Assessment 1.1: Portfolio Optimization Using Genetic Algorithm: Maximizing Return and Minimizing Risk under a Given Expected Return Constraint

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1. Introduction

Portfolio optimization is a crucial concept in finance, which involves assigning assets in a portfolio to create an optimal portfolio with the highest returns while minimizing risk. The primary goal is to determine the best possible combination of asset weights that results in an efficient portfolio, i.e., one that provides the highest return for a given risk level or the lowest risk for a given level of return (Markowitz, 1952). One of the main challenges in portfolio optimization is identifying the optimal combination of investments that yields the highest return for a given level of risk or lowest risk for a given level of return. This can be framed as an optimization problem with mathematical formulations of the objective function and constraints. Genetic algorithms (GAs) have emerged as a powerful optimization technique in finance, including portfolio optimization, due to their ability to explore complex search spaces and find near-optimal solutions effectively (Lu et al., 2014). As such, this report looks to address the portfolio optimization challenge using a genetic algorithm. The proposed approach is applied to historical financial market data to determine an optimal investment portfolio within specified risk and return constraints, maximize the return while minimizing the risk.

2. Problem Domain

In portfolio optimization, the problem is to select a combination of financial assets that will maximize returns while minimizing risks, taking into account the expected returns, risks, and inter-dependencies of the assets. The **problem instance** considered in this report involves a set of 10 financial assets with historical data on their returns and volatility over a 5-year period. The **dataset** is obtained from**Yahoo Finance** and is widely used in the literature to benchmark portfolio optimization algorithms (Bingul, 2007). Our goal is to determine the optimal allocation of capital across these assets that maximizes the portfolio return while keeping the portfolio volatility (**risk**) below 12%. To achieve this goal, we will estimate the expected returns and risks associated with each asset using the historical data and consider their correlations when constructing a portfolio. The optimization problem is to determine the proportions to invest in each asset such that the expected return is maximized subject to the volatility constraint.

$Let\ x1,\ x2,\ ...,\ x10$ denote the proportions to invest in each of the 10 stocks. The expected portfolio return (μp) and volatility (σp) can then be calculated as:

$$\mu p\ =\ x1\mu 1\ +\ x2\mu 2\ +\ \dots\ +\ x10\mu 10$$

$$\sigma p\ =\ \surd(x1^2\sigma 1^2\ +\ x2^2\sigma 2^2\ +\ ...+\ x10^2\sigma 10^2)$$

Where $\mu 1,\ \mu 2,\ ...,\ \mu 10\ and\ \sigma 1,\ \sigma 2,\ ...,\ \sigma 10$ are the expected returns and volatilities of the individual stocks. The optimization problem can thus be formulated as:

Maximize: $R(x)\ =\ \mu p$

Subject to: $E(x) =\ \sigma p\ \ \le\ 12\%$

$$x1,\ x2,\ ...,\ x10\ \ge\ 0\ (Non-negativity\ constraints)$$

$$x1 + x2 + ... + x10\ =\ 1\ (Full\ investment\ constraint)$$

2.1. Objective Function and Dependencies

The objective function for the portfolio optimization problem is to maximize returns while minimizing risks subject to a given expected return constraint. Formally, we can define the problem as follows:

$$maximize\ R(x)\ subject\ to\ E(x)\ \ge\ E\_min\ \sigma(x)\ \le\ \sigma\_max$$

where $R(x)$ is the expected return of the portfolio, $E(x)$ is the expected risk of the portfolio, $\sigma(x)$ is the standard deviation of the portfolio returns, E_min is the minimum expected return constraint, and σ_max is the maximum risk constraint.

The expected return of the portfolio can be computed as:

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$$R(x) = \sum i \, wi \, ri$$

where wi is the weight of asset i in the portfolio and ri is the expected return of asset i.

The expected risk of the portfolio can be computed as:

$$E(x) = \sum i, j \, wi \, wj \, \sigma ij$$

where σ1J is the covariance between asset i and asset j.

The standard deviation of the portfolio returns can be computed as:

$$\sigma(x) = \sqrt{}(wT \, \Sigma \, w)$$

where Σ is the covariance matrix of the asset returns.

2.2. Fitness Function

In order to evaluate the quality of the solutions generated by the genetic algorithm, we must define a fitness function that measures how well a solution meets the portfolio optimization criteria. The fitness function should take into account both the expected return and the risk associated with the portfolio. One common approach is to use the Sharpe ratio, which is defined as the ratio of the excess return of the portfolio over the risk-free rate to the portfolio's standard deviation. The Sharpe ratio serves as a measure of the risk-adjusted return of a portfolio.

Fitness Function: $F(x) = \frac{R(x) - R_f}{\sigma(x)}$

Where:

- $F(x)$ is the fitness of the portfolio,
- $R(x)$ is the expected return of the portfolio,
- R_f is the risk-free rate, and
- $\sigma(x)$ is the standard deviation (risk) of the portfolio.

By maximizing the Sharpe ratio, the genetic algorithm seeks to find a portfolio with the highest risk-adjusted return. This fitness function ensures that the algorithm balances the trade-off between risk and return while searching for an optimal solution. The fitness function will be used to evaluate and rank the solutions in the population, allowing the algorithm to select the best performing individuals for reproduction and mutation operations, ultimately converging towards an optimal solution.

3. Methodology

The methodology used to optimize the portfolio involves the use of the genetic algorithm. The genetic algorithm is a heuristic optimization technique that is inspired by the process of natural selection. The algorithm works by iteratively generating a population of potential solutions, evaluating their fitness based on their objective function value, and selecting the best solutions to generate a new population. The process continues until a satisfactory solution is found. The genetic algorithm has been extensively used in portfolio optimization, and several variations of the algorithm have been proposed in the literature. Some of the most popular variations include the multi-objective genetic algorithm, the bi-objective genetic algorithm, and the constrained genetic algorithm.

3.1. Optimisation Techniques

In this report, the optimization process will use the **crossover operator as the primary optimization technique**. The crossover operator involves exchanging genetic material between two parent solutions to create a new child solution. The process of crossover involves selecting two parent solutions from the population, selecting a crossover point, and swapping the genetic material between the two parents to create a new child solution (Ackora-Prah et al., 2014).

The effectiveness of crossover as an optimization method in genetic algorithms can be influenced by the population size. Therefore, the optimization process in this report will involve changing the population size three times to analyze the impact of population size on the effectiveness of the crossover operator.

1. The initial population size will be set at 50 portfolios, and the optimization process will be run for 100 iterations. At the end of the 100 iterations, the population will be evaluated, and the best portfolios will be selected to form the new population.
2. In the second iteration, the population size will be increased to 100 portfolios, and the optimization process will be run for 100 iterations. At the end of the 100 iterations, the population will be evaluated, and the best portfolios will be selected to form the new population.
3. In the final iteration, the population size will be decreased to 25 portfolios, and the optimization process will be run for 100 iterations. At the end of the 100 iterations, the population will be evaluated, and the best portfolios will be selected to form the final population.

The use of the crossover operator in combination with changing the population size will enable a comparative analysis of their effectiveness in solving the problem instance. To optimize the portfolio, we will first define our objective function (Vasiani et al., 2020). Let xi be the proportion of our total investment that is allocated to stock i , and let ri and σi be the expected return and standard deviation of returns for stock i, respectively. We can then define the expected return and risk of our portfolio as follows:

$$Expected\ return = \ R(x) \ \ = \sum i\ xi\ ri$$

$$Expected\ Risk = \ E(x) \ = \sum i, j\ wi\ wj\ \sigma ij$$

where σij is the correlation coefficient between the returns of stock i and stock j.

We want to maximize our expected return while keeping our risk below a given threshold, which we will call $Rmax$. We can express this as the following constrained optimization problem:

$$Maximize: \ Expected\ return \ = \sum i\ xi\ ri$$

$$Subject\ to: \ E(x) \ <= \ Rmax$$

$$\sum i\ xi \ = \ 1\ (entire\ investment\ is\ allocated)$$

$$0 \ <= \ xi \ <= \ 1\ (no\ short-selling\ or\ over-investment)$$

To solve this optimization problem using the genetic algorithm, we will represent a potential solution as a binary string of length n, where n is the number of stocks in the portfolio. Each element in the string represents whether or not to include the corresponding stock in the portfolio. We will then use crossover to generate a population of potential solutions, evaluate their fitness using the objective function, and evolve the population over generations until a satisfactory solution is found (Chen et al., 2019)

There are many optimization algorithms that can be used to solve this problem, such as the Markowitz (1952) mean-variance optimization and the Sharpe ratio optimization. However, the genetic algorithm has been shown to be effective in solving similar portfolio optimization problems, such as the one considered in the paper by (Hoklie and Zuhal 2010).

In conclusion, this report, we have described the problem instance of optimizing a portfolio of stocks from Yahoo Finance, a public available dataset using the genetic algorithm. We have defined the objective function, formulated the optimization problem, and explained how the genetic algorithm can be used to solve the problem.

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(Azad & Ryan, 2014) (Chen et al., 2019) (Markowitz, 1952) (Vasiani et al., 2020) (Suksonghong et al., 2014) (Reddy et al., 2021) (Hoklie & Zuhal, 2010) (Ackora-Prah et al., 2014)