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Adversarial Training for Neural Combinatorial Solvers

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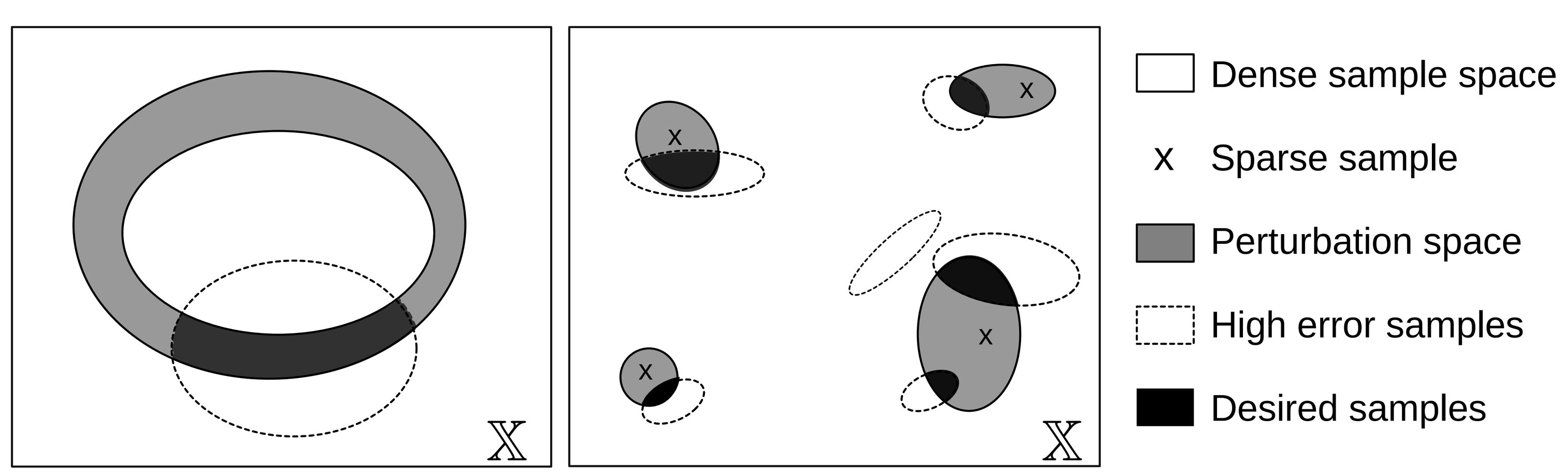
TL;DR

- We propose **adversarial training** to improve **generalization** and **robustness** on neural combinatorial solvers.
- We propose a **more efficient** data generation method for SAT and k-SAT.
- We show adversarial training **does not negatively impact** training performance.

Neural Combinatorial Solvers

- **Bad generalization:** from randomly generated data to other domains, and from small to large problem instances.
- **Bad robustness:** adversarial attacks reveal hard model-specific instances
- **Data generation** is either:
  ✗ Incomplete and efficient
  ✗ Complete and inefficient

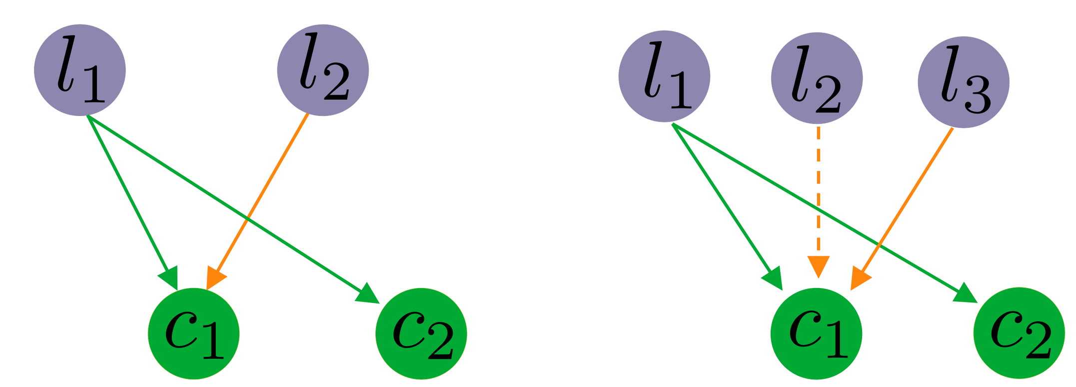


Use adversarial attacks to:
1) improve robustness and generalization
2) ease learning with costly generated data

Adversarial Training

- Exploit SAT **invariances** (satisfiability):

$$(l_1 \lor l_2) \land (l_1) = True \qquad \begin{cases} del \rightarrow (l_1 \lor \cancel{l_2}) \land (l_1) = True \\ add \rightarrow (l_1 \lor l_2 \lor l_3) \land (l_1) = True \end{cases}$$



- Given an instance x and a neural solver S_θ, we obtain the **perturbed instance** $\widetilde{x} = f_p(x)$ learning a perturbation matrix p s.t.

$$\max_p \mathcal{L}(S_\theta(\widetilde{x}))$$

$$f_p(x) = \begin{cases} x_{ij} - p_{ij}, & \text{if edge } i \rightarrow j \text{ exists} \\ x_{ij} + p_{ij}, & \text{otherwise} \end{cases}$$

$$x \left\{ \begin{array}{r|cc|} & c_1 & c_2 \\ l_1 & 1 & 1 \\ l_2 & 1 & 0 \\ \overline{l_1} = l_3 & 0 & 0 \\ \overline{l_2} = l_4 & 0 & 0 \end{array} \right. \overset{f_p}{\Longrightarrow} \widetilde{x} \left\{ \begin{array}{|cc|} c_1 & c_2 \\ 1-0 & 1-0 \\ 1-p_{21} & 0+p_{22} \\ 0+p_{31} & 0+p_{32} \\ 0+p_{41} & 0+p_{42} \end{array} \right.$$

- Adversarial training procedure: use the perturbed instances to train the neural solver S_θ s.t.

$$\min_\theta \mathcal{L}(S_\theta(\widetilde{x}))$$

Efficient Data Generator

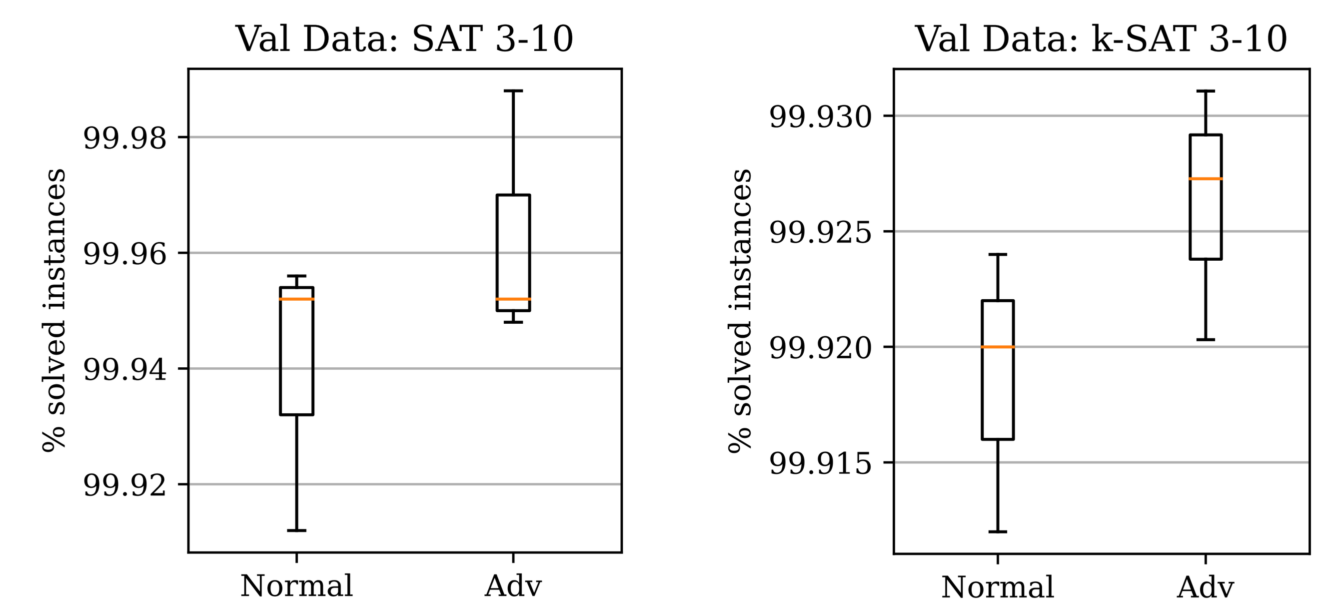
- **Former method:** randomly build x and get the solution y with a classical solver → Inefficient
- **Our method:** randomly build x starting from a solution y → No need for a solver

$$y = [x_1, \overline{x_1}, x_2, \overline{x_2}, x_3, \overline{x_3}]$$

$$x = \bigwedge_{i=1}^{n_c} (l_{i1} \lor l_{i2} \lor \cdots \lor l_{ik_i})$$

- For SAT $\forall i,\ k_i \sim 1 + Ber(0.7) + Geo(0.4)$ and $n_c \sim N(\mu, \sigma^2)$
- For k-SAT $\forall i,\ k_i = k$ and $n_c = n_v \cdot \alpha_k$

Training Performance



- **Adversarial training does not negatively impact** training performance since we have obtained same results compared with the normal training.