Corporate Finance: The Core

Fifth Edition, Global Edition

Corporate Finance
The Core

5th Edition

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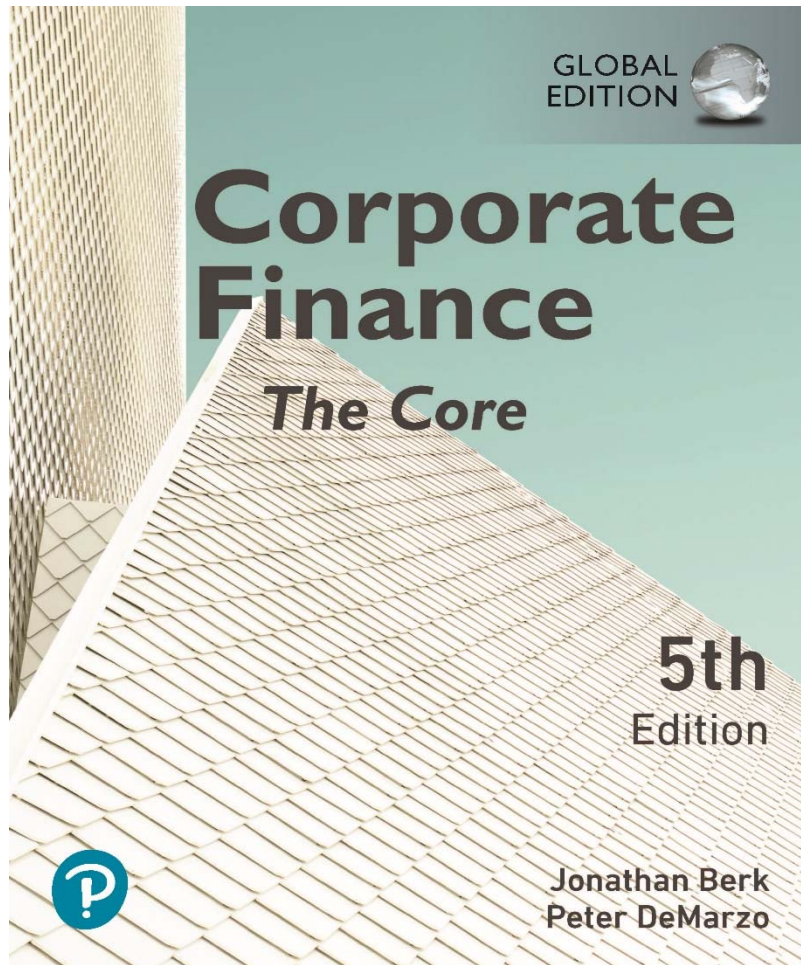
Chapter 3

Financial Decision Making and the Law of One Price

Chapter Outline

3.1 Valuing Decisions

3.2 Interest Rates and the Time Value of Money

3.3 Present Value and the NPV Decision Rule

3.4 Arbitrage and the Law of One Price

3.5 No-Arbitrage and Security Prices

Learning Objectives (1 of 3)

- Assess the relative merits of two-period projects using net present value.

- Define the term "competitive market," give examples of markets that are competitive and some that are not, and discuss the importance of a competitive market in determining the value of a good.

Learning Objectives (2 of 3)

- Explain why maximizing NPV is always the correct decision rule.

- Define arbitrage, and discuss its role in asset pricing. How does it relate to the Law of One Price?

- Calculate the no-arbitrage price of an investment opportunity.

Learning Objectives (3 of 3)

- Show how value additivity can be used to help managers maximize the value of the firm.

- Describe the Separation Principle.

3.1 Valuing Decisions

- Identify Costs and Benefits
  - May need help from other areas in identifying the relevant costs and benefits
    - Marketing
    - Economics
    - Organizational Behavior
    - Strategy
    - Operations

Analyzing Costs and Benefits

- Suppose a jewelry manufacturer has the opportunity to trade 10 ounces of gold and receive 20 ounces of palladium today.

- To compare the costs and benefits, we first need to convert them to a common unit.

Analyzing Costs and Benefits

- Similarly, if the current market price for palladium is $600 per ounce, then the 20 ounces of palladium we receive has a cash value of

$$(20 \text{ ounces of palladium}) \times \left(\frac{\$600}{\text{ounce}} \right) = \$12,000$$

Analyzing Costs and Benefits

- Suppose gold can be bought and sold for a current market price of $1,300 per ounce

- Then the 10 ounces of gold we give up has a cash value of

$$(10 \text{ ounces of gold}) \times \left(\frac{\$1,300}{\text{ounce}} \right) = \$13,000 \text{ today}$$

Analyzing Costs and Benefits

- Therefore, the jeweler's opportunity has a benefit of $12,000 today and a cost of $13,000 today.

- In this case, the net value of the project today is

$$\$12,000 - \$13,000 = -\$1,000$$

- Because it is negative, the costs exceed the benefits, and the jeweler should reject the trade.

Using Market Prices to Determine Cash Values

- Competitive Market
  - A market in which goods can be bought and sold at the same price
  - In a competitive market, the value of the good will not depend on the views or preferences of the decision maker

- In evaluating the jeweler's decision, we used the current market price to convert from ounces of platinum or gold to dollars.

Competitive Market Prices Determine Value

Problem

You have just won a radio contest and are disappointed to find out that the prize is four tickets to the Def Leppard reunion tour (face value $40 each). Not being a fan of 1980s power rock, you have no intention of going to the show. However, there is a second choice: two tickets to your favorite band's sold-out show (face value $45 each). You notice that on eBay, tickets to the Def Leppard show are being bought and sold for $30 apiece and tickets to your favorite band's show are being bought and sold at $50 each. Which prize should you choose?

Solution

Competitive market prices, not your personal preferences (nor the face value of the tickets), are relevant here:

> Four Def Leppard tickets at $30 apiece = $120 market value

> Two of your favorite band's tickets at $50 apiece = $100 market value

Instead of taking the tickets to your favorite band, you should accept the Def Leppard tickets, sell them on eBay, and use the proceeds to buy two tickets to your favorite band's show. You'll even have $20 left over to buy a T-shirt.

Textbook Example 3.2

Applying the Valuation Principle

Problem

You are the operations manager at your firm. Due to a pre-existing contract, you have the opportunity to acquire 200 barrels of oil and 3,000 pounds of copper for a total of $12,000. The current competitive market price of oil is $50 per barrel and for copper is $2 per pound. You are not sure you need all of the oil and copper, and are concerned that the value of both commodities may fall in the future. Should you take this opportunity?

Textbook Example 3.2

Solution

To answer this question, you need to convert the costs and benefits to their cash values using market prices:

$$(200 \text{ barrels of oil}) \times \left(\frac{\$50}{\text{barrel of oil today}} \right) = \$10,000 \text{ today}$$

$$(3000 \text{ pounds of copper}) \times \left(\frac{\$2}{\text{pound of copper today}} \right) = \$6000 \text{ today}$$

The net value of the opportunity is $\$10,000 + \$6,000 - \$12,000 - \$4,000$

today. Because the net value is positive, you should take it. This value depends only on the **current** market prices for oil and copper. Even if you do not need all the oil or copper, or expect their values to fall, you can sell them at current market prices and obtain their values of $16,000. Thus the opportunity is a good one for the firm, and will increase its value by $4,000.

3.2 Interest Rates and the Time Value of Money

- Time Value of Money
  - Consider an investment opportunity with the following certain cash flows
    - Cost: $100,000 today
    - Benefit: $105,000 in one year
  - The difference in value between money today and money in the future is due to the time value of money

The Interest Rate: An Exchange Rate Across Time

- The rate at which we can exchange money today for money in the future is determined by the current **interest rate**
  - Suppose the current annual interest rate is 7%. By investing or borrowing at this rate, we can exchange $1.07 in one year for each $1 today.
    - Risk-Free Interest Rate (Discount Rate), r_f :

      The interest rate at which money can be borrowed or lent without risk
      - Interest Rate Factor $= 1 + r_f$
      - Discount Factor $= \dfrac{1}{\left(1 + r_f\right)}$

- Value of Investment in One Year
  - If the interest rate is 7%, then we can express our costs as:

$$\text{Cost} = (\$100,000 \text{ today}) \times \left(\frac{1.07 \ \$ \text{ in one year}}{\$ \text{ today}}\right)$$

$$= \$107,000 \text{ in one year}$$

The Interest Rate: An Exchange Rate Across Time

- Value of Investment in One Year
  - Both costs and benefits are now in terms of "dollars in one year," so we can compare them and compute the investment's net value:

    $105,000 - $107,000 = -$2000 in one year

  - In other words, we could earn $2,000 more in one year by putting our $100,000 in the bank rather than making this investment.
  - We should reject the investment.

The Interest Rate: An Exchange Rate Across Time

- Value of Investment Today
  - Consider the benefit of $105,000 in one year. What is the equivalent amount in terms of dollars today?

$$\text{Benefit} = (\$105{,}000 \text{ in one year}) \div \left(\frac{1.07 \ \$ \text{ in one year}}{\$ \text{ today}} \right)$$

$$= (\$105{,}000 \text{ in one year}) \times \frac{1}{1.07} = \$98{,}130.84 \text{ today}$$

  - This is the amount the bank would lend to us today if we promised to repay $105,000 in one year.

The Interest Rate: An Exchange Rate Across Time

- Value of Investment Today
  - Now we are ready to compute the net value of the investment:

    $$\$98,130.84 - \$100,000 = -\$1869.16 \text{ today}$$

  - Once again, the negative result indicates that we should reject the investment.

The Interest Rate: An Exchange Rate Across Time

- Present Versus Future Value
  - This demonstrates that our decision is the same whether we express the value of the investment in terms of dollars in one year or dollars today.
  - If we convert from dollars today to dollars in one year,

    $$(-\$1869.16 \text{ today}) \times \left(\frac{1.07 \text{ \$ in one year}}{\text{\$ today}} \right) = -\$2000 \text{ in one year.}$$

  - The two results are equivalent but expressed as values at different points in time.

The Interest Rate: An Exchange Rate Across Time

- Present Versus Future Value
  - When we express the value in terms of dollars today, we call it the **present value** (PV) of the investment.
  - If we express it in terms of dollars in the future, we call it the **future value** of the investment.

- Discount Factors and Rate
    - We can interpret

$$\frac{1}{1+r} = \frac{1}{1.07} = 0.93458$$

as the price today of $1 in one year. The amount is called the one-year **discount factor.**

The risk-free rate is also referred to as the **discount rate** for a risk-free investment.

Comparing Costs at Different Points in Time

Problem

The cost of rebuilding the San Francisco Bay Bridge to make it earthquake-safe was approximately $3 billion in 2004. At the time, engineers estimated that if the project were delayed to 2005, the cost would rise by 10%. If the interest rate were 2%, what would be the cost of a delay in terms of dollars in 2004?

Textbook Example 3.3

Solution

- If the project were delayed, it would cost

  $3 billion $\times 1.10 \ = $ $3.3 billion in 2005.

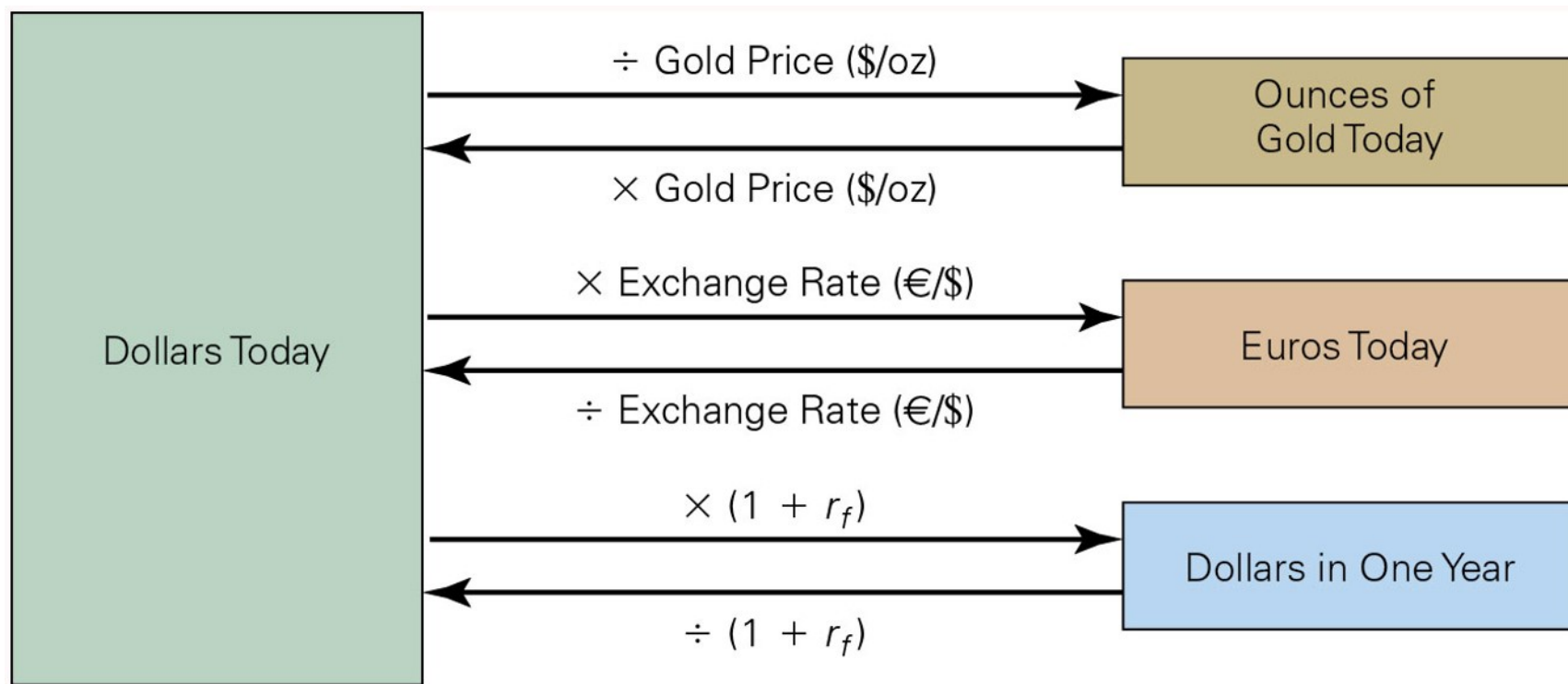 To compare this amount to the cost of $3 billion in 2004, we must convert it using the interest rate of 2%:

  $3.3 billion in 2005 $\div \left(\dfrac{\$1.02 \text{ in } 2005}{\$ \text{ in } 2004} \right) = $ $3.235 billion in 2004

  Therefore, the cost of a delay of one year was

  $3.235 billion $-$ $3 billion $=$ $235 million in 2004

That is, delaying the project for one year was equivalent to giving up $235 million in cash.

Figure 3.1 Converting Between Dollars Today and Gold, Euros, or Dollars in the Future

Dollars Today

$\div$ Gold Price (\$/oz)

$\times$ Gold Price (\$/oz)

Ounces of Gold Today

$\times$ Exchange Rate (€/\$)

$\div$ Exchange Rate (€/\$)

Euros Today

$\times (1 + r_f)$

$\div (1 + r_f)$

Dollars in One Year

3.3 Present Value and the NPV Decision Rule

- The **Net Present Value (NPV)** of a project or investment is the difference between the present value of its benefits and the present value of its costs.

  - Net Present Value

$$NPV = PV(\text{Benefits}) - PV(\text{Costs})$$

$$NPV = PV(\text{All project cash flows})$$

The NPV Decision Rule (1 of 3)

- When making an investment decision, take the alternative with the highest NPV.

- Choosing this alternative is equivalent to receiving its NPV in cash today.

The NPV Decision Rule (2 of 3)

- Accepting or Rejecting a Project
  - Accept those projects with positive NPV because accepting them is equivalent to receiving their NPV in cash today.
  - Reject those projects with negative NPV because accepting them would reduce the wealth of investors.

The NPV Is Equivalent to Cash Today

Problem

Your firm needs to buy a new $9,500 copier. As part of a promotion, the manufacturer has offered to let you pay $10,000 in one year, rather than pay cash today. Suppose the risk-free interest rate is 7% per year. Is this offer a good deal? Show that its NPV represents cash in your pocket.

Textbook Example 3.4

Solution

If you take the offer, the benefit is that you won't have to pay $9,500 today, which is already in PV terms. The cost, however, is $10,000 in one year. We therefore convert the cost to a present value at the risk-free interest rate:

$$\text{PV(Cost)}=(\$10{,}000 \text{ in one year})?\left(\frac{1.07 \text{ \$ in one year}}{\text{\$ today}}\right) \quad \$9345.79 \text{ today}$$

The NPV of the promotional offer is the difference between the benefits and the costs:

$$\text{NPV}=\$9500-\$9345.79=\$154.21 \text{ today}$$

The NPV is positive, so the investment is a good deal. It is equivalent to getting a cash discount today of $154.21, and only paying $9,345.79 today for the copier. To confirm our calculation, suppose you take the offer and invest $9,345.79 in a bank paying 7% interest. With interest, this amount will grow to

$$\$9{,}345.79 \times 1.07 = \$10{,}000 \text{ in one year, which you can use to pay for the copier.}$$

The NPV Decision Rule

- Choosing Among Alternatives
  - We can also use the NPV decision rule to choose among projects.
  - To do so, we must compute the NPV of each alternative, and then select the one with the highest NPV.
  - This alternative is the one which will lead to the largest increase in the value of the firm.

Choosing Among Alternative Plans

Problem

Suppose you started a Web site hosting business and then decided to return to school. Now that you are back in school, you are considering selling the business within the next year. An investor has offered to buy the business for $200,000 whenever you are ready. If the interest rate is 10%, which of the following three alternatives is the best choice?

1. Sell the business now.

2. Scale back the business and continue running it while you are in school for one more year, and then sell the business (requiring you to spend $30,000 on expenses now, but generating $50,000 in profit at the end of the year).

3. Hire someone to manage the business while you are in school for one more year, and then sell the business (requiring you to spend $50,000 on expenses now, but generating $100,000 in profit at the end of the year).

Solution

The cash flows and NPVs for each alternative are calculated in Table 3.1 faced with these three alternatives, the best choice is the one with highest NPV: Hire a manager and sell in one year. Choosing this alternative is equivalent to receiving $222,727 today.

Table 3.1 Cash Flows and NPVs for Web Site Business Alternatives

	Today	In One Year	NPV
Sell Now	$200,000	0	$200,000
Scale Back Operations	−$30,000	$50,000 $200,000	$-30{,}000 + \dfrac{\$250{,}000}{1.10} = \$197{,}273$
Hire a Manager	−$50,000	$100,000 $200,000	$-\$50{,}000 + \dfrac{\$300{,}000}{1.10} = \$222{,}727$

NPV and Cash Needs

- Regardless of our preferences for cash today versus cash in the future, we should always maximize NPV first.

- We can then borrow or lend to shift cash flows through time and find our most preferred pattern of cash flows.

3.4 Arbitrage and the Law of One Price

- Arbitrage
  - The practice of buying and selling equivalent goods in different markets to take advantage of a price difference
  - An **arbitrage opportunity** occurs when it is possible to make a profit without taking any risk or making any investment.

- Normal Market
  - A competitive market in which there are no arbitrage opportunities

Law of One Price

- If equivalent investment opportunities trade simultaneously in different competitive markets, then they must trade for the same price in both markets.

3.5 No-Arbitrage and Security Prices

- Valuing a Security with the Law of One Price
    - Assume a security promises a risk-free payment of $1,000 in one year. If the risk-free interest rate is 5%, what can we conclude about the price of this bond in a normal market?

$$\text{PV}(\$1000 \text{ in one year}) = (\$1000 \text{ in one year}) \div \left(\frac{1.05 \text{ \$ in one year}}{\$ \text{ today}} \right)$$

$$= \$952.38 \text{ today}$$

$$\text{Price (Bond)} = \$952.38$$

- Valuing a Security with the Law of One Price
  - Identifying Arbitrage Opportunities with Securities
    - What if the price of the bond is not $952.38?
      - Assume the price is $940

	Today($)	In One Year($)
Buy the bond	−940.00	+1,000.00
Borrow from the bank	+952.38	−1,000.00
Net cash flow	+12.38	0.00

  - The opportunity for arbitrage will force the price of the bond to rise until it is equal to $952.38.

3.5 No-Arbitrage and Security Prices (3 of 7)

- Valuing a Security with the Law of One Price
  - Identifying Arbitrage Opportunities with Securities
    - What if the price of the bond is not $952.38?
      - Assume the price is $960

	Today($)	In One Year($)
Sell the bond	+960.00	−1,000.00
Invest at the bank	−952.38	+1,000.00
Net cash flow	+7.62	0.00

  - The opportunity for arbitrage will force the price of the bond to fall until it is equal to $952.38.

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- Valuing a Security with the Law of One Price
  - Determining the No-Arbitrage Price
    - Unless the price of the security equals the present value of the security's cash flows, an arbitrage opportunity will appear.
    - No Arbitrage Price of a Security

Price (Security) = PV (All cash flows paid by the security)

Computing the No-Arbitrage Price

Problem

Consider a security that pays its owner $100 today and $100 in one year, without any risk. Suppose the risk-free interest rate is 10%. What is the no-arbitrage price of the security today (before the first $100 is paid)? If the security is trading for $195, what arbitrage opportunity is available?

Textbook Example 3.6

Solution

We need to compute the present value of the security's cash flows. In this case there are two cash flows: $100 today, which is already in present value terms, and $100 in one year. The present value of the second cash flow is

$$\$100 \text{ in one year} \div \left(\frac{1.10 \ \$ \text{ in one year}}{\$ \text{ year}} \right) = \$90.91 \text{ today}$$

Therefore, the total present value of the cash flows is $\$100 + \$90.91 = \$190.91$ today, which is the no-arbitrage price of the security.

If the security is trading for $195, we can exploit its overpricing by selling it for $195. we can then use $100 of the sale proceeds to replace the $100 we would have received from the security today and invest $90.91 of the sale proceeds at 10% to replace the $ 100 we would have received in one year. The remaining $\$195 - \$100 - \$90.91 = \$4.09$ in an arbitrage profit.

- Valuing a Security with the Law of One Price

  – Determining the Interest Rate from Bond Prices

    ▪ If we know the price of a risk-free bond, we can use

      Price (Security) = PV(All cash flows paid by the security)

      to determine what the risk-free interest rate must be if there are no arbitrage opportunities.

- Valuing a Security with the Law of One Price
  - Determining the Interest Rate from Bond Prices
    - Suppose a risk-free bond that pays $1,000 in one year is currently trading with a competitive market price of $929.80 today
    - The bond's price must equal the present value of the $1,000 cash flow it will pay.

- Valuing a Security with the Law of One Price
  - Determining the Interest Rate from Bond Prices

$$\$929.80 \text{ today} = (\$1000 \text{ in one year}) \div (1 + r_f \ \$ \text{ in one year}/\$ \text{ today})$$

$$1 + r_f = \frac{\$1000 \text{ in one year}}{\$929.80 \text{ today}} = \frac{1.0755 \ \$ \text{ in one year}}{\$ \text{ today}}$$

- The risk-free interest rate must be 7.55%.

The NPV of Trading Securities and Firm Decision Making

- In a normal market, the NPV of buying or selling a security is zero

$$\text{NPV (Buy security)} = \text{PV (All cash flows paid by the security)} - \text{Price (Security)}$$
$$= 0$$

$$\text{NPV (Sell security)} = \text{Price (Security)} - \text{PV (All cash flows paid by the security)}$$
$$= 0$$

The NPV of Trading Securities and Firm Decision Making

- Separation Principle
  - Security transaction in a normal market neither create nor destroy value on their own.
  - We can evaluate the NPV of an investment decision separately from the decision the firm makes regarding how to finance the investment or any other security transactions the firm is considering.

Separating Investment and Financing

Problem

Your firm is considering a project that will require an upfront investment of $10 million today and will produce $12 million in cash flow for the firm in one year without risk.

Rather than pay for the $10 million investment entirely using its own cash, the firm is considering raising additional funds by issuing a security that will pay investors $5.5 million in one year. Suppose the risk-free interest rate is 10%.

Is pursuing this project a good decision without issuing the new security? Is it a good decision with the new security?

Solution

Without the new security, the cost of the project is $10 million today and the benefit is $12 million in one year. Converting the benefit to a present value

$12 million in one year ÷ (1.10 $ in one year/$ today) = $10.91 million today

we see that the project has an NPV of

$10.91 million − $10 million = $0.91 million today.

Now suppose the firm issues the new security. In a normal market, the price of this security will be the present value of its future cash flow:

Price (Security)= $5.5 million÷1.10 = $5 million today

Textbook Example 3.7

Solution

Thus, after it raises $5 million by issuing the new security, the firm will only need to invest an additional $5 million to take the project.

To compute the project's NPV in this case, note that in one year the firm will receive the $12 million payout of the project, but owe $5.5 million to the investors in the new security, leaving $6.5 million for the firm. This amount has a present value of

$$\$6.5 \text{ million in one year} \div (1.10 \text{ $ in one year/$ today}) = \$5.91 \text{ million today}$$

Thus, the project has an NPV of $\$5.91 \text{ million} - \$5 \text{ million} = \$0.91 \text{million}$ today, as before.

In either case, we get the same result for the NPV. The separation principle indicates that we will get the same result for any choice of financing for the firm that occurs <u>in a normal market</u>. We can therefore evaluate the project without explicitly considering the <u>different financing possibilities</u> the firm might choose.

Valuing a Portfolio (1 of 2)

- The Law of One Price also has implications for packages of securities.

  – Consider two securities, A and B. Suppose a third security, C, has the same cash flows as A and B combined.

  – In this case, security C is equivalent to a portfolio, or combination, of the securities A and B.

- Value Additivity

$$\text{Price (C)} = \text{Price (A + B)} = \text{Price (A)} + \text{Price (B)}$$

Textbook Example 3.8

Valuing an Asset in a Portfolio

Problem

Holbrook Holdings is a publicly traded company with only two assets: It owns 60% of Harry's Hotcakes restaurant chain and an ice hockey team. Suppose the market value of Holbrook Holdings is $160 million, and the market value of the entire Harry's Hotcakes chain (which is also publicly traded) is $120 million. What is the market value of the hockey team?

Solution

We can think of Holbrook as a portfolio consisting of a 60% stake in Harry's Hotcakes and the hockey team. By value additivity, the sum of the value of the stake in Harry's Hotcakes and the hockey team must equal the $160 million market value of Holbrook. Because the 60% stake in Harry's

Hotcakes is worth 60%? 120 million = $72 million,

the hockey team has a value of

$160 million $-$ $72 million $=$ $88 million.

Valuing a Portfolio (2 of 2)

- Value Additivity and Firm Value
  - To maximize the value of the entire firm, managers should make decisions that maximize NPV.
  - The NPV of the decision represents its contribution to the overall value of the firm.

Where Do We Go from Here?

- Impact of Risk on Valuation
  - When cash flows are risky, we must discount them at a rate equal to the risk-free interest rate plus an appropriate <u>risk premium</u>.
  - The appropriate risk premium will be higher the more the project's returns tend to vary with overall risk in the economy.