Phase 4: Vacuum-to-FRW Consistency and Scale Sanity

A corridor-style test of the Phase 3 global-amplitude mechanism

Origin Axiom Program

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Abstract

Phase 4 tests whether the canonical global-amplitude mechanism defined in Phase 3 can be connected, in a structurally reasonable way, to FRW-style dynamics and vacuum-energy-like observables. The goal is to probe whether the non-cancellation floor that stabilises the toy vacuum can also support scale-sane behaviour when mapped into simple cosmological modules, without claiming a full cosmological fit or a theory of everything.

1 Introduction

Phase 4 sits between the mechanism-level vacuum work of Phase 3 and any future attempts to build a unified picture of vacuum, matter, and geometry. Its mission is narrow:

- take the canonical Phase 3 global-amplitude mechanism and non-cancellation floor as given;
- define simple, explicit mappings from the floor-enforced amplitude or residue into FRW-like dynamics and vacuum-energy-like observables; and
- study the resulting behaviour of the Origin-Axiom phase parameter θ in the corridor / ledger framework of Phase 0.

The guiding question is not whether we can reproduce the full ΛCDM model or fit precise cosmological parameters, but whether the Phase 3 mechanism can be made compatible with toy FRW modules in a way that is numerically stable, structurally sane, and expressible as a θ-filter in the sense of Phase 0.

Throughout this phase we distinguish carefully between:

- *binding* outputs, which may eventually define a Phase 4 θ-filter; and
- *non-binding* diagnostics and figures, which serve only as intuition and internal checks.

The present rung does not define any concrete mappings or filters. It only provides a minimal paper skeleton so that future rungs can add well-documented mechanisms and experiments without restructuring the front matter.

2 Mapping families: first pass (F1)

Phase 4 takes as input the Phase 3 global-amplitude mechanism: a toy vacuum with an unconstrained observable $A_0(\theta)$, a non-cancellation floor ε, and a floor-enforced amplitude $A(\theta) = \max(A_0(\theta), \varepsilon)$ defined on a grid $\theta \in [0, 2\pi)$. The present paper introduces a first, explicit mapping family, denoted **F1**, from this structure to a toy vacuum-energy-like scalar.

2.1 F1: direct scalar mapping from $A(\theta)$

The F1 family is intentionally simple. For a fixed Phase 3 vacuum configuration and floor ε (taken from the Phase 3 baseline diagnostics), we define a scalar

$$E_{\mathrm{vac}}(\theta) \;=\; \alpha\, A(\theta)^{\beta}, \tag{1}$$

where $\alpha > 0$ and $\beta > 0$ are explicit, configurable parameters. At this rung we adopt a conservative default, $\alpha = 1$ and $\beta = 2$, and focus on structural behaviour rather than numerical normalisation.

Operationally, Phase 4 reuses the Phase 3 baseline configuration `baseline_v1` and the floor ε recorded in `phase3/outputs/tables/mech_baseline_scan_diagnostics.json`. We then evaluate $A(\theta)$ and $E_{\mathrm{vac}}(\theta)$ on a uniform grid of $N_\theta = 2048$ points in $[0, 2\pi)$. The per-grid values and summary diagnostics are written to

`phase4/outputs/tables/phase4_F1_sanity_curve.csv`,

together with metadata describing the mapping parameters and the underlying Phase 3 diagnostics.

At this stage F1 is a *non-binding* mapping family: it does not yet define a θ-corridor or a Phase 4 θ-filter. Instead it serves as a concrete, auditable bridge between the Phase 3 mechanism and simple scalar observables that later rungs can connect to FRW-like toy modules and corridor construction.

3 Diagnostics and toy corridors

Phase 4 currently provides three layers of diagnostics built on the Phase 3 vacuum mechanism and the F1 mapping family. All of them are explicitly non-binding: they are used for internal checks and illustration, not as canonical θ-filters.

3.1 F1 sanity curve

The first layer is a direct sanity check of the F1 mapping. Using the Phase 3 baseline configuration and the quantile-based floor recorded in `phase3/outputs/tables/mech_baseline_scan_diagnostics.jso` we evaluate the floor-enforced amplitude $A(\theta)$ on a uniform grid of $N_\theta = 2048$ points in $[0, 2\pi)$ and construct a toy vacuum-energy-like scalar

$$E_{\mathrm{vac}}(\theta) = \alpha A(\theta)^{\beta},$$

with $\alpha = 1$ and $\beta = 4$ at this rung. The script `phase4/src/phase4/run_f1_sanity.py` writes the per-grid values and metadata to

`phase4/outputs/tables/phase4_F1_sanity_curve.csv`,

which establishes that $E_{\mathrm{vac}}(\theta)$ is strictly positive, numerically well-behaved, and tied cleanly to the Phase 3 baseline diagnostics.

3.2 F1 shape diagnostics and toy corridor

The second layer probes the *shape* of $E_{\mathrm{vac}}(\theta)$ by defining a toy, non-binding corridor in the space of scalar values. The script `phase4/src/phase4/run_f1_shape_diagnostics.py` constructs a mask based on

$$E_{\mathrm{vac}}(\theta) \le E_{\mathrm{vac,min}} + k_\sigma \sigma, \quad k_\sigma = 1,$$

where $E_{\mathrm{vac,min}}$ and σ are the global minimum and standard deviation of the F1 curve. This is intentionally weak: it retains a substantial fraction of the θ-grid while discarding the largest excursions.

The script writes a summary diagnostics file `phase4/outputs/tables/phase4_F1_shape_diagnostics.json`

and a per-grid mask

`phase4/outputs/tables/phase4_F1_shape_mask.csv`,

which later rungs and toy modules can reuse. At this stage the mask is explicitly described as a *toy corridor*: it does not define a canonical $\theta_\star$ and is not elevated to a Phase 4 filter.

3.3 FRW-inspired toy diagnostics

The third layer introduces a minimal FRW-inspired toy diagnostic that treats $E_{\text{vac}}(\theta)$ as a source for a *dimensionless* vacuum term in a simple Hubble-like quantity. From the sanity curve we define

$$\tilde{E}_{\text{vac}}(\theta) = \frac{E_{\text{vac}}(\theta)}{\langle E_{\text{vac}} \rangle}$$

and rescale it to a toy $\Omega_\Lambda(\theta)$ with mean ≈ 0.7. Together with fixed, dimensionless matter and radiation parameters $\Omega_m = 0.3$ and $\Omega_r = 0$, we evaluate

$$H^2(a;\theta) = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda(\theta)$$

on a late-time scale-factor grid $a \in [0.5, 1]$. No physical units are assigned and no claim is made that these numbers match observations.

The script `phase4/src/phase4/run_f1_frw_toy_diagnostics.py` writes:

- a JSON summary `phase4/outputs/tables/phase4_F1_frw_toy_diagnostics.json`, recording global extrema and moments of E_{vac}, $\Omega_\Lambda(\theta)$, the chosen Ω-parameters, and a simple FRW-sanity fraction; and
- a per-grid mask `phase4/outputs/tables/phase4_F1_frw_toy_mask.csv`, containing θ, $\Omega_\Lambda(\theta)$, and a Boolean indicator of whether $H^2(a;\theta)$ stays positive and within a bounded variation factor over the scale-factor grid.

Depending on the chosen toy parameters and sanity criterion, the resulting FRW-sanity fraction can be small or even zero; such empty-corridor outcomes are treated as informative negative results, recorded in the diagnostics and logs rather than hidden. In all cases the FRW-inspired module remains strictly non-binding: it does not fix a preferred $\theta_\star$ and does not, by itself, define a Phase 4 θ-filter.

FRW-inspired viability mask on F1

Beyond the FRW-inspired toy sanity check described above, we also define a simple FRW *viability* diagnostic on the F1 mapping. Here we keep the same background ansatz

$$H^2(a;\theta) = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda(\theta),$$

with fixed $\Omega_m = 0.3$, $\Omega_r = 0$, and a Hubble parameter $H_0 = 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. For every θ on the F1 grid we:

- rescale the F1 scalar $E_{\text{vac}}(\theta)$ into a toy $\Omega_\Lambda(\theta)$ whose mean matches a target $\langle \Omega_\Lambda \rangle \approx 0.7$;
- integrate a standard FRW age integral $t_0(\theta)$ over a scale-factor grid $a \in [a_{\min}, a_{\max}]$ and require the resulting cosmic age to lie in a coarse window $t_0 \in [10, 20]$ Gyr;
- check for a clear matter-dominated epoch and a late-time accelerating regime in which the deceleration parameter becomes negative; and

- enforce a smoothness criterion on $H^2(a;\theta)$ by bounding the ratio of its maximal to minimal value over the grid.

The script `phase4/src/phase4/run_f1_frw_viability.py` evaluates these conditions over the full F1 grid, writing

- a diagnostic summary `phase4/outputs/tables/phase4_F1_frw_viability_diagnostics.json`, including the age window, global moments of $t_0(\theta)$, and the fractions of θ-points that satisfy each FRW condition; and
- a per-grid mask `phase4/outputs/tables/phase4_F1_frw_viability_mask.csv`, with columns θ, $\Omega_\Lambda(\theta)$, and a Boolean viability indicator.

In the current baseline configuration we find that essentially all θ-points satisfy the age window, matter-era, and smoothness requirements, while a non-trivial subset (roughly half of the F1 grid) additionally exhibits late-time acceleration. The resulting viability mask therefore singles out a first, explicitly FRW-flavoured subset of θ-values as *toy-viable* cosmological histories. This mask is not yet promoted to a canonical Phase 4 θ-filter, but it plays a key role as a concrete, physics-facing bridge between the Phase 3 mechanism and simple FRW-like observables.

4 Limitations and scope of Phase 4

Phase 4 is intentionally modest in scope. Its purpose is to demonstrate how a simple mapping family and a small stack of diagnostics can be wired, in a fully reproducible way, on top of the Phase 3 vacuum mechanism. It is *not* a proposal for a definitive θ-corridor, a preferred $\theta_\star$, or a calibrated cosmological model. In this section we make explicit what the current Phase 4 construction does and does not claim.

4.1 Structural vs. physical content

The F1 mapping family, the toy corridor, and the FRW-inspired module are all *structural* devices. They are designed to test whether the Phase 3 amplitudes can be converted into simple scalar observables in a way that is numerically sane, auditable, and easy to reproduce.

No part of the present Phase 4 implementation is calibrated to observational data or to a concrete field-theoretic model. The choices of mapping exponents, normalisations, and diagnostic thresholds are deliberately simple and should be read as examples of the *workflow*, not as physically motivated parameter inference.

4.2 Status of θ-corridors and $\theta_\star$

The F1 toy corridor is defined by a k_σ-style condition on $E_{\rm vac}(\theta)$: points whose mapped scalar lies within a band of width k_σ times the empirical standard deviation around the minimum are marked as "inside". This is a purely diagnostic definition. It is:

- *not* justified by an underlying measure on θ-space;
- *not* tied to any observational likelihood; and
- *not* used to select a unique $\theta_\star$.

At this rung we do not claim that the corridor has any physical significance beyond being a compact way of summarising the shape of $E_{\rm vac}(\theta)$. The Phase 4 artifact therefore does not define a canonical $\theta_\star$, nor does it promote any specific subset of θ-values as physically preferred.

4.3 FRW-inspired toys and cosmology

The FRW-inspired module treats the F1 scalar as a proxy for a θ-dependent vacuum-energy density $\Omega_\Lambda(\theta)$, choosing the normalisation so that the mean of $\Omega_\Lambda(\theta)$ over the grid is close to a target value (e.g. ≈ 0.7). Together with fixed toy values of Ω_m and Ω_r, this defines a simple FRW-like quantity

$$H^2(a;\theta) = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda(\theta),$$

evaluated on a finite grid of scale factors a.

The diagnostic checks used here (positivity of H^2 on the grid and a bound on the ratio of maximal to minimal H^2 per θ) are deliberately crude. They are not meant to reproduce precision cosmology, nor to constrain θ from data. Their role is to illustrate how one could connect the Phase 3 mechanism to FRW-like sanity checks and to record both positive and negative outcomes (such as empty-corridor cases) in a structured way.

4.4 Numerical and implementation limitations

All Phase 4 results are computed on finite grids: a finite N_θ-grid for θ and a finite N_a-grid for the scale factor in the FRW-inspired module. The diagnostics such as "corridor fraction" or "FRW-sanity fraction" are therefore *grid-level* summaries and may change under refinement of these grids.

The current implementation uses a single mapping family (F1) and a single baseline configuration inherited from Phase 3. There is no systematic exploration of alternative mapping families, floors, normalisations, or FRW toy parameters. The artifact should therefore be read as documenting *one concrete, reproducible instance* of the workflow, not as a survey of the available design space.

4.5 Future extensions and non-claims

Phase 4 is explicitly positioned as a bridge between the Phase 3 mechanism and future corridor work, not as the end-point of that programme. In particular:

- we do not claim to have derived a physically meaningful θ-corridor;
- we do not claim to have identified a preferred $\theta_\star$; and
- we do not claim to have solved any cosmological fine-tuning problem.

What Phase 4 does claim is that:

1. the Phase 3 vacuum mechanism can be cleanly wired into a mapping family and scalar diagnostics;
2. the resulting workflow can be captured in a small, auditable code surface with explicit inputs and outputs; and
3. both positive and negative diagnostic outcomes (including empty-corridor cases) can be reported in a way that is aligned with the Phase 0 philosophy of structured, reproducible progress.

Any future tightening of the corridor notion, introduction of a candidate $\theta_\star$, or calibration to observational data would require additional rungs beyond the present Phase 4 artifact and would have to make their own assumptions and limitations explicit.

Appendix A: Phase 4 claims table (draft)

Table 1 summarises the intended Phase 4 claims. At this rung all entries are draft and non-binding.

Table 1: Draft Phase 4 claims. Binding status will be updated once the phase is complete and audited.

ID	Binding?	Summary
C4.1	no	Existence of at least one explicit mapping from the Phase 3 global amplitude or residue into an FRW-like or vacuum-energy-like observable with numerically stable behaviour.
C4.2	no	Existence of a non-empty, non-trivial θ-corridor for at least one such mapping.
C4.3	no	Structured negative result if all tested mappings yield empty or pathological corridors.

Appendix B: Reproducibility notes (draft)

At this rung, Phase 4 builds on the Phase 3 toy vacuum mechanism and its non-cancellation floor, introduces the F1 mapping family, and adds internal diagnostics and a toy, non-binding corridor. This appendix summarises how to rebuild the Phase 4 artifacts used in the paper.

Directory structure

Phase 4 lives in the top-level directory `phase4/` with the following relevant subdirectories:

- `phase4/paper/`: LaTeX sources for the Phase 4 paper (including this appendix).
- `phase4/src/phase4/`: Python source for the F1 mapping family and diagnostics.
- `phase4/outputs/`:
  - `outputs/paper/`: built Phase 4 PDF;
  - `outputs/tables/`: CSV/JSON artifacts from F1 diagnostics.
- `phase4/artifacts/`: canonical Phase 4 PDF artifact exported by the gate script.

Upstream Phase 3 prerequisites

Phase 4 assumes that the Phase 3 baseline scan and binding-certificate artifacts have been generated at least once. In practice this can be ensured by running, from the repository root:

```
bash scripts/phase3_gate.sh --level A
```

and, if needed, explicitly running the Phase 3 scripts:

```
python phase3/src/phase3_mech/run_baseline_scan.py
python phase3/src/phase3_mech/run_binding_certificate.py
```

These commands populate:

- phase3/outputs/tables/mech_baseline_scan.csv
- phase3/outputs/tables/mech_baseline_scan_diagnostics.json
- phase3/outputs/tables/mech_binding_certificate.csv
- phase3/outputs/tables/mech_binding_certificate_diagnostics.json

The F1 mapping family reads mech_baseline_scan_diagnostics.json to reuse the quantile-based floor and grid configuration.

Phase 4 F1 mapping and diagnostics

With the Phase 3 prerequisites in place, the current Phase 4 diagnostics can be rebuilt via:

```
python phase4/src/phase4/run_f1_sanity.py
python phase4/src/phase4/run_f1_shape_diagnostics.py
```

These scripts write:

- phase4/outputs/tables/phase4_F1_sanity_curve.csv (per-θ values of $E_{\mathrm{vac}}(\theta)$ and associated metadata);
- phase4/outputs/tables/phase4_F1_shape_diagnostics.json (summary statistics and toy corridor definition);
- phase4/outputs/tables/phase4_F1_shape_mask.csv (per-θ Boolean mask indicating membership in the toy corridor).

At this rung these diagnostics are explicitly non-binding: they do not define a canonical θ-filter, but they are used in the Phase 4 paper to illustrate the behaviour of the F1 mapping.

Building the Phase 4 paper and artifact

The Phase 4 paper and its canonical PDF artifact can be rebuilt from the repository root via:

```
bash scripts/phase4_gate.sh --level A
```

This gate script runs a Snakemake workflow in phase4/workflow/Snakefile that invokes latexmk on phase4/paper/main.tex and copies the resulting PDF to:

- phase4/outputs/paper/phase4_paper.pdf
- phase4/artifacts/origin-axiom-phase4.pdf

The gate currently covers the Phase 4 paper only; F1 diagnostics are rebuilt by the explicit Python commands above.

Planned extensions

The design note `phase4/FRW_TOY_DESIGN.md` specifies a minimal FRW-inspired toy module that may be implemented in later rungs. If such a module is added, this appendix will be extended to include:

- the relevant Python scripts and configuration files;
- any additional JSON/CSV outputs; and
- updated gate levels if FRW-like diagnostics become part of the canonical Phase 4 artifact.

Until then, reproducibility for Phase 4 consists of:

1. regenerating the Phase 3 baseline and binding-certificate artifacts;
2. regenerating the Phase 4 F1 diagnostics; and
3. rebuilding the Phase 4 paper and artifact via the Level A gate.

References