000. We start with the timeline (from the seller's perspective), where each period represents one month:

- Using Eq. 4.14, we can solve for the loan payment, *C*, as follows:

Textbook Example 4.15 (3 of 3)

$$C = \frac{P}{\frac{1}{r}\left(1 - \frac{1}{(1+r)^N}\right)} = \frac{4000,000}{\frac{1}{0.005}\left(1 - \frac{1}{(1.005)^{48}}\right)}$$
$$= \$9394$$

- Using the annuity spreadsheet:

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	48	0.50%	–400,000		0	
Solve for PMT				**9,394**		=PMT (0.005,48, –400000,0)

- Your firm will need to pay $9,394 each month to repay the loan.

4.9 The Internal Rate of Return

- In some situations, you know the present value and cash flows of an investment opportunity, but you do not know the **internal rate of return (IRR)**, the interest rate that sets the net present value of the cash flows equal to zero.

Textbook Example 4.16 (1 of 2)

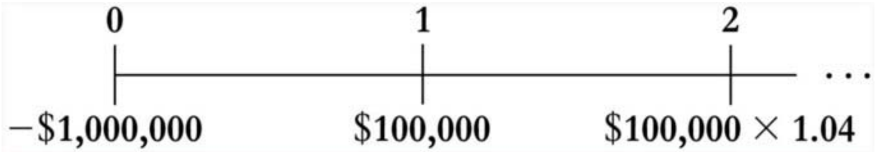
Computing the IRR for a Perpetuity

- **Problem**
  - Jessica has just graduated with her MBA . Rather than take the job she was offered at a prestigious investment bank—Baker, Bellingham, and Botts—she has decided to go into business for herself. She believes that her business will require an initial investment of $1 million. after that, it will generate a cash flow of $100,000 at the end of one year, and this amount will grow by 4% per year thereafter. What is the IRR of this investment opportunity?

Textbook Example 4.16 (2 of 2)

Solution

The timeline is



- The timeline shows that the future cash flows are a growing perpetuity with a growth rate of 4%. Recall from Eq. 4.11 that the PV of a growing perpetuity is $\frac{C}{(r-g)}$. Thus, the NPV of this investment would equal zero if

$$1{,}000{,}000 = \frac{1000{,}000}{r - 0.04}$$

We can solve this equation for r

$$r = \frac{1000{,}000}{1{,}000{,}000} + 0.04 = 0.14$$

So, the IRR on this investment is 14%.

Textbook Example 4.17 (1 of 3)

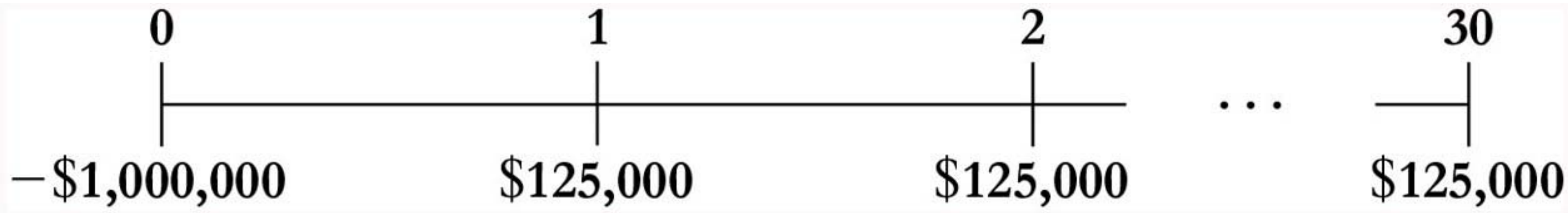
Computing the Internal Rate of return for an Annuity

- **Problem**
  - Baker, Bellingham, and Botts, was so impressed with Jessica that it has decided to fund her business. In return for providing the initial capital of $1 million, Jessica has agreed to pay them $125,000 at the end of each year for the next 30 years. What is the internal rate of return on Baker, Bellingham, and Botts's investment in Jessica's company, assuming she fulfills her commitment?

Textbook Example 4.17 (2 of 3)

Solution

- Here is the timeline (from Baker, Bellingham, and Botts's perspective):



- The timeline shows that the future cash flows are a 30-year annuity. Setting the NPV equal to zero requires

$$1{,}000{,}000 = 125, \times \frac{1}{r}\left(1 - \frac{1}{(1+r)^{30}}\right)$$

Textbook Example 4.17 (3 of 3)

- Using the annuity spreadsheet to solve for *r*,

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	30		−1,000,000	125,000	0	
Solve for Rate		**12.09%**				= RATE(30,125000, − 1000000,0)

- The IRR on this investment is 12.09%. In this case, we can interpret the IRR of 12.09% as the effective interest rate of the loan.