Statistics for Psychology A

Orin Levi

(Scatter Plot)

$\neq$

α

Z_c

(Type I Error)

$n\beta\alpha$

$\sigma\sigma$

(Z, B, C)

$(\Phi(z))$

$\Phi(z)$

$\rightarrow$ $\rightarrow$ $\rightarrow$

$$a \neq 0 y = ax + b$$

$$a \neq 0 y = ax$$

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\sum (x_i - c)^2 \Rightarrow c = \bar{X}$$

$$\sum |x_i - c| \Rightarrow c =$$

$$\Rightarrow c =$$

$$V = \frac{n - f_{Mo}}{n} \cdot 100$$

f_{Mo}

$$MAD = \frac{1}{n} \sum_{i=1}^{n} |x_i - |$$

$|b| b$

-1

$$S_n^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{X})^2$$

^{2}

$n - 1n$

$$S_n^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{X})^2 S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{X})^2$$

$nn - 1$

$$S_n = \sqrt{S_n^2}$$

a

a

b

b

b^2

$|b|$

$$Range = \max - \min$$

$$IQR = Q_3 - Q_1$$

$$Z_i = \frac{x_i - \bar{X}}{S_n}$$

$Z > 0$

$Z < 0$

$Z = 0$

$\bar{Z} = 0$

$S_Z = 1$

$S_n = 0$

Z

$p\%p$

(Scatter Plot)

(X, Y)

$$-1 \leq r \leq 1$$

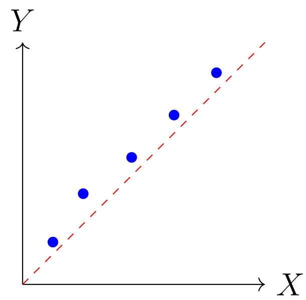
$r > 0$

$r < 0$

$r = 0$

$|r|$

(r)



$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n \cdot S_{nx} \cdot S_{ny}}$$

$$r = \frac{1}{n}\sum Z_{x_i} Z_{y_i}$$

$$r(x,y) = r(y,x)$$

$$r$$

$$r$$

$$r = 0$$

$$r \neq$$

$$Y = aX + b$$

$$(Z)$$

$$b \neq 0$$

$$a > 0$$

$$a < 0$$

$$(r)$$

$$z_i = \frac{x_i - \bar{x}}{S_x}$$

$$\bar{z} = \frac{1}{n}\sum_{i=1}^{n} z_i = \frac{1}{S_x} \cdot \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})$$

$$\sum_{i=1}^{n}(x_i - \bar{x}) = 0$$

$$\bar{z} = 0$$

$$S_z^2 = \frac{1}{n}\sum_{i=1}^{n} z_i^2 = \frac{1}{S_x^2} \cdot \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$S_x^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$S_z^2 = 1 \Rightarrow S_z = 1$$

1–1

$$r = \frac{1}{n}\sum_{i=1}^{n} z_{x_i} z_{y_i}$$

$$|r| \leq 1$$

$4 \cdot 10 = 40$

n

$$n! = n \cdot (n-1) \cdot \cdots \cdot 1$$

$5!$

n

$$\frac{n!}{k_1! \cdot k_2! \cdots}$$

k_i

$$\frac{5!}{2!}$$

nk

$$V(n,k) = \frac{n!}{(n-k)!}$$

nk

$$n^k$$

10^3

$\frac{26!}{23!}$

nk

$*$

$|$

$$\binom{k+n-1}{n-1}$$

nk

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$P() = 1 - P()$$

$$(A, B) \Rightarrow$$

Ω

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

$$P(A) = \frac{|A|}{|\Omega|}$$

$$P(\Omega) = 1$$
$$0 \leq P(A) \leq 1$$

$A \bar{A} A$

$$P(\bar{A}) = 1 - P(A)$$

$$A \cup B = \{\}$$

$$A \cap B = \{\}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = 0$$

$$A \cap B = \emptyset$$

$$A \cup \bar{A} = \Omega$$

$$\binom{7}{5} \cdot 0.8^5 \cdot 0.2^2$$

$\leftarrow$

$\leftarrow$

$$P() = 1 - P()$$

$$1 - P()$$

$$P =$$

BA

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, P(B) > 0$$

B

B

A

$$P(A \cap B) = P(A \mid B) \cdot P(B)$$

AB

BA

$$P(A \mid B) = P(A)$$

$$P(A \cap B) = P(A) \cdot P(B)$$

AB

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

$$P(B) = P(B \mid A)P(A) + P(B \mid \bar{A})P(\bar{A})$$

$P(\mid)$

$P(\mid)$

$$P(B \mid A)P(A \mid B)$$
$$P(B)$$

μ, σ, p

$\bar{X}, S, \hat{p}$

μ

$\bar{X}$

p

$\hat{p}$

$X : \Omega \to \mathbb{R}$

$P(X = x)$

$P(X = x) = 0$

$$\frac{f_i}{n}$$

$$\longrightarrow$$

$$E(X) = \sum x \cdot P(X = x)$$

7.6.3

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

$\bar{X}$

$$\bar{X}\mu$$

$$E(\hat{\theta}) = \theta$$

$n \to \infty$

$$\hat{\theta} \to \theta$$

$$\to \ \to \ \to$$

X

Y

X

$f_i w_i$

$$= f_i \cdot w_i$$

$$= \frac{f_i}{w_i}$$

$$= \times = f_i$$

n

$$= \frac{f_i}{n \cdot w_i}$$

$$= \frac{f_i}{n}$$

$f(x)$

8.8 A probability density function $f(x)$ satisfies:

$$f(x) \geq 0 \qquad \text{and} \qquad \int_{-\infty}^{\infty} f(x)\, dx = 1$$

$[a, b]$

$$P(a \leq X \leq b) = \int_a^b f(x)\, dx$$

$$P(X = x) = \int_x^x f(t)\, dt = 0$$

$$< \; <$$

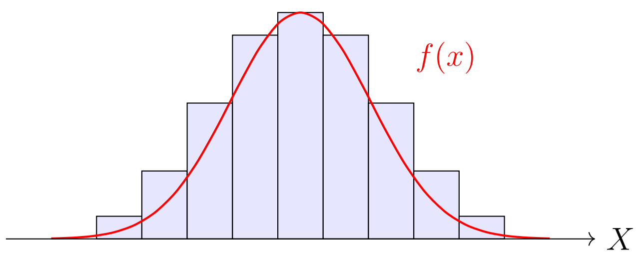
$$< \; <$$

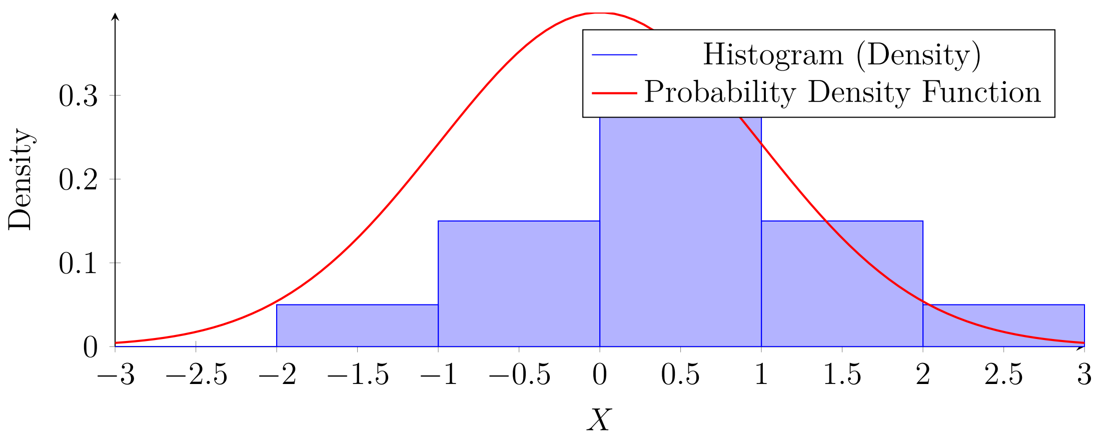
μ

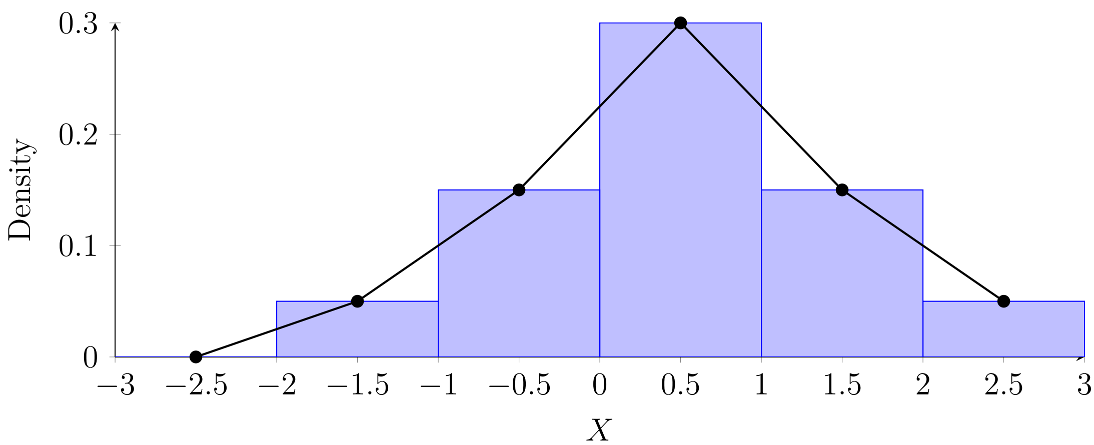
σ

Y

$\rightarrow \rightarrow \rightarrow$







μ

σ

μ

$X\mu$
σ

σ

σ

$f(x)$

$$f(x) = \lim_{\Delta x \to 0} \frac{P(x \le X < x + \Delta x)}{\Delta x}$$

$f(x)$

$f(x)$

$=$

$$P(X = x_0) = \int_{x_0}^{x_0} f(x)\, dx = 0$$

$$x_0 - \varepsilon \le X < x_0 + \varepsilon$$

$\leq <$

$$P(X \leq a) = P(X < a)$$

$$P(X \leq a) = P(X < a) + P(X = a)$$

$P(X = a) = 0$

$\mu \pm \sigma$

$\mu \pm 2\sigma$

$\mu \pm 3\sigma$

$$Z = \frac{X - \mu}{\sigma}$$

$$P(a \leq X \leq b) = P\left(\frac{a - \mu}{\sigma} \leq Z \leq \frac{b - \mu}{\sigma}\right)$$

X

$Z \sim N(0, 1)$

$Z Z 0$

$$P(Z < z) = 0.5 + B$$

$$P(Z < z) \Rightarrow$$

$$P(Z > z) = 1 - P(Z < z)$$

$$P(z_1 < Z < z_2) = P(Z < z_2) - P(Z < z_1)$$

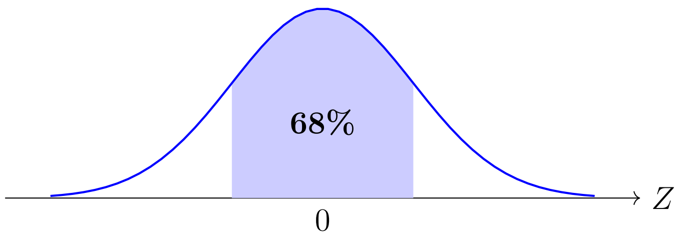
$$P(Z < -z) = 1 - P(Z < z)$$

$p\%$

p

Z

$$X = \mu + Z\sigma$$



n

n

n

n

X_iX_i

X_1

X_2

nX_n

X_i

$$\mu_{\bar{X}} = \mu$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

$$X_1, \dots, X_n$$

$$Cov(X_i, X_j) = 0 i \neq j$$

μn

$X_1, \dots, X_n$

$$E[X_i] = \mu, Var(X_i) = \sigma^2$$

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

$$E[\bar{X}] = \mu Var(\bar{X}) = \frac{\sigma^2}{n}$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

$$Var(X_1 + \cdots + X_n) = \sum_{i=1}^{n} Var(X_i) \ + \ 2 \sum_{i<j} Cov(X_i, X_j)$$

$X_1, \dots, X_n$

$$Cov(X_i, X_j) = 0 i \neq j$$

$$Var(X_1 + \cdots + X_n) = \sum_{i=1}^{n} Var(X_i)$$

$X_i E(X_i) = E(X)$

$E(\bar{X}) = \mu$

nnn

$X_j X_i$

$\sigma/\sqrt{n}\sigma$

n

$$X \sim N(\mu, \sigma^2)$$

n

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$n \geq 30$

n

$n \geq 30$

n

n

$n = 1n = 1$

$$\bar{X} = X \Rightarrow \sigma_{\bar{X}} = \sigma$$

$n \to \infty$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \to 0$$

n

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

Z

$$Z \sim N(0, 1)$$

σ

$\sigma/\sqrt{n}$

$\sqrt{n}$

n

H_0
H_1
H_0
α
H_0

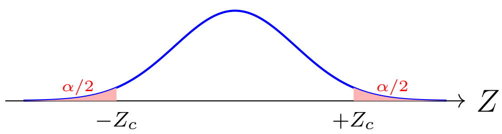
$$\alpha = 0.050.01$$

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

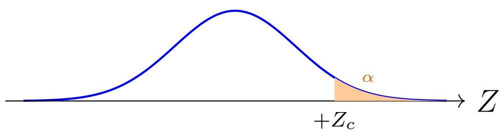
$H_0 \mu_0$

$(n \geq 30)$

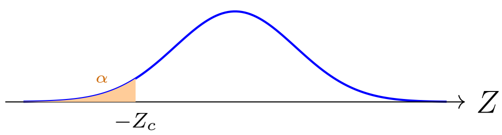
$H_0 \alpha Z_c$
$\alpha/2 (H_1 : \mu \neq \mu_0)$



$\alpha (H_1 : \mu > \mu_0)$



$\alpha (H_1 : \mu < \mu_0)$



H_0

pZ

$p \Longleftarrow$

$\alpha p \Longleftarrow Z_c Z$

Z

Z

$H_0 p \leq \alpha$

$H_0 p > \alpha$

$H_0 p$

p

p

$H_0 p$

Z

αp

$\bar{X}$

$H_0 p$

H_0

$p \alpha$

$H_1 H_0$

H_0

$H_1 H_0$

H_0

H_0

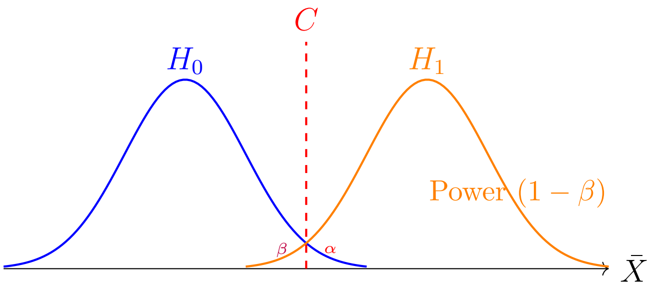
α

$(1-\beta)$	H_0	(H_1)
(α)	H_0	(H_0)
(β)	H_0	(H_1)
$(1-\alpha)$	H_0	(H_0)

$H_1 H_0$

α

		(α)
1.96	1.645	$\alpha = 0.05$
2.58	2.33	$\alpha = 0.01$



$H_1 H_0$

H_0

H_0

(Type I Error)
H_0

$$P() = \alpha$$

α

α

α

$H_1 H_0$

$$P() = \beta$$

β

β

$H_1 \beta$

$H_1 H_0$

$$= 1 - \beta$$

$H_1 H_0$

α

n

$n\beta\alpha$

α

n

α

$\beta\alpha$

α

H_1

Reality ↓ / Decision →	**Retain** H_0	**Reject** H_0
H_0 **is True**	Correct $(1-\alpha)$ ✓	Type I (α) ✗
H_1 **is True**	Type II (β) ✗	Power $(1-\beta)$ ✓

Table 1: Statistical Decision Matrix

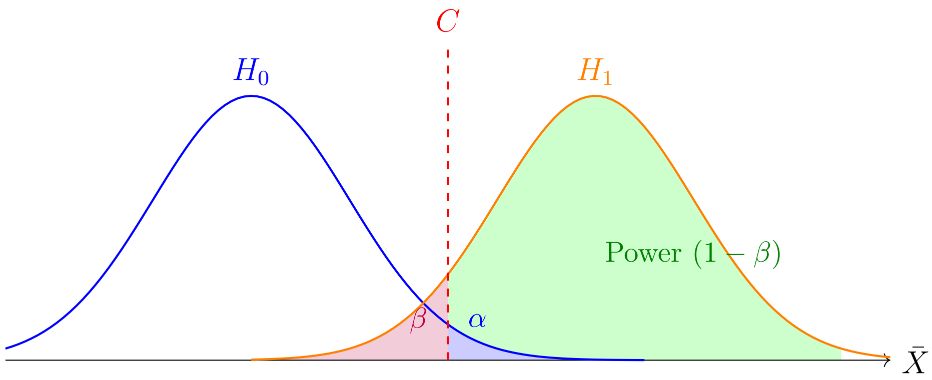
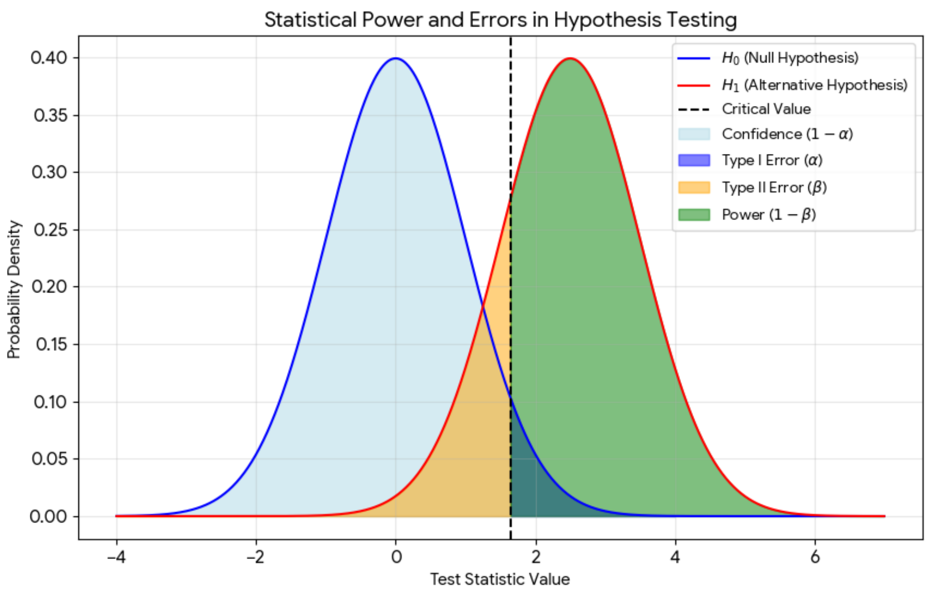


Figure 7: Visual representation of Type I/II errors and Statistical Power



α

β

n

$$n = \left(\frac{(Z_{1-\alpha} + Z_{1-\beta}) \cdot \sigma}{\mu_1 - \mu_0} \right)^2$$

$Z_{1-\alpha} Z_{1-\alpha/2}$

H_0

n

$\alpha = 0.05$

$p = 0.051 p = 0.049$

H_0

H_0

H_0

H_0

H_0

d

d

$$d = \frac{\bar{X}_1 - \bar{X}_2}{s}$$

s

$$s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

μ_0

$$d = \frac{\bar{X} - \mu_0}{s}$$

d

dp

$d \approx 0.2$

$d \approx 0.5$

$d \approx 0.8$

pd

pd

Cohen's d

$d \approx 0.2$

$d \approx 0.5$

$d \approx 0.8$

$\pm$

$\sigma\sigma$

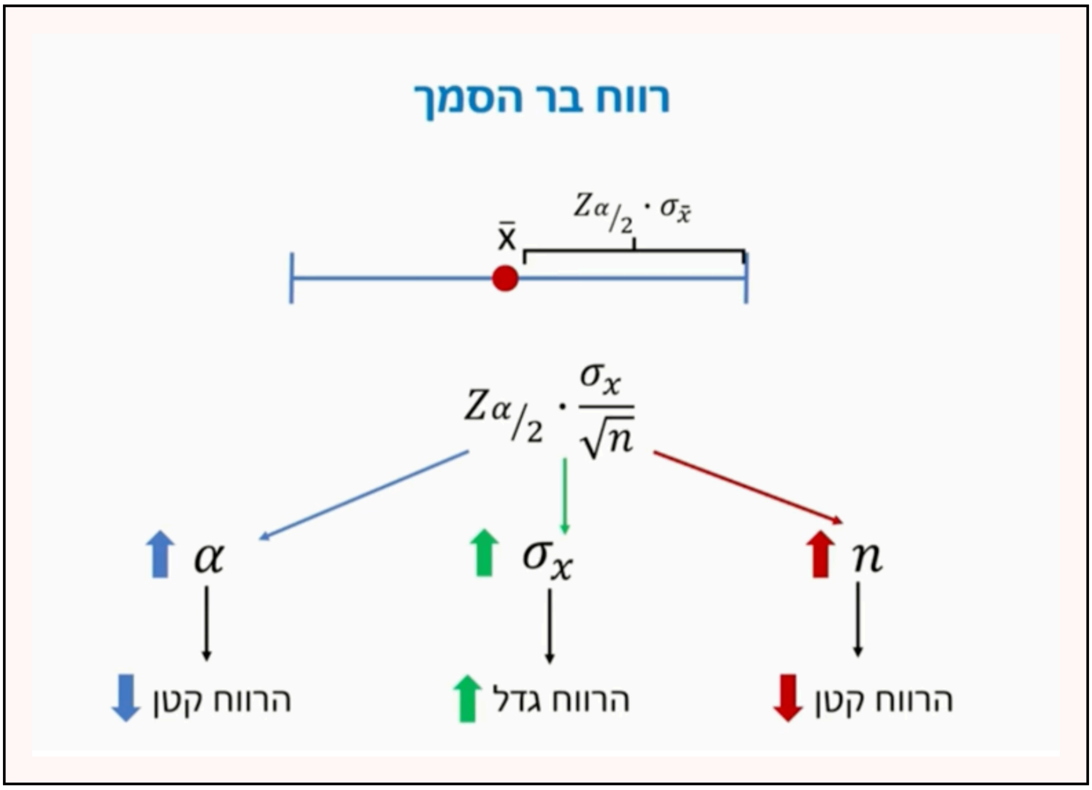
$$\bar{X} \pm Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

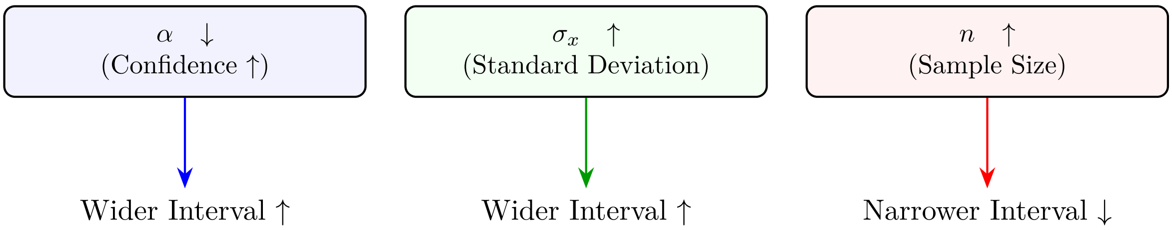
$Z_{\alpha/2} \cdot \frac{\sigma_x}{\sqrt{n}}$

$\rightarrow n$

$\rightarrow \sigma$

$\rightarrow$





$1 - \alpha$

α

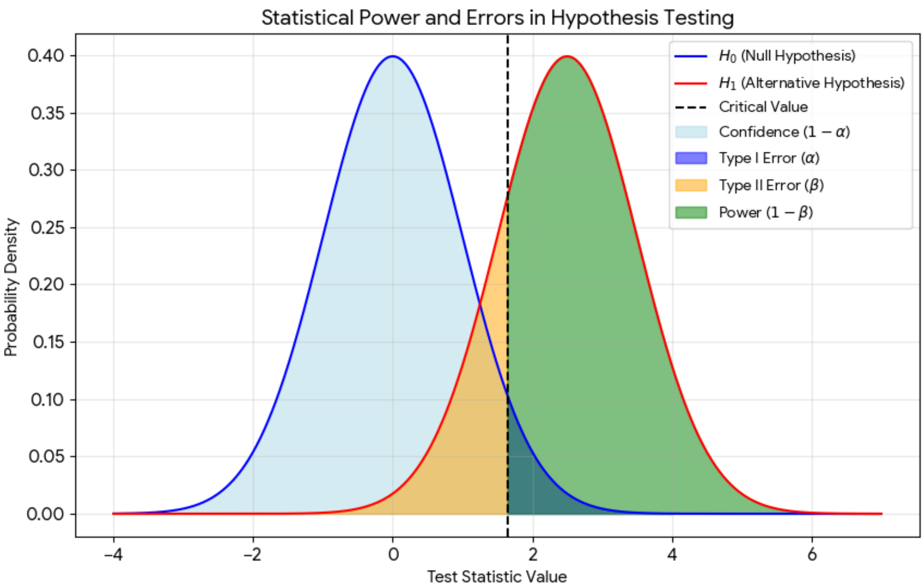
$H_0 \rightarrow H_0$

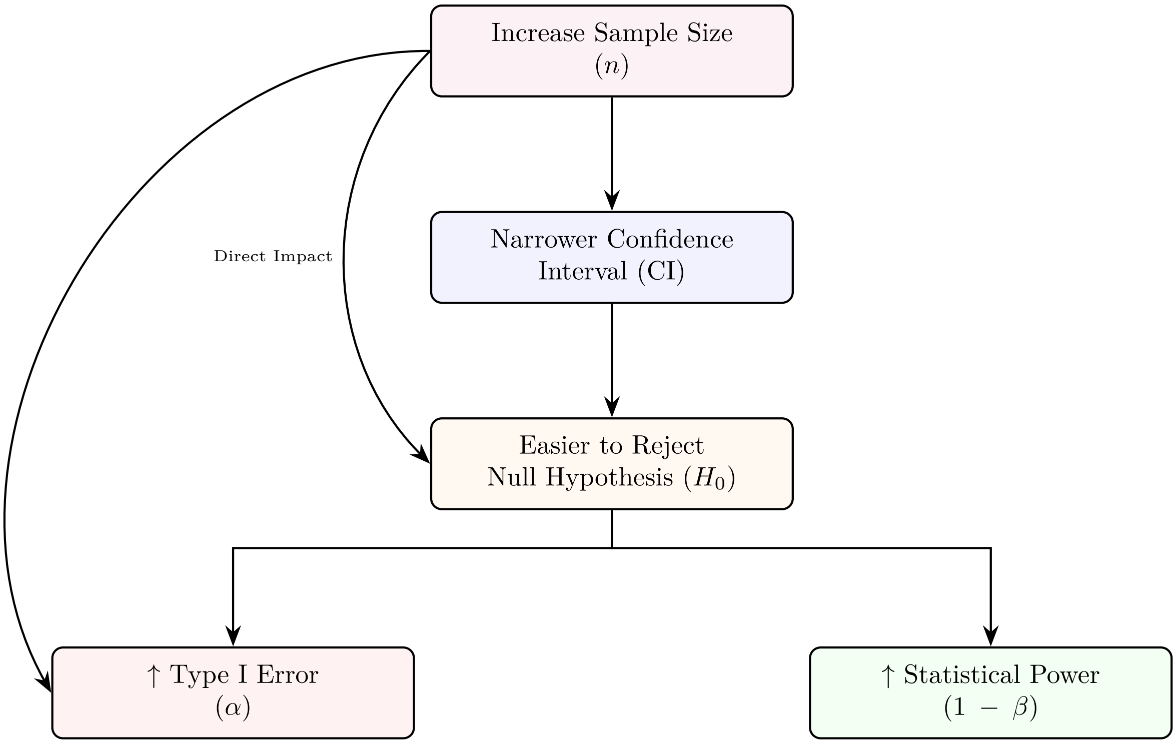
$H_0 \rightarrow H_0$

$$(n)$$

$$H_0\alpha$$

$$(1-\beta)H_0$$





Increase Sample Size (n)
Direct Impact
Narrower Confidence Interval (CI)
Easier to Reject Null Hypothesis (H_0)
↑ Type I Error (α)
↑ Statistical Power ($1 - \beta$)

$$Var(X) = E[(X - \mu)^2]$$

$$X \sim N(\mu, \sigma^2)$$

$\sigma\mu$

$$Z = \frac{X - \mu}{\sigma} \sim N(0, 1)$$

Z

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$X_1, \ldots, X_n$

$$E[\bar{X}] = \mu Var(\bar{X}) = \frac{\sigma^2}{n}$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

n

n

σ

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

p

$$p = P(|Z| \geq |z| \mid H_0)$$

αH_0

$\beta H_1 H_0$

$$= 1 - \beta$$

H_1

$$d = \frac{\mu_1 - \mu_0}{\sigma}$$

$$\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

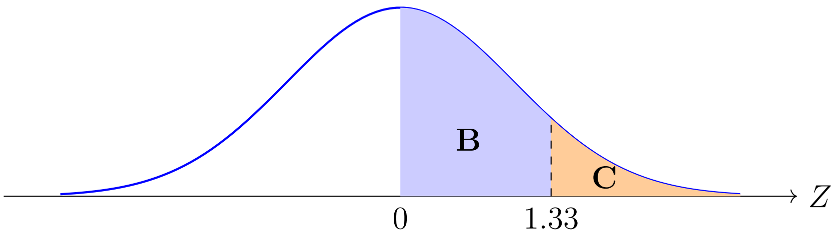
$\alpha 1 - \alpha$

(Z, B, C)

$z \geq 0$

$Z 0$

Z



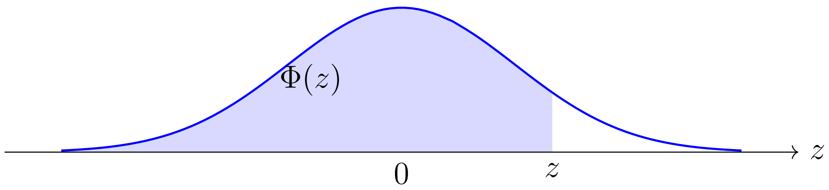
Z	B	C	Z	B	C	Z	B	C	Z	B	C
0.00	.0000	.5000	0.30	.1179	.3821	0.60	.2257	.2743	0.90	.3159	.1841
0.01	.0040	.4960	0.31	.1217	.3783	0.61	.2291	.2709	0.91	.3186	.1814
0.02	.0080	.4920	0.32	.1255	.3745	0.62	.2324	.2676	0.92	.3212	.1788
0.03	.0120	.4880	0.33	.1293	.3707	0.63	.2357	.2643	0.93	.3238	.1762
0.04	.0160	.4840	0.34	.1331	.3669	0.64	.2389	.2611	0.94	.3264	.1736
0.05	.0199	.4801	0.35	.1368	.3632	0.65	.2422	.2578	0.95	.3289	.1711
0.06	.0239	.4761	0.36	.1406	.3594	0.66	.2454	.2546	0.96	.3315	.1685
0.07	.0279	.4721	0.37	.1443	.3557	0.67	.2486	.2514	0.97	.3340	.1660
0.08	.0319	.4681	0.38	.1480	.3520	0.68	.2517	.2483	0.98	.3365	.1635
0.09	.0359	.4641	0.39	.1517	.3483	0.69	.2549	.2451	0.99	.3389	.1611
0.10	.0398	.4602	0.40	.1554	.3446	0.70	.2580	.2420	1.00	.3413	.1587
0.11	.0438	.4562	0.41	.1591	.3409	0.71	.2611	.2389	1.10	.3643	.1357
0.12	.0478	.4522	0.42	.1628	.3372	0.72	.2642	.2358	1.20	.3849	.1151
0.20	.0793	.4207	0.50	.1915	.3085	0.80	.2881	.2119	1.30	.4032	.0968

$Z = 1.96 0.4750$

$Z = 1 0 Z = -1$

$0.5 + B Z \Phi$

$z\big(\Phi(z)\big)$



$\Phi(z)$

	z