Representative democracy and marginal rate progressive income taxation

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Abstract

This paper develops a political economy model that is consistent with the fact that democracies have a preference for increasing marginal tax rates on income. We present a model in which there is an exogenous set of political parties with preferences over the set of admissible tax schedules. This set contains virtually any increasing and piecewise linear continuous function. Each party decides whether or not to present a candidate for election. There is a fixed cost of running. The elected candidate implements one of her preferred tax policies. Our main results provide conditions under which a Strong Nash Equilibrium exists, and a tax schedule with increasing marginal tax rates is implemented in some Nash Equilibria and in any Strong Nash Equilibrium.
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1. Introduction

A common feature of tax systems in all industrial democracies is the progressivity of statutory income taxation; that is, in these countries, the amount of

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income tax paid as a proportion of income rises with income. More surprisingly, in an overwhelming majority of these countries the statutory income tax schedule is progressive at the margin; not only the average tax rate increases with income, but so does the marginal tax rate (this is so for all OECD countries; see OECD, 1997).[1]

Marginal rate progressivity can be justified from a normative perspective through its connection with inequality aversion (cf. Le Breton et al., 1996) and the principle of equal sacrifice (see Mitra and Ok, 1997).[2] While the normative approach may be viewed as a compelling one, it is unclear whether its influence on the actual practice of tax design is significant. Moreover, it is natural to suppose that in a democracy the choice of a tax schedule should be related to the preferences of a majority of citizens. Yet a general positive theory of progressive statutory income taxation has proven elusive. A major reason for this is that modeling income taxation as the outcome of some voting mechanism suffers from the well-known multidimensionality problem of voting theory. Indeed, choices of tax schedules from general sets of admissible functions result in situations that are entirely unstable in nature. In general, voting over tax schedules leads to 'gross instability and cycling over tax structures, with new majority coalitions perpetually emerging and overturning the existing tax code in favor of a new one which favors them' (Kramer, 1983, p. 226).

Consequently, the models in the related literature typically restrict the shape of the tax functions to achieve low dimensions. Romer (1975), Roberts (1977), and Meltzer and Richard (1981), among others, restrict the set of feasible tax schedules to linear functions. Cukierman and Meltzer (1991) and Roemer (1999) analyze models in which the tax functions are quadratic in income. Snyder and Kramer (1988) place fewer restrictions on the shape of allowable tax schedules but assume that parties may only propose tax functions that are ideal for some voter. The results in these studies run into serious difficulties when nonlinear tax schedules with increasing marginal rates are viewed as admissible members of political agendas. In fact, once marginal rate progressive tax schedules are taken into consideration, voting cycles become inescapable. The result is a highly counter-factual picture of perpetual chaos. This is dampened by a result in Marhuenda and Ortuño-Ortín (1995), which says that if the median voter's income is below the mean income and voting is self-interested, any marginal rate progressive (convex) tax schedule defeats any marginal rate regressive (concave) one under pairwise majority voting. However, this result provides only a partial account of a possible

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[1] The *effective* tax schedule, in contrast with the *statutory* tax schedule, takes into account many special deductions and exemptions provided in the tax code. It is much less progressive than the statutory tax schedule, and might be even regressive in some countries.

[2] Other normative results, however, contradict marginal progressive income taxation. Among them is the famous *end-point theorem* from the theory of optimal income taxation, which states that, under quite general conditions, the optimal marginal tax rate at the top of the income distribution should be zero (Seade, 1977).

connection between democracy and marginal rate progressive taxation, since, given any tax schedule, it is always possible to design an alternative one that hurts a minority of agents, benefiting the rest of the population, therefore beating the first tax schedule in pairwise majority voting.[3]

While nonexistence of equilibria seems to be inescapable in models of *direct* democracy, this is not the case in models of *representative* democracy.[4] In the context of income taxation, a model of representative democracy features citizens that do not vote directly on alternative tax schedules. Rather, they vote for representatives, and delegate decisions on taxation to these elected delegates, who will have different attitudes toward taxation and redistribution.

In this paper, we adapt a model of representative democracy developed by Feddersen et al. (1990) to a taxation environment in which the policy space contains virtually *any* statutory tax function. This model provides a political framework friendly to stable equilibria, even though it was not especially built to solve taxation problems. We argue that general sets of admissible tax schedules are compatible with existence of equilibrium. What is more, we find that only tax functions with increasing marginal tax rates (i.e., convex functions) are implemented in any equilibrium under some qualifications. This is consistent with the observed stability of tax schedules and demand for progressivity in developed democracies.

Essentially, the model in Feddersen et al. (1990) proposes a game of electoral competition in which political parties choose a policy location and whether or not to present a candidate; each citizen then chooses a candidate and the winner is determined by the plurality rule. In our model, there is an exogenous set of political parties with lexicographic preferences over the set of tax functions. Their main concern is to implement tax schedules whose corresponding post-tax income distributions are desirable in terms of their own ideology.[5] Whenever this criterion yields indifference between two tax functions, preference is determined by the 'simplicity' of each tax structure.[6] Each party decides whether or not to present a candidate for election. There is a fixed cost of running, while the only benefit is

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[3]In a second paper, Marhuenda and Ortuño-Ortín (1998) find that this instability problem can be solved by adding small amounts of uncertainty to the tax schedule proposed against the status quo. Their main result is that a flat-rate tax policy is a majority winner status quo. Therefore, the explanation of the attraction of tax schedules progressive at the margin remains open.

[4]See Lindbeck and Weibull (1987) for an example of a model of representative democracy that resolves the multidimensionality problem in the context of balanced-budget redistribution with two exogenously given political parties. Besley and Coate (1997) provide a more general discussion of this point.

[5]Feddersen et al. (1990) assume that candidates are interested solely in winning. Here, following Osborne and Slivinski (1996) and Besley and Coate (1997), we assume that political parties' preferences are defined over the policy space. See Wittman (1990) for a good discussion of this topic and a survey of the literature.

[6]The simplicity of a tax schedule will be measured by the number of brackets; that is, the lower the number of brackets, the simpler the tax.

that, if elected, a candidate imposes one of her preferred tax policies. The bulk of society consists of citizens that are not ideologically motivated; rather, each citizen is egoistical and votes for some candidate who is expected to implement, if elected, tax schedules that minimize this citizen's tax liability (the voting is thus modeled through sincere voting). Like Osborne and Slivinski (1996), we compare the results under the plurality rule with those under a two-ballot runoff system. The main results of the present article provide conditions under which a Strong Nash Equilibrium exists. Furthermore, a tax schedule with increasing marginal tax rates is always implemented in some Nash Equilibria and in any Strong Nash Equilibrium. The latter concept is relevant in our framework since it is related to the formation of coalitions by political parties.

The idea of income distribution as a pure public good was first developed by Thurow (1971) and used in the context of income taxation by Hamada (1973). In our paper, in contrast to Hamada's, only political parties (not the rest of the agents) derive utility from the income distribution of the society. This captures the idea that, in a representative democracy, it is not the whole society, but politically and socially active individuals, who are willing to sacrifice some of their own resources to decide on national policies. The idea that politicians regard simplicity of a tax system as important is discussed in Hall and Rabushka (1983, ch. 2) and Atkinson (1995, ch. 1) for the cases of the US and UK, respectively.

In the next section we present the model in further detail. In Section 3, we solve for the set of tax schedules that are preferred by each candidate. In Section 4 we provide conditions under which an equilibrium exists. Our main results are stated in Section 5. We conclude in the last section. Appendix A contains the proofs of some of the results stated in the paper.

2. An endowment economy model

2.1. Preliminaries

We consider an endowment economy with a continuum of agents, each of which is identified by an exogenous pre-tax income level in [0,1]. An **income distribution** is defined as a distribution function $F: \mathbf{R}_+ \to [0,1]$ with $F(1) = 1$; given an income level x in [0,1], $F(x)$ measures the proportion of income recipients with incomes of at most x. The initial income distribution of the economy is denoted by F and is assumed to be continuous and strictly increasing. Define μ and med as the mean and median income of F respectively, that is, $\mu = \int_0^1 x \, dF$ and $med = F^{-1}(1/2)$. We assume throughout that the pre-tax income distribution is right skewed, i.e., $\mu > med$.

A **B-bracket tax schedule** is defined as a tuple $(\alpha_1, \ldots, \alpha_B; b_1, \ldots, b_{B+1})$, where $B \in \mathbf{N}$, $\alpha_i \in [0, 1]$ and $\alpha_{i+1} \neq \alpha_i$ for every i, and $0 \equiv b_1 < b_2 < \ldots < b_B < b_{B+1} \equiv 1$. We will often use the term *tax schedule* in place of *B-bracket tax*

schedule. A tax schedule $(\alpha, b) \equiv (\alpha_1, \ldots, \alpha_B; b_1, \ldots, b_{B+1})$ uniquely determines a function $t_{\alpha,b}: \mathbf{R}_+ \to \mathbf{R}$ given by

$$
t_{\alpha,b}(x) = \begin{cases}
\alpha_1 x & \text{if } b_1 \leq x \leq b_2, \\
\alpha_2(x - b_2) + \alpha_1 b_2 & \text{if } b_2 < x \leq b_3, \\
\vdots & \vdots \\
\alpha_B(x - b_B) + \alpha_{B-1}(b_B - b_{B-1}) + \cdots + \alpha_1 b_2 & \text{if } b_B < x.
\end{cases}
$$

An agent x is required to pay $t_{\alpha,b}(x)$ units of income. Henceforth, we will treat each tax schedule (α, b) and its corresponding function $t_{\alpha,b}$ as identical objects. We restrict attention to the set of all tax schedules (α, b) with $\alpha_i \in (0, 1)$ for every i that satisfies

$$
\int t_{\alpha,b}(x) \, \mathrm{d}F = R, \tag{1}
$$

where $R \in (0, \mu)$. Let $\mathcal{T}$ denote this set.

A number of observations about $\mathcal{T}$ is in order. First, notice that for any t in $\mathcal{T}$ we have $t(x) < x$ for all x; that is, the tax payed by any agent is less than her pre-tax income. Second, every tax schedule in $\mathcal{T}$ has slope less than one everywhere on its domain. This is a natural condition that rules out situations in which the agents' post-tax income is negative at the margin. Finally, Eq. (1) requires that the total tax collected meet the target R.[7]

Note that the set of feasible tax functions achieves a significant level of generality, compared with the policy space in all previous related literature. $\mathcal{T}$ contains all linear and two-bracket tax functions. Moreover, any quadratic, concave, or convex function can be uniformly approximated by an element of $\mathcal{T}$. We should also mention that personal income taxes of all OECD countries except Germany do belong to $\mathcal{T}$ (OECD, 1997).

2.2. Political parties' preferences

Let $P \equiv \{1, \ldots, p\}$ be the set of political parties in the society. Political parties are policy-motivated; their preferences depend on certain features of tax schedules and their corresponding post-tax income distributions. In order to present the politicians' preferences in detail, we need to introduce the following definitions.

For each tax schedule t in $\mathcal{T}$, define

$$
F_t(x) = F(r_t^{-1}(x)), \quad x \geq 0, \tag{2}
$$

where $r_t(x) = x - t(x)$ for all x. We call F_t the *net income distribution* of the economy after taxes have been payed in accord with t. Let

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[7]As in Mitra et al. (1998), the revenue constraint condition can be relaxed to $\int t(x) \, \mathrm{d}F \geq R$. On the other hand, we could as well accomodate negative taxation (i.e. subsidies). See footnote 13.

$$\mathscr{F} = \{F_t : t \in \mathscr{T}\}.$$

In words, $\mathscr{F}$ is the set of all possible net income distributions given the set of feasible taxes.

The *Lorenz curve* associated to income distribution G, denoted by L_G, is a real function on $[0, 1]$ such that

$$L_G(q) = \frac{1}{\mu_G} \int_0^q G^{-1}(s)\,\mathrm{d}s, \tag{3}$$

where $\mu_G \equiv \int_0^1 x\,\mathrm{d}G$.[8] $L_G(q)$ is interpreted as the share of total income that is held by the poorest cumulative share q of the population under distribution G. For any two income distributions G and G^*, we say that $G \succeq_L G^*$ (or, in words, distribution G *Lorenz dominates* distribution G^*) if

$$L_G(q) \geq L_{G^*}(q) \text{ for all } q \in [0, 1].$$

The asymmetric part of $\succeq_L$, $\succ_L$, is defined as usual: $G \succ_L G^*$ if $G \succeq_L G^*$ but not $G^* \succeq_L G$.

An ***inequality measure*** I is defined as a real map on the set of all income distributions that is continuous (with respect to the sup metric) and satisfies the following property:

(***S-concavity***) $G \succ_L G^* \Rightarrow I(G) < I(G^*)$ $\forall G, G^*$

(see Dasgupta et al., 1973). $I(G)$ is interpreted as the degree of inequality associated to distribution G. The notion of continuity on an inequality measure says that small perturbations of an income distribution do not cause drastic movements of the corresponding degree of inequality. The latter condition says that when a ranking between two distributions can be derived by the Lorenz criterion, this ranking will be agreed by all S-concave inequality measures. Henceforth, I will denote a fixed inequality measure such that $I(\mathscr{F}) = (\inf_{G \in \mathscr{F}} I(G), \sup_{G \in \mathscr{F}} I(G))$. Without loss of generality, we set $I(\mathscr{F}) = (0,1)$ for convenience.

Each political party i is endowed with an ideal inequality point m_i in $(0, 1)$. Ideal points are permanent attributes of parties, in the sense that they cannot be changed at will.[9] One can show that I is not injective, and that the same degree of

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[8] The definition provided here is suitable for income distributions G whose inverse mapping G^{-1} is well-defined on $[0, 1)$. The definition can be extended to incorporate cases where the inverse mapping may fail to exist. However, since G^{-1} exists on $[0, 1)$ for all $G \in \mathscr{F}$, the above definition will be enough for our purposes.

[9] An interesting extension of this model would take into account that each party's ideal inequality point must somehow reflect the preferences of the agents that comprise this party. This extension, which complicates matters significantly, is left for future research.

inequality as evaluated by I can be obtained from a number of net income distributions. Therefore, associated with any m_i there may be several feasible tax schedules t such that $I(F_t) = m_i$. It turns out that these tax schedules will often be numerous and very different from one another. One differentiating attribute is the complexity of a tax schedule, which is naturally measured by the number of brackets; i.e., the bigger the number of brackets, the more complex the tax.[10]

Party i's preferences over tax schedules are described by means of a binary relation $\succeq_i$ on $\mathcal{T}$ such that

$$t \succeq_i t^* \Leftrightarrow \begin{cases} |I(F_t) - m_i| < |I(F_{t*}) - m_i| \\ \text{or} \\ |I(F_t) - m_i| = |I(F_{t*}) - m_i| \text{ and } B \leq B^*, \end{cases} \tag{4}$$

where B and B^* are the number of brackets of t and t^* respectively.

Several aspects of this preference relation are worth noting. The adoption of lexicographic preferences allows us to differentiate between two aspects of a political party's attitude towards taxation; while simplicity is a desirable property of a tax schedule, higher priority in determining the preference ordering is given to 'ideology'. Since a party may be indifferent among several tax schedules in terms of the latter concern, the preference for simplicity is nothing but a tie breaker.

We contend that the idea that politicians regard simplicity of a tax system as important is reasonable in the environment considered here. Moreover, it is supported by the fact that all tax reforms in OECD countries since the early 1980s have reduced the number of brackets of the statutory income tax schedule. We should also note that our results remain valid even without assuming that political parties have a preference for simplicity, if we restrict the set of feasible taxes to concave and convex functions.

2.3. The game

Following Feddersen et al. (1990), Osborne and Slivinski (1996), and Besley and Coate (1997), we consider a model of electoral competition in which each political party must decide whether or not to present a candidate that runs for office. Each party is allowed to run, although there is a fixed cost $c > 0$ of candidacy. After all parties have made their entry decisions, all members of society cast their votes. Under the *plurality rule* the winner of the election is the candidate who obtains the most votes. Under a *runoff system* the winner is determined as follows. If some candidate obtains more than half of the votes, she is the winner. Otherwise, the winner is the candidate who obtains a majority in a second election between the two candidates who obtained the most votes in the first round. In both cases, ties are broken by an equal-probability rule. An elected

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[10]Hettich and Winer (1999, p. 90) provide the same characterization of the complexity of a tax schedule.

candidate implements a tax schedule from $\mathcal{T}$. We assume that, if none of the political parties presents a candidate, the payoff of each party is dominated by any other entry profile.

For each $i \in P$, let $\mathcal{T}_i$ be the set of tax schedules $t \in \mathcal{T}$ such that $t \succeq_i t^*$ for all $t^* \in \mathcal{T}$. Clearly, an elected candidate i will implement a tax schedule from $\mathcal{T}_i$, the set of those tax schedules from $\mathcal{T}$ that are preferred by i; any other promise is not credible. Although citizens realize that party i's optimal set is $\mathcal{T}_i$, they are uncertain about their disposable income regardless of the elected candidate, because in general $\mathcal{T}_i$ is not a singleton.[11] It is assumed that citizens have prior beliefs on an elected candidate's final choice and that candidates are unable to affect citizens' perceptions through campaigning.[12] Thus, voting behavior is governed solely by prior beliefs. Let π_i be a probability measure on the Borel σ-algebra on $\mathcal{T}_i$ describing the beliefs citizens have on an elected candidate i's final choice from $\mathcal{T}_i$. According to π_i, candidate i assigns probability $\pi_i(\mathcal{T}')$ to collection of tax schedules $\mathcal{T}'$. Citizens' voting behavior is assumed to be driven by purely egoistical motivations. A citizen x votes for a candidate who would minimize, if elected, x's expected tax burden. To be concrete, let

$$T_i(x) = \int t(x)\pi_i(\mathrm{d}t), \quad 0 \leq x \leq 1, \quad i \in P. \tag{5}$$

Note that the image of each x under T_i is agent x's expected tax liability if party i is elected. It is natural to refer to each T_i as the expected tax schedule conditional on party i's victory.[13] Given a candidate set $C \subset P$, citizen x will vote for some $j \in C$ such that $T_j(x) \leq T_k(x)$ for all $k \in C$.[14,15]

Entry decisions by political parties are described by tuples in $\{0,1\}^p$. Specifically, entry decision $a = (a_1, \ldots, a_p)$ describes the situation in which each political party i with $a_i = 1$ decides to present a candidate and each party i with $a_i = 0$ decides otherwise. For each $i \in P$ with $a_i = 1$, let $E_i(a)$ be the set of all agents x that vote for candidate i under profile a (according to the rule specified earlier). The number of votes that accrue to party i under profile a is

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[11]We are departing here from Besley and Coate (1997) and Osborne and Slivinski (1996), where, by assumption, each candidate's set of preferred policies is a singleton.

[12]Admittedly, the inability of candidates to influence voting behavior may seem somewhat unnatural. Yet the case in which candidates can commit to a tax schedule in their optimal set is compatible with all the results obtained in this paper.

[13]If the post-tax income distribution induced by i's expected tax schedule Lorenz dominates that induced by j's expected tax schedule for any two parties i and j such that i's ideal point lies to the left of j's ideal point, then our results remain true if the set of admissible tax functions admits negative taxation.

[14]If more than one candidate satisfies this requirement, x chooses one at random.

[15]The voting is thus modeled through *sincere voting*.

$$\omega_i(a) = \int_{E_i(a)} dF. \tag{6}$$

Thus, the set of parties that obtain the most votes under a is

$$W(a) = \{i \in P : \omega_i(a) = \max_{k \in P} \omega_k(a)\}.$$

Let c and θ be positive reals and define $\chi_i : \{0, 1\}^p \to \mathbf{R}$ by

$$\chi_i(a) = \begin{cases} 1 & \text{if } a_i = 1, \\ 0 & \text{otherwise} \end{cases}$$

for each $i \in P$. We study two strategic games, one for the plurality rule, denoted by $\mathcal{G}_{PR}$, and one for a runoff system, denoted by $\mathcal{G}_{RS}$. In both games, the set of players is the set of political parties and the action space for each party consists of entry decisions in $\{0, 1\}$. Under the plurality rule, party i's payoff given an entry profile $a \in \{0, 1\}^p$ is

$$u_i(a) = -\frac{\theta}{|W(a)|} \sum_{j \in W(a)} |m_j - m_i| - c\chi_i(a)$$

if $a \neq 0$, $u_i(a) < -c - \theta$ otherwise.

In words, each party i evaluates an entry profile a in terms of i's expenditure under a (0 if i does not run, c otherwise) and the expected gap between i's ideal inequality point and the winners' ideal points, θ being the marginal rate of substitution between the two. If no party presents a candidate, the outcome is dominated by any other entry profile.[16] The payoff function for the game under a runoff system is defined similarly.[17] The solution concepts that we use are Nash Equilibrium and Strong Nash Equilibrium.

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[16] We assume this for simplicity only. Had we assumed instead that either a convex or a concave status quo tax schedule is implemented in this case, none of the results of this paper would change.

[17] Let $W_J(a') = \{i \in P : \omega_i(a') = \max_{k \in J} \omega_k(a')\}$ be the set of most popular parties within $J \subset P$ under entry profile $a' \neq 0$. Party i's payoff under entry profile a for the game under a runoff system is

$$v_i(a) = \begin{cases} -\theta|m_i - m_j| - c\chi_i(a) & \text{if } W(a) = \{j\} \text{ and } \omega_j(a) > 1/2, \\ \dfrac{1}{|W_{P\backslash j}(a)|} \displaystyle\sum_{k \in W_{P\backslash j}(a)} u_i(a^{k,j}) & \text{if } W(a) = \{j\} \text{ and } \omega_j(a) \le 1/2, \\ \dfrac{2}{|W(a)|(|W(a)|-1)} \displaystyle\sum_{\{k,l\} \subset W(a)} u_i(a^{k,l}) & \text{if } |W(a)| \ge 2, \end{cases}$$

and $v_i(a) < -c - \theta$ if $a = 0$, where, for each pair k, $l \in P$, $a^{k,l}$ denotes the action profile where only parties k and l present candidates.

3. Political parties' ideal tax schedules

In this section we identify the set of preferred tax schedules of each party. To do so, we first introduce a critical cut-off value of the inequality measure. This value helps us classify political parties as equality lovers or equality averse. We conclude the section by showing that the expected tax schedule of each party is either concave or convex. The proofs of the three lemmata are in Appendix A.

Note that $\mathcal{T}$ contains a unique *linear* tax schedule, t_l, given by

$$t_l(x) = (R/\mu)x, \quad x \geq 0.$$

This particular tax defines a unique cut-off value $I_l \equiv I(F_{t_l})$ in the image of I. The importance of that specific value is shown in the following lemma.

Lemma 1. *For every $m < I_l$ ($m > I_l$), there exists a nonlinear convex (concave) tax schedule $t \in \mathcal{T}$ such that $I(F_t) = m$.*

The following lemma provides a crucial finding, according to which the set of preferred tax schedules of each party is composed by either convex or concave functions, depending on the party's ideal inequality point.

Lemma 2. *For every political party $j \in P$, if $m_j < I_l$ ($m_j > I_l$) all the elements in the set of preferred tax schedules for party j, $\mathcal{T}_j$, are nonlinear convex (concave) functions. If $m_j = I_l$, then $\mathcal{T}_j = \{t_l\}$.*

Although there exist many tax schedules that attain any given value of I, it turns out that any value of I (except I_l) is attainable with a two-bracket tax schedule. Therefore, given the parties' preferences for simplicity, no tax schedule with three or more brackets will belong to the set of preferred tax schedules for any party.[18] According to Lemma 2, parties can be classified into three categories: a party is egalitarian, neutral, or inegalitarian as its ideal point can be reached by a convex, linear, or concave tax schedule respectively. As a consequence of Lemma 2, the expected tax schedule of each political party is either concave or convex.

Lemma 3. *For every political party $j \in P$,*

(i) $0 \leq T_j(x) \leq x$ *for all x,*

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[18]The fact that each party's set of preferred tax schedules consists only of two-bracket tax schedules may be regarded as an awkward feature of the model. However, we would like to mention that one can derive a generalization of the preferences in (4) such that the corresponding optimal sets need not contain only two-bracket tax schedules (see Carbonell-Nicolau, 2000). The results in this paper would remain unaltered if this generalization were used in the present setting.

(ii) T_j is continuous and nondecreasing, and
(iii) $\int_0^1 T_j(x) \, dF = R$.

Moreover, if $m_j < I_l$ ($m_j > I_l$), then T_j is a nonlinear convex (concave) function, and $m_j = I_l$ implies $T_j = t_l$.

Lemma 3 implies that actual competition among candidates depends on functions that are either concave or convex, even though the set of feasible tax functions $\mathcal{T}$ is fairly unrestricted.

The following section analyzes existence of equilibrium.

4. A note on existence of equilibrium

While the model presented so far is fairly general, existence of Nash Equilibrium can not be guaranteed without imposing some restrictions. Indeed, there are instances in which the configuration of ideal points leads to voting cycles and an equilibrium fails to exist.[19] In this section, we provide sufficient conditions on the distribution of political parties' ideal inequality points and the cost of presenting a candidate under which an equilibrium exists.

The main results of this paper involve a condition on the ratio c/θ. Recall that c is the monetary cost of candidacy and θ measures how intensely parties value ideology relative to money. This suggests that c/θ may be interpreted as the 'effective' cost of presenting a candidate.

On the one hand, we require that the effective cost of presenting a candidate be sufficiently high (greater than $I_l/2$). This excludes situations where two parties with a similar orientation engage in competition to hold office, even if one could win the elections for sure if it would present a candidate.

On the other hand, it is assumed that the distribution of ideological tendencies is sufficiently rich in the sense that ideal points are dispersed all along [0, 1]. To express this idea formally, we introduce the following definitions.

For $\varepsilon > 0$, let $0 \equiv N_1(\varepsilon) < N_2(\varepsilon) < \cdots < N_n(\varepsilon) \equiv 1$ be a finite partition of [0, 1] such that $N_{k+1}(\varepsilon) - N_k(\varepsilon) = \varepsilon$ for all $k \in \{1, 2, \ldots, n-2\}$. We shall say that $\{m_i : i \in P\}$ ***covers an ε-partition*** of the ideological space if, for each $k \in \{1, \ldots, n-1\}$, $(N_k(\varepsilon), N_{k+1}(\varepsilon))$ contains at least one element of $\{m_i : i \in P\}$.

Formally, sufficient ideological variety is expressed by requiring that P cover an ε^*-partition for a certain ε^*. This condition will allow us to construct an

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[19] The Condorcet paradox may arise, for example, if there are only three political parties with ideal inequality points smaller than I_l. Notice that the existence of a Condorcet cycle is a necessary but not a sufficient condition for nonexistence of equilibrium. Given that there is a positive cost of presenting a candidate, an equilibrium may exist even in the presence of cycles among winners.

appropriate entry profile to show existence in Proposition 1. The plausibility of the condition becomes more apparent if we think as the elements of *P* not just as political parties, but also as organizations of individuals with a certain ideological tendency that may incur the cost of having an active participation in the electoral system.

Suppose that $I_l/2 < c/\theta$ and there is at least one party in *P* whose ideal point is greater than I_l. Define

$$\varepsilon^* \equiv \frac{1}{2} \min\left\{ m^* - I_l, \frac{c}{\theta} - \frac{I_l}{2} \right\}, \tag{7}$$

where

$$m^* \equiv \min\{m_i : i \in P \text{ and } m_i > I_l\}. \tag{8}$$

The following proposition states that, under the aforesaid conditions, the game under the plurality rule has not only a Nash Equilibrium, but also a Strong Nash Equilibrium (Aumann, 1959).[20]

Proposition 1. *If $\mu > med$, $I_l/2 < c/\theta$, and $\{m_i : i \in P\}$ covers an ε^*-partition, $\mathcal{G}_{PR}$ has at least one Strong Nash Equilibrium (and so at least one Nash Equilibrium).*

This proposition is based on a result first obtained by Marhuenda and Ortuño-Ortín (1995). Therefore we find it worthwhile to recapitulate it here as an adaptation to the present setting.

Lemma 4. (*Marhuenda and Ortuño-Ortín*, 1995). *Let entry profile a be such that only political parties i and j, with ideal inequality points $m_i < I_l \leq m_j$, present candidates. If $\mu > med$, then $\omega_i(a) > \omega_j(a)$.*

According to this lemma, whenever citizens are voting over two different expected tax functions T_i and T_j, one convex and the other concave, more than half of the population will prefer the former to the latter. While this is an important result, it remains silent with respect to the equilibrium outcome of the games introduced in Section 2.3. The following Corollary (which will be useful in what follows) extends Lemma 4 in that it also considers profiles with more than two entries. It simply says that the share of the votes obtained by candidates favoring marginal rate progressivity must be greater than one half.

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[20]An entry profile a^* is a Strong Nash Equilibrium of $\mathcal{G}_{PR}$ if for no coalition $J \subset P$ and entry subprofile a_J,

$$u_i(a_J, a^*_{-J}) > u_i(a^*) \text{ for all } i \in J,$$

where $a^*_{-J} \equiv \{a^*_i\}_{i \notin J}$.

Corollary 1. *Let a be an entry profile, $C = \{i \in P : a_i = 1\}$, and $J = \{i \in C : m_i < I_l\}$. If $\mu > med$ and J is nonempty, $\Sigma_{k \in J} \omega_k(a) > 1/2$.*

Proof. See Appendix A.   □

We are now ready to proceed with the proof of Proposition 1.

Proof of Proposition 1. If $c/\theta > I_l$ any entry profile such that only one party i with $m_i < I_l$ enters is a Strong Nash Equilibrium. Suppose that $c/\theta \leq I_l$. By Lemma A.2 (see Appendix A), there exists $i^* \in P$ such that

$$m_{i*} \in \left[I_l - \frac{c}{\theta}, \min\left\{ m^* - \frac{c}{\theta}, \frac{I_l}{2} \right\} \right],$$

where m^* is as in (8).

Consider entry profile a^*, where $a_{i*}^* = 1$ and $a_i^* = 0$ for all $i \in P\backslash\{i^*\}$. Take any nonempty coalition $J \subset P$ and let a_J be an entry subprofile for coalition J. It will be shown that there exists $j \in J$ such that $u_j(a_J, a_{-J}^*) \leq u_j(a^*)$. Two cases will be considered.

Suppose first that $a_j = 1$ for some $j \in J\backslash\{i^*\}$ with $m_j \leq I_l$. If $m_j = m_{i*}$, it is clear that $u_j(a_J, a_{-J}^*) \leq -c < 0 = u_j(a^*)$. Suppose that $m_j < m_{i*}$. We have $0 \leq m_j < m_{i*} \leq c/\theta$ (recall that $m_{i*} \leq I_l/2$ and, by assumption, $I_l/2 \leq c/\theta$). Hence $\theta(m_{i*} - m_j) \leq \theta(c/\theta - 0) = c$, and so $u_j(a_J, a_{-J}^*) \leq -c \leq -\theta(m_{i*} - m_j) = u_j(a^*)$, as desired. Next, suppose that $m_j > m_{i*}$. Then $\theta(m_j - m_{i*}) \leq \theta(I_l - m_{i*}) \leq \theta[I_l - (I_l - c/\theta)] = c$ (the last inequality follows from the relation $m_{i*} \in A^*$), and so the same result follows.

Next, suppose that $a_j = 0$ for all $j \in J\backslash\{i^*\}$ with $m_j \leq I_l$. Either $i^* \notin J$ or $i^* \in J$. If $i^* \notin J$, it follows from Corollary 1 that $W(a_J, a_{-J}^*) = \{i^*\}$. Take $j \in J$. If $a_j = 0$ then $u_j(a_J, a_{-J}^*) = u_j(a^*)$, and if $a_j = 1$ then $u_j(a_J, a_{-J}^*) < u_j(a^*)$. If, on the other hand, $i^* \in J$, we have $u_{i*}(a_J, a_{-J}^*) \leq u_{i*}(a^*)$. In fact, this is clearly the case if $a_{i*} = 1$. Otherwise (if $a_{i*} = 0$), observe that $u_{i*}(a_J, a_{-J}^*) = u_{i*}(0) < -c - \theta < -c = u_{i*}(a^*)$ if $(a_J, a_{-J}^*) = 0$ (the first inequality is true by assumption). It remains to show that $u_{i*}(a_J, a_{-J}^*) \leq u_{i*}(a^*)$ whenever $a_j = 0$ for all $j \in J$ with $m_j \leq I_l$, $i^* \in J$, and $(a_J, a_{-J}^*) \neq 0$. Note that, in this case,

$$
\begin{aligned}
u_{i*}(a_J, a_{-J}^*) &= -\frac{\theta}{|W(a_J, a_{-J}^*)|} \sum_{k \in W(a_J, a_{-J}^*)} (m_k - m_{i*}) \\
&\leq -\theta(m^* - m_{i*}) \\
&\leq -\theta\left[m^* - \left(m^* - \frac{c}{\theta} \right) \right] \\
&= -c = u_{i*}(a^*).
\end{aligned}
$$

This establishes the result.   □

Note that in this proof existence is shown by constructing the simplest kind of equilibrium, a one-candidate equilibrium. Since for entry profiles with a single entrant the payoff functions of $\mathcal{G}_{PR}$ and $\mathcal{G}_{RS}$ coincide, a Strong Nash Equilibrium of the game under a runoff system exists as well.

The following section analyzes equilibria with different numbers of candidates.

5. Equilibrium tax schedules

Given the results of the previous sections, we are now ready to analyze the equilibria of the political game. Results are first provided for Nash Equilibrium under the plurality rule and a runoff system, and then for Strong Nash Equilibrium.

5.1. Nash equilibrium

Different numbers and configurations of candidates can arise under the plurality rule. In the following proposition, however, we state that when a mild condition is satisfied ($c/\theta < I_l - m_i$ for some $i \in P$), the implemented tax schedule in any Nash Equilibrium with one or two candidates is a nonlinear convex function. This condition says that the most egalitarian party in P will always prefer to incur the cost of candidacy and choose a tax schedule over saving the cost and facing regressive taxation.

Proposition 2. *Suppose that* $\mu > med$. *If* $c/\theta < I_l - m_i$ *for some* $i \in P$, *then a nonlinear convex tax schedule is implemented in any Nash Equilibrium of* $\mathcal{G}_{PR}$ *with less than three candidates.*

Proof. Let a be an entry profile. If $a = 0$ (that is, action profile a is such that no party presents a candidate), the assumption that $u_i(0) < -c - \theta$ for all $i \in P$ implies that a is not a Nash Equilibrium. Suppose that $a \neq 0$ is a profile such that at most two parties propose a candidate and there exists a winner in $W(a)$, denoted by j, that implements a concave tax schedule. By Lemma 2, $m_j \geq I_l$. If $a_k = 1$ for some k with $m_k < I_l$, Corollary 1 implies $W(a) = \{k\}$. Therefore, $a_k = 1$ implies $m_k \geq I_l$. By assumption, there exists $i \in P$ such that $c/\theta < I_l - m_i$. Let a^* be a profile with $a_k^* = 1$ if $k = i$, $a_k^* = a_k$ otherwise. Corollary 1 implies $W(a^*) = \{i\}$. Therefore, $u_i(a^*) = -c > -\theta(I_l - m_i) \geq u_i(a)$, whence a is not a Nash Equilibrium. $\square$

When the number of candidates is greater than or equal to three, there may exist equilibria in which some candidates are sure to lose. The reason why such parties decide to pay the cost of presenting a candidate, as Osborne and Slivinski (1996, p. 74) explained it for the three-candidate case, is that they prefer the resulting equal-probability lottery over their rivals' positions to certain victory by the

candidate who would win if they withdrew. In our framework, Corollary 1 implies that, in any equilibrium with three or more candidates, at least two of them must have ideal points below I_l. Still, Nash equilibria leading to regressive taxation are possible when there are more than two candidates. As an example, suppose that there are three political parties with ideal points m_1, $m_2 < I_l \leq m_3$ that are located along the unit interval as in Fig. 1. Take the entry profile in which all three parties present a candidate. Suppose that according to this profile the first and third candidates win with equal probability and the second candidate loses for sure. Suppose further that if candidate 3 withdraws from the competition, 1 wins for sure. It is easy to see that there is a range for the ratio c/θ such that the aforesaid profile constitutes a Nash Equilibrium. Take, for example, party 2. If this party withdraws, it will save the cost of candidacy. However, we know that withdrawal by candidate 2 benefits candidate 1, who will then defeat party 3 by Lemma 4. If the cost of candidacy is sufficiently low, party 2 will not have incentives to withdraw. A similar argument works for the rest of the candidates.

If one is willing to impose a certain condition on the cost of candidacy, the conclusion in Proposition 2 can be generalized to any number of candidates, provided that at most two candidates have ideal points smaller than I_l. This observation is formalized in the following proposition.

Proposition 3. *Suppose that $\mu > med$. Let $I_l/2 \leq c/\theta < I_l - m_i$ for some $i \in P$. Then a nonlinear convex tax schedule is implemented in any Nash Equilibrium of $\mathcal{G}_{PR}$ with at most two candidates i and j such that m_i, $m_j < I_l$.*

Proof. Let a be an entry profile. If $a = 0$, the assumption that $u_i(0) < -c - \theta$ for all $i \in P$ implies that a is not a Nash Equilibrium. Let $C = \{i \in P : a_i = 1\}$ and $J = \{i \in C : m_i < I_l\}$. Suppose that $a \neq 0$ is a profile such that at most two progressive parties enter the electoral competition and there is a winner in $W(a)$ that implements a concave tax schedule. By Lemma 2, this winner must be in $W(a) \backslash J$. It will be shown that a is not a Nash Equilibrium. If there is one single party k in J, Corollary 1 implies that k obtains more than half of the votes. Therefore, if there are at most two parties in J and $W(a) \backslash J$ is nonempty, either J is empty or J contains exactly two parties. If J is empty, we can use the assumption that there exists $i \in P$ such that $c/\theta < I_l - m_i$ as in the proof of Proposition 2 to see that a is not a Nash Equilibrium. Also, it is clear that a cannot be a Nash Equilibrium if J contains exactly two parties with the same ideal point (one of the parties in J is unambiguously better off by withdrawing from the competition). So
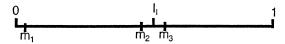

Fig. 1. Example of a multiple candidate Nash equilibrium consistent with regressive taxation.

suppose that J contains exactly two parties i and j with distinct ideal points. Say $m_i < m_j$ and let $a_i^* = 0$. Note that if $i \notin W(a)$, party i has incentives to withdraw, since $u_i(a) \leq -\theta(m_j - m_i) - c < -\theta(m_j - m_i) = u_i(a_i^*, a_{-i})$ (the last equality follows from Corollary 1), so a is not a Nash Equilibrium. If $i \in W(a)$, the same conclusion is obtained. To see this, we distinguish two cases. Suppose first that $i \in W(a)$ and $j \notin W(a)$. Then

$$u_i(a) < -\frac{\theta}{2}(m_j - m_i) - c \leq -\theta(m_j - m_i) = u_i(a_i^*, a_{-i}),$$

where the last inequality holds because, by assumption, $I_l/2 \leq c/\theta$. If, on the other hand, $i, j \in W(a)$, suppose that k is a winner in $W(a) \backslash J$ such that $m_k = \min_{l \in W(a) \backslash J} m_l$ and let $m' = (m_i + m_j + m_k)/3$. Note that

$$u_i(a) \leq -\frac{\theta}{3}(m_j - m_i) - \frac{\theta}{3}(m_k - m_i) - c = -\theta(m' - m_i) - c. \tag{9}$$

Since $m_j < I_l$ and (as is easily verified) $m_j = 3/2(m_j - m') + 1/2(m_i + m_k)$, we have $3/2(m_j - m') < I_l - 1/2(m_i + m_k)$. The right hand side of this last inequality is less than or equal to $I_l/2$ because $I_l \leq m_i + m_k$. It follows that $m_j - m' < I_l/3 \leq c/\theta$ (recall that $I_l/2 \leq c/\theta$). Hence

$$u_i(a_i^*, a_{-i}) = -\theta(m_j - m_i) > -\theta\left(\frac{c}{\theta} + m' - m_i\right) = -\theta(m' - m_i) - c.$$

Combine this equation with (9) to obtain $u_i(a_i^*, a_{-i}) > u_i(a)$. $\square$

Nash equilibria leading to regressive taxation with positive probability are possible when there are three or more candidates favoring marginal rate progressivity, even when the ratio c/θ satisfies the condition in Proposition 3. In fact, an entry profile may be such that withdrawal by any candidate may end up favoring ideologies that are more unpleasant (for the party that withdraws) than the ideology induced by the initial distribution of votes. Nevertheless, a Nash Equilibrium leading to regressive taxation may not be regarded as self-enforceable if it is possible for some coalition of parties to engage in some collective strategy that improves the payoff of all the members of the coalition. This idea is exploited in the next section.

Under a runoff system, the set of equilibria featuring marginal rate progressive taxation is expanded. In fact, if the conditions in Proposition 2 are satisfied, regressive taxation cannot be the outcome of a Nash Equilibrium of $\mathcal{G}_{RS}$ with less than five candidates. This result is stated formally next.

Proposition 4. *Suppose that $\mu > med$. If $c/\theta < I_l - m_i$ for some $i \in P$, then a*

nonlinear convex tax schedule is implemented in any Nash Equilibrium of $\mathcal{G}_{RS}$ with less than five candidates.

Proof. Let a^* be an entry profile. If $a^* = 0$, the assumption that $v_i(0) < -c - \theta$ for all $i \in P$ implies that a^* is not a Nash Equilibrium.

Suppose that $a^* \neq 0$ is such that at most four parties propose a candidate. Define $C = \{i \in P : a_i^* = 1\}$ and $J = \{i \in C : m_i < I_l\}$. By Lemma 2, any winner in C who chooses a tax schedule that is concave must belong to $C \backslash J$. If $J = \emptyset$, we can proceed as in the proof of Proposition 2 to conclude that a^* is not a Nash Equilibrium. Suppose that $J = \emptyset$. By Corollary 1, all parties in $C \backslash J$ obtain less than half of the votes. Thus, if a^* is to be consistent with the implementation of a concave tax schedule under a runoff system, at least one party in $C \backslash J$ makes it to the second round and wins. Since a party in $C \backslash J$ loses in the second round against a party in J by Lemma 4, a party in $C \backslash J$ makes it to the second round and wins only if the two parties that make it to the second round belong to $C \backslash J$. If there is only one party j in J, j obtains more than half of the votes in the first round by Corollary 1. If there are exactly two parties j and k in J, more than half of the citizens vote for either j or k by Corollary 1, whence at least one of them obtains more than one fourth of the votes. Hence, if there are exactly two parties in J, at least one of them reaches the second round. Since at most four parties enter under a^*, we conclude that the two parties that reach the second round belong to $C \backslash J$ only if $J = \emptyset$. Thus, a^* is consistent with the implementation of a concave tax schedule under a runoff system only if J is empty. Suppose that J is empty. It will be shown that a^* is not a Nash Equilibrium. Suppose that $i \in P$ satisfies $m_i < I_l - c/\theta$ (such an i exists by assumption), and let $a_i = 1$. By Corollary 1, $W(a_i, a_{-i}^*) = \{i\}$ and $\omega_i(a_i, a_{-i}^*) > 1/2$. It is then easy to verify that $v_i(a_i, a_{-i}^*) = -c > -\theta(I_l - m_i) \geq v_i(a^*)$, and so a^* is not a Nash Equilibrium. $\square$

Proposition 4 extends the results obtained in Proposition 2 to a greater number of candidates for a simple reason: under a runoff system, to ensure progressive taxation, one only needs to make sure that a candidate implementing a convex tax schedule reaches the second round. In fact, even in the case where a candidate j with $m_j \geq I_l$ obtains a plurality in the first round, this candidate will lose against any other candidate i with $m_i < I_l$ in a pairwise election in the second round. As an illustration, take our previous example. According to that example, even if candidates 1 and 3 win with equal probability in the first round, under a runoff system, candidate 1 defeats candidate 3 in the second round. As a consequence of this, party 2 will not present a candidate because it cannot affect the outcome of the election. Hence, the aforementioned strategy profile is not a Nash Equilibrium of $\mathcal{G}_{RS}$.

To conclude this section, we mention that the argument in the proof of Proposition 4 may be altered slightly to prove the analogue of Proposition 3 for a runoff system, namely that regressive taxation is not compatible with any Nash

Equilibrium of $\mathscr{G}_{RS}$ with at most two candidates favoring marginal rate progressivity, regardless of the number of candidates with ideal points greater than I_l.

5.2. Strong Nash equilibrium

While the Nash best-response property is certainly a requirement for stability, one can argue that the present setting favors the formation of coalitions by political parties, which are likely to arrange plausible and mutually beneficial deviations from Nash agreements. The notion of Strong Nash Equilibrium captures this idea from a noncooperative angle. We strengthen the results obtained in the preceding section by applying this concept to the games being analyzed. The following proposition states that no Strong Nash Equilibrium of either the game under the plurality rule or the game under a runoff system is compatible with regressive taxation, regardless of the number of candidates.

Proposition 5. *If $\mu > med$, $c/\theta < I_l - m_i$ for some $i \in P$, and $\{m_i : i \in P\}$ covers a (c/θ)-partition, then a nonlinear convex tax schedule is implemented in any Strong Nash Equilibrium of either $\mathscr{G}_{PR}$ or $\mathscr{G}_{RS}$.*

Proof. We shall only provide an argument for $\mathscr{G}_{PR}$, the game for the plurality rule. The argument for $\mathscr{G}_{RS}$ is similar and available from the authors upon request. Let a^* be an entry profile. If $a^* = 0$, the assumption that $u_i(0) < -c - \theta$ for all $i \in P$ implies that a^* is not a Strong Nash Equilibrium.

Define $C = \{i \in P : a_i^* = 1\}$ and $J = \{i \in C : m_i < I_l\}$, and suppose that a^* is consistent with the implementation of a regressive (concave) tax schedule. By Lemma 2, this means that there is at least one winner in $C \setminus J$, or, equivalently, $W(a^*) \setminus J \neq \emptyset$. We shall show that there exists an entry subprofile for a certain coalition (a subset of P) that improves the payoff of all its members, which implies that a^* is not a Strong Nash Equilibrium. Let $\overline{m}$ be the expected inequality point induced by a^*; that is,

$$\overline{m} = \frac{1}{|W(a^*)|} \sum_{l \in W(a^*)} m_l.$$

Observe that

$$
\begin{aligned}
|\overline{m} - m_k| &= \left| \frac{1}{|W(a^*)|} \sum_{l \in W(a^*)} (m_l - m_k) \right| \\
&\leq \frac{1}{|W(a^*)|} \sum_{l \in W(a^*)} |m_l - m_k| \quad \forall k \in P.
\end{aligned}
\tag{10}
$$

We consider four cases.

First, suppose that $J = \emptyset$. Let $j \in P$ satisfy $m_j < I_l - c/\theta$ (such a j exists by

assumption) and let $a_j = 1$. Since $J = \emptyset$, $c/\theta < I_l - m_j$, and (by Corollary 1) $W(a_j, a^*_{-j}) = \{j\}$, we have

$$u_j(a_j, a^*_{-j}) = -c > -\theta(I_l - m_j) \geq -\frac{\theta}{|W(a^*)|} \sum_{l \in W(a^*)} (m_l - m_j) = u_j(a^*),$$

and so a^* is not a Strong Nash Equilibrium.

Next, suppose that $J \neq \emptyset$ and $\overline{m} \geq I_l$. Let a_J be an entry subprofile for coalition J such that $a_i = 1$ for some $i \in J$ with $m_i = \max_{k \in J} m_k$ and $a_k = 0$ for all $k \in J \setminus \{i\}$. By Corollary 1, $W(a_J, a^*_{-J}) = \{i\}$. Take any $k \in J \setminus \{i\}$ and observe that

$$
\begin{aligned}
u_k(a_J, a^*_{-J}) &= -\theta(m_i - m_k) > -\theta(\overline{m} - m_k) - c \\
&\geq -\frac{\theta}{|W(a^*)|} \sum_{l \in W(a^*)} |m_l - m_k| - c = u_k(a^*),
\end{aligned}
$$

where the last inequality follows from (10). Moreover,

$$u_i(a_J, a^*_{-J}) = -c > -\frac{\theta}{|W(a^*)|} \sum_{l \in W(a^*)} |m_l - m_i| - c = u_i(a^*),$$

since $\sum_{l \in W(a^*)} |m_l - m_i| > 0$. Thus, $u_k(a_J, a^*_{-J}) > u_k(a^*)$ for all $k \in J$, whence a^* is not a Strong Nash Equilibrium.

Suppose now that $J \neq \emptyset$, $\overline{m} < I_l$, and there exists an element of C, say i^*, such that $m_{i^*} \in (\overline{m} - c/\theta, \overline{m} + c/\theta)$. Let a_C be an entry subprofile for coalition C such that $a_{i^*} = 1$ and $a_k = 0$ for all $k \in C \setminus \{i^*\}$. Then $W(a_C, a^*_{-C}) = \{i^*\}$. Pick any $k \in C \setminus \{i^*\}$ and note that we can write $m_{i^*} = \overline{m} + \gamma$, where $|\gamma| < c/\theta$. We have

$$
\begin{aligned}
u_k(a_C, a^*_{-C}) &= -\theta|m_{i^*} - m_k| = -\theta|\overline{m} + \gamma - m_k| \geq -\theta(|\overline{m} - m_k| + |\gamma|) \\
&> -\theta|\overline{m} - m_k| - c \geq -\frac{\theta}{|W(a^*)|} \sum_{l \in W(a^*)} |m_l - m_k| - c = u_k(a^*),
\end{aligned}
$$

where the last inequality follows from (10). Further, $u_{i^*}(a_C, a^*_{-C}) > u_{i^*}(a^*)$. Indeed, if $i^* \in J$, we have

$$\frac{1}{|W(a^*)|} \sum_{l \in W(a^*)} |m_l - m_{i^*}| > 0 \tag{11}$$

since $W(a^*) \setminus J \neq \emptyset$ by assumption, and if $i^* \in C \setminus J$, (11) also holds because $W(a^*) \cap J \neq \emptyset$ by the inequality $\overline{m} < I_l$. Therefore

$$u_{i^*}(a_C, a^*_{-C}) = -c > -\frac{\theta}{|W(a^*)|} \sum_{l \in W(a^*)} |m_l - m_{i^*}| - c = u_{i^*}(a^*).$$

Hence $u_k(a_C, a^*_{-C}) > u_k(a^*)$ for every $k \in C$, and so a^* is not a Strong Nash Equilibrium.

Finally, suppose that $J \neq \emptyset$, $\overline{m} < I_l$, and $m_k \notin (\overline{m} - c/\theta, \overline{m} + c/\theta)$ for all $k \in C$. Let $a \equiv \overline{m} - c/\theta$ and $b \equiv \overline{m} + c/\theta$. Since $\{m_i : i \in P\}$ covers a (c/θ)-partition, there exists a party in P, say j^*, such that $m_{j^*} \in (a, b)$. By Lemma A.3 (see Appendix A),

$$\frac{1}{|W(a^*)|} \sum_{l \in W(a^*)} |m_l - m_{j^*}| \geq c/\theta. \tag{12}$$

Let N_1 be the set of all i in $W(a^*)$ with $m_i < a$, and let N_2 be the set of all i in $W(a^*)$ with $m_i > b$. If (12) holds as an equality, it must be that $m_k = a$ for every $k \in N_1$ and $m_k = b$ for every $k \in N_2$. This, along with $\overline{m} < I_l$ and $W(a^*) \backslash J \neq \emptyset$, implies $N_1 = W(a^*) \cap J$ and $N_2 = W(a^*) \backslash J$. All these observations imply that if a_J is an entry subprofile for coalition J such that $a_{k^*} = 1$ for some $k^* \in J$ and $a_k = 0$ for all $k \in J \backslash \{k^*\}$, then

$$u_k(a_J, a^*_{-J}) \geq -c > -\frac{\theta}{|W(a^*)|} \sum_{l \in W(a^*)} |m_l - m_k| - c = u_k(a^*)$$

for all $k \in J$, which implies that a^* is not a Strong Nash Equilibrium. If the inequality in (12) is strict, let $K \equiv C \cup \{j^*\}$ and take the entry subprofile a_K for coalition K, where $a_{j^*} = 1$ and $a_k = 0$ for all $k \in C$. Then $W(a_K, a^*_{-K}) = \{j^*\}$. Noting that we can write $m_{j^*} = \overline{m} + \gamma$, where $|\gamma| < c/\theta$, we have

$$
\begin{aligned}
u_k(a_K, a^*_{-K}) &= -\theta |m_{j^*} - m_k| = -\theta |\overline{m} + \gamma - m_k| \geq -\theta(|\overline{m} - m_k| + |\gamma|) \\
&> -\theta |\overline{m} - m_k| - c \geq -\frac{\theta}{|W(a^*)|} \sum_{l \in W(a^*)} |m_l - m_k| - c = u_k(a^*)
\end{aligned}
$$

for any $k \in C$, where the last inequality follows from (10). Moreover,

$$u_{j^*}(a_K, a^*_{-K}) = -c > -\frac{\theta}{|W(a^*)|} \sum_{l \in W(a^*)} |m_l - m_{j^*}| = u_{j^*}(a^*)$$

because the inequality in (12) is strict. Thus $u_k(a_K, a^*_{-K}) > u_k(a^*)$ for every $k \in K$, whence a^* is not a Strong Nash Equilibrium. $\square$

According to the Strong Nash Equilibrium concept, political parties can make non binding mutually beneficial agreements. Therefore, parties with sufficiently distant ideal inequality points that are presenting a candidate can be better off by withdrawing if they 'find' a party with an ideal point in-between that is willing to present a candidate. Several entry profiles are consistent with the notion of Strong

Nash Equilibrium. And in all such equilibrium profiles, the implemented income tax schedule must be a nonlinear convex function.

6. Conclusion and extensions

This paper analyzes the relationship between a representative democracy and marginal rate progressive income taxation. Essentially, in our model there is an exogenous set of political parties with given preferences over tax schedules. The model proposes a game of electoral competition in which political parties decide whether or not to present a candidate; each citizen then chooses a candidate. There is a fixed cost of running, while the only benefit is that, if elected, each candidate imposes one of her preferred tax policies. We analyze the cases where the winner is determined under the plurality rule or a two-ballot runoff system.

While defining the set of feasible taxes as virtually any non-decreasing and piecewise linear continuous function, we provide conditions under which Nash and Strong Nash Equilibria exist. Moreover, in some Nash Equilibria and in any Strong Nash Equilibrium, a tax schedule with increasing marginal tax rates (i.e., a nonlinear convex tax function) is always implemented. While the related literature seems to suggest that rich sets of admissible tax schedules generate serious instability problems that are highly counterfactual, we argue that, on the contrary, general sets of admissible tax schedules are compatible with existence of equilibrium. What is more, equilibrium outcomes favor marginal rate progressive taxation and are thus consistent with the observed demand for progressivity.

As an interesting extension of our model we would like to conduct a similar analysis when agents experience disutility from labor. Also, we would like to see if there is a relation between the initial distribution of income and the progressivity of the income tax schedule; this could allow us to conduct comparisons among countries. Finally, allowing for a benefit of being elected (besides the right to choose the implemented tax function) seems a more realistic way of modeling the political parties' preferences. These tasks (by no means trivial) are left for future research.

Appendix A

In order to prove Lemma 1 we first introduce a preliminary result.

Lemma A.1. *Let t, $t' \in \mathcal{T}$ and suppose that there exists x such that $t' < t$ on $(0, x)$ and $t' > t$ on $(x, 1)$. Then $F_{t'} >_L F_t$.*

Proof. Let t, t' satisfy the conditions of the lemma. Since $\int_0^1 t(x)\, dF = \int_0^1 t'(x)\, dF = R$, it follows from Proposition 5.2 of Le Breton et al. (1996) that $F_{t'} \succeq_L F_t$. Since $t' < t$ on $(0, x)$ implies $F_{t'}^{-1} > F_t^{-1}$ on $(0, F_t(x))$, $F_{t'} >_L F_t$. $\square$

We can now proceed with the proof of Lemma 1.

Proof of Lemma 1. Take $j \in P$ and suppose that $m_j < I_l$. Because $I(\mathcal{F}) = (0, 1)$, there exists $t^* \in \mathcal{T}$ such that $I(F_{t^*}) = m_j$. Let

$$g(x) = \int_x^1 (z - x)\, dF(z) = \int_x^1 z\, dF(z) - x(1 - F(x))$$

for all x in $[0, 1]$. By continuity of F, g is continuous. Also, $g(0) = \mu > R$ and $g(1) = 0 < R$. By the Intermediate Value Theorem, therefore, there exists $\underline{x} \in (0, 1)$ such that $g(\underline{x}) = R$. Since F is strictly increasing, g is strictly decreasing. Therefore, there exists a unique such $\underline{x}$. For every $\beta \in (0, R/\mu]$, let

$$\varphi(\beta) = \frac{R - \beta\left[\displaystyle\int_0^{\underline{x}} z\, dF(z) + \underline{x}(1 - F(\underline{x}))\right]}{\displaystyle\int_{\underline{x}}^1 (z - \underline{x})\, dF(z)}.$$

Define tax schedules $\underline{t} = (0, 1; 0, \underline{x}, 1)$ and $t_\beta = (\beta, \varphi(\beta); 0, \underline{x}, 1)$. It is readily observed that $t_\beta \in \mathcal{T}$ for each β and $\lim_{\beta \to 0} t_\beta = \underline{t}$ (where the limit is defined with respect to the sup metric). Further, for some x^*, $\underline{t} < t^*$ on $(0, x^*)$ and $\underline{t} > t^*$ on $(x^*, 1)$. It follows that there exist β' and x' such that $t_{\beta'} < t^*$ on $(0, x')$ and $t_{\beta'} > t^*$ on $(x', 1)$. By Lemma A.1, then, $F_{t_{\beta'}} >_L F_{t^*}$, whence, by S-concavity of I, $I(F_{t_{\beta'}}) < I(F_{t^*}) = m_j$.

Define $f : [\beta', R/\mu] \to \mathbf{R}$ by $f(\beta) = I(F_{t_\beta})$. It will be shown that f is continuous.

Choose $\beta \in [\beta', R/\mu]$ and let $\{\beta_n\}$ be a sequence from $[\beta', R/\mu]$ converging to β. We shall only consider the case in which $\beta_n \geq \beta$ for all n, for the other cases are handled similarly. Suppose then that $\beta_n \geq \beta$ for all n. Fix $\epsilon > 0$. Since F is continuous on a compact set, it is uniformly continuous. It follows that there exists $\delta > 0$ such that $|F(y) - F(z)| < \epsilon$ for all y and z with $|y - z| < \delta$. Since $\beta_n \to \beta$ and $\varphi(\beta_n) \to \varphi(\beta)$, we can find N such that

$$|\beta_n - \beta|, |\varphi(\beta_n) - \varphi(\beta)| < \min\left\{\frac{\delta(\mu - R)}{2\mu}, \frac{\delta(1 - \varphi(\beta'))}{2}\right\} \quad \forall n \geq N \quad (A.1)$$

and

$$\frac{(1 - \beta_n)\underline{x}}{1 - \beta} > \underline{x} - \frac{\delta}{2} \quad \forall n \geq N. \tag{A.2}$$

To simplify notation, let $t = t_\beta$ and $t_n = t_{\beta_n}$. Take $n \geq N$ and $x \in [0, 1]$, and let $y = r_t^{-1}(x)$ and $y_n = r_{t_n}^{-1}(x)$. Three cases are possible: $y, y_n \in [0, \underline{x}]$, $y, y_n > \underline{x}$, and $y \in [0, \underline{x}]$ and $y_n > \underline{x}$.[21] If $y, y_n \in [0, \underline{x}]$, we have

$$|y_n - y| = \left|\frac{(\beta_n - \beta)y}{1 - \beta_n}\right| \leq \frac{\beta_n - \beta}{1 - (R/\mu)} < \delta/2,$$

where the last inequality follows from (A.1). If $y, y_n > \underline{x}$,

$$|y_n - y| = \frac{|\beta_n - \beta|\underline{x} + (y - \underline{x})|\varphi(\beta_n) - \varphi(\beta)|}{1 - \varphi(\beta_n)} \leq \frac{\max\{|\beta_n - \beta|, |\varphi(\beta_n) - \varphi(\beta)|\}y}{1 - \varphi(\beta')},$$

where the last inequality holds true because φ is decreasing. By (A.1), therefore, $|y_n - y| < \delta/2$. Suppose that $y \in [0, \underline{x}]$ and $y_n > \underline{x}$. Then $y > (1 - \beta_n)\underline{x}/1 - \beta$, for $y \leq (1 - \beta_n)\underline{x}/1 - \beta$ implies $y_n - y = (\beta_n - \beta)y/1 - \beta_n \leq \underline{x} - y$. Therefore $y > \underline{x} - \delta/2$ by (A.2). Let $x_n = r_{t_n}^{-1}(\underline{x} - t(\underline{x}))$. Since $z - t(z)$ is increasing and $y \leq \underline{x}$, $y - t(y) \leq \underline{x} - t(\underline{x})$. Since $r_{t_n}^{-1}$ is increasing, we obtain

$$y_n = r_{t_n}^{-1}(y - t(y)) \leq r_{t_n}^{-1}(\underline{x} - t(\underline{x})) = x_n.$$

Hence $y_n - y \leq \underline{x} - y + x_n - \underline{x}$. Since $y_n - y \geq 0$, $|x_n - \underline{x}| < \delta/2$, and $|\underline{x} - y| < \delta/2$, we have $|y_n - y| < \delta$. Thus $|y_n - y| < \delta$ in all cases. It follows that

$$|F_t(x) - F_{t_n}(x)| = |F(y) - F(y_n)| < \epsilon.$$

We conclude that, given $\epsilon > 0$, there exists N such that, for all $n \geq N$,

$$|F_t(x) - F_{t_n}(x)| < \epsilon \quad \forall x.$$

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[21] The case $y_n \in [0, \underline{x}]$ and $y > \underline{x}$ cannot occur because we are assuming $\beta_n \geq \beta$ for all n.

This implies $F_{t_n} \to F_t$. By continuity of I, therefore, $I(F_{t_n}) \to I(F_t)$, or, equivalently, $f(\beta_n) \to f(\beta)$. Thus, f is continuous.

Since $f(\beta') < m_j < f(R/\mu) = I_l$ and f is continuous, the Intermediate Value Theorem implies the existence of $\beta^* \in (\beta', R/\mu)$ such that $f(\beta^*) = m_j$. Because t_{β^*} is nonlinear convex and belongs to $\mathcal{T}$, the desired conclusion follows. The argument for the case $m_j > I_l$ is analogous. ☐

Proof of Lemma 2. Take $j \in P$ and suppose that $m_j < I_l$. Clearly, (4) implies that any tax schedule in $\mathcal{T}_j$ is nonlinear. It will be shown that if $t \in \mathcal{T}_j$ then t is convex. Suppose that $t \in \mathcal{T}$ is not convex. The proof of Lemma 1 makes it clear that there exists a two-bracket convex tax schedule in $\mathcal{T}$, say t^*, such that $I(F_{t^*}) = m_j$. It is therefore sufficient to prove that $t^* >_j t$, that is, $t \notin \mathcal{T}_j$. By (4), it is plain that this is the case if t has more than two brackets. If t has one bracket, that is, $t = t_l$, we have $I(F_{t_l}) = I_l > m_j$, and so $t^* >_j t$. Suppose that t has two brackets. We claim that in this case we also have $I(F_t) \neq m_j$. By S-concavity, it suffices to show that $F_{t^*} >_L F_t$. Since t has two brackets and is nonlinear concave, there exists x such that $t^* < t$ on $(0, x)$ and $t^* > t$ on $(x, 1)$. It follows from Lemma A.1 that $F_{t^*} >_L F_t$, as desired. The case $m_j > I_l$ is handled similarly. That $\mathcal{T}_j = \{t_l\}$ whenever $m_j = I_l$ follows directly from (4). ☐

Proof of Lemma 3. Take $j \in P$. Condition (i) follows from the fact that every t in $\mathcal{T}$ satisfies $0 \leq t(x) \leq x$ for all x. T_j is nondecreasing because each t in $\mathcal{T}$ is nondecreasing. To see that T_j is continuous, take any $x \in [0, 1]$ and let $\{x_n\}$ be a sequence from $[0, 1]$ converging to x. For each $y \in [0, 1]$, let τ_y be a real function on $\mathcal{T}$ such that $\tau_y(t) = t(y)$, and let $\tau_{y,j}$ be the restriction of τ_y to $\mathcal{T}_j$. Because each t in $\mathcal{T}$ is continuous, the sequence $\{\tau_{x_n}\}$ converges to τ_x pointwise. Further, since $|t(x)| \leq 1$ on $[0, 1]$ for each $t \in \mathcal{T}$, τ_{x_n} is uniformly bounded for each n. By the Lebesgue Convergence Theorem, therefore, $\{\int \tau_{x_n,j} \, d\pi_j\}$ converges to $\int \tau_{x,j} \, d\pi_j$, or, equivalently, $\{T_j(x_n)\}$ converges to $T_j(x)$. Thus, T_j is continuous. To establish (iii), let μ' be the (unique) measure on the Borel σ-algebra $\mathcal{B}$ on $(0,1]$ such that $\mu'((a, b]) = F(b) - F(a)$. We have $\int T_j(x) \, dF = \int \int \tau_{x,j} \, d\pi_j \, d\mu'$. Define $\phi: \mathcal{T}_j \times (0, 1] \to \mathbf{R}$ by $\phi(t, x) = t(x)$. It is easily seen that ϕ is measurable with respect to the σ-algebra of subsets of $\mathcal{T}_j \times (0, 1]$ generated by rectangles $\mathcal{T}' \times A$ with $\mathcal{T}' \in \mathcal{A}_j$ and $A \in \mathcal{B}$. It follows from Fubini's Theorem that $\int \int \tau_{x,j} \, d\pi_j \, d\mu' = \int \tau \, d\pi_j$, where τ is a map on $\mathcal{T}_j$ such that $\tau(t) = \int t \, d\mu'$. Thus, $\int T_j(x) \, dF = \int \tau \, d\pi_j$. Noting that $\tau(t) = \int t \, d\mu' = R$ for each $t \in \mathcal{T}_j$ (see (1)), we obtain (iii).

It remains to show that if $m_j < I_l$ ($m_j > I_l$), then T_j is a nonlinear convex (concave) function, and $m_j = I_l$ implies $T_j = t_l$. If $m_j = I_l$, we have $\mathcal{T}_j = \{t_l\}$ by Lemma 2, and so (5) yields $T_j = t_l$. Let $m_j < I_l$. Suppose by way of contradiction that T_j is not convex. Then there are x, $y \in [0, 1]$ and $\lambda \in (0, 1)$ such that $T_j(\lambda x + (1 - \lambda)y) > \lambda T_j(x) + (1 - \lambda)T_j(y)$. This is equivalent to

$$\int t(\lambda x + (1 - \lambda)y)\pi_j(dt) > \lambda \int t(x)\pi_j(dt) + (1 - \lambda)\int t(y)\pi_j(dt).$$

Therefore

$$\int t(\lambda x + (1-\lambda)y)\,\pi_j(\mathrm{d}t) > \int [\lambda t(x) + (1-\lambda)t(y)]\,\pi_j(\mathrm{d}t).$$

It follows that there exists a finite measurable partition $\{\mathscr{P}_1, \ldots, \mathscr{P}_n\}$ of $\mathscr{T}_j$ such that

$$\sum_{k=1}^{n}[\inf_{t\in\mathscr{P}_k}t(\lambda x + (1-\lambda)y)]\pi_j(\mathscr{P}_k) > \sum_{k=1}^{n}[\inf_{t\in\mathscr{P}_k}\lambda t(x) + (1-\lambda)t(y)]\pi_j(\mathscr{P}_k). \tag{A.3}$$

By Lemma 2, every element of $\mathscr{T}_j$ is convex. Thus, $t(\lambda x + (1-\lambda)y) \le \lambda t(x) + (1-\lambda)t(y)$ for each $t \in \mathscr{T}_j$. It is readily observed that this implies

$$\inf_{t\in\mathscr{P}_k} t(\lambda x + (1-\lambda)y) \le \inf_{t\in\mathscr{P}_k} \lambda t(x) + (1-\lambda)t(y) \quad \forall k,$$

which contradicts (A.3). It remains to show that T_j is nonlinear. Since $\mathscr{T}_j$ is a subset of $\mathscr{T}$ whose elements are nonlinear convex tax schedules (see Lemma 2) there exist y_0 and y_1 in $[0, 1]$ such that $t(y_0) < t_l(y_0)$ and $t(y_1) > t_l(y_1)$ for all $t \in \mathscr{T}_j$. It follows that $T_j(y_0) < t_l(y_0)$ and $T_j(y_1) > t_l(y_1)$, so T_j is nonlinear. The case $m_j > I_l$ is handled similarly. $\square$

Proof of Corollary 1. For simplicity, we provide the proof for the case where J is a singleton. The general case is proven similarly. Take any entry profile a such that $J = \{i\}$. If $C\backslash J$ is empty, there is nothing to prove, so let $C\backslash J \ne \emptyset$. By hypothesis, $m_k \ge I_l$ for all $k \in C\backslash\{i\}$. By Lemma 3, T_k satisfies conditions (i) to (iii) for each $k \in C$, T_i is convex, and T_k is concave for each $k \in C\backslash\{i\}$. Therefore, there exists a unique $x_k \in (0, 1]$ such that $T_i(x_k) = T_k(x_k)$ for each $k \in C\backslash\{i\}$. Say $1 \in C\backslash\{i\}$ and $x_1 = \min\{x_k : k \in C\backslash\{i\}\}$. By Lemma 4, $\omega_i(a') > 1/2$, where $a_1' = a_i' = 1$ and $a_k' = 0$ for $k \in P\backslash\{1, i\}$. This inequality can be rewritten as $\int_{E_i(a')} \mathrm{d}F > 1/2$ (see (6)). Since $E_i(a') = [0, x_1] = E_i(a)$, we have

$$\int_{E_i(a)} \mathrm{d}F = \int_{E_i(a')} \mathrm{d}F > 1/2,$$

and so $\omega_i(a) > 1/2$. $\square$

Lemma A.2. *Suppose that $I_l/2 < c/\theta$. If $\{m_i : i \in P\}$ covers an ε^*-partition, there exists $i^* \in P$ such that $m_{i*} \in [I_l - c/\theta,\ \min\{m^* - c/\theta,\ I_l/2\}]$.*

Proof. Let

$$A^* \equiv \left[I_l - \frac{c}{\theta},\ \min\left\{m^* - \frac{c}{\theta}, \frac{I_l}{2}\right\}\right]$$

and define $\underline{k} = \min A$, where

$$A \equiv \{k:[N_k(\varepsilon^*), N_{k+1}(\varepsilon^*)] \cap A^* \neq \emptyset\}$$

and ε^* is as in (7). We have $N_k(\varepsilon^*) \leq I_l - c/\theta$, for $N_k(\varepsilon^*) > I_l - c/\theta$ implies the existence of $k' \in A$ with $k' < k$. Since $\{m_i : i \in P\}$ covers an ε^*-partition, $N_{k+1}(\varepsilon^*) - N_k(\varepsilon^*) \leq (1/2) \min\{m^* - I_l, c/\theta - I_l/2\}$ for all k. Therefore, $N_{\underline{k}}(\varepsilon^*) \leq I_l - c/\theta$ implies

$$N_{\underline{k}+1}(\varepsilon^*) \leq I_l - \frac{c}{\theta} + \frac{1}{2} \min\left\{m^* - I_l, \frac{c}{\theta} - \frac{I_l}{2}\right\}$$

and

$$N_{\underline{k}+2}(\varepsilon^*) \leq I_l - \frac{c}{\theta} + \min\left\{m^* - I_l, \frac{c}{\theta} - \frac{I_l}{2}\right\}.$$

Arranging terms, we obtain

$$N_{\underline{k}+1}(\varepsilon^*) \leq \frac{1}{2} \min\left\{m^* - \frac{c}{\theta}, \frac{I_l}{2}\right\} + \frac{1}{2}\left(I_l - \frac{c}{\theta}\right) \tag{A.4}$$

and

$$N_{\underline{k}+1}(\varepsilon^*) \leq \min\left\{m^* - \frac{c}{\theta}, \frac{I_l}{2}\right\}. \tag{A.5}$$

Further, $I_l - \frac{c}{\theta} \leq N_{\underline{k}+1}(\varepsilon^*)$ by definition of $\underline{k}$. This, along with (A.4) and (A.5), implies $[N_{\underline{k}+1}(\varepsilon^*), N_{\underline{k}+2}(\varepsilon^*)] \subset A^*$. By assumption, there exists an element of $\{m_i : i \in P\}$ in $(N_{\underline{k}+1}(\varepsilon^*), N_{\underline{k}+2}(\varepsilon^*))$, and so there exists $i^* \in P$ such that $m_{i^*} \in A^*$. $\square$

Lemma A.3. *Let a^*, C, and $\overline{m}$ be as in the proof of Proposition 5. If $a^* \neq 0$, $m_k \notin (\overline{m} - c/\theta, \overline{m} + c/\theta)$ for all $k \in C$, and $m_{j*} \in (\overline{m} - c/\theta, \overline{m} + c/\theta)$, then*

$$\frac{1}{|W(a^*)|} \sum_{l \in W(a^*)} |m_l - m_{j*}| \geq c/\theta.$$

Proof. Suppose that

$$\frac{1}{|W(a^*)|} \sum_{l \in W(a^*)} |m_l - m_{j*}| < c/\theta. \tag{A.6}$$

Let $a \equiv \overline{m} - c/\theta$ and $b \equiv \overline{m} + c/\theta$. Let N_1 be the set of all i in $W(a^*)$ with $m_i < a$, and let N_2 be the set of all i in $W(a^*)$ with $m_i > b$. Either $|N_1| \geq |N_2|$ or vice versa. Say $|N_1| \geq |N_2|$ (the other case is similar). Then we can rewrite (A.6) as

$$\frac{1}{|W(a^*)|} \left[\sum_{l \in N_1} (m_{j*} - m_l) + \sum_{l \in N_2} (m_l - m_{j*}) \right] < c/\theta.$$

After some manipulations of this inequality, we obtain

$$\frac{|N_1| - |N_2|}{|W(a^*)|} m_{j*} + \overline{m} - \frac{2 \sum\limits_{l \in N_1} m_l}{|W(a^*)|} < c/\theta.$$

If $m_{j*} \in (a, b)$, we have

$$\frac{|N_1| - |N_2|}{|W(a^*)|} a + \overline{m} - \frac{2 \sum\limits_{l \in N_1} m_l}{|W(a^*)|} < c/\theta.$$

Therefore,

$$\left(\overline{m} - \frac{c}{\theta} \right) \left(1 + \frac{|N_1| - |N_2|}{|W(a^*)|} \right) < \frac{2 \sum\limits_{l \in N_1} m_l}{|W(a^*)|}. \tag{A.7}$$

Observe that the condition $m_k \notin (\overline{m} - c/\theta, \overline{m} + c/\theta)$ for all $k \in C$ implies $\sum_{l \in N_1} m_l < |N_1|(\overline{m} - c/\theta)$. Further, we have $|N_1| + |N_2| = |W(a^*)|$. Therefore, we obtain

$$\left(\overline{m} - \frac{c}{\theta} \right) \frac{2|N_1|}{|N_1| + |N_2|} = \left(\overline{m} - \frac{c}{\theta} \right) \left(1 + \frac{|N_1| - |N_2|}{|W(a^*)|} \right)$$

and

$$\frac{2 \sum\limits_{l \in N_1} m_l}{|W(a^*)|} < \frac{2|N_1|}{|N_1| + |N_2|} \left(\overline{m} - \frac{c}{\theta} \right).$$

Combining these two equations with (A.7), we obtain a contradiction. $\square$

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