

# Phase 4: Vacuum-to-FRW Consistency and Scale Sanity

A corridor-style test of the Phase 3 global-amplitude mechanism

Origin Axiom Program

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## Abstract

Phase 3 of the origin-axiom programme defined a reproducible scalar vacuum diagnostic  $E_{\text{vac}}(\theta)$  over an abstract  $\theta$ -grid. Phase 4 tests whether that diagnostic can be connected to simple Friedmann–Robertson–Walker (FRW) histories in a structurally reasonable way. We introduce a first mapping family (F1) from  $E_{\text{vac}}(\theta)$  to a toy  $\Omega_\Lambda(\theta)$ , and we stack several FRW-inspired diagnostics: a sanity check on the shape of  $\Omega_\Lambda(\theta)$ , a viability scan for FRW histories with matter, radiation and late-time acceleration, an automated extraction of FRW-viable  $\theta$ -corridors, and a coarse  $\Lambda$ CDM-like probe centred on  $\Omega_\Lambda \approx 0.7$  and  $t_0 \approx 13.8$  Gyr. For the baseline toy parameters used here, roughly half of the  $\theta$ -grid yields FRW-viable histories and a small but non-zero subset falls inside the  $\Lambda$ CDM-like window. We emphasise that these results are *non-binding*: Phase 4 does not propose a unique  $\theta_*$  or an observational fit, but instead provides a reproducible FRW-facing pipeline and a concrete worked example of how Phase 3 outputs can be tested against cosmological sanity criteria.

## 1 Introduction

Phase 3 of the origin–axiom programme constructed a concrete lattice mechanism for scanning a dimensionless parameter  $\theta$  and produced a baseline diagnostic of the resulting amplitude landscape. In particular, it delivered a reproducible pipeline which, given a fixed mechanism configuration, yields a numerically stable *amplitude*  $A(\theta)$ , a global floor, and a summary JSON file with basic moments and quantiles of the scan. Phase 3 is deliberately agnostic about cosmology: it stops at a well-controlled  $\theta \mapsto A(\theta)$  map together with a mechanism-facing diagnostic layer.

Phase 4 takes this Phase 3 output as a starting point and asks a deliberately modest but physically oriented question: *can we systematically stress-test the Phase 3 landscape through simple, Friedmann–Robertson–Walker (FRW) inspired summaries without over-claiming physical predictions or selecting a unique  $\theta_*$ ?* The guiding philosophy, inherited from Phase 0, is to separate (i) the construction of reproducible mechanism- and mapping-level objects from (ii) any stronger claims about the structure of the real Universe.

At this rung we therefore introduce a single, explicit mapping family

$$E_{\text{vac}}(\theta) = \alpha A(\theta)^\beta, \quad (1)$$

denoted “F1”, which turns the Phase 3 amplitude  $A(\theta)$  into a non-negative scalar  $E_{\text{vac}}(\theta)$  intended to play the role of a toy vacuum-energy density. We adopt a simple baseline choice  $\alpha = 1$ ,  $\beta = 4$  and treat the mapping as a tunable, mechanism-facing object rather than as a physical law. On top of F1 we build a stack of FRW-inspired diagnostics: a toy FRW sanity check, a more structured FRW “viability” scan, an extraction of contiguous  $\theta$ -corridors, and a broad  $\Lambda$ CDM-like probe, together with an explicit hook for future data-level tests.

Crucially, all of these layers are *non-binding*. Phase 4 does *not* attempt to fit cosmological data, does *not* claim a preferred value of  $\theta$ , and does *not* promote any FRW diagnostic into a

hard filter on the origin–axiom mechanism. Instead, the goal is to show, in a fully auditable way, that the Phase 3 landscape can be connected to simple cosmology-facing summaries and that both positive and negative outcomes (including empty or narrow corridors) are recorded and reasoned about explicitly. Subsequent phases are free to refine the mapping family, the FRW summaries, and the data interfaces, but they do so on top of the reproducible infrastructure established here.

In the concrete baseline implementation reported in this paper, Phase 4 *does* instantiate a specific mapping family and a stack of FRW-inspired diagnostics. The F1 mapping takes the Phase 3 scalar as input and, via a simple normalisation, produces a toy  $\Omega_\Lambda(\theta)$ ; the associated scripts implement FRW-facing sanity, viability, corridor, and  $\Lambda$ CDM-like probes as explicit, reproducible computations with per-grid masks and JSON summaries. What this rung does *not* yet provide is a binding Phase 4  $\theta$ -filter or any claim of a fit to observational data. All FRW-facing results are deliberately presented as non-binding worked examples and structured sanity checks, rather than as evidence for a preferred  $\theta_*$ .

## 2 Mapping family and scalar $E_{\text{vac}}(\theta)$

Phase 3 furnishes a fixed, mechanism-level map  $\theta \mapsto A(\theta)$  together with a global floor and a summary diagnostics file. Concretely, a baseline configuration of the Phase 3 mechanism produces a table of amplitudes  $A(\theta_j)$  on a uniform  $\theta$ -grid and writes, among other outputs, a JSON file

$$\text{phase3/outputs/tables/mech_baseline_scan_diagnostics.json}, \quad (2)$$

which records the global minimum and maximum, basic moments, and quantiles of the scan. Phase 3 itself stops at this level: it does not interpret  $A(\theta)$  as a cosmological quantity.

Phase 4 introduces a simple, explicit mapping family that turns the Phase 3 amplitude into a non-negative scalar intended to play the role of a toy vacuum-energy density:

$$E_{\text{vac}}(\theta) = \alpha A(\theta)^\beta, \quad \alpha > 0, \beta > 0. \quad (3)$$

We refer to this family as “F1”. The two parameters  $\alpha$  and  $\beta$  are treated as tunable, mechanism-facing hyperparameters: they shape the relative weighting of amplitude variations across the  $\theta$ -grid but do not correspond to any claimed physical constants.

In this Phase 4 baseline rung we adopt the concrete choice

$$\alpha = 1, \quad \beta = 4, \quad (4)$$

motivated by three simple considerations:

1.  $A(\theta)$  is already normalised at the mechanism level, so it is natural to absorb any overall scale into a separate FRW toy-model normalisation rather than into  $\alpha$ ;
2. an even power  $\beta$  guarantees that  $E_{\text{vac}}$  remains non-negative wherever  $A(\theta)$  is defined;
3. a quartic choice  $\beta = 4$  accentuates contrasts between low- and high-amplitude regions without being numerically extreme on the Phase 3 baseline scan.

These points are *pragmatic* rather than physical: they select a well-behaved default for the present rung while keeping the door open to future exploration of alternative exponents and normalisations.

Given a fixed Phase 3 amplitude table, the Phase 4 script `phase4/src/phase4/run_f1_sanity.py` constructs the scalar curve  $E_{\text{vac}}(\theta)$  according to Eq. (3) with  $\alpha = 1$  and  $\beta = 4$ , reusing the global minimum and quantile-based floor reported in `mech_baseline_scan_diagnostics.json`. It writes:

- a per- $\theta$  CSV `phase4/outputs/tables/phase4_F1_sanity_curve.csv` containing  $\theta$  and  $E_{\text{vac}}(\theta)$ ; and
- a JSON summary `phase4/outputs/tables/phase4_F1_sanity_curve_diagnostics.json` with global extrema, basic moments, and configuration metadata.

This establishes a reproducible, one-dimensional scalar landscape on top of the Phase 3 mechanism. All subsequent Phase 4 diagnostics—including the toy corridor, FRW-inspired sanity checks, FRW viability scan, corridor extraction, and  $\Lambda$ CDM-facing probes—are built *on this F1 scalar*, and their outputs always reference `phase4_F1_*` tables rather than the raw Phase 3 amplitudes.

Throughout this rung, F1 is treated as a *diagnostic mapping* rather than a physical law. In particular, we do not claim that  $E_{\text{vac}}(\theta)$  is the actual vacuum energy of the real Universe; instead, we use it as a controlled, reproducible proxy for asking whether the Phase 3 landscape can coherently feed into FRW-like summaries without generating obviously pathological behaviour (e.g. negative or wildly oscillatory  $H^2(a; \theta)$ ) across the  $\theta$ -grid. Future work is free to vary  $\alpha, \beta$ , or even the functional form of the mapping, but such variations will sit on top of the baseline pipeline documented here.

At this stage the F1 family should be read strictly as a structural, toy choice. The exponents and normalisation  $(\alpha, \beta) = (1, 4)$  are selected for numerical convenience and to accentuate contrast in the Phase 3 amplitude; they are *not* proposed as a physically motivated relation between the toy amplitude and any real vacuum energy density. Any future, data-facing use of the Phase 4 pipeline would be expected to revisit and possibly replace this mapping.

The Phase 4 diagnostics are deliberately layered. The goal is to place the F1 mapping family on a numerically solid footing and to expose a sequence of increasingly structured, but still non-binding, probes that later work can refine or replace. All layers are wired through explicit on-disk artefacts in `phase4/outputs/tables/` so that their behaviour can be inspected without touching the code.

We summarise the main layers here.

**(1) F1 sanity curve.** The first rung is a minimal “vacuum curve” sanity check. We reuse the Phase 3 baseline mechanism diagnostics stored in `phase3/outputs/tables/mech_baseline_scan_diagnostics` to define a quantile-based floor  $\varepsilon$  for the unconstrained amplitude  $A(\theta)$ . The F1 family then defines a scalar

$$E_{\text{vac}}(\theta) = \alpha A(\theta)^\beta$$

with fixed toy parameters  $\alpha = 1$  and  $\beta = 4$  in the baseline configuration. The script `phase4/src/phase4/run_f1` evaluates  $E_{\text{vac}}(\theta)$  on a uniform grid of  $N_\theta = 2048$  points in  $[0, 2\pi]$ , writing a per-grid CSV

`phase4/outputs/tables/phase4_F1_sanity_curve.csv,`

together with a metadata block recording global minima, maxima, quantiles, and the exact Phase 3 diagnostics that were reused. This layer only checks that the F1 map is numerically sane (strictly positive, small, and well-resolved) and cleanly wired to Phase 3; it does not attempt any physical calibration.

**(2) F1 shape diagnostics and a toy corridor.** The second rung looks at the *shape* of the F1 curve rather than individual values. The script `phase4/src/phase4/run_f1_shape_diagnostics.py` consumes the F1 sanity CSV and constructs a toy “low- $E_{\text{vac}}$ ” corridor via a simple threshold

$$E_{\text{vac}}(\theta) \leq E_{\text{vac}, \min} + k_\sigma \sigma,$$

with a baseline choice  $k_\sigma = 1$ . It produces

- a JSON summary `phase4/outputs/tables/phase4_F1_shape_diagnostics.json`, recording amplitude statistics, the chosen threshold, and the resulting corridor fraction; and

- a per-grid mask `phase4/outputs/tables/phase4_F1_shape_mask.csv`, listing  $\theta$ ,  $E_{\text{vac}}(\theta)$ , and an indicator of membership in this toy corridor.

This is explicitly labelled as exploratory and non-binding. It shows how a mechanically defined scalar can induce a corridor-like subset of the  $\theta$ -grid, but it is *not* used to define any Phase 4  $\theta$ -filter or to suggest a preferred  $\theta_*$ .

**(3) FRW-inspired toy sanity checks.** The third rung embeds  $E_{\text{vac}}(\theta)$  into a simple, late-time FRW-inspired background. The script `phase4/src/phase4/run_f1_frw_toy_diagnostics.py` rescales the F1 vacuum curve into a toy dark-energy parameter  $\Omega_\Lambda(\theta)$  with  $\langle \Omega_\Lambda \rangle \approx 0.7$  at fixed  $\Omega_m = 0.3$ ,  $\Omega_r = 0$ . It then evaluates a schematic FRW quantity

$$H^2(a; \theta) = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda(\theta)$$

on a late-time scale-factor grid  $a \in [0.5, 1]$  and applies a simple sanity criterion:  $H^2(a; \theta)$  must stay positive and bounded in variation across that grid. This produces

- a JSON diagnostics file `phase4/outputs/tables/phase4_F1_frw_toy_diagnostics.json`, recording moments of  $E_{\text{vac}}$ ,  $\Omega_\Lambda(\theta)$ , the FRW parameters, and a “FRW-sanity fraction”; and
- a per-grid mask `phase4/outputs/tables/phase4_F1_frw_toy_mask.csv` with  $\theta$ ,  $\Omega_\Lambda(\theta)$ , and a Boolean indicator of FRW sanity.

This module is intentionally modest and strictly non-binding. It is used as a structural probe of how the F1 scalar can feed into FRW-like checks, and as an example of how empty-corridor outcomes (when the sanity fraction is small or zero) are recorded and analysed rather than hidden.

**(4) FRW viability scan with age and era constraints.** The fourth rung tightens the FRW-facing analysis into a more cosmology-like viability scan, while still remaining at the level of a toy model. The script `phase4/src/phase4/run_f1_frw_viability.py` again uses a mapping from  $E_{\text{vac}}(\theta)$  to  $\Omega_\Lambda(\theta)$  with  $\langle \Omega_\Lambda \rangle \approx 0.7$  at fixed  $\Omega_m = 0.3$ ,  $\Omega_r = 0$  and a toy Hubble parameter  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . For each grid point in  $\theta$  it then

1. computes a dimensionless FRW age integral  $t_0 H_0 = \int_0^1 da / (a \sqrt{H^2(a; \theta)})$  and converts it to a cosmic age  $t_0(\theta)$  in Gyr;
2. checks for the existence of a “matter era” in which the  $\Omega_m a^{-3}$  term dominates over both radiation and  $\Omega_\Lambda(\theta)$  on part of the  $a$ -grid;
3. checks for a “late-time acceleration” regime where the effective deceleration parameter becomes negative near  $a = 1$ ;
4. requires a smooth, positive  $H^2(a; \theta)$  on the chosen grid; and
5. demands that the resulting age lies in a broad window  $10 \text{ Gyr} \leq t_0(\theta) \leq 20 \text{ Gyr}$ .

The output is

- a diagnostics file `phase4/outputs/tables/phase4_F1_frw_viability_diagnostics.json` summarising the distribution of ages and the fractions of grid points satisfying each individual condition and the combined viability cut; and
- a per- $\theta$  mask `phase4/outputs/tables/phase4_F1_frw_viability_mask.csv` with columns `theta`, `E_vac`, `omega_lambda`, `age_Gyr`, individual Boolean flags (`has_matter_era`, `has_late_accel`, `smooth_H2`), and a combined `frw_viable` flag.

At the baseline settings used here, a non-trivial fraction of the grid ( $\approx 0.5$ ) satisfies all of these toy viability criteria. The exact numbers are not tuned to any observational data; they are treated as diagnostics of how the F1 mapping behaves under a cosmology-flavoured set of constraints.

**(5) Corridor extraction from the FRW viability mask.** The final rung in the current Phase 4 diagnostics compresses the FRW viability mask into contiguous “corridors” in  $\theta$ -space. The script `phase4/src/phase4/run_f1_frw_corridors.py` reads `phase4_F1_frw_viability_mask.csv`, interprets the `frw_viable` flag as a Boolean viability indicator, and groups successive viable grid points into connected components, allowing for a small tolerance on the grid spacing. For each corridor it records

- $\theta_{\min}, \theta_{\max}$ ;
- the number of included grid points; and
- an optional mean  $\langle \Omega_\Lambda \rangle$  over that corridor when the corresponding column is present.

The resulting corridors are written to

```
phase4/outputs/tables/phase4_F1_frw_corridors.json,
phase4/outputs/tables/phase4_F1_frw_corridors.csv.
```

The JSON file additionally highlights a *principal corridor* (selected as the one with the largest number of grid points) together with basic metadata such as the total grid size, the overall viable fraction, and the total number of corridors.

Even at this rung the construction remains explicitly non-binding: the corridors are treated as candidate bands emerging from a cosmology-flavoured viability scan, not as a canonical Phase 4  $\theta$ -filter or a claimed prediction of a preferred  $\theta_*$ . Their role is to show, in a fully reproducible way, how the Phase 3 mechanism, the F1 mapping, and simple FRW-inspired constraints can be chained into corridor-like structures that later, more physically ambitious work can interrogate or replace.

## FRW $\Lambda$ CDM-like probe

As a final FRW-facing diagnostic we place a broad “ $\Lambda$ CDM-like” window on top of the viability mask from the previous rung. Concretely, we take the per-grid FRW-viable rows from `phase4/outputs/tables/phase4_F1_frw_viability_mask.csv` and require, in addition:

- a vacuum fraction  $\Omega_\Lambda(\theta)$  within a toy tolerance window around a target  $\Omega_\Lambda^{\text{target}} \approx 0.7$ ; and
- a cosmological age in a toy window around  $t_0^{\text{target}} \approx 13.8$  Gyr, using the same FRW toy model as the viability rung ( $\Omega_m = 0.3$ ,  $\Omega_r = 0$ ,  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ).

The script `phase4/src/phase4/run_f1_frw_lcdm_probe.py` implements this probe, reading the FRW-viability mask and writing:

- a JSON summary `phase4/outputs/tables/phase4_F1_frw_lcdm_probe.json`, which records the target windows, the  $\theta$ -extent of the  $\Lambda$ CDM-like subset, and the corresponding grid fraction;
- a per-grid mask `phase4/outputs/tables/phase4_F1_frw_lcdm_probe_mask.csv`, which augments the FRW-viability columns with a Boolean `lcdm_like` flag.

In the current baseline configuration, this  $\Lambda$ CDM-like window selects a small but non-zero subset of the FRW-viable  $\theta$ -grid points. We stress that this remains a toy-level probe: it is not a fit to observational data and does not fix a preferred  $\theta_\star$ . Instead, it serves to demonstrate that the Phase 3 → Phase 4 pipeline admits FRW histories that are, at least in a broad sense, compatible with late-time  $\Lambda$ CDM-like cosmology.

Finally, we introduce a joined F1/FRW “shape probe” that ties together the toy F1 shape corridor, the FRW-viability scan, and the  $\Lambda$ CDM-like FRW probe on a common  $\theta$ -grid. The script `phase4/src/phase4/run_f1_frw_shape_probe.py` reads

- the toy F1 shape mask `phase4/outputs/tables/phase4_F1_shape_mask.csv` (columns `theta`, `E_vac`, `in_toy_corridor`);
- the FRW-viability mask `phase4/outputs/tables/phase4_F1_frw_viability_mask.csv` (columns `theta`, `E_vac`, `omega_lambda`, `age_Gyr` and the Boolean flags `has_matter_era`, `has_late_accel`, `smooth_H2`, `frw_viable`);
- the  $\Lambda$ CDM-like FRW probe mask `phase4/outputs/tables/phase4_F1_frw_lcdm_probe_mask.csv`, which adds a `lcdm_like` flag on top of the FRW-viability columns.

It joins these by the  $\theta$ -grid and writes

- a joined per- $\theta$  mask `phase4/outputs/tables/phase4_F1_frw_shape_probe_mask.csv` with columns `theta`, `E_vac`, `omega_lambda`, `age_Gyr`, `in_toy_corridor`, `frw_viable`, `lcdm_like`, and the composite flags `shape_and_viable`, `shape_and_lcdm`; and
- a JSON diagnostics summary `phase4/outputs/tables/phase4_F1_frw_shape_probe.json` recording the corresponding fractions and  $\theta$ -ranges.

In the current baseline configuration this reveals a small but non-zero overlap region where the toy F1 corridor, FRW-viable histories, and the  $\Lambda$ CDM-like windows coexist. As with the other FRW-facing diagnostics, this *remains non-binding*: it does not fix a unique  $\theta_\star$  and is used to make the Phase 3-to-Phase 4 interface auditable rather than to claim a physical fit to cosmological data.

To give a compact view of these overlaps, we also generate a summary plot using `phase4/src/phase4/plot_1` which reads the joined mask and produces a  $\theta-\Omega_\Lambda(\theta)$  scatter. Figure 1 shows all grid points, the FRW-viable subset, the  $\Lambda$ CDM-like subset, and the intersection where the toy F1 corridor and the  $\Lambda$ CDM-like conditions coincide.

**FRW data probe (optional).** Finally, we implemented a simple data-facing FRW probe that reuses the FRW viability mask and, when provided with external binned distance-redshift data, compares the model distance modulus  $\mu_{\text{model}}(z; \theta)$  to the data via a  $\chi^2$  statistic. The implementation lives in `phase4/src/phase4/run_f1_frw_data_probe.py`. It writes diagnostics to `phase4/outputs/tables/phase4_F1_frw_data_probe.json` and a per-grid mask to `phase4/outputs/tables/phase4_F1_frw_data_probe.csv` which extends the FRW viability mask with `chi2_data`, `chi2_per_dof`, and a Boolean `data_ok` flag. In the baseline repository configuration no external dataset is bundled at `phase4/data/external/frw_data.csv` the current run therefore records `data_available = false`, `n_data_points = 0`, and sets `data_ok = 0` for all grid points. This preserves full reproducibility while leaving a single, explicit hook for later data-level experiments. Consistent with the Phase 0 claim-discipline philosophy, this layer is strictly non-binding and does not introduce any new  $\theta$ -filter.

The data-facing probe implemented at this rung is therefore an explicit stub: if no external FRW distance data are provided under `phase4/data/external/`, the code records a structured negative result and sets `data_ok = 0` for all grid points rather than silently fabricating constraints. This makes it clear that the current Phase 4 layer exercises the shape and FRW-viability machinery, but does *not* yet attempt a genuine comparison to observational data.

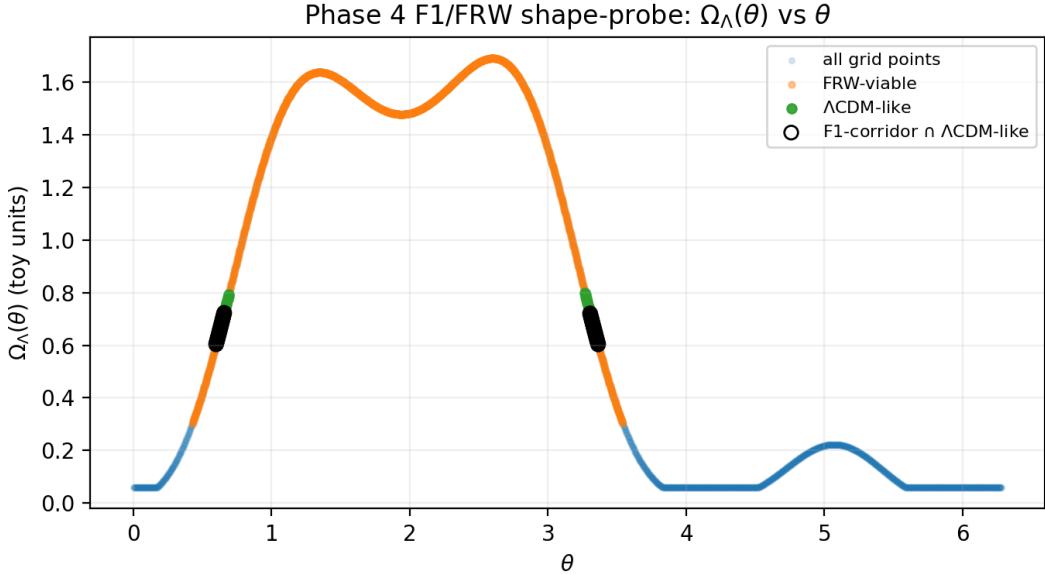


Figure 1: Phase 4 F1/FRW shape probe: toy F1 corridor, FRW-viable histories, and  $\Lambda$ CDM-like points on the common  $\theta$ -grid. The plot is generated from the joined mask `phase4_F1_frw_shape_probe_mask.csv`.

### 3 Limitations and scope of Phase 4

Phase 4 is intentionally modest in scope. Its purpose is to demonstrate how a simple mapping family and a small stack of diagnostics can be wired, in a fully reproducible way, on top of the Phase 3 vacuum mechanism. It is *not* a proposal for a definitive  $\theta$ -corridor, a preferred  $\theta_*$ , or a calibrated cosmological model. In this section we make explicit what the current Phase 4 construction does and does not claim.

#### 3.1 Structural vs. physical content

The F1 mapping family, the toy corridor, and the FRW-inspired module are all *structural* devices. They are designed to test whether the Phase 3 amplitudes can be converted into simple scalar observables in a way that is numerically sane, auditable, and easy to reproduce.

No part of the present Phase 4 implementation is calibrated to observational data or to a concrete field-theoretic model. The choices of mapping exponents, normalisations, and diagnostic thresholds are deliberately simple and should be read as examples of the *workflow*, not as physically motivated parameter inference.

#### 3.2 Status of $\theta$ -corridors and $\theta_*$

The F1 toy corridor is defined by a  $k_\sigma$ -style condition on  $E_{\text{vac}}(\theta)$ : points whose mapped scalar lies within a band of width  $k_\sigma$  times the empirical standard deviation around the minimum are marked as “inside”. This is a purely diagnostic definition. It is:

- *not* justified by an underlying measure on  $\theta$ -space;
- *not* tied to any observational likelihood; and
- *not* used to select a unique  $\theta_*$ .

At this rung we do not claim that the corridor has any physical significance beyond being a compact way of summarising the shape of  $E_{\text{vac}}(\theta)$ . The Phase 4 artifact therefore does not define a canonical  $\theta_*$ , nor does it promote any specific subset of  $\theta$ -values as physically preferred.

### 3.3 FRW-inspired toys and cosmology

The FRW-inspired module treats the F1 scalar as a proxy for a  $\theta$ -dependent vacuum-energy density  $\Omega_\Lambda(\theta)$ , choosing the normalisation so that the mean of  $\Omega_\Lambda(\theta)$  over the grid is close to a target value (e.g.  $\approx 0.7$ ). Together with fixed toy values of  $\Omega_m$  and  $\Omega_r$ , this defines a simple FRW-like quantity

$$H^2(a; \theta) = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda(\theta),$$

evaluated on a finite grid of scale factors  $a$ .

The diagnostic checks used here (positivity of  $H^2$  on the grid and a bound on the ratio of maximal to minimal  $H^2$  per  $\theta$ ) are deliberately crude. They are not meant to reproduce precision cosmology, nor to constrain  $\theta$  from data. Their role is to illustrate how one could connect the Phase 3 mechanism to FRW-like sanity checks and to record both positive and negative outcomes (such as empty-corridor cases) in a structured way.

### 3.4 Numerical and implementation limitations

All Phase 4 results are computed on finite grids: a finite  $N_\theta$ -grid for  $\theta$  and a finite  $N_a$ -grid for the scale factor in the FRW-inspired module. The diagnostics such as “corridor fraction” or “FRW-sanity fraction” are therefore *grid-level* summaries and may change under refinement of these grids.

The current implementation uses a single mapping family (F1) and a single baseline configuration inherited from Phase 3. There is no systematic exploration of alternative mapping families, floors, normalisations, or FRW toy parameters. The artifact should therefore be read as documenting *one concrete, reproducible instance* of the workflow, not as a survey of the available design space.

### 3.5 Future extensions and non-claims

Phase 4 is explicitly positioned as a bridge between the Phase 3 mechanism and future corridor work, not as the end-point of that programme. In particular:

- we do not claim to have derived a physically meaningful  $\theta$ -corridor;
- we do not claim to have identified a preferred  $\theta_*$ ; and
- we do not claim to have solved any cosmological fine-tuning problem.

What Phase 4 does claim is that:

1. the Phase 3 vacuum mechanism can be cleanly wired into a mapping family and scalar diagnostics;
2. the resulting workflow can be captured in a small, auditable code surface with explicit inputs and outputs; and
3. both positive and negative diagnostic outcomes (including empty-corridor cases) can be reported in a way that is aligned with the Phase 0 philosophy of structured, reproducible progress.

Any future tightening of the corridor notion, introduction of a candidate  $\theta_*$ , or calibration to observational data would require additional rungs beyond the present Phase 4 artifact and would have to make their own assumptions and limitations explicit.

**FRW-facing limitations.** The FRW-facing layers of Phase 4 (toy sanity checks, the viability scan, corridor construction, and the  $\Lambda$ CDM-like probe) are deliberately simple. They fix  $\Omega_m$ ,  $\Omega_r$ , and  $H_0$  by hand, impose broad box-shaped windows in age and  $\Omega_\Lambda$  rather than using likelihoods, and ignore radiation, baryons, curvature, and observational systematics. As such, they are best read as structured plausibility checks on the F1 mapping and on the Phase 3 mechanism, not as competitive cosmological fits. A full FRW analysis would require a more complete treatment of the background and of data, and lies beyond the scope of this phase.

## Appendix A: Phase 4 claims table (draft)

Table 1 summarises the intended Phase 4 claims. At this rung all entries are draft and non-binding.

Table 1: Draft Phase 4 claims. Binding status will be updated once the phase is complete and audited.

ID	Binding?	Summary
C4.1	no	Existence of at least one explicit mapping from the Phase 3 global amplitude or residue into an FRW-like or vacuum-energy-like observable with numerically stable behaviour.
C4.2	no	Existence of a non-empty, non-trivial $\theta$ -corridor for at least one such mapping.
C4.3	no	Structured negative result if all tested mappings yield empty or pathological corridors.

## A Reproducibility notes for Phase 4

This appendix records the concrete steps required to reproduce the Phase 4 diagnostics and the main paper artefact from a fresh clone of the repository. The emphasis is on (i) identifying the precise scripts and tables that underpin the claims in the main text and (ii) clarifying the logical flow from Phase 3 outputs to Phase 4 FRW-facing summaries.

### B.1. Prerequisites

Phase 4 is designed to sit on top of the Phase 3 mechanism. The following assumptions are therefore required before running the Phase 4 scripts:

- The repository is checked out at a Phase 4-compatible commit and a Python environment satisfying the documented dependencies (NumPy, SciPy, Matplotlib, etc.) is available.
- The Phase 3 baseline scan has been executed, producing the amplitude table and diagnostics JSON `phase3/outputs/tables/mech_baseline_scan_diagnostics.json` as in the Phase 3 paper.
- The `phase3_gate` script (or its Phase 3 equivalent) has been run to ensure that the baseline mechanism artefacts are present and consistent.

In a typical workflow one would run

```
bash scripts/phase3_gate.sh --level A
```

before entering the Phase 4 stage, so that all Phase 3 tables are present and up to date.

## B.2. Phase 4 mapping and diagnostics scripts

Phase 4 introduces a single mapping family (F1) and layered diagnostics built on top of it. The core scripts are:

### (1) F1 scalar construction and basic diagnostics.

- **Script:** `phase4/src/phase4/run_f1_sanity.py`
- **Inputs:** Phase 3 diagnostics JSON `phase3/outputs/tables/mech_baseline_scan_diagnostics.json` (for the amplitude range and quantile-based floor) and the corresponding amplitude table.
- **Outputs:**
  - `phase4/outputs/tables/phase4_F1_sanity_curve.csv`, containing  $\theta$  and  $E_{\text{vac}}(\theta)$  for the F1 mapping with  $\alpha = 1$ ,  $\beta = 4$ ;
  - `phase4/outputs/tables/phase4_F1_sanity_curve_diagnostics.json`, recording global extrema, basic moments, and configuration metadata.

### (2) F1 shape diagnostics and toy corridor.

- **Script:** `phase4/src/phase4/run_f1_shape_diagnostics.py`
- **Inputs:** `phase4_F1_sanity_curve.csv`.
- **Outputs:**
  - `phase4/outputs/tables/phase4_F1_shape_diagnostics.json`, a JSON summary of the F1 scalar, including a toy one- $\sigma$ -style corridor definition;
  - `phase4/outputs/tables/phase4_F1_shape_mask.csv`, a per- $\theta$  CSV with Boolean `in_toy_corridor` flags.

### (3) FRW-inspired toy diagnostics.

- **Script:** `phase4/src/phase4/run_f1_frw_toy_diagnostics.py`
- **Inputs:** `phase4_F1_sanity_curve.csv` and the chosen toy parameters  $\Omega_m$ ,  $\Omega_r$ , a target mean  $\langle \Omega_\Lambda \rangle$ , and a scale-factor window  $[a_{\min}, a_{\max}]$ .
- **Outputs:**
  - `phase4/outputs/tables/phase4_F1_frw_toy_diagnostics.json`, which records global extrema and moments of  $E_{\text{vac}}(\theta)$  and the toy  $\Omega_\Lambda(\theta)$ , along with a simple FRW-sanity fraction;
  - `phase4/outputs/tables/phase4_F1_frw_toy_mask.csv`, a per- $\theta$  mask with  $\Omega_\Lambda(\theta)$  and a Boolean FRW-sanity indicator. In the baseline configuration the FRW toy is treated as a non-binding diagnostic only.

### (4) FRW viability scan.

- **Script:** `phase4/src/phase4/run_f1_frw_viability.py`
- **Inputs:** `phase4_F1_sanity_curve.csv` and a choice of FRW parameters ( $\Omega_m$ ,  $\Omega_r$ ,  $H_0$ ) as well as an age window  $[t_{\min}, t_{\max}]$ .
- **Outputs:**

- `phase4/outputs/tables/phase4_F1_frw_viability_diagnostics.json`, recording basic statistics of the FRW histories across the  $\theta$ -grid, the fraction of points with a matter-dominated era, late-time acceleration, smooth  $H^2(a; \theta)$ , and an age within the specified window;
- `phase4/outputs/tables/phase4_F1_frw_viability_mask.csv`, a per- $\theta$  mask with columns `theta`, `E_vac`,  $\Omega_\Lambda(\theta)$ , the inferred age in Gyr, Boolean flags (`has_matter_era`, `has_late_accel`, `smooth_H2`), and a combined `frw_viable` flag.

## (5) Corridor extraction on the FRW-viable subset.

- **Script:** `phase4/src/phase4/run_f1_frw_corridors.py`
- **Inputs:** `phase4_F1_frw_viability_mask.csv`.
- **Outputs:**
  - `phase4/outputs/tables/phase4_F1_frw_corridors.json`, a JSON summary of contiguous “corridors” of FRW-viable  $\theta$ -values, including a principal corridor with its  $\theta$ -extent and basic statistics of  $\Omega_\Lambda(\theta)$ ;
  - `phase4/outputs/tables/phase4_F1_frw_corridors.csv`, a per-corridor CSV listing the  $\theta$ -ranges and sizes.

## (6) $\Lambda$ CDM-like probe.

- **Script:** `phase4/src/phase4/run_f1_frw_lcdm_probe.py`
- **Inputs:** `phase4_F1_frw_viability_mask.csv` and a broad target window for  $\Omega_\Lambda$  (e.g.  $0.7 \pm 0.1$ ) and the age of the Universe (e.g.  $13.8 \pm 1.0$  Gyr), with the same FRW parameters as the viability scan.
- **Outputs:**
  - `phase4/outputs/tables/phase4_F1_frw_lcdm_probe.json`, which records the fraction of FRW-viable grid points that fall into this broad  $\Lambda$ CDM-like window, together with the  $\theta$ -extent and the ranges of  $\Omega_\Lambda(\theta)$  and age across the selected subset;
  - `phase4/outputs/tables/phase4_F1_frw_lcdm_probe_mask.csv`, a per- $\theta$  mask augmenting the FRW-viability columns with a Boolean `lcdm_like` flag.

In the baseline configuration this probe is explicitly non-binding: it is used to demonstrate the existence of  $\theta$ -regions with broadly  $\Lambda$ CDM-like FRW histories, not to select a unique  $\theta_*$  or to fit observational data.

## (7) Joint shape/FRW probe.

- **Script:** `phase4/src/phase4/run_f1_frw_shape_probe.py`
- **Inputs:** `phase4_F1_shape_mask.csv`, `phase4_F1_frw_viability_mask.csv`, and `phase4_F1_frw_lcdm_pr...`
- **Outputs:**
  - `phase4/outputs/tables/phase4_F1_frw_shape_probe.json`, which reports the fractions and  $\theta$ -ranges associated with the toy shape corridor, the FRW-viable subset, the  $\Lambda$ CDM-like subset, and their intersections;
  - `phase4/outputs/tables/phase4_F1_frw_shape_probe_mask.csv`, a joined mask that carries the shape, FRW-viability, and  $\Lambda$ CDM-like flags on a common  $\theta$ -grid.

Figure ?? in the main text is derived from these joined tables.

## (8) Data-facing probe (stub).

- **Script:** `phase4/src/phase4/run_f1_frw_data_probe.py`
- **Inputs:** `phase4_F1_frw_viability_mask.csv` and an optional external binned-distance data file `phase4/data/external/frw_distance_binned.csv`.
- **Outputs:**
  - `phase4/outputs/tables/phase4_F1_frw_data_probe.json`, a diagnostics file recording whether external data were found, the number of data points, and—if present—summary  $\chi^2$  statistics;
  - `phase4/outputs/tables/phase4_F1_frw_data_probe_mask.csv`, a per- $\theta$  mask with a Boolean `data_ok` flag.

In the baseline repository checkout no external FRW distance data are bundled, so the script records `data_available = false` and sets `data_ok = 0` for all grid points. This is an explicit, structured negative result rather than an implicit limitation: it demonstrates how data-level tests will be integrated in later work without making any present data-driven claims.

## B.3. Recommended execution flow

Putting the pieces together, a minimal end-to-end reproduction of the Phase 4 artefact proceeds as follows:

1. Run the Phase 3 gate at an appropriate level, e.g.

```
bash scripts/phase3_gate.sh --level A
```

to ensure the baseline mechanism scan and diagnostics are present.

2. Execute the Phase 4 scripts in the logical order described above, from F1 sanity (`run_f1_sanity.py`) through shape diagnostics, FRW toy, FRW viability, FRW corridors, the  $\Lambda$ CDM-like probe, the joint shape/FRW probe, and finally the optional data-facing stub.
3. Regenerate the Phase 4 paper and canonical artefact via

```
bash scripts/phase4_gate.sh --level A
```

which runs `latexmk` on `phase4/paper/main.tex` and copies the resulting `main.pdf` to `phase4/outputs/paper/phase4_paper.pdf` and `phase4/artifacts/origin-axiom-phase4.pdf`.

The claims made in the main text and in the Phase 4 claims table are deliberately modest: they refer only to the existence and behaviour of these diagnostics and corridors *within this pipeline*. No attempt is made to extract precise cosmological parameter values or to promote any diagnostic into a binding filter on the origin–axiom mechanism.

## References