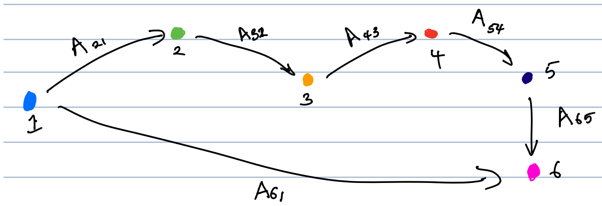
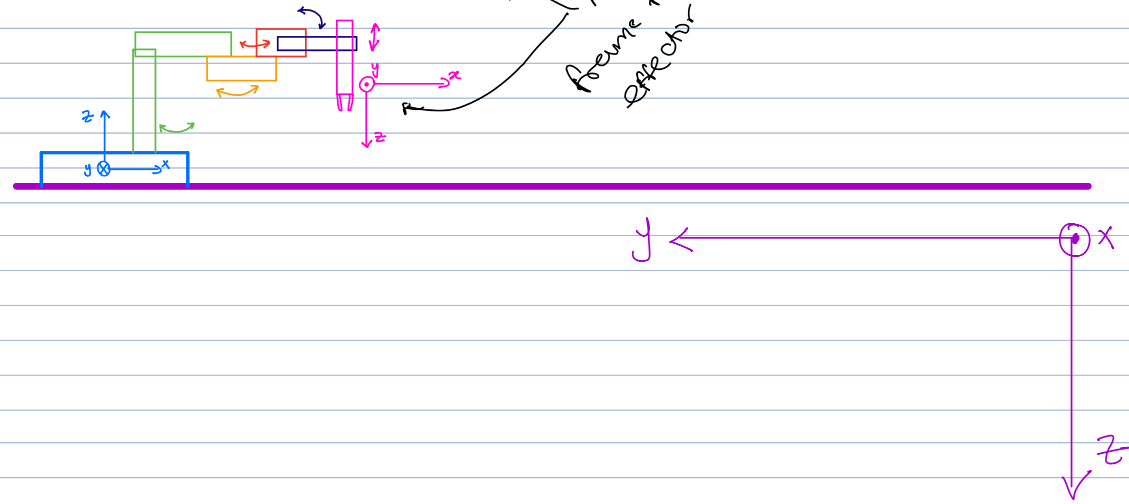
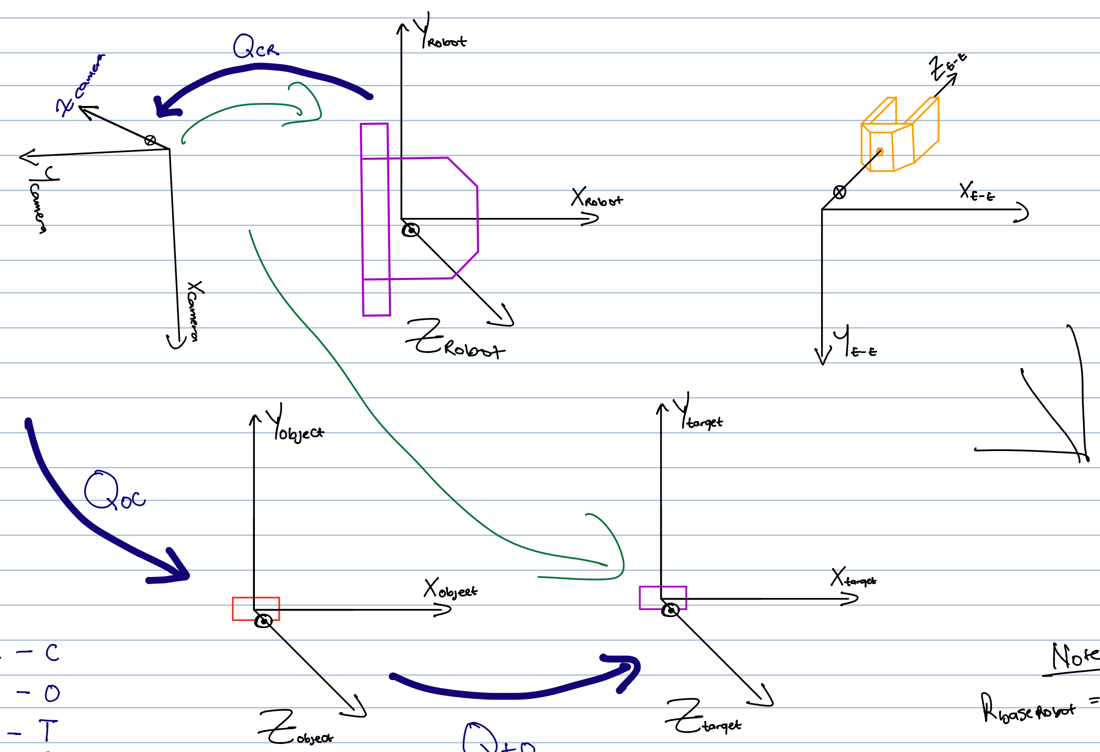


Remember 8 add frame for end effector





Camera – C
Object – O
Target – T
Robot – R
END-effector – E

Note From Class

$R_{baseRobot} = R_{world}$

$$Q_{CR} = \begin{bmatrix} 1 & 0 & 0 & X_T \\ 0 & 1 & 0 & Y_T \\ 0 & 0 & 1 & Z_T \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_{OC} = \begin{bmatrix} 1 & 0 & 0 & X_T \\ 0 & 1 & 0 & Y_T \\ 0 & 0 & 1 & Z_T \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

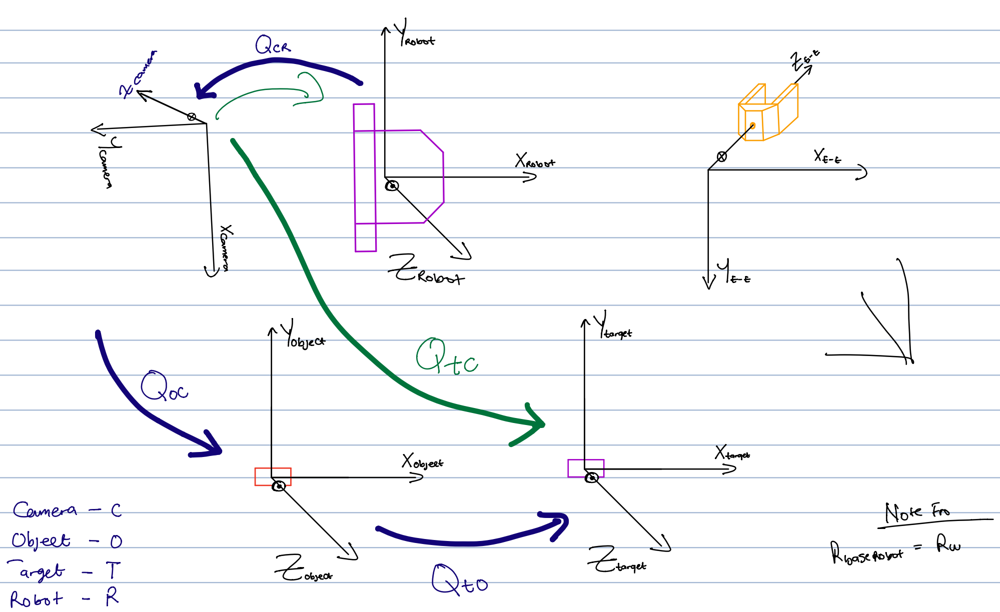
$$Q_{tc} = \begin{bmatrix} 1 & 0 & 0 & X_T \\ 0 & 1 & 0 & Y_T \\ 0 & 0 & 1 & Z_T \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_{to} = \begin{bmatrix} 1 & 0 & 0 & X_T \\ 0 & 1 & 0 & Y_T \\ 0 & 0 & 1 & Z_T \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

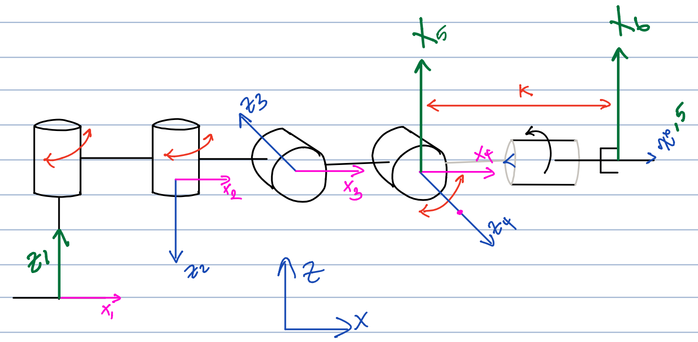
This transformation matrix is not accurate, since the position and orientations of both the object and target could be anything

Instead.

$$Q_{to} = Q_{oc}^{-1} \cdot Q_{tc}$$



FORWARD KINEMATICS

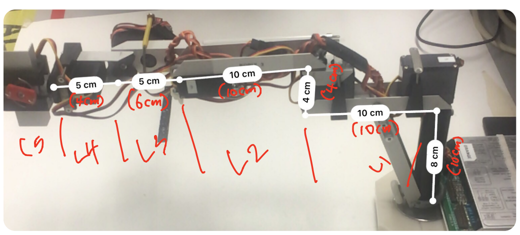


USE THIS

i	l(i)	d(i)	α(i)	θ(i)
1	10 cm	12.5	180	θ_1
2	10 cm	0	90	θ_2
3	6 cm	0	180°	θ_3
4	0	0	90	90 + θ_4
5	0	K	0	θ_5

$$A_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & l_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & l_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$A_3 = \begin{vmatrix} \cos\theta_3 & -\sin\theta_3\cos(180) & \sin\theta\sin(180) & 6\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3\cos(180) & -\cos\theta\sin(180) & 6\sin\theta_3 \\ 0 & \sin(180) & \cos(180) & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$A_3(\theta_3) = \begin{vmatrix} \cos\theta_3 & \sin\theta_3 & 0 & 6\cos\theta_3 \\ \sin\theta_3 & -\cos\theta_3 & 0 & 6\sin\theta_3 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

For A_1

$$A_1 = \begin{vmatrix} \cos\theta_1 & -\sin\theta_1\cos(180) & \sin\theta_1\sin(180) & 10\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1\cos(180) & -\cos\theta_1\sin(180) & 10\sin\theta_1 \\ 0 & \sin(180) & \cos(180) & 12.5 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow A_1(\theta_1) = \begin{vmatrix} \cos\theta_1 & \sin\theta_1 & 0 & 10\cos\theta_1 \\ \sin\theta_1 & -\cos\theta_1 & 0 & 10\sin\theta_1 \\ 0 & 0 & -1 & 12.5 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$A_4 = \begin{vmatrix} \cos(90+\theta_4) & -\sin(90+\theta_4)\cos(90) & \sin(90+\theta_4)\sin(90) & 0\cdot\cos\theta_4 \\ \sin(90+\theta_4) & \cos(90+\theta_4)\cos(90) & -\cos(90+\theta_4)\sin(90) & 0\cdot\sin\theta_4 \\ 0 & \sin(90) & \cos(90) & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$A_4(\theta_4) = \begin{vmatrix} \cos(90+\theta_4) & 0 & \sin(90+\theta_4) & 0 \\ \sin(90+\theta_4) & 0 & -\cos(90+\theta_4) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$A_2 = \begin{vmatrix} \cos\theta_2 & -\sin\theta_2\cos(90) & \sin\theta_2\sin(90) & 10\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2\cos(90) & -\cos\theta_2\sin(90) & 10\sin\theta_2 \\ 0 & \sin(90) & \cos(90) & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$A_2(\theta_2) = \begin{vmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 10\cos\theta_2 \\ \sin\theta_2 & 0 & -\cos\theta_2 & 10\sin\theta_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$A_5 = \begin{vmatrix} \cos\theta_5 & -\sin\theta_5\cos(0) & \sin\theta_5\sin(0) & 0\cdot\cos\theta_5 \\ \sin\theta_5 & \cos\theta_5\cos(0) & -\cos\theta_5\sin(0) & 0\cdot\sin\theta_5 \\ 0 & \sin(0) & \cos(0) & K \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$A_5(\theta_5) = \begin{vmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ \sin\theta_5 & \cos\theta_5 & 0 & 0 \\ 0 & 0 & 1 & K \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Slope eqns for Servos S_i

$S_1 \Rightarrow (1400, 77.5^\circ) \rightarrow (-1400, -77.5^\circ)$ $y_1 = 0.0554 x_1$

$S_2 \Rightarrow (1400, 60^\circ) \rightarrow (-1400, -60^\circ)$ $y_2 = 0.0429 x_2$

$S_3 \Rightarrow (1400, 77.5^\circ) \rightarrow (-1400, -77.5^\circ)$ $y_3 = 0.0554 x_3$

$S_4 \Rightarrow (1400, 82.5^\circ) \rightarrow (-1400, -82.5^\circ)$ $y_4 = 0.0589 x_4$

$S_5 \Rightarrow (1400, 75^\circ) \rightarrow (-1400, -75^\circ)$ $y_5 = 0.0536 x_5$

Note: In the code, y_1 and y_5 will flip signs because our axes definition (z) is opposite of the robots for motors 1 and 5.

$$\Rightarrow A_1(\theta_1) = \begin{vmatrix} \cos\theta_1 & \sin\theta_1 & 0 & 10\cos\theta_1 \\ \sin\theta_1 & -\cos\theta_1 & 0 & 10\sin\theta_1 \\ 0 & 0 & -1 & 12.5 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad A_2(\theta_2) = \begin{vmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 10\cos\theta_2 \\ \sin\theta_2 & 0 & -\cos\theta_2 & 10\sin\theta_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$A_3(\theta_3) = \begin{vmatrix} \cos\theta_3 & \sin\theta_3 & 0 & 6\cos\theta_3 \\ \sin\theta_3 & -\cos\theta_3 & 0 & 6\sin\theta_3 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad A_4(\theta_4) = \begin{vmatrix} \cos(270+\theta_4) & 0 & \sin(270+\theta_4) & 0 \\ \sin(270+\theta_4) & 0 & -\cos(270+\theta_4) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$A_5(\theta_5) = \begin{vmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ \sin\theta_5 & \cos\theta_5 & 0 & 0 \\ 0 & 0 & 1 & K \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$A_1 \cdot A_2 = \begin{vmatrix} \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 & 0 & \cos\theta_1\sin\theta_2 - \sin\theta_1\cos\theta_2 & 10\cos\theta_1\cos\theta_2 + 10\sin\theta_1\sin\theta_2 + 10\cos\theta_1 \\ \sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2 & 0 & \sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2 & 10\sin\theta_1\cos\theta_2 - 10\cos\theta_1\sin\theta_2 + 10\sin\theta_1 \\ 0 & -1 & 0 & 12.5 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \cos(\theta_1-\theta_2) & 0 & -\sin(\theta_1-\theta_2) & 10\cos(\theta_1-\theta_2) + 10\cos\theta_1 \\ \sin(\theta_1-\theta_2) & 0 & \cos(\theta_1-\theta_2) & 10\sin(\theta_1-\theta_2) + 10\sin\theta_1 \\ 0 & -1 & 0 & 12.5 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} \cos(\theta_1-\theta_2) & 0 & -\sin(\theta_1-\theta_2) & 10\cos(\theta_1-\theta_2) + 10\cos\theta_1 \\ \sin(\theta_1-\theta_2) & 0 & \cos(\theta_1-\theta_2) & 10\sin(\theta_1-\theta_2) + 10\sin\theta_1 \\ 0 & -1 & 0 & 12.5 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \cos\theta_3 & \sin\theta_3 & 0 & 6\cos\theta_3 \\ \sin\theta_3 & -\cos\theta_3 & 0 & 6\sin\theta_3 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow A_1A_2A_3 = \begin{vmatrix} \cos(\theta_1-\theta_2)\cos\theta_3 & \cos(\theta_1-\theta_2)\sin\theta_3 & \sin(\theta_1-\theta_2) & 6\cos(\theta_1-\theta_2)\cos\theta_3 + 10\cos(\theta_1-\theta_2) + 10\cos\theta_1 \\ \sin(\theta_1-\theta_2)\cos\theta_3 & \sin(\theta_1-\theta_2)\sin\theta_3 & -\cos(\theta_1-\theta_2) & 6\sin(\theta_1-\theta_2)\cos\theta_3 + 10\sin(\theta_1-\theta_2) + 10\sin\theta_1 \\ -\sin\theta_3 & \cos\theta_3 & 0 & -6\sin\theta_3 + 12.5 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$A_4(\theta_4) = \begin{vmatrix} \cos(90+\theta_4) & 0 & \sin(90+\theta_4) & 0 \\ \sin(90+\theta_4) & 0 & -\cos(90+\theta_4) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$A_1 A_2 A_3 A_4 =$$

$$\begin{vmatrix} \cos(\theta_1-\theta_2)\cos(\theta_3-90-\theta_4) & \sin(\theta_1-\theta_2) & \cos(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3) & 6\cos(\theta_1-\theta_2)\cos\theta_3 + 10\cos(\theta_1-\theta_2) + 10\cos\theta_1 \\ \sin(\theta_1-\theta_2)\cos(\theta_3-90-\theta_4) & -\cos(\theta_1-\theta_2) & \sin(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3) & 6\sin(\theta_1-\theta_2)\cos\theta_3 + 10\sin(\theta_1-\theta_2) + 10\sin\theta_1 \\ \sin(90+\theta_4-\theta_3) & 0 & -\cos(\theta_3-90-\theta_4) & -6\sin\theta_3 + 12.5 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$A_5(\theta_5) = \begin{vmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ \sin\theta_5 & \cos\theta_5 & 0 & 0 \\ 0 & 0 & 1 & K \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$A = A_1 A_2 A_3 A_4 A_5$$

$$A = \begin{vmatrix} \cos(\theta_1-\theta_2)\cos(\theta_3-90-\theta_4)\cos\theta_5 + \sin(\theta_1-\theta_2)\sin\theta_5 & -\cos(\theta_1-\theta_2)\cos(\theta_3-90-\theta_4)\sin\theta_5 + \sin(\theta_1-\theta_2)\cos\theta_5 & \cos(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3) & \boxed{K\cdot\cos(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3) + 6\cos(\theta_1-\theta_2)\cos\theta_3 + 10\cos(\theta_1-\theta_2) + 10\cos\theta_1} \\ \sin(\theta_1-\theta_2)\cos(\theta_3-90-\theta_4)\cos\theta_5 - \cos(\theta_1-\theta_2)\sin\theta_5 & -\sin(\theta_1-\theta_2)\cos(\theta_3-90-\theta_4)\sin\theta_5 - \cos(\theta_1-\theta_2)\cos\theta_5 & \sin(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3) & \boxed{K\cdot\sin(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3) + 6\sin(\theta_1-\theta_2)\cos\theta_3 + 10\sin(\theta_1-\theta_2) + 10\sin\theta_1} \\ \sin(90+\theta_4-\theta_3)\cos\theta_5 & -\sin(90+\theta_4-\theta_3)\sin\theta_5 & -\cos(\theta_3-90-\theta_4) & \boxed{-K\cdot\cos(\theta_3-90-\theta_4) + -6\sin\theta_3 + 12.5} \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

INVERSE KINEMATICS

$$Q_{spec} = Q_T(x_T, y_T, z_T)\cdot Q_{Rz}(\theta)\cdot Q_{Ry}(\phi)\cdot Q_{Rx}(\psi) =$$

For example,

$$\text{let } Q_{spec} = Q_T(10, 20, 10)\cdot Q_{Rz}(30°)\cdot Q_{Ry}(30°)\cdot Q_{Rx}(30°) = \begin{vmatrix} \overset{a_x}{0.75} & \overset{b_x}{-0.2165} & \overset{c_x}{0.6250} & \overset{p_x}{10} \\ \overset{a_y}{0.433} & \overset{b_y}{0.8750} & \overset{c_y}{-0.2165} & \overset{p_y}{20} \\ \overset{a_z}{-0.5} & \overset{b_z}{0.4330} & \overset{c_z}{0.7500} & \overset{p_z}{10} \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

FOR θ_3

$$P_z = -K\cos(\theta_3-90-\theta_4) - 6\sin\theta_3 + 12.5$$

$$C_z = -\cos(\theta_3-90-\theta_4)$$

$$\Rightarrow P_z = K\cdot C_z - 6\sin\theta_3 + 12.5$$

$$\Rightarrow \sin\theta_3 = \frac{K\cdot C_z + 12.5 - P_z}{6}$$

$$\theta_3 = a\sin\left|\frac{K\cdot C_z + 12.5 - P_z}{6}\right|$$

Proof:

for $(K, C_z, P_z) = (10, 0.75, 10) \rightarrow \theta_3 = 66.4$ or 113.6

Check (for $\theta_3 = 66.4$)

$P_z = (4)(0.75) - 6\sin(66.4) + 12.5 = 10$

$\theta_4 = a\cos\left(\pm\sqrt{C_x^2+C_y^2}\right) + \theta_3 = a\cos(\pm 0.66) + 66.4 = 114.99$ or 197.809 or 17.7 or -64.9

$\Rightarrow C_z = -\cos(\theta_3-90-\theta_4) = -\cos(66.4-90-114.99) = 0.75$ (for $\theta_4 = 114.99$)

$= -\cos(66.4-90-197.809) = 0.75$ (for $\theta_4 = 197.809$)

also, $\theta_5 = 40.89$

$\Rightarrow a_z = \sin(90+114.99-66.4)\cos(40.89) = 0.5$

$b_z = -\sin(90+114.99-66.4)\sin(40.89) = -0.4329$

$$C_x = \cos(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3)$$

$$C_y = \sin(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3)$$

$$\theta_4 = \theta_3 - 90 - (\pm 138.6)$$

then,

$$C_x^2 + C_y^2 = \sin^2(90+\theta_4-\theta_3)\left(\cos^2(\theta_1-\theta_2) + \sin^2(\theta_1-\theta_2)\right)$$

$$\theta_3 - 90 - \cos^{-1}(-C_z) \qquad 115.04$$

$$= \sin^2(90+\theta_4-\theta_3) \qquad -162.16$$

$$\Rightarrow C_x^2 + C_y^2 = \cos^2(\theta_4-\theta_3)$$

$$= 66.4 - 90 -$$

$$\Rightarrow \theta_4 = a\cos\left(\pm\sqrt{C_x^2+C_y^2}\right) + \theta_3$$

$$\cos^{-1}(-0.75)$$

For θ_5

$a_z = \sin(90+\theta_4-\theta_3)\cos\theta_5$

$b_z = -\sin(90+\theta_4-\theta_3)\sin\theta_5$

$$\frac{b_z}{a_z} = \frac{-\sin(90+\theta_4-\theta_3)\sin\theta_5}{\sin(90+\theta_4-\theta_3)\cos\theta_5} = \frac{-\sin\theta_5}{\cos\theta_5} = -\tan\theta_5$$

$$\Rightarrow \theta_5 = \tan^{-1}\left[\frac{b_z}{-a_z}\right]$$

Proof: for $(a_z, b_z) = (-0.5, 0.433)$ $= \theta_5 = \mathrm{atan2}\left(0.433/0.5\right) = 40.89$

Check: for $\theta_5 = 40.89$,

$a_z = \sin(90+73.19-24.6)\cos(40.89) = 0.5$

For θ_2

$$P_x = K\cdot\cos(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3) + 6\cos(\theta_1-\theta_2)\cos\theta_3 + 10\cos(\theta_1-\theta_2) + 10\cos\theta_1$$

$$P_y = K\cdot\sin(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3) + 6\sin(\theta_1-\theta_2)\cos\theta_3 + 10\sin(\theta_1-\theta_2) + 10\sin\theta_1$$

$$\begin{aligned}P_x^2 = {} & K^2\cos^2(\theta_1-\theta_2)\sin^2(90+\theta_4-\theta_3) + 12K\cos^2(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3)\cos\theta_3 + 20K\cos^2(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3) \\ & + 20K\cos(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3)\cos\theta_1 + 36\cos^2(\theta_1-\theta_2)\cos^2\theta_3 + 120\cos^2(\theta_1-\theta_2)\cos\theta_3 + 120\cos(\theta_1-\theta_2)\cos\theta_3\cos\theta_1 \\ & + 100\cos^2(\theta_1-\theta_2) + 100\cos(\theta_1-\theta_2)\cos\theta_1 + 100\cos^2\theta_1\end{aligned}$$

$$\begin{aligned}P_y^2 = {} & K^2\sin^2(\theta_1-\theta_2)\sin^2(90+\theta_4-\theta_3) + 12K\sin^2(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3)\cos\theta_3 + 20K\sin^2(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3) \\ & + 20K\sin(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3)\sin\theta_1 + 36\sin^2(\theta_1-\theta_2)\cos^2\theta_3 + 120\sin^2(\theta_1-\theta_2)\cos\theta_3 + 120\sin(\theta_1-\theta_2)\cos\theta_3\sin\theta_1 \\ & + 100\sin^2(\theta_1-\theta_2) + 100\sin(\theta_1-\theta_2)\sin\theta_1 + 100\sin^2\theta_1\end{aligned}$$

$$\begin{aligned}P_x^2+P_y^2 = {} & K^2\sin^2(90+\theta_4-\theta_3) + 12K\sin(90+\theta_4-\theta_3)\cos\theta_3 + 20K\sin(90+\theta_4-\theta_3) + 20K\cos(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3)\cos\theta_1 \\ & + 20K\sin(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3)\sin\theta_1 + 36\cos^2\theta_3 + 120\cos\theta_3 + 120\cos(\theta_1-\theta_2)\cos\theta_3\cos\theta_1 \\ & + 120\sin(\theta_1-\theta_2)\cos\theta_3\sin\theta_1 + 200 + 100\cos(\theta_1-\theta_2)\cos\theta_1 + 100\sin(\theta_1-\theta_2)\sin\theta_1\end{aligned}$$

$$\begin{aligned}P_x^2+P_y^2 = {} & K^2\sin^2(90+\theta_4-\theta_3) + 12K\sin(90+\theta_4-\theta_3)\cos\theta_3 + 20K\sin(90+\theta_4-\theta_3) + 20K\sin(90+\theta_4-\theta_3)\cos\theta_2 \\ & + 36\cos^2\theta_3 + 120\cos\theta_3 + 120\cos\theta_3\cos\theta_2 + 200 + 100\cos\theta_2 \\ = {} & K^2[\sin(90+\theta_4)\cos\theta_3 - \cos(90+\theta_4)\sin\theta_3]^2 + 12K\cos\theta_3[\sin(90+\theta_4)\cos\theta_3 - \cos(90+\theta_4)\sin\theta_3] \\ & + 20K[\sin(90+\theta_4)\cos\theta_3 - \cos(90+\theta_4)\sin\theta_3] + 20K\cos\theta_2[\sin(90+\theta_4)\cos\theta_3 - \cos(90+\theta_4)\sin\theta_3] \\ & + 36\cos^2\theta_3 + 120\cos\theta_3 + 120\cos\theta_3\cdot\cos\theta_2 + 100\cos\theta_2 + 200 \\ = {} & K^2\cos^2(\theta_4-\theta_3) + 12K\cos(\theta_4-\theta_3)\cos\theta_3 + 20K\cos(\theta_4-\theta_3) + 20K\cos\theta_2\cos(\theta_4-\theta_3) + 36\cos^2\theta_3 \\ & + 120\cos\theta_3 + 120\cos\theta_3\cos\theta_2 + 100\cos\theta_2 + 200 \\ = {} & K^2\cos^2(\theta_4-\theta_3) + 12K\cos(\theta_4-\theta_3)\cos\theta_3 + 20K\cos(\theta_4-\theta_3) + 36\cos^2\theta_3 + 120\cos\theta_3 \\ & + 200 + \cos\theta_2\left(20K\cos(\theta_4-\theta_3) + 120\cos\theta_3 + 100\right)\end{aligned}$$

note: $\cos(x) = \cos(-x)$

now,

$$\cos\theta_2 = \frac{P_x^2+P_y^2 - K^2\cos^2(\theta_4-\theta_3) - 12K\cos(\theta_4-\theta_3)\cos\theta_3 - 20K\cos(\theta_4-\theta_3) - 36\cos^2\theta_3 - 120\cos\theta_3 - 200}{20K\cos(\theta_4-\theta_3) + 120\cos\theta_3 + 100} = \Delta_1$$

$\Rightarrow \theta_2 = \mathrm{acos}\,\Delta_1$

For θ_1

$$\frac{b_y}{b_x} = \frac{-\sin(\theta_1-\theta_2)\cos(\theta_3-90-\theta_4)\sin\theta_5 - \cos(\theta_1-\theta_2)\cos\theta_5}{-\cos(\theta_1-\theta_2)\cos(\theta_3-90-\theta_4)\sin\theta_5 + \sin(\theta_1-\theta_2)\cos\theta_5}$$

$$= \frac{\tan(\theta_1-\theta_2) + \dfrac{1}{\cos(\theta_3-90-\theta_4)\tan\theta_5}}{1 - \dfrac{\tan(\theta_1-\theta_2)}{\cos(\theta_3-90-\theta_4)\tan\theta_5}}$$

$$\Rightarrow \frac{b_y}{b_x} = \tan(\alpha+\beta)$$

$$\Rightarrow \operatorname{atan2}(b_y/b_x) = \alpha + \beta$$

$$\Rightarrow \alpha = \theta_1 - \theta_2 = \operatorname{atan2}\left|\frac{b_y}{b_x}\right| - \operatorname{atan2}\left|\frac{1}{\cos(\theta_3 - 90 - \theta_4)\tan\theta_5}\right|$$

$$\Rightarrow \tan\alpha = \tan(\theta_1 - \theta_2) \Rightarrow \alpha = \theta_1 - \theta_2$$

$$\tan\beta = \frac{1}{\cos(\theta_3 - 90 - \theta_4)\tan\theta_5}$$

$$\Rightarrow \beta = \operatorname{atan2}\left|\frac{1}{\cos(\theta_3 - 90 - \theta_4)\tan\theta_5}\right|$$

but notice that $\cos(\theta_3 - 90 - \theta_4) = -C_z$

then we have

$$\theta_1 = \operatorname{atan2}\left|\frac{b_y}{b_x}\right| - \operatorname{atan2}\left|\frac{1}{(-C_z)\tan\theta_5}\right| + \theta_2$$

$$P_x = K\cdot\cos(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3) + 6\cos(\theta_1-\theta_2)\cos\theta_3 + 10\cos(\theta_1-\theta_2) + 10\cos\theta_1$$

$$P_y = K\cdot\sin(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3) + 6\sin(\theta_1-\theta_2)\cos\theta_3 + 10\sin(\theta_1-\theta_2) + 10\sin\theta_1$$

$$C_x = \cos(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3)$$

$$C_y = \sin(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3)$$

then

$$P_x = K\cdot C_x + 6\cos(\theta_1-\theta_2)\cos\theta_3 + 10\cos(\theta_1-\theta_2) + 10\cos\theta_1$$

$$P_y = K\cdot C_x + 6\sin(\theta_1-\theta_2)\cos\theta_3 + 10\sin(\theta_1-\theta_2) + 10\sin\theta_1$$

let $m = K\cdot C_x$

$n = K\cdot C_y$

then

$$P_x^2 = m^2 + 12m\cos(\theta_1-\theta_2)\cos\theta_3 + 20m\cos(\theta_1-\theta_2) + 20m\cos\theta_1$$
$$+ 36\cos^2(\theta_1-\theta_2)\cos^2\theta_3 + 120\cos^2(\theta_1-\theta_2)\cos\theta_3 + 120\cos(\theta_1-\theta_2)\cos\theta_1\cos\theta_3$$
$$+ 100\cos^2(\theta_1-\theta_2) + 200\cos(\theta_1-\theta_2)\cos\theta_1 + 100\cos^2\theta_1$$

$$P_y^2 = n^2 + 12n\sin(\theta_1-\theta_2)\cos\theta_3 + 20n\sin(\theta_1-\theta_2) + 20n\sin\theta_1$$
$$+ 36\sin^2(\theta_1-\theta_2)\cos^2\theta_3 + 120\sin^2(\theta_1-\theta_2)\cos\theta_3 + 120\sin(\theta_1-\theta_2)\sin\theta_1\cos\theta_3$$
$$+ 100\sin^2(\theta_1-\theta_2) + 200\sin(\theta_1-\theta_2)\sin\theta_1 + 100\sin^2\theta_1$$

then,

$$P_x^2 + P_y^2 = m^2 + n^2 + 12m\cos(\theta_1-\theta_2)\cos\theta_3 + 12n\sin(\theta_1-\theta_2)\cos\theta_3 + 20m\cos(\theta_1-\theta_2) + 20n\sin(\theta_1-\theta_2) + 20m\cos\theta_1$$
$$+ 20n\sin\theta_1 + 36\cos^2\theta_3 + 120\cos\theta_3 + 120\cos\theta_3\cos\theta_2 + 200 + 200\cos\theta_2$$

$$P_z^2 = k^2\cos^2(90+\theta_4-\theta_3) - 12k\cos(90+\theta_4-\theta_3)\sin\theta_3 + 25k\cos(90+\theta_4-\theta_3) + 36\sin^2\theta_3 - 150\sin\theta_3 + 156.25$$
$$= k^2\sin^2(\theta_3-\theta_4) - 12k\sin(\theta_3-\theta_4)\sin\theta_3 + 25k\sin(\theta_3-\theta_4) + 36\sin^2\theta_3 - 150\sin\theta_3 + 156.25$$

now,

$$P_x^2 + P_y^2 + P_z^2 = k^2 + 12k\left[\cos(\theta_3-\theta_4)\cos\theta_3 - \sin(\theta_3-\theta_4)\sin\theta_3\right] + 20k\cos(\theta_2-\theta_1) + 25k\sin(\theta_3-\theta_4)$$

$$+ 20K\cos\theta_2 \cos(\theta_4-\theta_3) + 36 + 120\cos\theta_3 - 150\sin\theta_3 + 120\cos\theta_3\cos\theta_2 + 100\cos\theta_2 + 456.25$$

$$\frac{P_y}{P_x} = \frac{K\cdot\sin(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3) + 6\sin(\theta_1-\theta_2)\cos\theta_3 + 10\sin(\theta_1-\theta_2) + 10\sin\theta_1}{K\cdot\cos(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3) + 6\cos(\theta_1-\theta_2)\cos\theta_3 + 10\cos(\theta_1-\theta_2) + 10\cos\theta_1}$$

$$= \frac{(K\sin(90+\theta_4-\theta_3) + 6\cos\theta_3 + 10)\sin(\theta_1-\theta_2) + 10\sin\theta_1}{(K\sin(90+\theta_4-\theta_3) + 6\cos\theta_3 + 10)\cos(\theta_1-\theta_2) + 10\cos\theta_1}$$

$$= \frac{\dfrac{\sin(\theta_1-\theta_2)}{\cos(\theta_1-\theta_2)} + \dfrac{10\sin\theta_1}{(K\sin(90+\theta_4-\theta_3) + 6\cos\theta_3 + 10)\cos(\theta_1-\theta_2)}}{1 - \dfrac{-10\sin\theta_1}{(K\sin(90+\theta_4-\theta_3) + 6\cos\theta_3 + 10)\cos(\theta_1-\theta_2)}\cdot \tan^{-1}(\theta_1)}$$

now,

$$\cos\theta_2 = \frac{P_x^2 + P_y^2 - K^2\cos^2(\theta_4-\theta_3) - 12K\cos(\theta_4-\theta_3)\cos\theta_3 - 20K\cos(\theta_4-\theta_3) - 36\cos^2\theta_3 - 120\cos\theta_3 - 200}{20K\cos(\theta_4-\theta_3) + 120\cos\theta_3 + 100}$$

$\Rightarrow \theta_2 = a\cos\Delta_1$

$K = 4$ | $\cos(\theta_4-\theta_3) = 0.6614$
$P_x^2 = 10^2 = 100$ | $\cos\theta_3 = 0.4$
$P_y^2 = 20^2 = 400$
$\theta_3 = 66.4435$
$\theta_4 = 17.8532, -64.9661$

θ_4 $\Rightarrow$ alpha $= 360 - 7 - 12.69 - 52.912 - 5.79 - 48 = 173.608$

beta $= 52.912 + 48 + 100 = 200.912$

result $= \dfrac{173.608}{200.912} = 0.8642$

$\Rightarrow \theta_2 = \cos^{-1}(0.8642) = 30.$

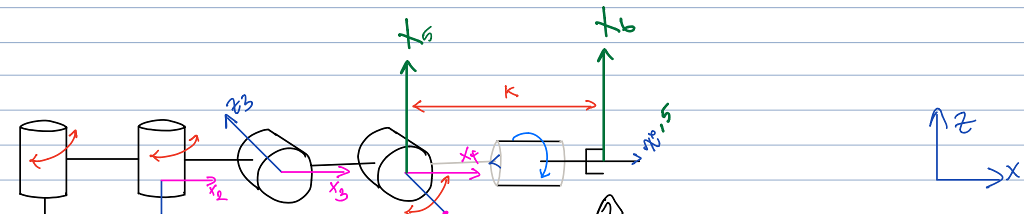
$\cos(\theta_4-\theta_3) \approx 1 - 0.66$
$\cos\theta_3 = 0.4$

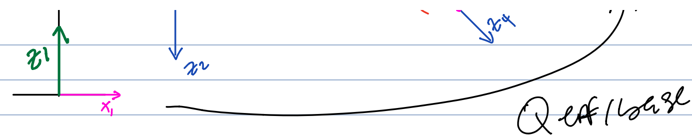
θ_4

alpha =
beta =

.24

$\theta_2 =$





$Q_{spec} = Q_T(x_T, y_T, z_T) \cdot Q_{Rz}(\theta) \cdot Q_{Ry}(\phi) \cdot Q_{Rx}(\psi) =$

For example,

let $Q_{spec} = Q_T(10, 20, 10) \cdot Q_{Rz}(30°) \cdot Q_{Ry}(30°) \cdot Q_{Rx}(30°) =$

$$\left|\begin{array}{cccc} 0.75\ ^{a_x} & -0.2165\ ^{b_x} & 0.6250\ ^{c_x} & ^{p_x}\ 10 \\ 0.433\ ^{a_y} & 0.8750\ ^{b_y} & -0.2165\ ^{c_y} & ^{p_y}\ 20 \\ -0.5\ ^{a_z} & 0.4330\ ^{b_z} & 0.7500\ ^{c_z} & ^{p_z}\ 10 \\ 0 & 0 & 0 & 1 \end{array}\right|$$

$$\left(K \cdot \sin(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3)\right) + \left(6\sin(\theta_1-\theta_2)\cos\theta_3\right) + \left(10\sin(\theta_1-\theta_2) + 10\sin\theta_1\right)$$

$$\left(K \cdot \cos(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3)\right) + \left(6\cos(\theta_1-\theta_2)\cos\theta_3\right) + \left(10\cos(\theta_1-\theta_2) + 10\cos\theta_1\right)$$

$$\Rightarrow \frac{\sin(\theta_1-\theta_2)\left(K\sin x + 6\cos\theta_3 + 10\right) + 10\sin\theta_1}{\cos(\theta_1-\theta_2)\left(K\sin x + 6\cos\theta_3 + 10\right) + 10\cos\theta_1}$$

$$\Rightarrow \frac{\sin(\theta_1-\theta_2)(K\sin x + 6\cos\theta_3 + 10) + 10\sin(\theta_1-\theta_2)\cos\theta_2 + 10\sin\theta_2\cos(\theta_1-\theta_2)}{\cos(\theta_1-\theta_2)(K\sin x + 6\cos\theta_3 + 10) + 10\cos(\theta_1-\theta_2)\cos\theta_2 - 10\sin(\theta_1-\theta_2)\sin\theta_2}$$

$$\Rightarrow \frac{\sin(\theta_1-\theta_2)(K\sin x + 6\cos\theta_3 + 10 + 10\cos\theta_2) + 10\sin\theta_2\cos(\theta_1-\theta_2)}{\cos(\theta_1-\theta_2)(K\sin x + 6\cos\theta_3 + 10 + 10\cos\theta_2) - 10\sin\theta_2\sin(\theta_1-\theta_2)}$$

$$\Rightarrow \frac{\tan(\theta_1-\theta_2) + \dfrac{10\sin\theta_2}{(K\sin x + 6\cos\theta_3 + 10 + 10\cos\theta_2)}}{1 - \dfrac{10\sin\theta_2}{(K\sin x + 6\cos\theta_3 + 10 + 10\cos\theta_2)}\tan(\theta_1-\theta_2)}$$

$$\tan\alpha = \tan(\theta_1-\theta_2) \Rightarrow \alpha = \theta_1-\theta_2$$

$$\tan\beta = \frac{10\sin\theta_2}{K\sin x + 6\cos\theta_3 + 10 + 10\cos\theta_2} \Rightarrow \beta = \mathrm{atan2}\left\{\frac{10\sin\theta_2}{K\sin x + 6\cos\theta_3 + 10 + 10\cos\theta_2}\right\}$$

$$\tan(\alpha+\beta) = \frac{C_y}{C_x} \Rightarrow \alpha+\beta = \mathrm{atan2}\left(\frac{C_y}{C_x}\right)$$

$$\alpha = \mathrm{atan2}\left(\frac{C_y}{C_x}\right) - \mathrm{atan2}\left\{\frac{10\sin\theta_2}{K\sin x + 6\cos\theta_3 + 10 + 10\cos\theta_2}\right\}$$

$$\theta_1 = \mathrm{atan2}\left(\frac{C_y}{C_x}\right) - \mathrm{atan2}\left\{\frac{10\sin\theta_2}{K\sin x + 6\cos\theta_3 + 10 + 10\cos\theta_2}\right\} + \theta_2$$

$$P_y = \sin(\theta_1-\theta_2)(K\sin x + 6\cos\theta_3 + 10 + 10\cos\theta_2) + 10\sin\theta_2\cos(\theta_1-\theta_2)$$
$$P_x = \cos(\theta_1-\theta_2)(K\sin x + 6\cos\theta_3 + 10 + 10\cos\theta_2) - 10\sin\theta_2\sin(\theta_1-\theta_2)$$

$$P_y^2 = \sin^2(\theta_1-\theta_2)(K\sin x + 6\cos\theta_3 + 10 + 10\cos\theta_2)^2 + 20\sin(\theta_1-\theta_2)\cos(\theta_1-\theta_2)\sin\theta_2(K\sin x + 6\cos\theta_3 + 10 + 10\cos\theta_2) + 100\sin^2\theta_2\cos^2(\theta_1-\theta_2)$$
$$P_x^2 = \cos^2(\theta_1-\theta_2)(K\sin x + 6\cos\theta_3 + 10 + 10\cos\theta_2)^2 - 20\sin(\theta_1-\theta_2)\cos(\theta_1-\theta_2)\sin\theta_2(K\sin x + 6\cos\theta_3 + 10 + 10\cos\theta_2) + 100\sin^2\theta_2\sin^2(\theta_1-\theta_2)$$

$$P_x^2 + P_y^2 = (K\sin x + 6\cos\theta_3 + 10 + 10\cos\theta_2)^2 + 100\sin^2\theta_2$$
$$= K^2\sin^2 x + 12K\cos\theta_3\sin x + 20K\sin x + 20K\sin x\cos\theta_2 + 36\cos^2\theta_3 + 120\cos\theta_3 + 120\cos\theta_3\cos\theta_2 + 100 + 200\cos\theta_2 + 100\cos^2\theta_2 + 100\sin^2\theta_2$$
$$= K^2\sin^2 x + 12K\cos\theta_3\sin x + 20K\sin x + 20K\sin x\cos\theta_2 + 36\cos^2\theta_3 + 120\cos\theta_3 + 120\cos\theta_3\cos\theta_2 + 200 + 200\cos\theta_2$$
$$= K^2\sin^2 x + 12K\cos\theta_3\sin x + 20K\sin x + 36\cos^2\theta_3 + 120\cos\theta_3 + 200 + \cos\theta_2(20K\sin x + 120\cos\theta_3 + 200)$$
$$\cos\theta_2 = \frac{P_x^2 + P_y^2 - K^2\sin^2 x - 12K\cos\theta_3\sin x - 20K\sin x - 36\cos^2\theta_3 - 120\cos\theta_3 - 200}{(20K\sin x + 120\cos\theta_3 + 200)}$$
$$= \frac{P_x^2 + P_y^2 - K^2\cos^2(\theta_4-\theta_3) - 12K\cos\theta_3\cos(\theta_4-\theta_3) - 20K\cos(\theta_4-\theta_3) - 36\cos^2\theta_3 - 120\cos\theta_3 - 200}{(20K\cos(\theta_4-\theta_3) + 120\cos\theta_3 + 200}$$

$$\theta_3 = \cos^{-1}(-C_z) + 90 + \theta_4$$

$$\sin(90+\theta_4-\theta_3)$$

$$\sin(90+\theta_4)\cos\theta_3 - \cos(90+\theta_4)\sin\theta_3$$
$$\Rightarrow (\sin 90\cos\theta_4 + \sin\theta_4\cos 90)\cos\theta_3 - (\cos 90\cos\theta_4 - \sin 90\sin\theta_4)\sin\theta_3$$
$$\Rightarrow \cos\theta_4\cos\theta_3 + \sin\theta_4\sin\theta_3$$
$$\Rightarrow \cos(\theta_4-\theta_3)$$

$$\cos^2\theta_4\cos^2\theta_3 + 2\cos\theta_4\cos\theta_3\sin\theta_4\sin\theta_3 + \sin^2\theta_4\sin^2\theta_3$$

$$C_x = \cos(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3)$$
$$C_y = \sin(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3)$$

$$\cos\theta_1 = \cos(\theta_1-\theta_2)\cos\theta_2 - \sin(\theta_1-\theta_2)\sin\theta_2$$
$$\sin\theta_1 = \sin(\theta_1-\theta_2)\cos\theta_2 + \sin\theta_2\cos(\theta_1-\theta_2)$$

$$a_z = \sin(90+\theta_4-\theta_3)\cos\theta_5$$
$$b_z = -\sin(90+\theta_4-\theta_3)\sin\theta_5$$

$$\sin^2(90+\theta_4-\theta_3) = a_z^2+b_z^2 = \cos^2(\theta_4-\theta_3)$$
$$\sin^2(90+\theta_4-\theta_3) = C_x^2+C_y^2 = \cos^2(\theta_4-\theta_3)$$

$$\theta_4 = \cos^{-1}\left(\sqrt{a_z^2+b_z^2}\right)+\theta_3$$

$$C_x = \cos(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3)$$
$$C_y = \sin(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3)$$

then,

$$C_x^2+C_y^2 = \sin^2(90+\theta_4-\theta_3)\left(\cos^2(\theta_1-\theta_2)+\sin^2(\theta_1-\theta_2)\right)$$
$$= \sin^2(90+\theta_4-\theta_3)$$
$$\Rightarrow C_x^2+C_y^2 = \cos^2(\theta_4-\theta_3)$$
$$\Rightarrow \theta_4 = a\cos\left(\pm\sqrt{C_x^2+C_y^2}\right)+\theta_3$$

$$a_x = \cos(\theta_1-\theta_2)\cos(\theta_3-90-\theta_4)\cos\theta_5 + \sin(\theta_1-\theta_2)\sin\theta_5$$
$$a_y = \sin(\theta_1-\theta_2)\cos(\theta_3-90-\theta_4)\cos\theta_5 - \cos(\theta_1-\theta_2)\sin\theta_5$$
$$a_z = \sin(90+\theta_4-\theta_3)\cos\theta_5$$

$$a_x^2 = \cos^2(\theta_1-\theta_2)\cos^2(\theta_3-90-\theta_4)\cos^2\theta_5 + 2\cos(\theta_1-\theta_2)\sin(\theta_1-\theta_2)\cos(\theta_3-90-\theta_4)\cos\theta_5\sin\theta_5 + \sin^2(\theta_1-\theta_2)\sin^2\theta_5$$
$$a_y^2 = \sin^2(\theta_1-\theta_2)\cos^2(\theta_3-90-\theta_4)\cos^2\theta_5 - 2\cos(\theta_1-\theta_2)\sin(\theta_1-\theta_2)\cos(\theta_3-90-\theta_4)\cos\theta_5\sin\theta_5 + \cos^2(\theta_1-\theta_2)\sin^2\theta_5$$
$$a_z^2 = \sin^2(90+\theta_4-\theta_3)\cos^2\theta_5$$
$$a_x^2+a_y^2 = \cos^2(\theta_3-90-\theta_4)\cos^2\theta_5+\sin^2\theta_5$$
$$= C_z^2\cos^2\theta_5+\sin^2\theta_5$$
$$= C_z^2\cos^2\theta_5+(1-\cos^2\theta_5)$$
$$= (C_z^2-1)\cos^2\theta_5+1$$
$$\frac{a_x^2+a_y^2-1}{(C_z^2-1)} = \cos^2\theta_5$$
$$\theta_5 = a\cos\left[\pm\sqrt{\frac{a_x^2+a_y^2-1}{(C_z^2-1)}}\right]$$

Proof

$$\theta_5 = a\cos\pm\sqrt{\frac{-0.25}{-0.4375}} = a\cos(\pm 0.75595) \begin{cases} \xrightarrow{+} 40.89 \\ \xrightarrow{-} 139.1080 \end{cases}$$

$$\theta_3 = 66.44$$
$$\theta_4 = a\cos(\pm 0.6614) \begin{cases} \xrightarrow{+} \pm 48.59 + \theta_3 \begin{cases} \xrightarrow{-} 17.849 \\ \xrightarrow{+} 115.03 \end{cases} \\ \xrightarrow{-} \pm 131.409 + \theta_3 \begin{cases} \xrightarrow{+} 197.85 \\ \xrightarrow{-} -64.969 \end{cases} \end{cases}$$

for $(\theta_3, \theta_4) = (66.44, 17.849)$, θ_2:

$$\theta_2 = \begin{cases} +30.1556 \\ -30.1556 \end{cases}$$

for $(\theta_3, \theta_4, \theta_2) = (66.44, 17.849, 30.1556)$, $\theta_1 = -0.9232$

now check for $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) = (-0.9232, 30.1556, 66.44, 17.849, 40.89)$

$$a_x = \cos(\theta_1-\theta_2)\cos(\theta_3-90-\theta_4)\cos\theta_5 + \sin(\theta_1-\theta_2)\sin\theta_5$$
$$= \cos(-31.0788)\cos(-41.409)\cos(40.89) + \sin(-31.0788)\sin(40.89)$$
$$= 0.4856 + (-0.3379) =$$

$\theta_3 = 66.44$, $\theta_4 = 115.0339$ $\xrightarrow{\theta_2}$ 30.1552 $\xrightarrow{\theta_1}$ -0.9232

$\qquad\qquad\qquad\qquad\qquad\;\to$ -30.1552 $\to$ -37.2900

197.8532 $\to$ Complex ✗

✗ 17.8532 $\to$ complex* ✗

✗ -64.9661

$\theta_5 = 40.8934$

$\cos(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3)$

$\sin(\theta_1-\theta_2)\sin(90+\theta_4-\theta_3)$

For Q_{spec} =
$$\begin{bmatrix} 0.640^{a_x} & -0.76313^{b_x} & -0.08716^{c_x} & 23.6294^{p_x} \\ -0.766^{a_y} & -0.64279^{b_y} & 0^{c_y} & 4.226^{p_y} \\ -0.056^{a_z} & 0.0668^{b_z} & -0.996^{c_z} & 5.074^{p_z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$K = 4$

$$\theta_3 = \operatorname{asin}\left|\frac{K\cdot C_z + 12.5 - P_z}{6}\right| = \operatorname{asin}\left(\frac{4(-0.996) + 12.5 - 5.074}{6}\right) = 35.006^\circ \text{ or } 114.99 \text{ ✗}$$

$$\theta_4 = \operatorname{acos}\left(\pm\sqrt{C_x^2+C_y^2}\right) + \theta_3 = \left[\operatorname{acos}(\pm 0.08716)\right] + 35^\circ$$

$\theta_5 = 50$

① $= \pm 85 + 35^\circ =$ $+$ 120 ✗ ; $-$ -50

② $= \pm 95 + 35^\circ =$ $+$ 130 ✗ ; $-$ -60

$$\cos(\theta_2) = \frac{P_x^2 + P_y^2 - K^2\cos^2(\theta_4-\theta_3) - 12K\cos(\theta_4-\theta_3)\cos\theta_3 - 20K\cos(\theta_4-\theta_3) - 36\cos^2\theta_3 - 120\cos\theta_3 - 200}{20K\cos(\theta_4-\theta_3) + 120\cos\theta_3 + 200} = \Delta_1$$

for $(\theta_3, \theta_4) = (35, -50)$, $\cos(\theta_4-\theta_3) = 0.087156$

$\cos(\theta_2) = 376.2 - 0.1215 - 4.18347$

$$\theta_5 = \operatorname{acos}\left[\pm\sqrt{\frac{a_x^2 + a_y^2 - 1}{(C_z^2 - 1)}}\right]$$

$$\theta_1 = \operatorname{atan2}\left|\frac{b_y}{b_x}\right| - \operatorname{atan2}\left|\frac{1}{(-C_z)\tan\theta_5}\right| + \theta_2$$

$$= \operatorname{atan2}\left(\frac{-0.64279}{-0.76313}\right) - \operatorname{atan2}\left(\frac{1}{1.18699}\right) + 25^\circ$$

$$= \frac{-0.003644}{-0.007984}$$

$= -139.89 - 40 + 25$

$acos(\pm 0.675)$

± 47.5

$$\theta_1 = atan2\left(\frac{C_y}{C_x}\right) - atan2\left\{\frac{10\sin\theta_2}{K\sin x + 6\cos\theta_3 + 10 + 10\cos\theta_2}\right\} + \theta_2$$

$180 \quad - \quad 10 \quad + 25$

$x = 90 + \theta_4 - \theta_3$

$$atan2\left(\frac{4.226}{-0.3486 + 4.9149 + 10 + 9}\right)$$

$$atan2\left(\frac{4.226}{23.5663}\right)$$

$$\tan(\theta_1 - \theta_2) = \frac{C_y}{C_x}$$

$$\theta_1 - \theta_2 = atan2\left(C_y / C_x\right)$$

$$\theta_1 = atan2\left(\frac{C_y}{C_x}\right) + \theta_2$$