2025 Asia and Pacific Mathematical Contest in Modeling

Problem A

Optimization of Robot's Performance

Hangzhou Yushu Technology is a world-renowned civilian robot company, focusing on the independent research and development, production, and sales of consumer grade, industry grade high-performance universal foot/humanoid robots and dexterous robotic arms.

Among them, the safety of the robot motor is the core of operation, and the stability of its power output directly affects the safety of the entire machine. If the motor brake fails, the speed suddenly changes, or the load matching is unbalanced, it is easy to cause the risk of losing control; However, robot falls are often related to motor abnormalities and unreasonable center of gravity design, such as insufficient motor power leading to obstacle crossing failure, excessive start stop impact disrupting posture balance, or excessively high center of gravity or center of gravity deviation beyond the support range, all of which can cause the robot to lose stability; To avoid falling, the key is to optimize the motor control algorithm to achieve smooth power output, while reasonably allocating the weight of the aircraft and reducing the height of the center of gravity, ensuring that the center of gravity always falls within the support surface, and building a dual guarantee from both motor safety and center of gravity stability.

The humanoid robot Unitree G1 of Yushu is divided into an upper body and a lower body, with multiple degrees of freedom. The schematic diagram of the front and back of Yushu's humanoid robot is shown in Figure 1.

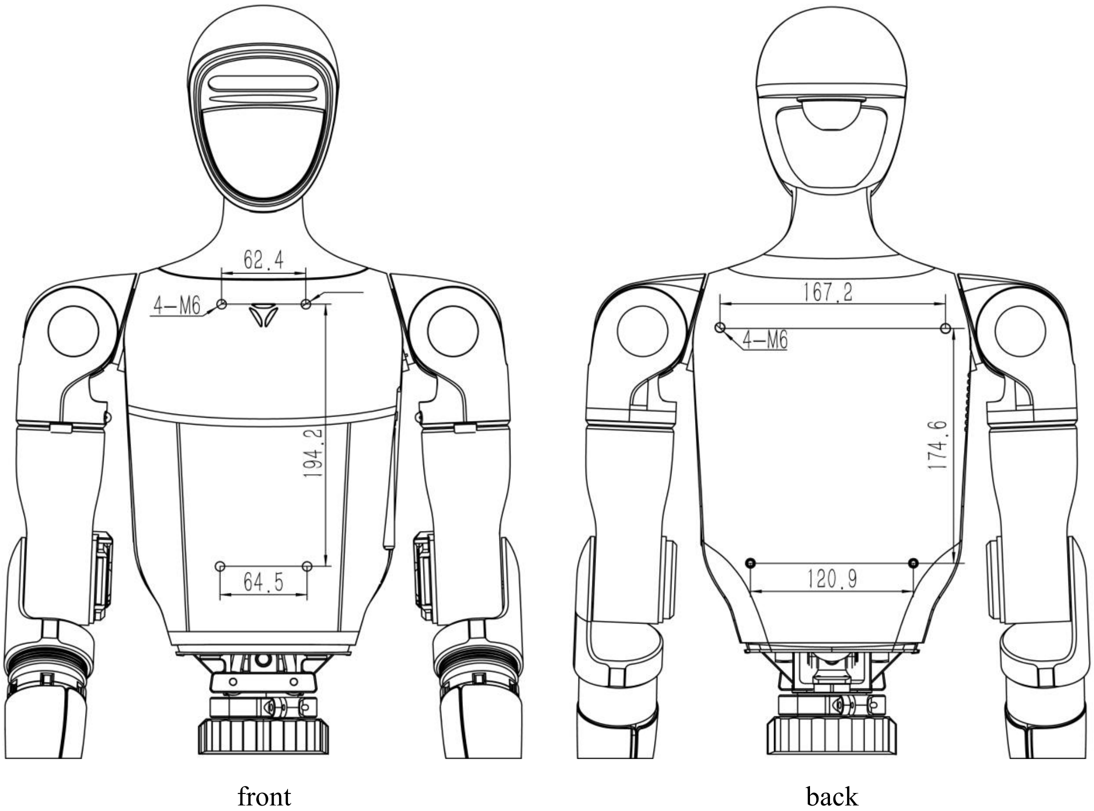


Figure 1. Schematic diagram of the front and back of Yushu's humanoid robot.

The G1 is equipped with a D435i depth camera on its head, providing the robot with excellent visual perception capabilities. It can more accurately perceive and understand the surrounding environment, achieve precise spatial perception and obstacle detection, and enable the robot to interact with the environment and respond to various scenarios more intelligently and flexibly, as shown in Figure 2.

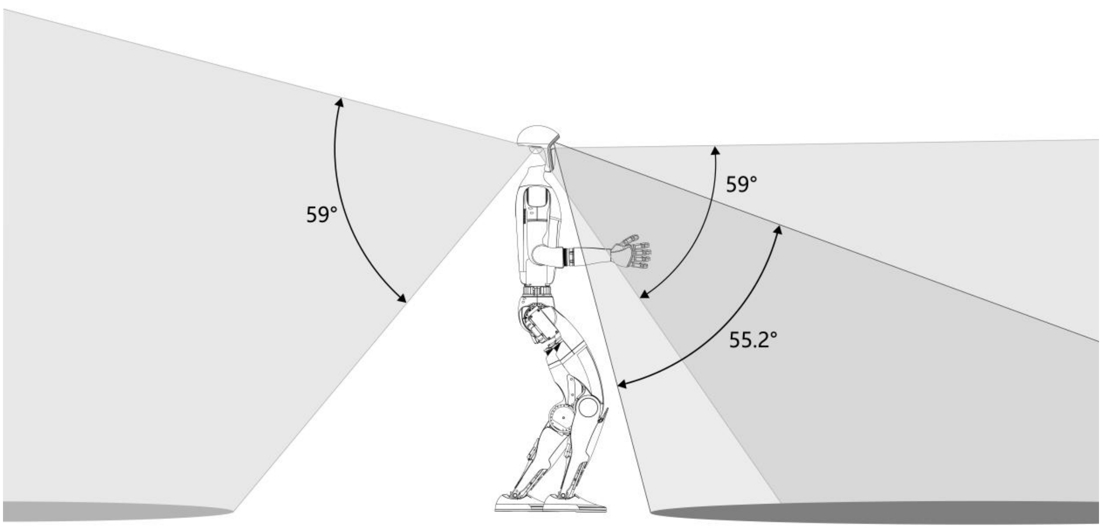


Figure 2. Schematic diagram of the maximum range and joint distribution of different visual

sensors in Yushu's humanoid robot.

When all joints are at zero degrees, the coordinate systems are shown in Figure 3.

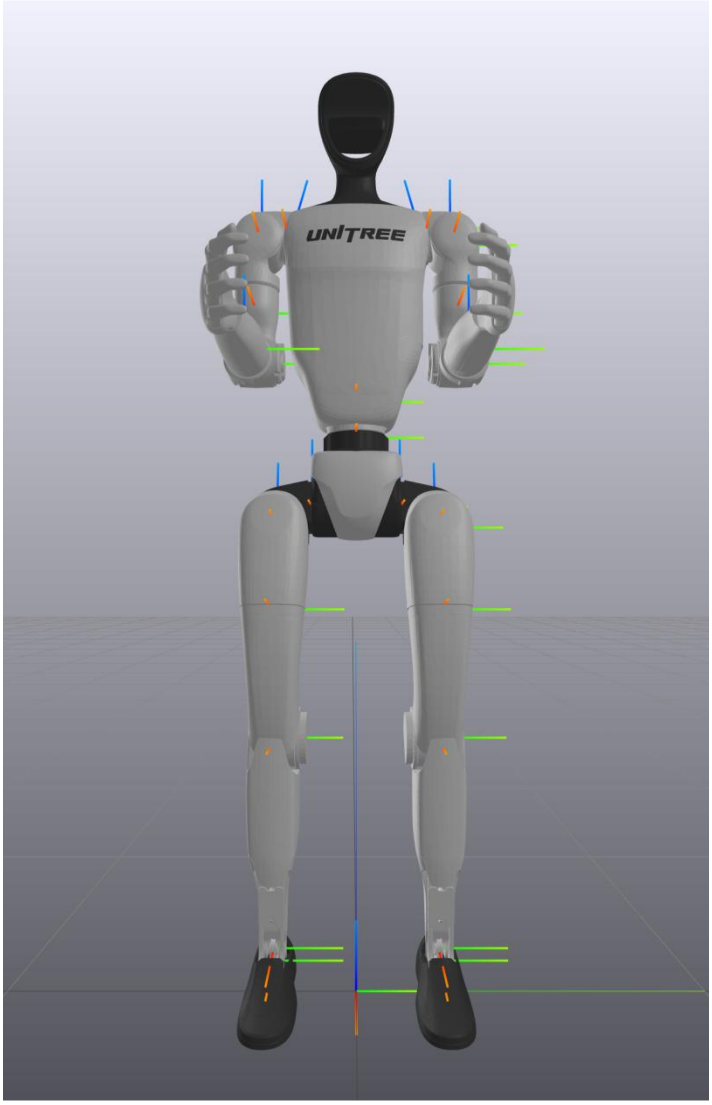


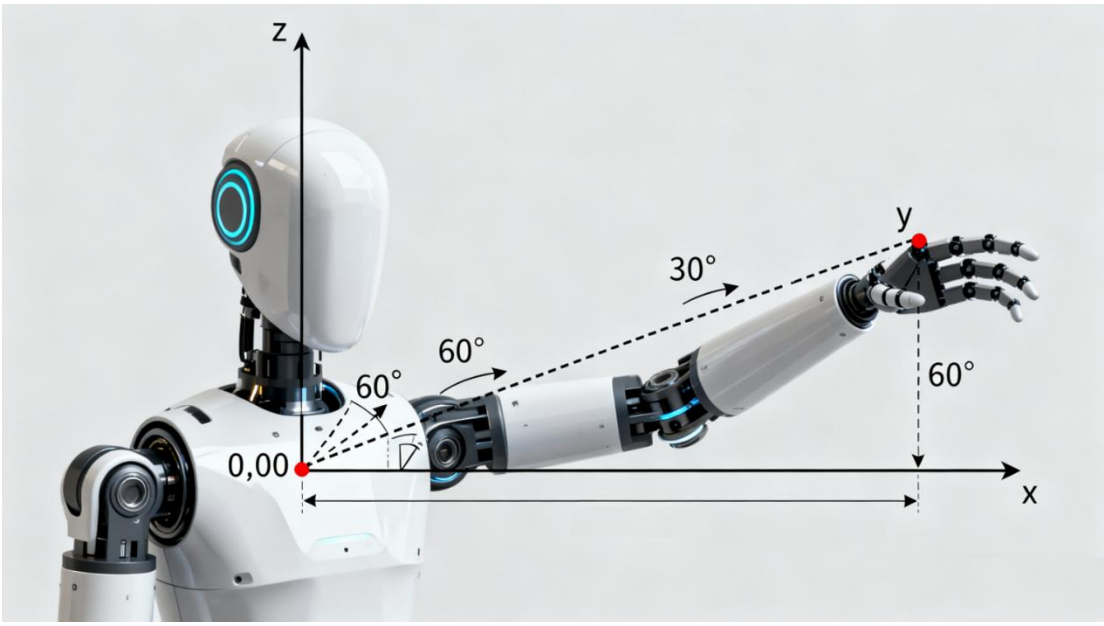
Figure 3. Schematic diagram of the coordinate system, joint rotation axis, and joint zero point of Yushu's humanoid robot.

It is known that at the opening ceremony of an important technology exhibition, the Uyghur humanoid robot Unitree G1 will perform a wonderful dance performance. To ensure the smoothness and enjoyment of the performance, precise planning of the robot's movements is necessary. The initial position of the robot is known to be at the center of the stage, and the initial posture is that all joints are at zero degrees. The performance venue is a rectangular area that is 20 meters long and 15 meters wide. Please have your team establish a corresponding mathematical model based on the above information to solve the following problems:

Problem 1. Calculation of Angles and Positions of Robot's Joints

At the beginning of the performance, the Yushu humanoid robot needs to complete a simple action: extend and lift its left hand forward from its initial position, forming a 60° angle with its body, while rotating its arm 30° to the left of its body. Assuming an arm length of 338 mm and ignoring hand dimensions, a Cartesian coordinate system is established with the robot shoulder as the coordinate origin (0,0,0), with the x-axis pointing directly in front of the stage, the y-axis pointing horizontally to the left, and the z-axis perpendicular to the stage facing upwards, as shown in Figure 4.

Please calculate the final coordinate position of the robot's left endpoint in this coordinate system. When completing this action, it is necessary to ensure that the motor operates within a safe range and verify whether the action meets the safety requirements of the motor.



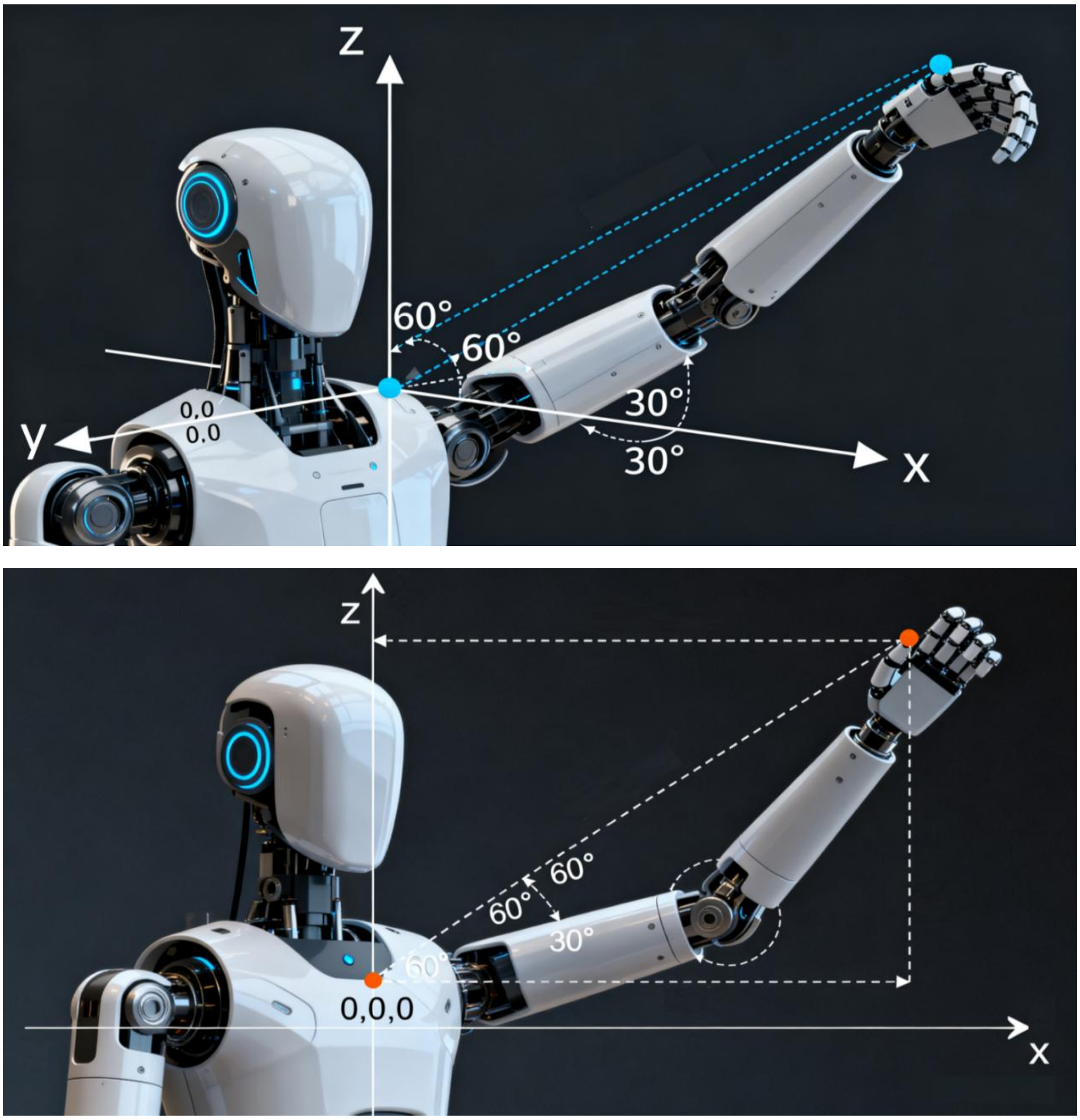


Figure 4. Schematic diagram of angle and position calculation of robot joints.

Problem 2. Modeling and Time Planning of Robot's Motion Trajectory

The Yushu humanoid robot will move along a straight line from the center of the stage (0,0, 0) to point (10, 0, 0) (unit: meter) for non-uniform motion, with an average speed of 2 m/s for straight walking. Assuming the robot starts moving from its initial position, starts at time $t = 0$, and reaches the target point at $t = T$, as shown in Figure 5.

Please establish a mathematical model between the rotation angles of each joint of the single leg of the Yushu humanoid robot and time t ($0 \leq t \leq T$), and calculate the time with maximum change in the rotation angle of the robot's knee joint during the movement from the starting point to the target point. Additionally, determine the time

T required to complete all actions.

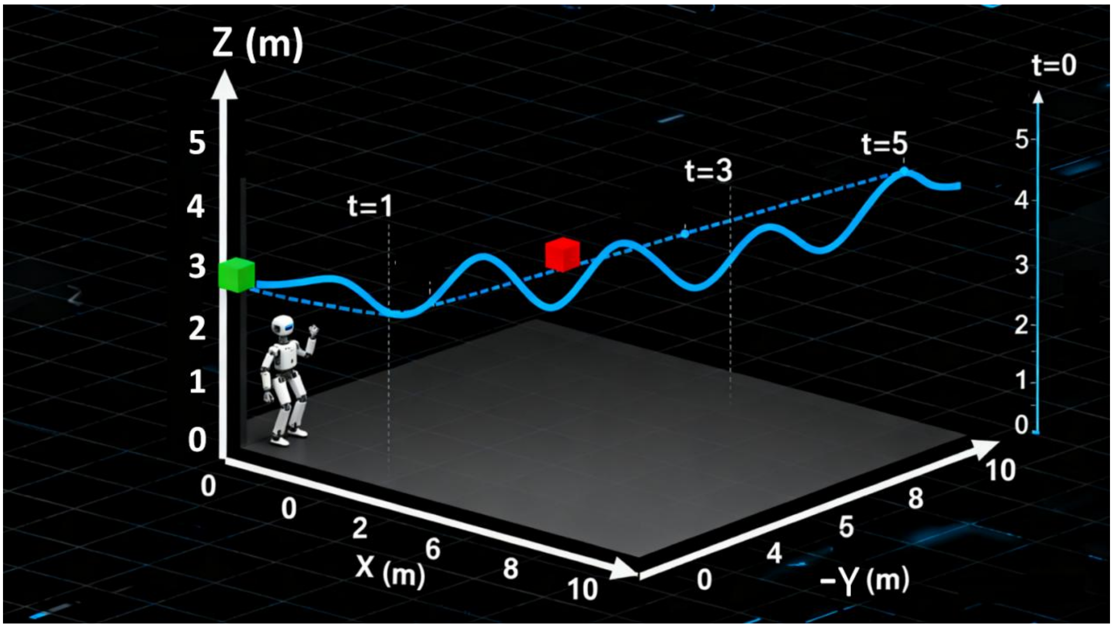


Figure 5. Schematic diagram of robot motion trajectory modeling and time planning.

Problem 3. Multi Joint Collaborative Motion Planning for Robots

At the climax of the dance performance, the Yushu humanoid robot needs to perform a complex combination of movements: the body first rotates 45° to the left, while the arms move in circles around the shoulders, with a circle radius of 300 mm and a circle drawing period of 4 seconds, and the arms move in opposite directions. In this process, to ensure the balance of the robot, corresponding adjustments need to be made to the legs, as shown in Figure 6.

Please establish a mathematical model for the angles of each joint of the arms and each joint of the legs of the Yushu humanoid robot over time t ($0 \le t \le 4$).

Figure 6. Schematic diagram of multi joint collaborative motion planning for robots.

Problem 4: Optimization of Robot's Performance Actions and Energy Consumption

Throughout the performance, the energy consumption of the Yushu humanoid robot was an important factor. It is known that the battery capacity of the robot is 15 Ah and the maximum voltage is 67.2 V. The power consumption of each joint motor of the robot during motion is related to the torque and speed of the motor, and relevant references can be consulted.

Please calculate the total energy consumed by the Yushu humanoid robot during the process of completing all complex actions from above problem one to problem three. Then, based on the calculation results, a model for the optimized action plan should be proposed to reduce energy consumption without affecting the performance, and the optimized energy consumption is required to be recalculated again.

References:

[1] https://www.unitree.com/cn/g1

[2] https://support.unitree.com/home/zh/G1_developer/about_G1

[3] Liang Liang, Wu Chengdong, Liu Shichang Absolute Position Accuracy Calibration Algorithm for Robots Based on Joint Geometric Errors [J]. Journal of Northeastern University (Natural Science Edition), 2025, 46 (4): 1-7

[4] Zhang Xiuli, Zhao Haoyu, Wu Jianing, etc Trajectory optimization and control method for quadruped robot jumping in the air [J]. Journal of Beijing Jiaotong University, 2024, 48 (3): 161-170

[5] Zhang Xinhao Research on Motion Control Method of Four legged Robot Based on Trajectory Optimization [D]. University of Electronic Science and Technology of China two thousand and twenty-three

[6] Zhang Xiuli, Sun Guokang, Zhou Hongmiao, Liu Ying, Li Wei A series parallel hybrid bionic robotic arm with flexible driving joints [J]. Journal of Beijing Jiaotong University, 2024, 48 (6): 154-161

[7] Zhu Haohui Optimization control method for motion trajectory accuracy of quadruped robot based on joint angle compensation [J]. Modeling and Simulation, 2024, 13 (3): 2305-2314

[8] Liu Qingyun, You Xiong, Zhang Xin, Zuo Jiwei, Li Jia Overview of Path Planning Algorithms for Mobile Robots [J]. Computer Science, 2025, 52 (6A): 240900074-10

[9] Xie Cunxi, Zheng Shixiong, Qu Hanhua Calculation of Position and Attitude of Industrial Robots and Determination of Joint Motion Parameters [J]. Mechanical Industry Automation, 1980 (4): 50-56

[10] Fu Haitao, Zhong Senming, Luo Zhida Development and Application of Joint Angle Compensation Algorithm for Industrial Robots [J]. Metrology and Testing

Technology, 2021 (010): 048

[11] Wu Wenxiang Friction analysis and low-speed high-precision motion control of multi degree of freedom serial robot joints [D]. Zhejiang University, 2013

[12] Wu Yongqiang, Tang Xianzhi, Song Wei, etc Power Equivalent Model and Parameter Identification of Industrial Robots [J]. Journal of Chongqing University, 2021 (044-010)

[13] Bai Liping Robot dynamics analysis and motor selection calculation [J]. Knowledge output of Shenyang Institute of Automation: 2000 years ago, 2000

[14] Yuan Bing Research on Jump Control Method of Four legged Robot Based on Deep Reinforcement Learning [D]. Wuhan University of Science and Technology [25-11-10]

[15] Zhang Bin Multi constraint based robot joint space trajectory planning [J]. Journal of Mechanical Engineering, 2011, 47 (21): 6

[16] Wang Mei, Wu Tiejun Research on Multi Robot Collaborative Motion Planning and Related Issues [J]. Manufacturing Automation, 2005, 27 (5): 6

[17] Zeng Qiang, Fang Yuefa Integrated coordinated motion planning for bipedal walking robots [J]. Mechanical, 2007, 34 (2): 5

[18] Chen Anjun, Liu Daohua Joint trajectory planning for coordinated motion of dual arm robots [J]. Journal of Xinyang Normal University: Natural Science Edition, 2001, 14 (1): 4

[19] Wang Mei, Wu Tiejun, Tu Dawei, etc Distributed Collaborative Motion Planning for Multiple Joint Robots [J]. Information and Control, 2011.0.2011-04-011

[20] Wang Ruimao, Yu Yueqing Terminal positioning planning for coordinated operation of flexible robots [J]. Mechanical Design and Research, 2004 (z1): 3

[21] Zhang Man, Gao Yubo, Deng Jiayu, etc Research on Linear Walking Gait Planning and Joint Trajectory of bipedal Walking Robot [J]. Forestry Machinery and

Woodworking Equipment, 2017 (006): 045

[22] Jiang Yujie, Li Jingchun, Zhang Guozhong Motion Planning of Joint type Redundant Robots [J]. Combination Machine Tool and Automation Processing Technology, 2005 (3): 2

[23] Qiu Binquan, Chen Silu, Gu Yingkui, etc Synchronization planning of robot joint trapezoidal velocity trajectory for optimal energy consumption [J]. Mechanical Design and Research, 2022 (004): 038

[24] Chen Anjun, Cai Jianle Joint trajectory planning for dual arm robots based on optimal load allocation [J]. Journal of Hebei University: Natural Science Edition, 2001, 21 (3): 6

[25] Ma Junwei Motion Planning for Complex Robots with Series Parallel Joints [D]. Harbin University of Science and Technology, 2009