

# Phase 4: Vacuum-to-FRW Consistency and Scale Sanity

A corridor-style test of the Phase 3 global-amplitude mechanism

Origin Axiom Program

January 7, 2026

## Abstract

Phase 4 tests whether the canonical global-amplitude mechanism defined in Phase 3 can be connected, in a structurally reasonable way, to FRW-style dynamics and vacuum-energy-like observables. The goal is to probe whether the non-cancellation floor that stabilises the toy vacuum can also support scale-sane behaviour when mapped into simple cosmological modules, without claiming a full cosmological fit or a theory of everything.

## 1 Introduction

Phase 4 sits between the mechanism-level vacuum work of Phase 3 and any future attempts to build a unified picture of vacuum, matter, and geometry. Its mission is narrow:

- take the canonical Phase 3 global-amplitude mechanism and non-cancellation floor as given;
- define simple, explicit mappings from the floor-enforced amplitude or residue into FRW-like dynamics and vacuum-energy-like observables; and
- study the resulting behaviour of the Origin-Axiom phase parameter  $\theta$  in the corridor / ledger framework of Phase 0.

The guiding question is not whether we can reproduce the full  $\Lambda$ CDM model or fit precise cosmological parameters, but whether the Phase 3 mechanism can be made compatible with toy FRW modules in a way that is numerically stable, structurally sane, and expressible as a  $\theta$ -filter in the sense of Phase 0.

Throughout this phase we distinguish carefully between:

- *binding* outputs, which may eventually define a Phase 4  $\theta$ -filter; and
- *non-binding* diagnostics and figures, which serve only as intuition and internal checks.

The present rung does not define any concrete mappings or filters. It only provides a minimal paper skeleton so that future rungs can add well-documented mechanisms and experiments without restructuring the front matter.

## 2 Mapping families: first pass (F1)

Phase 4 takes as input the Phase 3 global-amplitude mechanism: a toy vacuum with an unconstrained observable  $A_0(\theta)$ , a non-cancellation floor  $\varepsilon$ , and a floor-enforced amplitude  $A(\theta) = \max(A_0(\theta), \varepsilon)$  defined on a grid  $\theta \in [0, 2\pi)$ . The present paper introduces a first, explicit mapping family, denoted **F1**, from this structure to a toy vacuum-energy-like scalar.

## 2.1 F1: direct scalar mapping from $A(\theta)$

The F1 family is intentionally simple. For a fixed Phase 3 vacuum configuration and floor  $\varepsilon$  (taken from the Phase 3 baseline diagnostics), we define a scalar

$$E_{\text{vac}}(\theta) = \alpha A(\theta)^\beta, \quad (1)$$

where  $\alpha > 0$  and  $\beta > 0$  are explicit, configurable parameters. At this rung we adopt a conservative default,  $\alpha = 1$  and  $\beta = 2$ , and focus on structural behaviour rather than numerical normalisation.

Operationally, Phase 4 reuses the Phase 3 baseline configuration `baseline_v1` and the floor  $\varepsilon$  recorded in `phase3/outputs/tables/mech_baseline_scan_diagnostics.json`. We then evaluate  $A(\theta)$  and  $E_{\text{vac}}(\theta)$  on a uniform grid of  $N_\theta = 2048$  points in  $[0, 2\pi)$ . The per-grid values and summary diagnostics are written to

`phase4/outputs/tables/phase4_F1_sanity_curve.csv`,

together with metadata describing the mapping parameters and the underlying Phase 3 diagnostics.

At this stage F1 is a *non-binding* mapping family: it does not yet define a  $\theta$ -corridor or a Phase 4  $\theta$ -filter. Instead it serves as a concrete, auditable bridge between the Phase 3 mechanism and simple scalar observables that later rungs can connect to FRW-like toy modules and corridor construction.

## 3 Diagnostics and toy corridors

Phase 4 currently provides three layers of diagnostics built on the Phase 3 vacuum mechanism and the F1 mapping family. All of them are explicitly non-binding: they are used for internal checks and illustration, not as canonical  $\theta$ -filters.

### 3.1 F1 sanity curve

The first layer is a direct sanity check of the F1 mapping. Using the Phase 3 baseline configuration and the quantile-based floor recorded in `phase3/outputs/tables/mech_baseline_scan_diagnostics.json` we evaluate the floor-enforced amplitude  $A(\theta)$  on a uniform grid of  $N_\theta = 2048$  points in  $[0, 2\pi)$  and construct a toy vacuum-energy-like scalar

$$E_{\text{vac}}(\theta) = \alpha A(\theta)^\beta,$$

with  $\alpha = 1$  and  $\beta = 4$  at this rung. The script `phase4/src/phase4/run_f1_sanity.py` writes the per-grid values and metadata to

`phase4/outputs/tables/phase4_F1_sanity_curve.csv`,

which establishes that  $E_{\text{vac}}(\theta)$  is strictly positive, numerically well-behaved, and tied cleanly to the Phase 3 baseline diagnostics.

### 3.2 F1 shape diagnostics and toy corridor

The second layer probes the *shape* of  $E_{\text{vac}}(\theta)$  by defining a toy, non-binding corridor in the space of scalar values. The script `phase4/src/phase4/run_f1_shape_diagnostics.py` constructs a mask based on

$$E_{\text{vac}}(\theta) \leq E_{\text{vac},\min} + k_\sigma \sigma, \quad k_\sigma = 1,$$

where  $E_{\text{vac},\min}$  and  $\sigma$  are the global minimum and standard deviation of the F1 curve. This is intentionally weak: it retains a substantial fraction of the  $\theta$ -grid while discarding the largest excursions.

The script writes a summary diagnostics file

`phase4/outputs/tables/phase4_F1_shape_diagnostics.json`

and a per-grid mask

`phase4/outputs/tables/phase4_F1_shape_mask.csv`,

which later rungs and toy modules can reuse. At this stage the mask is explicitly described as a *toy corridor*: it does not define a canonical  $\theta_*$  and is not elevated to a Phase 4 filter.

### 3.3 FRW-inspired toy diagnostics

The third layer introduces a minimal FRW-inspired toy diagnostic that treats  $E_{\text{vac}}(\theta)$  as a source for a *dimensionless* vacuum term in a simple Hubble-like quantity. From the sanity curve we define

$$\tilde{E}_{\text{vac}}(\theta) = \frac{E_{\text{vac}}(\theta)}{\langle E_{\text{vac}} \rangle}$$

and rescale it to a toy  $\Omega_{\Lambda}(\theta)$  with mean  $\approx 0.7$ . Together with fixed, dimensionless matter and radiation parameters  $\Omega_m = 0.3$  and  $\Omega_r = 0$ , we evaluate

$$H^2(a; \theta) = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_{\Lambda}(\theta)$$

on a late-time scale-factor grid  $a \in [0.5, 1]$ . No physical units are assigned and no claim is made that these numbers match observations.

The script `phase4/src/phase4/run_f1_frw_toy_diagnostics.py` writes:

- a JSON summary `phase4/outputs/tables/phase4_F1_frw_toy_diagnostics.json`, recording global extrema and moments of  $E_{\text{vac}}$ ,  $\Omega_{\Lambda}(\theta)$ , the chosen  $\Omega$ -parameters, and a simple FRW-sanity fraction; and
- a per-grid mask `phase4/outputs/tables/phase4_F1_frw_toy_mask.csv`, containing  $\theta$ ,  $\Omega_{\Lambda}(\theta)$ , and a Boolean indicator of whether  $H^2(a; \theta)$  stays positive and within a bounded variation factor over the scale-factor grid.

Depending on the chosen toy parameters and sanity criterion, the resulting FRW-sanity fraction can be small or even zero; such empty-corridor outcomes are treated as informative negative results, recorded in the diagnostics and logs rather than hidden. In all cases the FRW-inspired module remains strictly non-binding: it does not fix a preferred  $\theta_*$  and does not, by itself, define a Phase 4  $\theta$ -filter.

## 4 Limitations and scope of Phase 4

Phase 4 is intentionally modest in scope. Its purpose is to demonstrate how a simple mapping family and a small stack of diagnostics can be wired, in a fully reproducible way, on top of the Phase 3 vacuum mechanism. It is *not* a proposal for a definitive  $\theta$ -corridor, a preferred  $\theta_*$ , or a calibrated cosmological model. In this section we make explicit what the current Phase 4 construction does and does not claim.

### 4.1 Structural vs. physical content

The F1 mapping family, the toy corridor, and the FRW-inspired module are all *structural* devices. They are designed to test whether the Phase 3 amplitudes can be converted into simple scalar observables in a way that is numerically sane, auditable, and easy to reproduce.

No part of the present Phase 4 implementation is calibrated to observational data or to a concrete field-theoretic model. The choices of mapping exponents, normalisations, and diagnostic thresholds are deliberately simple and should be read as examples of the *workflow*, not as physically motivated parameter inference.

## 4.2 Status of $\theta$ -corridors and $\theta_*$

The F1 toy corridor is defined by a  $k_\sigma$ -style condition on  $E_{\text{vac}}(\theta)$ : points whose mapped scalar lies within a band of width  $k_\sigma$  times the empirical standard deviation around the minimum are marked as “inside”. This is a purely diagnostic definition. It is:

- *not* justified by an underlying measure on  $\theta$ -space;
- *not* tied to any observational likelihood; and
- *not* used to select a unique  $\theta_*$ .

At this rung we do not claim that the corridor has any physical significance beyond being a compact way of summarising the shape of  $E_{\text{vac}}(\theta)$ . The Phase 4 artifact therefore does not define a canonical  $\theta_*$ , nor does it promote any specific subset of  $\theta$ -values as physically preferred.

## 4.3 FRW-inspired toys and cosmology

The FRW-inspired module treats the F1 scalar as a proxy for a  $\theta$ -dependent vacuum-energy density  $\Omega_\Lambda(\theta)$ , choosing the normalisation so that the mean of  $\Omega_\Lambda(\theta)$  over the grid is close to a target value (e.g.  $\approx 0.7$ ). Together with fixed toy values of  $\Omega_m$  and  $\Omega_r$ , this defines a simple FRW-like quantity

$$H^2(a; \theta) = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda(\theta),$$

evaluated on a finite grid of scale factors  $a$ .

The diagnostic checks used here (positivity of  $H^2$  on the grid and a bound on the ratio of maximal to minimal  $H^2$  per  $\theta$ ) are deliberately crude. They are not meant to reproduce precision cosmology, nor to constrain  $\theta$  from data. Their role is to illustrate how one could connect the Phase 3 mechanism to FRW-like sanity checks and to record both positive and negative outcomes (such as empty-corridor cases) in a structured way.

## 4.4 Numerical and implementation limitations

All Phase 4 results are computed on finite grids: a finite  $N_\theta$ -grid for  $\theta$  and a finite  $N_a$ -grid for the scale factor in the FRW-inspired module. The diagnostics such as “corridor fraction” or “FRW-sanity fraction” are therefore *grid-level* summaries and may change under refinement of these grids.

The current implementation uses a single mapping family (F1) and a single baseline configuration inherited from Phase 3. There is no systematic exploration of alternative mapping families, floors, normalisations, or FRW toy parameters. The artifact should therefore be read as documenting *one concrete, reproducible instance* of the workflow, not as a survey of the available design space.

## 4.5 Future extensions and non-claims

Phase 4 is explicitly positioned as a bridge between the Phase 3 mechanism and future corridor work, not as the end-point of that programme. In particular:

- we do not claim to have derived a physically meaningful  $\theta$ -corridor;
- we do not claim to have identified a preferred  $\theta_*$ ; and
- we do not claim to have solved any cosmological fine-tuning problem.

What Phase 4 does claim is that:

1. the Phase 3 vacuum mechanism can be cleanly wired into a mapping family and scalar diagnostics;
2. the resulting workflow can be captured in a small, auditable code surface with explicit inputs and outputs; and
3. both positive and negative diagnostic outcomes (including empty-corridor cases) can be reported in a way that is aligned with the Phase 0 philosophy of structured, reproducible progress.

Any future tightening of the corridor notion, introduction of a candidate  $\theta_*$ , or calibration to observational data would require additional rungs beyond the present Phase 4 artifact and would have to make their own assumptions and limitations explicit.

## Appendix A: Phase 4 claims table (draft)

Table 1 summarises the intended Phase 4 claims. At this rung all entries are draft and non-binding.

Table 1: Draft Phase 4 claims. Binding status will be updated once the phase is complete and audited.

ID	Binding?	Summary
C4.1	no	Existence of at least one explicit mapping from the Phase 3 global amplitude or residue into an FRW-like or vacuum-energy-like observable with numerically stable behaviour.
C4.2	no	Existence of a non-empty, non-trivial $\theta$ -corridor for at least one such mapping.
C4.3	no	Structured negative result if all tested mappings yield empty or pathological corridors.

## Appendix B: Reproducibility notes (draft)

At this rung, Phase 4 builds on the Phase 3 toy vacuum mechanism and its non-cancellation floor, introduces the F1 mapping family, and adds internal diagnostics and a toy, non-binding corridor. This appendix summarises how to rebuild the Phase 4 artifacts used in the paper.

### Directory structure

Phase 4 lives in the top-level directory `phase4/` with the following relevant subdirectories:

- `phase4/paper/`: LaTeX sources for the Phase 4 paper (including this appendix).
- `phase4/src/phase4/`: Python source for the F1 mapping family and diagnostics.
- `phase4/outputs/`:
  - `outputs/paper/`: built Phase 4 PDF;
  - `outputs/tables/`: CSV/JSON artifacts from F1 diagnostics.
- `phase4/artifacts/`: canonical Phase 4 PDF artifact exported by the gate script.

## Upstream Phase 3 prerequisites

Phase 4 assumes that the Phase 3 baseline scan and binding-certificate artifacts have been generated at least once. In practice this can be ensured by running, from the repository root:

```
bash scripts/phase3_gate.sh --level A
```

and, if needed, explicitly running the Phase 3 scripts:

```
python phase3/src/phase3_mech/run_baseline_scan.py
python phase3/src/phase3_mech/run_binding_certificate.py
```

These commands populate:

- `phase3/outputs/tables/mech_baseline_scan.csv`
- `phase3/outputs/tables/mech_baseline_scan_diagnostics.json`
- `phase3/outputs/tables/mech_binding_certificate.csv`
- `phase3/outputs/tables/mech_binding_certificate_diagnostics.json`

The F1 mapping family reads `mech_baseline_scan_diagnostics.json` to reuse the quantile-based floor and grid configuration.

## Phase 4 F1 mapping and diagnostics

With the Phase 3 prerequisites in place, the current Phase 4 diagnostics can be rebuilt via:

```
python phase4/src/phase4/run_f1_sanity.py
python phase4/src/phase4/run_f1_shape_diagnostics.py
```

These scripts write:

- `phase4/outputs/tables/phase4_F1_sanity_curve.csv` (per- $\theta$  values of  $E_{\text{vac}}(\theta)$  and associated metadata);
- `phase4/outputs/tables/phase4_F1_shape_diagnostics.json` (summary statistics and toy corridor definition);
- `phase4/outputs/tables/phase4_F1_shape_mask.csv` (per- $\theta$  Boolean mask indicating membership in the toy corridor).

At this rung these diagnostics are explicitly non-binding: they do not define a canonical  $\theta$ -filter, but they are used in the Phase 4 paper to illustrate the behaviour of the F1 mapping.

## Building the Phase 4 paper and artifact

The Phase 4 paper and its canonical PDF artifact can be rebuilt from the repository root via:

```
bash scripts/phase4_gate.sh --level A
```

This gate script runs a Snakemake workflow in `phase4/workflow/Snakefile` that invokes `latexmk` on `phase4/paper/main.tex` and copies the resulting PDF to:

- `phase4/outputs/paper/phase4_paper.pdf`
- `phase4/artifacts/origin-axiom-phase4.pdf`

The gate currently covers the Phase 4 paper only; F1 diagnostics are rebuilt by the explicit Python commands above.

## Planned extensions

The design note `phase4/FRW_TOY_DESIGN.md` specifies a minimal FRW-inspired toy module that may be implemented in later rungs. If such a module is added, this appendix will be extended to include:

- the relevant Python scripts and configuration files;
- any additional JSON/CSV outputs; and
- updated gate levels if FRW-like diagnostics become part of the canonical Phase 4 artifact.

Until then, reproducibility for Phase 4 consists of:

1. regenerating the Phase 3 baseline and binding-certificate artifacts;
2. regenerating the Phase 4 F1 diagnostics; and
3. rebuilding the Phase 4 paper and artifact via the Level A gate.

## References