Phase 4: Vacuum-to-FRW Consistency and Scale Sanity

A corridor-style test of the Phase 3 global-amplitude mechanism

Origin Axiom Program

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Abstract

Phase 4 tests whether the canonical global-amplitude mechanism defined in Phase 3 can be connected, in a structurally reasonable way, to FRW-style dynamics and vacuum-energy-like observables. The goal is to probe whether the non-cancellation floor that stabilises the toy vacuum can also support scale-sane behaviour when mapped into simple cosmological modules, without claiming a full cosmological fit or a theory of everything.

1 Introduction

Phase 4 sits between the mechanism-level vacuum work of Phase 3 and any future attempts to build a unified picture of vacuum, matter, and geometry. Its mission is narrow:

- take the canonical Phase 3 global-amplitude mechanism and non-cancellation floor as given;
- define simple, explicit mappings from the floor-enforced amplitude or residue into FRW-like dynamics and vacuum-energy-like observables; and
- study the resulting behaviour of the Origin-Axiom phase parameter θ in the corridor / ledger framework of Phase 0.

The guiding question is not whether we can reproduce the full ΛCDM model or fit precise cosmological parameters, but whether the Phase 3 mechanism can be made compatible with toy FRW modules in a way that is numerically stable, structurally sane, and expressible as a θ-filter in the sense of Phase 0.

Throughout this phase we distinguish carefully between:

- *binding* outputs, which may eventually define a Phase 4 θ-filter; and
- *non-binding* diagnostics and figures, which serve only as intuition and internal checks.

The present rung does not define any concrete mappings or filters. It only provides a minimal paper skeleton so that future rungs can add well-documented mechanisms and experiments without restructuring the front matter.

2 Mapping families: first pass (F1)

Phase 4 takes as input the Phase 3 global-amplitude mechanism: a toy vacuum with an unconstrained observable $A_0(\theta)$, a non-cancellation floor ε, and a floor-enforced amplitude $A(\theta) = \max(A_0(\theta), \varepsilon)$ defined on a grid $\theta \in [0, 2\pi)$. The present paper introduces a first, explicit mapping family, denoted **F1**, from this structure to a toy vacuum-energy-like scalar.

2.1 F1: direct scalar mapping from $A(\theta)$

The F1 family is intentionally simple. For a fixed Phase 3 vacuum configuration and floor ε (taken from the Phase 3 baseline diagnostics), we define a scalar

$$E_{\mathrm{vac}}(\theta) \;=\; \alpha\, A(\theta)^{\beta}, \tag{1}$$

where $\alpha > 0$ and $\beta > 0$ are explicit, configurable parameters. At this rung we adopt a conservative default, $\alpha = 1$ and $\beta = 2$, and focus on structural behaviour rather than numerical normalisation.

Operationally, Phase 4 reuses the Phase 3 baseline configuration `baseline_v1` and the floor ε recorded in `phase3/outputs/tables/mech_baseline_scan_diagnostics.json`. We then evaluate $A(\theta)$ and $E_{\mathrm{vac}}(\theta)$ on a uniform grid of $N_\theta = 2048$ points in $[0, 2\pi)$. The per-grid values and summary diagnostics are written to

`phase4/outputs/tables/phase4_F1_sanity_curve.csv`,

together with metadata describing the mapping parameters and the underlying Phase 3 diagnostics.

At this stage F1 is a *non-binding* mapping family: it does not yet define a θ-corridor or a Phase 4 θ-filter. Instead it serves as a concrete, auditable bridge between the Phase 3 mechanism and simple scalar observables that later rungs can connect to FRW-like toy modules and corridor construction.

The Phase 4 diagnostics are deliberately layered. The goal is to place the F1 mapping family on a numerically solid footing and to expose a sequence of increasingly structured, but still non-binding, probes that later work can refine or replace. All layers are wired through explicit on-disk artefacts in `phase4/outputs/tables/` so that their behaviour can be inspected without touching the code.

We summarise the main layers here.

(1) F1 sanity curve. The first rung is a minimal "vacuum curve" sanity check. We reuse the Phase 3 baseline mechanism diagnostics stored in `phase3/outputs/tables/mech_baseline_scan_diagnostics` to define a quantile-based floor ε for the unconstrained amplitude $A(\theta)$. The F1 family then defines a scalar

$$E_{\mathrm{vac}}(\theta) \;=\; \alpha\, A(\theta)^{\beta}$$

with fixed toy parameters $\alpha = 1$ and $\beta = 4$ in the baseline configuration. The script `phase4/src/phase4/run_f1` evaluates $E_{\mathrm{vac}}(\theta)$ on a uniform grid of $N_\theta = 2048$ points in $[0, 2\pi)$, writing a per-grid CSV

`phase4/outputs/tables/phase4_F1_sanity_curve.csv`,

together with a metadata block recording global minima, maxima, quantiles, and the exact Phase 3 diagnostics that were reused. This layer only checks that the F1 map is numerically sane (strictly positive, small, and well-resolved) and cleanly wired to Phase 3; it does not attempt any physical calibration.

(2) F1 shape diagnostics and a toy corridor. The second rung looks at the *shape* of the F1 curve rather than individual values. The script `phase4/src/phase4/run_f1_shape_diagnostics.py` consumes the F1 sanity CSV and constructs a toy "low-E_{vac}" corridor via a simple threshold

$$E_{\mathrm{vac}}(\theta) \;\leq\; E_{\mathrm{vac,min}} + k_\sigma\, \sigma,$$

with a baseline choice $k_\sigma = 1$. It produces

- a JSON summary `phase4/outputs/tables/phase4_F1_shape_diagnostics.json`, recording amplitude statistics, the chosen threshold, and the resulting corridor fraction; and

- a per-grid mask `phase4/outputs/tables/phase4_F1_shape_mask.csv`, listing θ, $E_{\rm vac}(\theta)$, and an indicator of membership in this toy corridor.

This is explicitly labelled as exploratory and non-binding. It shows how a mechanically defined scalar can induce a corridor-like subset of the θ-grid, but it is *not* used to define any Phase 4 θ-filter or to suggest a preferred $\theta_\star$.

(3) FRW-inspired toy sanity checks. The third rung embeds $E_{\rm vac}(\theta)$ into a simple, late-time FRW-inspired background. The script `phase4/src/phase4/run_f1_frw_toy_diagnostics.py` rescales the F1 vacuum curve into a toy dark-energy parameter $\Omega_\Lambda(\theta)$ with $\langle\Omega_\Lambda\rangle \approx 0.7$ at fixed $\Omega_m = 0.3$, $\Omega_r = 0$. It then evaluates a schematic FRW quantity

$$H^2(a;\theta) \;=\; \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda(\theta)$$

on a late-time scale-factor grid $a \in [0.5, 1]$ and applies a simple sanity criterion: $H^2(a;\theta)$ must stay positive and bounded in variation across that grid. This produces

- a JSON diagnostics file `phase4/outputs/tables/phase4_F1_frw_toy_diagnostics.json`, recording moments of $E_{\rm vac}$, $\Omega_\Lambda(\theta)$, the FRW parameters, and a "FRW-sanity fraction"; and
- a per-grid mask `phase4/outputs/tables/phase4_F1_frw_toy_mask.csv` with θ, $\Omega_\Lambda(\theta)$, and a Boolean indicator of FRW sanity.

This module is intentionally modest and strictly non-binding. It is used as a structural probe of how the F1 scalar can feed into FRW-like checks, and as an example of how empty-corridor outcomes (when the sanity fraction is small or zero) are recorded and analysed rather than hidden.

(4) FRW viability scan with age and era constraints. The fourth rung tightens the FRW-facing analysis into a more cosmology-like viability scan, while still remaining at the level of a toy model. The script `phase4/src/phase4/run_f1_frw_viability.py` again uses a mapping from $E_{\rm vac}(\theta)$ to $\Omega_\Lambda(\theta)$ with $\langle\Omega_\Lambda\rangle \approx 0.7$ at fixed $\Omega_m = 0.3$, $\Omega_r = 0$ and a toy Hubble parameter $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. For each grid point in θ it then

1. computes a dimensionless FRW age integral $t_0 H_0 = \int_0^1 \mathrm{d}a/\left(a\sqrt{H^2(a;\theta)}\right)$ and converts it to a cosmic age $t_0(\theta)$ in Gyr;
2. checks for the existence of a "matter era" in which the $\Omega_m a^{-3}$ term dominates over both radiation and $\Omega_\Lambda(\theta)$ on part of the a-grid;
3. checks for a "late-time acceleration" regime where the effective deceleration parameter becomes negative near $a = 1$;
4. requires a smooth, positive $H^2(a;\theta)$ on the chosen grid; and
5. demands that the resulting age lies in a broad window $10\,\mathrm{Gyr} \le t_0(\theta) \le 20\,\mathrm{Gyr}$.

The output is

- a diagnostics file `phase4/outputs/tables/phase4_F1_frw_viability_diagnostics.json` summarising the distribution of ages and the fractions of grid points satisfying each individual condition and the combined viability cut; and
- a per-θ mask `phase4/outputs/tables/phase4_F1_frw_viability_mask.csv` with columns `theta`, `E_vac`, `omega_lambda`, `age_Gyr`, individual Boolean flags (`has_matter_era`, `has_late_accel`, `smooth_H2`), and a combined `frw_viable` flag.

At the baseline settings used here, a non-trivial fraction of the grid (≈ 0.5) satisfies all of these toy viability criteria. The exact numbers are not tuned to any observational data; they are treated as diagnostics of how the F1 mapping behaves under a cosmology-flavoured set of constraints.

(5) Corridor extraction from the FRW viability mask. The final rung in the current Phase 4 diagnostics compresses the FRW viability mask into contiguous "corridors" in θ-space. The script `phase4/src/phase4/run_f1_frw_corridors.py` reads `phase4_F1_frw_viability_mask.csv`, interprets the `frw_viable` flag as a Boolean viability indicator, and groups successive viable grid points into connected components, allowing for a small tolerance on the grid spacing. For each corridor it records

- $\theta_{\min}$, $\theta_{\max}$;
- the number of included grid points; and
- an optional mean $\langle \Omega_\Lambda \rangle$ over that corridor when the corresponding column is present.

The resulting corridors are written to

`phase4/outputs/tables/phase4_F1_frw_corridors.json`,
`phase4/outputs/tables/phase4_F1_frw_corridors.csv`.

The JSON file additionally highlights a *principal corridor* (selected as the one with the largest number of grid points) together with basic metadata such as the total grid size, the overall viable fraction, and the total number of corridors.

Even at this rung the construction remains explicitly non-binding: the corridors are treated as candidate bands emerging from a cosmology-flavoured viability scan, not as a canonical Phase 4 θ-filter or a claimed prediction of a preferred $\theta_\star$. Their role is to show, in a fully reproducible way, how the Phase 3 mechanism, the F1 mapping, and simple FRW-inspired constraints can be chained into corridor-like structures that later, more physically ambitious work can interrogate or replace.

FRW ΛCDM-like probe

As a final FRW-facing diagnostic we place a broad "ΛCDM-like" window on top of the viability mask from the previous rung. Concretely, we take the per-grid FRW-viable rows from `phase4/outputs/tables/phase4_F1_frw_viability_mask.csv` and require, in addition:

- a vacuum fraction $\Omega_\Lambda(\theta)$ within a toy tolerance window around a target $\Omega_\Lambda^{\mathrm{target}} \approx 0.7$; and
- a cosmological age in a toy window around $t_0^{\mathrm{target}} \approx 13.8\,\mathrm{Gyr}$, using the same FRW toy model as the viability rung ($\Omega_m = 0.3$, $\Omega_r = 0$, $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$).

The script `phase4/src/phase4/run_f1_frw_lcdm_probe.py` implements this probe, reading the FRW-viability mask and writing:

- a JSON summary `phase4/outputs/tables/phase4_F1_frw_lcdm_probe.json`, which records the target windows, the θ-extent of the ΛCDM-like subset, and the corresponding grid fraction;
- a per-grid mask `phase4/outputs/tables/phase4_F1_frw_lcdm_probe_mask.csv`, which augments the FRW-viability columns with a Boolean `lcdm_like` flag.

In the current baseline configuration, this ΛCDM-like window selects a small but non-zero subset of the FRW-viable θ-grid points. We stress that this remains a toy-level probe: it is not a fit to observational data and does not fix a preferred $\theta_\star$. Instead, it serves to demonstrate that the Phase 3 $\rightarrow$ Phase 4 pipeline admits FRW histories that are, at least in a broad sense, compatible with late-time ΛCDM-like cosmology.

3 Limitations and scope of Phase 4

Phase 4 is intentionally modest in scope. Its purpose is to demonstrate how a simple mapping family and a small stack of diagnostics can be wired, in a fully reproducible way, on top of the Phase 3 vacuum mechanism. It is *not* a proposal for a definitive θ-corridor, a preferred $\theta_\star$, or a calibrated cosmological model. In this section we make explicit what the current Phase 4 construction does and does not claim.

3.1 Structural vs. physical content

The F1 mapping family, the toy corridor, and the FRW-inspired module are all *structural* devices. They are designed to test whether the Phase 3 amplitudes can be converted into simple scalar observables in a way that is numerically sane, auditable, and easy to reproduce.

No part of the present Phase 4 implementation is calibrated to observational data or to a concrete field-theoretic model. The choices of mapping exponents, normalisations, and diagnostic thresholds are deliberately simple and should be read as examples of the *workflow*, not as physically motivated parameter inference.

3.2 Status of θ-corridors and $\theta_\star$

The F1 toy corridor is defined by a k_σ-style condition on $E_{\mathrm{vac}}(\theta)$: points whose mapped scalar lies within a band of width k_σ times the empirical standard deviation around the minimum are marked as "inside". This is a purely diagnostic definition. It is:

- *not* justified by an underlying measure on θ-space;
- *not* tied to any observational likelihood; and
- *not* used to select a unique $\theta_\star$.

At this rung we do not claim that the corridor has any physical significance beyond being a compact way of summarising the shape of $E_{\mathrm{vac}}(\theta)$. The Phase 4 artifact therefore does not define a canonical $\theta_\star$, nor does it promote any specific subset of θ-values as physically preferred.

3.3 FRW-inspired toys and cosmology

The FRW-inspired module treats the F1 scalar as a proxy for a θ-dependent vacuum-energy density $\Omega_\Lambda(\theta)$, choosing the normalisation so that the mean of $\Omega_\Lambda(\theta)$ over the grid is close to a target value (e.g. ≈ 0.7). Together with fixed toy values of Ω_m and Ω_r, this defines a simple FRW-like quantity

$$H^2(a;\theta) = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda(\theta),$$

evaluated on a finite grid of scale factors a.

The diagnostic checks used here (positivity of H^2 on the grid and a bound on the ratio of maximal to minimal H^2 per θ) are deliberately crude. They are not meant to reproduce precision cosmology, nor to constrain θ from data. Their role is to illustrate how one could connect the Phase 3 mechanism to FRW-like sanity checks and to record both positive and negative outcomes (such as empty-corridor cases) in a structured way.

3.4 Numerical and implementation limitations

All Phase 4 results are computed on finite grids: a finite N_θ-grid for θ and a finite N_a-grid for the scale factor in the FRW-inspired module. The diagnostics such as "corridor fraction" or "FRW-sanity fraction" are therefore *grid-level* summaries and may change under refinement of these grids.

The current implementation uses a single mapping family (F1) and a single baseline configuration inherited from Phase 3. There is no systematic exploration of alternative mapping families, floors, normalisations, or FRW toy parameters. The artifact should therefore be read as documenting *one concrete, reproducible instance* of the workflow, not as a survey of the available design space.

3.5 Future extensions and non-claims

Phase 4 is explicitly positioned as a bridge between the Phase 3 mechanism and future corridor work, not as the end-point of that programme. In particular:

- we do not claim to have derived a physically meaningful θ-corridor;
- we do not claim to have identified a preferred $\theta_\star$; and
- we do not claim to have solved any cosmological fine-tuning problem.

What Phase 4 does claim is that:

1. the Phase 3 vacuum mechanism can be cleanly wired into a mapping family and scalar diagnostics;
2. the resulting workflow can be captured in a small, auditable code surface with explicit inputs and outputs; and
3. both positive and negative diagnostic outcomes (including empty-corridor cases) can be reported in a way that is aligned with the Phase 0 philosophy of structured, reproducible progress.

Any future tightening of the corridor notion, introduction of a candidate $\theta_\star$, or calibration to observational data would require additional rungs beyond the present Phase 4 artifact and would have to make their own assumptions and limitations explicit.

Appendix A: Phase 4 claims table (draft)

Table 1 summarises the intended Phase 4 claims. At this rung all entries are draft and non-binding.

Table 1: Draft Phase 4 claims. Binding status will be updated once the phase is complete and audited.

ID	Binding?	Summary
C4.1	no	Existence of at least one explicit mapping from the Phase 3 global amplitude or residue into an FRW-like or vacuum-energy-like observable with numerically stable behaviour.
C4.2	no	Existence of a non-empty, non-trivial θ-corridor for at least one such mapping.
C4.3	no	Structured negative result if all tested mappings yield empty or pathological corridors.

Appendix B: Reproducibility notes (draft)

At this rung, Phase 4 builds on the Phase 3 toy vacuum mechanism and its non-cancellation floor, introduces the F1 mapping family, and adds internal diagnostics and a toy, non-binding corridor. This appendix summarises how to rebuild the Phase 4 artifacts used in the paper.

Directory structure

Phase 4 lives in the top-level directory `phase4/` with the following relevant subdirectories:

- `phase4/paper/`: LaTeX sources for the Phase 4 paper (including this appendix).
- `phase4/src/phase4/`: Python source for the F1 mapping family and diagnostics.
- `phase4/outputs/`:
  - `outputs/paper/`: built Phase 4 PDF;
  - `outputs/tables/`: CSV/JSON artifacts from F1 diagnostics.
- `phase4/artifacts/`: canonical Phase 4 PDF artifact exported by the gate script.

Upstream Phase 3 prerequisites

Phase 4 assumes that the Phase 3 baseline scan and binding-certificate artifacts have been generated at least once. In practice this can be ensured by running, from the repository root:

```
bash scripts/phase3_gate.sh --level A
```

and, if needed, explicitly running the Phase 3 scripts:

```
python phase3/src/phase3_mech/run_baseline_scan.py
python phase3/src/phase3_mech/run_binding_certificate.py
```

These commands populate:

- `phase3/outputs/tables/mech_baseline_scan.csv`
- `phase3/outputs/tables/mech_baseline_scan_diagnostics.json`
- `phase3/outputs/tables/mech_binding_certificate.csv`
- `phase3/outputs/tables/mech_binding_certificate_diagnostics.json`

The F1 mapping family reads `mech_baseline_scan_diagnostics.json` to reuse the quantile-based floor and grid configuration.

Phase 4 F1 mapping and diagnostics

With the Phase 3 prerequisites in place, the current Phase 4 diagnostics can be rebuilt via:

```
python phase4/src/phase4/run_f1_sanity.py
python phase4/src/phase4/run_f1_shape_diagnostics.py
```

These scripts write:

- `phase4/outputs/tables/phase4_F1_sanity_curve.csv` (per-θ values of $E_{\mathrm{vac}}(\theta)$ and associated metadata);

- `phase4/outputs/tables/phase4_F1_shape_diagnostics.json` (summary statistics and toy corridor definition);
- `phase4/outputs/tables/phase4_F1_shape_mask.csv` (per-θ Boolean mask indicating membership in the toy corridor).

At this rung these diagnostics are explicitly non-binding: they do not define a canonical θ-filter, but they are used in the Phase 4 paper to illustrate the behaviour of the F1 mapping.

Building the Phase 4 paper and artifact

The Phase 4 paper and its canonical PDF artifact can be rebuilt from the repository root via:

```
bash scripts/phase4_gate.sh --level A
```

This gate script runs a Snakemake workflow in `phase4/workflow/Snakefile` that invokes `latexmk` on `phase4/paper/main.tex` and copies the resulting PDF to:

- `phase4/outputs/paper/phase4_paper.pdf`
- `phase4/artifacts/origin-axiom-phase4.pdf`

The gate currently covers the Phase 4 paper only; F1 diagnostics are rebuilt by the explicit Python commands above.

Planned extensions

The design note `phase4/FRW_TOY_DESIGN.md` specifies a minimal FRW-inspired toy module that may be implemented in later rungs. If such a module is added, this appendix will be extended to include:

- the relevant Python scripts and configuration files;
- any additional JSON/CSV outputs; and
- updated gate levels if FRW-like diagnostics become part of the canonical Phase 4 artifact.

Until then, reproducibility for Phase 4 consists of:

1. regenerating the Phase 3 baseline and binding-certificate artifacts;
2. regenerating the Phase 4 F1 diagnostics; and
3. rebuilding the Phase 4 paper and artifact via the Level A gate.

References