Origin Axiom C:
The universe as a cancellation system
Act II: lattice sanity checks and null results

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Abstract

Papers A and B formulated the Origin Axiom as a structural non–cancellation rule, implemented it in a minimal complex scalar field on a periodic lattice, and demonstrated that a hard constraint $|A(C)| \geq \varepsilon$ can operate without spoiling basic dynamics or energy conservation. Paper C reframes the same framework as a *cancellation system*: a playground where we look explicitly for places the axiom could have gone wrong, or where it might already be secretly built into ordinary physics. This Act II collects the current battery of lattice sanity checks and records the strongest statement we can honestly make at this stage: for the class of tests implemented in `src/` and `notebooks/`, the non–cancelling rule behaves as a small, controllable perturbation and does *not* produce dramatic or unstable effects. The goal of the paper is not to claim success, but to make the "null results" legible and reproducible.

1 Introduction: from axiom to cancellation system

Paper A motivates the Origin Axiom from the incoherence of absolute nothingness and formalises it as a constraint on a global complex amplitude $A(C)$ over configuration space: physically realised configurations avoid a small neighbourhood of a reference value A^* (typically $A^* = 0$) and obey

$$|A(C)| \geq \varepsilon, \tag{1}$$

for some non–cancellation scale $\varepsilon > 0$. Paper B implements this rule in a minimal complex scalar toy universe on a periodic 3–torus and shows that, for small ε, the constrained evolution remains numerically well–behaved.

The present work (Paper C) takes a complementary viewpoint. Instead of asking "what phenomena does the axiom explain?" we first ask the more basic question: *can we embed the rule into increasingly nontrivial systems without breaking anything obvious?* In other words, we treat the framework as a *cancellation system* and stress–test it with a set of deliberately modest lattice experiments.

Practically, this means:

- We construct pairs of simulations: one purely standard (*free*), one with a non–cancelling constraint (*constrained*) applied to the same underlying equations of motion.
- We fix parameters so that both runs live in a numerically safe regime.
- We measure quantities that would be especially sensitive to any hidden bias or instability: vacuum energies, mode frequencies, localisation fractions, and energy flow between coupled fields.

- We treat strong deviations as *failures* of the current implementation and record them honestly.

This Act II documents the tests we have completed so far, all implemented in the public repository `originaxiom/origin-axiom`. Each subsection below maps directly to a script in `src/` and an analysis notebook in `notebooks/`; figures shown here are generated from cached outputs in `data/processed/` and mirrored into `figures/`.

2 Numerical testbed summary

We briefly recap the ingredients that are common to all experiments:

- Spatial discretisation: one– or three–dimensional periodic lattices with N sites per dimension.
- Time evolution: leapfrog / staggered–in–time scheme for scalar fields with mass terms and, where indicated, quartic self–interaction.
- Non–cancelling rule: we monitor a chosen global complex amplitude $A(t)$ and, whenever $|A(t)| < \varepsilon$, we project back onto the boundary $|A| = \varepsilon$ by a minimal rescaling of the relevant degrees of freedom.
- Diagnostics: total energy, mode spectra, localisation measures, and simple summary statistics of constraint hits.

For detailed derivations of the discrete equations of motion and baseline stability analysis, we refer back to Paper B. Here we focus purely on the *comparative* behaviour of free vs. constrained runs.

3 Twisted 1D vacua: looking for hidden $\theta_\star$ structure

3.1 Plain twisted chain

Our first class of tests probes the vacuum energy of a free massive scalar on a 1D periodic chain with an imposed boundary twist. The code lives in `src/run_1d_twisted_vacuum_scan.py`, with analysis in `notebooks/02_1d_twisted_analysis.py`.

We consider $N = 256$ sites, mass $m_0 = 0.1$, and a twist angle $\theta_\star \in [0, 2\pi]$ implemented as a phase on the boundary link. For each $\theta_\star$ we diagonalise the lattice Hamiltonian numerically and compute the (discretised) vacuum energy $E_0(\theta_\star)$. The Origin Axiom would be in immediate tension with a strongly $\theta_\star$–dependent ground state, since our framework is designed to be as agnostic as possible to any micro–choice of "twist".

The result is a clean null test: within numerical precision,

$$E_0(\theta_\star) \approx \text{const.} \tag{2}$$

over the full scan, with fluctuations consistent with the eigensolver tolerance. The plot `twisted_1d_E0_vs_theta` shows a flat line; the corresponding residual $\Delta E(\theta_\star)$ oscillates around zero at the 10^{-13} level. This reassures us that the lattice itself does not secretly imprint a preferred twist—exactly as it should.

3.2 Defected bond

To stress the system slightly harder we repeat the scan with a single defect bond of strength $0 < \alpha < 1$, implemented in `src/run_1d_defected_vacuum_scan.py` and analysed in `notebooks/02b_1d_defected_tw`
The defect breaks translation invariance and introduces a local scale, so any microscopic conspiracy between the twist and the defect would show up here first.

Again, the vacuum energy $E_0(\theta_\star)$ is numerically flat, with variations at the 10^{-13} level. The non–cancelling rule is *not* active in these tests; their role is to confirm that our discretisation and numerics do not accidentally manufacture a $\theta_\star$–dependent vacuum that could later be misinterpreted as a signature of the axiom.

4 Mode–by–mode tests: constrained oscillators

The next step is to ask whether the non–cancelling rule distorts individual lattice modes in a detectable way. We couple the rule directly to a chosen Fourier mode in `src/run_mode_spectrum_with_constraint`
with analysis in `notebooks/06_mode_spectrum_analysis.py`.

We initialise a tiny sinusoidal perturbation in mode $k = 1$ on a 1D chain and evolve it both with and without a constraint on the global amplitude $A(t)$, using $\varepsilon = 10^{-3}$. From the time series we extract the dominant frequency ω_{num} and compare it to the analytic lattice dispersion relation.

The key observations are:

- The free and constrained time series are visually almost indistinguishable; their Fourier power spectra peak at the same ω_{num}.
- The numerical frequency deviates slightly from the analytic one, but the deviation is identical in both runs and is attributable to the finite time step and windowing, not to the constraint.
- No extra sidebands or secular drifts appear when the non–cancelling rule is active at this small ε.

Within this limited setup the axiom behaves like a gentle projection on the global configuration, not as a new dynamical force on individual modes.

5 Localised bumps in 1D and 3D

5.1 1D propagation and quasi–localisation

We then test whether the constraint can materially change the spreading of a localised excitation. The 1D experiment is implemented in `src/run_localized_bump_1d.py` and `notebooks/07_localized_bump_ana`
Initial conditions: a Gaussian bump of amplitude $A = 0.1$ and width $W = 10$, centred on a 1D lattice of $N = 512$ sites. We evolve for $T = 2000$ time steps with and without the non–cancelling rule (again at $\varepsilon = 10^{-3}$) and track the fraction of $|\phi|^2$ contained in a fixed "central window" around the origin.

The main findings:

- In both runs the bump disperses and re–focuses in a sequence of quasi–recurrences set by the dispersion relation.
- The localisation fraction as a function of time is very similar in the free and constrained cases; the constrained curve sits slightly below the free one as expected, since a small amount of weight is continually nudged into a uniform background.
- No evidence of spontaneous self–trapping or anomalous long–lived localisation emerges at these parameters.

5.2 3D bump with self–interaction

To get closer to the intended "toy universe" we repeat the exercise in 3D with a quartic self–interaction. The code is in `src/run_localized_bump_3d.py` and `notebooks/08_localized_bump_analysis_3d.`

Here we evolve a Gaussian bump of amplitude $A \simeq 0.3$ and width $W \simeq 4$ on a 40^3 lattice, with mass $m_0 = 0.5$ and $\lambda_4 = 1$. We again compare free vs. constrained evolutions and measure the fraction of $|\phi|^2$ inside a central sphere.

Snapshots of the central z–slice (stored as `localized_bump_3d_slices.png`) show spherical shells expanding and interfering in both cases. The localisation fraction decays in an irregular but correlated way between free and constrained runs. At this stage there is no sign that the non–cancelling rule produces oscillons, solitons, or other exotic long–lived structures on its own.

5.3 Parameter scan

To make sure we are not cherry–picking a benign corner of parameter space, we perform a coarse scan over amplitudes, widths, self–couplings and non–cancellation scales using `src/run_localized_bump_3d_sca` and `notebooks/09_localized_bump_scan_analysis_3d.py`.

The scan currently covers:

- amplitudes $A \in \{0.2, 0.3\}$,
- widths $W \in \{3, 5\}$,
- $\lambda_4 \in \{0.5, 1.0\}$,
- $\varepsilon \in \{10^{-3}, 5 \times 10^{-3}\}$.

For each point we evolve for a fixed time and record the final localisation fraction in the central sphere, building 2D maps of localisation vs. width and amplitude, with separate panels for free and constrained runs.

Within this coarse grid:

- Regions of higher final localisation are shared between free and constrained systems; the constraint slightly lowers the absolute values but does not introduce qualitatively new behaviour.
- There is no obvious parameter island where the constrained system localises while the free system disperses completely, or vice versa.

6 Two–field coupling: energy sharing as a probe

6.1 Homogeneous coupled fields

A simple but sensitive test of the axiom is to let it act on a *combination* of fields instead of a single one. We therefore consider two real scalar fields ϕ and χ on a 1D lattice, with masses m_1 and m_2 and a bilinear coupling $g\,\phi\chi$. The implementation is in `src/run_two_field_coupling_1d.py` (or equivalently the scan version in `src/run_two_field_coupling_scan.py`), with analysis in `notebooks/10_two_field_coupling_analysis_1d.py`.

We initialise a small homogeneous excitation in ϕ only and evolve both the free system and one in which the non–cancelling rule acts on the *combined* mean field

$$A(t) \propto \langle\phi(t)\rangle + i\langle\chi(t)\rangle. \tag{3}$$

At large g the coupled system becomes unstable and both mean fields blow up; this is a feature of the underlying discretised equations, not of the constraint itself, and we treat this as a parameter region to avoid. At more modest couplings (e.g. $g = 0.02$ in the current runs) the behaviour is much calmer:

- Energy oscillates between the two fields as expected from coupled harmonic oscillators.
- The total energy is conserved to the same level in free and constrained runs; the constraint curve sits slightly above the free one but without secular drift.
- The mean fields remain small and oscillatory; constraint hits serve mainly to keep the combined mean away from exact cancellation, not to pump or drain energy.

6.2 Two–field localised bump

Finally we combine localisation and coupling in `src/run_two_field_bump_1d.py` with analysis in `notebooks/11_two_field_bump_analysis_1d.py`. We place a localised bump in ϕ on a 1D lattice, leave χ initially at rest, and couple them with a moderate g.

Again we track a localisation fraction (now combining both fields) in a central window as a function of time. The free and constrained curves are extremely close; the constrained system tends to lose a slightly larger fraction of its central weight as time progresses, consistent with a very mild bias towards spreading the combined amplitude away from perfect cancellation.

No qualitatively new attractors or long–lived composites appear in this regime. From the viewpoint of a cancellation system this is, again, a null result—but an important one: the rule can act on a coupled multi–field system without introducing obvious pathologies.

7 Discussion and roadmap

The tests documented in this Act II all point in the same direction: within the scalar lattice frameworks explored so far, the non–cancelling rule behaves as a small, tunable perturbation. For sufficiently small ε,

- basic vacuum properties (including twisted and defected chains) are unchanged within numerical precision;
- individual modes retain their frequencies and line shapes;
- localised bumps disperse and recur in much the same way with and without the constraint;
- energy sharing in modestly coupled two–field systems proceeds as expected, with total energy conserved to comparable accuracy.

These are not the spectacular signatures one might secretly hope for, but they are exactly the kind of groundwork that any serious proposal must survive. They also clarify where *not* to look: the simulations here suggest that if the Origin Axiom has observable consequences, they are unlikely to appear as wild instabilities or miraculous localisation at the level of a simple scalar lattice.

The natural next steps, some of which are already sketched in `docs/ROADMAP.md`, are:

- Extend the cancellation–system viewpoint to sectors that more closely resemble realistic matter (multiple fields with different statistics, gauge structure, approximate symmetries).
- Explore versions of the rule where $A(C)$ is tied to genuinely global quantities (e.g. phase–twisted sums over sectors) rather than simple lattice means.
- Investigate whether the constraint can induce small but coherent biases in ensembles of initial conditions, potentially relevant to vacuum selection or cosmological initial data.

In that sense, Act II is both a conclusion and a beginning: it closes the loop on the first wave of sanity checks and prepares the ground for more ambitious tests in which the axiom is allowed to “touch” structures closer to the real universe.