Poverty-Reducing Income Taxation with Endogenous Income

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Abstract

This paper develops nonlinear income tax systems designed to reduce poverty within a framework where income is endogenously determined. Our approach to poverty reduction is applicable across a wide range of poverty measures and remains robust under varying economic conditions, including heterogeneous wage distributions, non-labor endowments, and diverse individual preferences. When negative marginal tax rates are deemed infeasible—potentially due to concerns about vertical equity—the optimal policy simplifies to a fixed lump-sum subsidy for individuals whose after-tax incomes fall below the poverty line. Even when negative marginal tax rates are permissible, effective poverty-reducing tax policies must still rely on subsidization for the poor and non-positive marginal tax rates as essential features.

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JEL classifications: D63, D71, I32.

1. Introduction

Unlike the traditional utilitarian approach prevalent in neoclassical economics, this paper employs a non-welfarist framework for policy evaluation. We focus on the design of nonlinear income tax systems specifically aimed at reducing poverty, examining this question within a rich and multi-faceted economic context.

The framework we develop emphasizes three core considerations of the underlying economic problem. First, we treat income as fundamentally endogenous, recognizing that individuals adjust their earnings in response to tax policies, available non-labor income, and labor market opportunities. Second, we demonstrate that the impact of tax policy on poverty remains qualitatively consistent across varying economic conditions and heterogeneous individual preferences—elements that are either dynamic or inherently unobservable. Third, our theoretical framework applies to a broad spectrum of established poverty measures, enhancing its practical relevance.

Focusing on poverty reduction represents a significant departure from classical welfare economics. While the traditional welfarist approach requires the government to aggregate

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individual utility functions—inherently unobservable theoretical constructs—into a social welfare function that makes explicit judgments about interpersonal utility comparisons (see Boadway, 2012), our poverty-focused framework provides a more concrete and measurable policy objective. This shift toward an observable and widely understood policy target offers theoretical clarity and practical advantages for policy design and evaluation.

The analysis builds on the classical Mirrlees model of endogenous income (Mirrlees, 1971), extending it to incorporate two additional dimensions of heterogeneity beyond the standard variation in individual wage rates.

First, the framework allows for heterogeneity in individual preferences. Admissible preferences are represented by utility functions defined over labor-consumption bundles, which are assumed to satisfy standard regularity conditions. A central assumption is the normality of consumption, which implies that consumption increases in response to lump-sum subsidies, holding other factors constant.

Second, the model introduces heterogeneity in non-labor income through individual-specific endowments. This variation captures differences in unearned income across individuals, further enriching the economic environment.

Poverty measurement has generated extensive scholarly literature encompassing diverse methodologies for complete and incomplete poverty orderings of income distributions. Despite this methodological variety, economists broadly agree that poverty measures must conform to certain fundamental principles.[1] Our theoretical framework builds upon four essential axioms from this literature: *Focus*, *Symmetry*, *Continuity*, and *Monotonicity*.

The Focus axiom stipulates that poverty measurement should respond exclusively to income changes among the poor population, remaining invariant to changes in non-poor incomes. Symmetry ensures that poverty measurement remains independent of individual characteristics beyond income, treating all individuals with equal income identically. The Continuity axiom requires that marginal changes in income levels produce correspondingly small changes in poverty measurements. Finally, the Monotonicity axiom establishes that ceteris paribus, any decrease in a poor person's income must result in an increased poverty measure.

These four foundational axioms, first formally articulated in Sen (1976)'s seminal work, form the minimal requirements for our theoretical analysis. More precisely, we demonstrate that our results hold for *any* poverty measure adhering to these principles.

We consider a multidimensional tax policy space, with an *income tax schedule* defined as a piecewise linear mapping from gross incomes to tax liabilities that must satisfy three fundamental properties: continuity, rank preservation, and monotonicity, while ensuring that tax liabilities never exceed gross incomes. Each of these properties serves an important economic purpose in tax system design.

Continuity of the tax schedule ensures that small changes in income lead to correspondingly small changes in tax liability, preventing cliff effects that could distort economic behavior and violate principles of horizontal equity. Rank preservation guarantees that the relative ordering of individuals by income remains unchanged after taxation. This property helps maintain the perceived fairness of the tax system.

The monotonicity requirement embodies a fundamental principle of vertical equity: those with greater ability to pay, as measured by income, face weakly higher tax liabilities. This property precludes negative marginal tax rates, a constraint whose implications have

[1] For a comprehensive review of these principles, see Zheng (2000)

been extensively studied in the optimal taxation literature. While our main characterization of poverty-reducing tax schedules maintains this monotonicity requirement, we complement it with an extended analysis that relaxes this assumption. This generalization reveals that negative marginal tax rates can emerge in the broader framework, aligning with findings from related studies discussed later in this section.

The concept of a poverty-reducing tax schedule is central to our analysis. A tax schedule is deemed *poverty-reducing* if, across all possible economic environments—regardless of the distributions of non-labor endowments or wage rates or the configuration of individual preference profiles—the resulting after-tax income distribution exhibits no more poverty than the tax-free income distribution. This robustness ensures that poverty reduction remains a consistent and achievable policy objective despite dynamic or unobservable economic factors.

We provide a comprehensive characterization of poverty-reducing tax schedules: For a given poverty line $z > 0$, which serves as the threshold below which individuals are classified as "poor," a tax schedule is poverty-reducing if and only if it provides a fixed lump-sum subsidy to individuals whose after-tax incomes fall below the poverty line.

This characterization rests on the following insight about how tax schedules interact with poverty levels. Consider any tax schedule that either fails to provide subsidies to the poor or implements a positive marginal tax rate. Such a schedule can potentially increase poverty relative to a tax-free environment. This possibility can be illustrated by constructing a homogeneous economy where all individuals share identical endowments, preferences, and wage rates. In this scenario, individuals achieve higher consumption levels in the absence of taxation, thereby experiencing lower levels of poverty.

Beyond offering a clear and implementable criterion for tax policy design, this characterization yields several insights.

First, the poverty reduction property defined in this paper is context-free. As such, it cannot be contingent on resource constraints, which fluctuate based on the underlying model fundamentals. Our analysis focuses on identifying the essential characteristics of poverty-reducing tax policies, which serve as a robust theoretical basis upon which more specific, context-dependent policies can be built.

Notably, the universality of our results extends beyond context independence with respect to endowment and wage rate distributions to encompass the entire universe of preference profiles. This independence from preference specifications contrasts with normative criteria such as standard welfare maximization or inequality-reducing tax policy design, which often rely on particular subclasses of preferences.

Second, we can establish a complete ordering of all poverty-reducing tax schedules based on a single criterion: the size of their lump-sum subsidies. Tax schedules that provide larger subsidies consistently yield lower poverty measurements, regardless of the specific poverty measure used, as long as it adheres to the four fundamental axioms of Focus, Symmetry, Continuity, and Monotonicity. This complete ordering enables straightforward comparisons between different poverty-reducing tax schedules beyond their poverty dominance relative to a tax-free environment.

Finally, when the monotonicity requirement for tax schedules is relaxed, a broader class of poverty-reducing policies emerges: any tax schedule that subsidizes the poor and maintains a non-increasing marginal tax rate for all after-tax incomes below the poverty line qualifies as poverty-reducing.

Removing the monotonicity constraint opens the door to subsidizing the poor through negative marginal tax rates. Under such a system, subsidies can increase as individuals earn more income, creating an incentive structure that rewards additional work. This approach aligns with findings in the broader literature, which we explore in greater detail below.

However, despite this expanded policy space, one fundamental principle from our earlier analysis remains unchanged: attempting to subsidize the poor while simultaneously imposing positive marginal tax rates is ineffective at reducing poverty.

The academic literature examining the relationship between income taxation and poverty has evolved along two distinct research trajectories.

The first approach integrates poverty considerations into the classical utilitarian framework of optimal taxation. This line of inquiry is part of a broader scholarly effort to incorporate principles of fairness and equity into traditional social welfare frameworks, as extensively reviewed by Fleurbaey and Maniquet (2018).

Saez and Stantcheva (2016) develop a refined approach to optimal taxation using social marginal welfare weights, which measure the social value of providing an additional dollar to any given individual. These weights can be specified to incorporate various equity considerations, including poverty concerns, while preserving the Pareto principle. By concentrating the weights among those individuals below the poverty line, their framework explicitly prioritizes poverty reduction within the utilitarian tradition. The authors find that even when heavily weighting poverty reduction, optimal marginal tax rates below the poverty line remain positive. This result aligns with standard optimal tax theory.[2]

A fundamental tension exists between two approaches to measuring the well-being of the poor: while purely income-based metrics are easily observable, they ignore the welfare costs of work effort. Optimal taxation models account for labor's disutility, but they typically struggle to accommodate preference heterogeneity without relying on strong assumptions about interpersonal utility comparisons.

In a framework that excludes non-labor income, Maniquet and Neumann (2021) address this challenge by developing specific Paretian social preferences that favor progressive transfers across the poverty line between individuals working equal hours. This approach makes income levels comparable from a welfare perspective by controlling for labor time. The authors can partially rank tax schedules based on the minimum earnings needed to escape poverty. They also show that optimal marginal tax rates should be negative on average for after-tax earnings below the poverty line—contrasting with Saez and Stantcheva's finding of positive rates.

Similarly, Henry de Frahan and Maniquet (2021) analyze income tax schemes that maximize social welfare functions based on axioms of responsibility—ensuring equal treatment of individuals with identical wages—and poverty reduction. To address the limitations of income-based poverty measures, which overlook the welfare effects of work effort, they define poverty as consumption on indifference curves lying entirely below the poverty line. In a model with preference heterogeneity and no non-labor income, they find that optimal marginal tax rates on low incomes are non-positive. In the special case of iso-elastic preferences, they derive optimal tax formulas and calibrate them to the U.S. economy.

[2]Other welfarist studies have shown that negative marginal tax rates can emerge under specific conditions, such as fixed labor force participation costs (see, e.g., Saez, 2002) or present bias in work effort (Lockwood, 2020).

The second research trajectory takes a fundamentally different approach by placing poverty alleviation at the center of its analysis. This approach explicitly centers on poverty reduction as a primary policy objective, reflecting a shift toward more targeted analyses and a departure from traditional welfarist frameworks.

While scholars within this trajectory share a common commitment to poverty alleviation, they employ a range of normative criteria and policy objectives in their analyses.

Bourguignon and Fields (1990) examine the features of a poverty-minimizing anti-poverty budget using complete orderings of income distributions. They find that the optimal tax policy is sensitive to the specific poverty index adopted as the policy goal and the initial income distribution. In contrast, our framework inherently avoids such dependencies due to the generality of our class of poverty measures and the context-free nature of our approach to poverty reduction.

Kanbur et al. (1994) provide a partial characterization of optimal tax schedules aimed at minimizing an income-based poverty index of the additively separable form considered by Atkinson (1987). They find that optimal marginal tax rates should be negative at the lower end of the income distribution, suggesting a potential for poverty alleviation through targeted wage subsidy schemes.

Building on this work, Pirttilä and Tuomala (2004) extend the analysis to encompass mixed tax systems, which involve the simultaneous taxation of income and commodities, while maintaining the explicit social objective of poverty minimization. Their findings demonstrate that this approach justifies differentiated tax rates across commodities, challenging the conventional wisdom established by Atkinson and Stiglitz (1976), which advocates for uniform commodity taxation.

Besley and Coate (1992) and Besley and Coate (1995) weigh in on the distinction between income-based and welfare-based poverty measures, advocating for the former. Adopting a positive perspective, they argue that poverty programs, which are funded by the general population, reflect the priorities of taxpayers, who are likely more concerned with reducing visible manifestations of poverty—captured by income-based measures—than with the leisure choices of the poor. The authors characterize cost-minimizing income maintenance programs designed to ensure all individuals exceed a predetermined income threshold. They propose work requirements for individuals with lower wages, encouraging private-sector employment as a means to reduce their public-sector requirements.

Chambers and Moreno-Ternero (2017) develop an axiomatic approach to poverty-sensitive taxation by building on Young's equal sacrifice principle (Young, 1987, 1988, 1990). They selectively remove certain axioms from Young's original framework—specifically, those axioms that prevent tax systems from exempting low-income individuals—enabling a formal characterization of generalized equal-sacrifice tax rules that integrate poverty alleviation goals.

Finally, Carbonell-Nicolau (2025b) explores a concept of universal poverty reduction similar to the one examined in the present paper but focuses on a class of distribution-sensitive poverty measures that need not comply with the monotonicity axiom in poverty measurement. This paper completely characterizes poverty-reducing tax schedules within an exogenous income framework.

The paper is structured as follows. Section 2 defines the class of poverty measures for which our results are valid. Section 3 introduces the endogenous income model and formulates the tax policy space. The main findings are presented in Section 4, where we discuss the implications of our results and develop heuristic insights into the logic behind the characterization of poverty-reducing tax schedules. Finally, Section 5 summarizes the main contributions and suggests directions for future research.

2. Poverty criterion

The poverty criterion we employ in this analysis is the most general one found in the poverty measurement literature. It is based on first-order stochastic dominance and encompasses all conventional poverty measures in the field.

Our analysis uses a poverty pre-ordering that applies first-order stochastic dominance to compare income distributions. To proceed with the formal definition, we must first establish some terminology.

An *income distribution* is represented by an n-dimensional vector

$$\boldsymbol{x} = (x_1, \ldots, x_n),$$

where each coordinate is a nonnegative real number and $n \in \mathbb{N}$ represents the population size. Here, x_i denotes the income of individual i.

Let $(x_{[1]}, \ldots, x_{[n]})$ be a non-decreasing rearrangement of the coordinates in $\boldsymbol{x}$:

$$x_{[1]} \leq \cdots \leq x_{[n]}.$$

Given an income distribution $\boldsymbol{x} = (x_1, \ldots, x_n)$ and a *poverty line* $z > 0$, we define the *poor* as the set of individuals i whose income x_i is less than or equal to z.

The distribution $\boldsymbol{x} = (x_1, \ldots, x_n)$ *censored at* z, denoted as

$$\boldsymbol{x}^z = (x_1^z, \ldots, x_n^z),$$

is defined by

$$x_i^z = \begin{cases} x_i & \text{if } x_i \leq z, \\ z & \text{if } x_i > z, \end{cases}$$

for each $i \in \{1, \ldots, n\}$.

First-order stochastic dominance is formalized as a binary relation $\succcurlyeq_D$ over the set of income distributions. For any two distributions $\boldsymbol{x}$ and $\boldsymbol{y}$, we say that $\boldsymbol{x}$ dominates $\boldsymbol{y}$ (written as $\boldsymbol{x} \succcurlyeq_D \boldsymbol{y}$) if and only if

$$x_{[i]} \geq y_{[i]} \text{ for each } i \in 1, \ldots, n,$$

where $x_{[i]}$ and $y_{[i]}$ represent the ith ordered elements of the respective distributions.

For a fixed poverty line $z > 0$, we establish a ranking system for income distributions using a poverty pre-order $\succcurlyeq_z$. This binary relation over the set of distributions is defined as follows:

$$\boldsymbol{x} \succcurlyeq_z \boldsymbol{y} \Leftrightarrow \boldsymbol{y}^z \succcurlyeq_D \boldsymbol{x}^z,$$

where $\boldsymbol{x}^z$ and $\boldsymbol{y}^z$ represent the censored distributions of $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively, at the poverty line z.

The relation "$\boldsymbol{x} \succcurlyeq_z \boldsymbol{y}$" should be interpreted as "the distribution $\boldsymbol{x}$ exhibits no less poverty than the distribution $\boldsymbol{y}$," relative to the poverty line z.

The poverty pre-order $\succcurlyeq_z$ has been characterized in the poverty literature through a set of fundamental principles. Specifically, for fixed-size distributions, $\succcurlyeq_z$ is uniquely defined as the pre-order that simultaneously satisfies the following principles:

1. *Focus*: Changes in non-poor incomes do not affect poverty measurement.
2. *Symmetry*: Poverty measurement remains unchanged when individuals exchange incomes.
3. *Continuity*: Poverty measurement varies continuously with income levels.
4. *Monotonicity*: A decrease in a poor person's income increases poverty.

A fundamental result regarding the characterization of $\succcurlyeq_z$ states that for any two income distributions $\boldsymbol{x}$ and $\boldsymbol{y}$, we have $\boldsymbol{x} \succcurlyeq_z \boldsymbol{y}$ if and only if $\boldsymbol{x}$ exhibits no less poverty than $\boldsymbol{y}$ under all poverty measures that satisfy focus, symmetry, continuity, and monotonicity. This result was first established by Atkinson (1987) for the subclass of additively separable poverty measures. For a comprehensive discussion of these axioms and their implications, see Zheng (2000).

We conclude this section with an instrumental result on the poverty pre-order $\succcurlyeq_z$:

Lemma 1. *For any two income distributions $\boldsymbol{x} = (x_1, \ldots, x_n)$ and $\boldsymbol{y} = (y_1, \ldots, y_n)$, $x_i \geq y_i$ for each i implies $\boldsymbol{x} \succcurlyeq_z \boldsymbol{y}$.*

Proof. We must show that $x_i \geq y_i$ for each i implies $x_{[i]} \geq y_{[i]}$ for each i.

We proceed by induction on n. When $n = 1$, we have $x_1 \geq y_1$, and since these are the only values, $x_{[1]} = x_1$ and $y_{[1]} = y_1$. Therefore, $x_{[1]} \geq y_{[1]}$.

Assume the statement is true for some $n \geq 1$. Consider vectors of length $n + 1$: $\boldsymbol{x} = (x_1, \ldots, x_{n+1})$ and $\boldsymbol{y} = (y_1, \ldots, y_{n+1})$ where $x_i \geq y_i$ for all i.

Let

$$\overline{x} = \max\{x_1, \ldots, x_{n+1}\} \quad \text{and} \quad \overline{y} = \max\{y_1, \ldots, y_{n+1}\}.$$

Then $x_{[n+1]} = \overline{x}$ and $y_{[n+1]} = \overline{y}$.

Since $x_i \geq y_i$ for each i, we must have $\overline{x} \geq \overline{y}$, so $x_{[n+1]} \geq y_{[n+1]}$.

Now consider the n-dimensional vectors obtained by removing $\overline{x}$ and $\overline{y}$ from their respective vectors. Since

$$\overline{x} = x_i \quad \text{and} \quad \overline{y} = y_j, \quad \text{some } i, j,$$

we can write these n-dimensional vectors as

$$(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_{n+1}) \quad \text{and} \quad (y_1, \ldots, y_{j-1}, y_{j+1}, \ldots, y_{n+1}).$$

Since

$$x_j \geq y_j = \overline{y} \geq y_i \quad \text{and} \quad x_\iota \geq y_\iota \text{ for all } \iota \neq i, j,$$

the induction hypothesis gives

$$x_{[\iota]} \geq y_{[\iota]}, \quad \text{for all } \iota \in \{1, \ldots, n\}.$$

Hence, $x_{[\iota]} \geq y_{[\iota]}$ for all i from 1 to $n + 1$, completing the inductive step.

Thus, the statement of the lemma is true for all positive integers n. ∎

3. Endogenous income framework

Our model extends the foundational Mirrlees (1971) income taxation framework by incorporating both labor and non-labor income sources, as well as heterogeneous preferences.

Individual i's preferences over labor-consumption pairs $(l, x) \in [0,1] \times \mathbb{R}_+$ are represented by a utility function $u_i(l, x)$, which is non-increasing in labor, l, and non-decreasing in consumption, x, and strictly decreasing in l and strictly increasing in x in $(0,1) \times \mathbb{R}_{++}$. The quantity of labor hours, l, is normalized to range between zero and one, where $l = 0$ represents no work and $l = 1$ represents full-time work.

The utility function u_i is assumed to be continuous over its domain $[0,1] \times \mathbb{R}_+$ and differentiable and strictly quasiconcave on the interior of this domain.

Furthermore, consumption, x, is assumed to be a normal good (i.e., it increases with income).

The set of all such utility functions is denoted by $\mathscr{U}$.

Each individual i is uniquely defined by their non-labor endowment, $\omega_i \geq 0$, and wage rate, $a_i > 0$, which together determine their gross income,

$$\omega_i + a_i l_i,$$

where l_i represents the hours worked by i. Thus, an individual's gross income is calculated as the sum of their non-labor endowment and earnings from labor, reflecting the interaction between unearned income and labor market participation.[3]

An *income tax schedule* is defined as a continuous, rank-preserving, and non-decreasing function $t : \mathbb{R}_+ \to \mathbb{R}$, where $t(y)$ represents the tax liability associated with income level y, subject to the fundamental constraint that $t(y) \leq y$ for every $y \geq 0$, which ensures that tax liabilities never exceed pre-tax income levels, maintaining economic feasibility.

The monotonicity requirement embodies a weak form of vertical equity: individuals with higher income cannot face lower tax burdens. By design, this requirement explicitly precludes negative marginal tax rates, a concept that has garnered significant attention in the literature on nonlinear income taxation with endogenous income, both in welfarist and non-welfarist frameworks (see, e.g., Saez, 2002; Saez and Stantcheva, 2016; Boadway et al., 2002; Lockwood, 2020; Kanbur et al., 1994; Fleurbaey and Maniquet, 2006, 2007; Maniquet and Neumann, 2021; Henry de Frahan and Maniquet, 2021). Nevertheless, we demonstrate that this monotonicity constraint can be relaxed without compromising our core results. A detailed analysis of the implications of removing the monotonicity assumption is presented in Appendix B (see also the discussion following the statement of Theorem 1).

Rank preservation means that post-tax income $y - t(y)$ must be non-decreasing in pre-tax income y, preventing income rankings from being reversed through taxation.

Negative tax liabilities represent subsidies.

In this paper, we concentrate on *piecewise linear* tax schedules. Specifically, these schedules partition the domain $\mathbb{R}_+$ into a finite number of intervals, with the tax function assuming a linear form within each interval.

The set of all piecewise linear tax schedules is denoted by $\mathscr{T}$.

An individual i earning a wage rate $a_i > 0$, facing an income tax schedule t, and endowed with non-labor income $\omega_i \geq 0$, chooses the optimal quantity of labor l_i to solve

[3] This formulation can be equivalently expressed in terms of wealth rather than income.

the following utility maximization problem:

$$\max_{l_i \in [0,1]} u_i(l_i, \omega_i + a_i l_i - t(\omega_i + a_i l_i)), \tag{1}$$

where $\omega_i + a_i l_i - t(\omega_i + a_i l_i)$ represents i's income net of taxes.

The problem in (1) has at least one solution due to the continuity of the objective function and the compactness of the feasible set.

We define a *solution function* $l_i^{u_i}(\omega_i, a_i, t)$ as a function that represents a solution to (1) for given values of endowment ω_i, wage rate a_i, and tax schedule t.

From this solution function, we can derive both gross (before-tax) and net (after-tax) *income functions*. These are defined as follows:

$$\begin{aligned} y_i^{u_i}(\omega_i, a_i, t) &= \omega_i + a_i l_i^{u_i}(\omega_i, a_i, t), \\ x_i^{u_i}(\omega_i, a_i, t) &= \omega_i + a_i l_i^{u_i}(\omega_i, a_i, t) - t(\omega_i + a_i l_i^{u_i}(\omega_i, a_i, t)). \end{aligned}$$

where $y_i^{u_i}(\omega_i, a_i, t)$ represents the gross income function and $x_i^{u_i}(\omega_i, a_i, t)$ represents the net income function.

In an economy consisting of $n \in \mathbb{N}$ individuals, this framework generates net income distributions represented by the vector

$$(x_1^{u_1}(\omega_1, a_1, t), \ldots, x_n^{u_n}(\omega_n, a_n, t)).$$

This distribution is parameterized by three elements: the wage distribution $(a_1, \ldots, a_n)$, the endowment distribution $(\omega_1, \ldots, \omega_n)$, and the income tax schedule t. Each component $x_i^{u_i}(\omega_i, a_i, t)$ represents the net income of individual i, which is a function of their specific endowment ω_i, wage rate a_i, and the applied tax schedule t.

The income distribution in the absence of taxation is denoted by

$$(x_1^{u_1}(\omega_1, a_1, 0), \ldots, x_n^{u_n}(\omega_n, a_n, 0)).$$

This baseline distribution serves as a reference point for assessing the impact of various tax policies on poverty.

4. Poverty-reducing tax schedules

This paper characterizes income tax schedules that guarantee robust poverty reduction across all possible economic environments. Specifically, we identify tax structures that decrease poverty—as measured by the poverty pre-order $\succcurlyeq_z$ defined in Section 2—regardless of the underlying distribution of endowments and wages, or the heterogeneity in individual preferences.

In formal terms, given a poverty pre-order $\succcurlyeq$, a tax schedule t is $\succcurlyeq$*-poverty-reducing* if

$$(x_1^{u_1}(\omega_1, a_1, 0), \ldots, x_n^{u_n}(\omega_n, a_n, 0)) \succcurlyeq (x_1^{u_1}(\omega_1, a_1, t), \ldots, x_n^{u_n}(\omega_n, a_n, t))$$

for all endowment distributions $(\omega_1, \ldots, \omega_n) \in \mathbb{R}^n_+$, all wage distributions $(a_1, \ldots, a_n) \in \mathbb{R}^n_{++}$, all preference profiles $(u_1, \ldots, u_n) \in \mathscr{U}^n$, and all income solution functions $x_1^{u_1}, \ldots, x_n^{u_n}$.[4]

Our main result provides a complete characterization of the set of $\succcurlyeq_z$-poverty-reducing tax schedules.

Theorem 1. *A tax schedule $t \in \mathscr{T}$ is $\succcurlyeq_z$-poverty-reducing if and only if it $t(y) = -s$ for all y with $y + s \leq z$ and some $s \geq 0$.*

This result states a tax schedule t is $\succcurlyeq_z$-poverty-reducing if and only if it provides a fixed subsidy s to those individuals whose post-tax incomes fall below the poverty line z.

Before informally discussing the proof of Theorem 1, we examine significant ramifications of this result and explore the consequences of relaxing the monotonicity assumption in the definition of a tax schedule.

4.1. Discussion on Theorem 1

Our analysis notably omits explicit budgetary constraints on admissible tax schedules. Such constraints typically depend on contextual economic factors—specifically, the distribution of wage rates, the resulting gross income distribution, and the government's resource capacity. Our analysis intentionally leaves these variables unspecified, as our universal poverty reduction framework applies across all economic environments.

Perhaps most remarkably, this context-independence extends beyond economic variables to preference specifications. Intuition might suggest that a tax schedule's effectiveness in poverty reduction would depend on restricted preference profiles—that is, constraints limiting the universe of admissible preference profiles, $\mathscr{U}^n$, to a proper subset.[5] Yet Theorem 1 holds independently of consumers' preferences.

Beyond this preference-independence property, our framework yields another significant finding: the set of universally poverty-reducing tax schedules exhibits a complete ordering under our poverty measure. This ordering naturally corresponds to the subsidy magnitude, s. More precisely, let $\mathscr{T}_{\succcurlyeq_z}$ be the set of all $\succcurlyeq_z$-poverty-reducing tax schedules characterized by Theorem 1:

$$\mathscr{T}_{\succcurlyeq_z} = \{t_s \in \mathscr{T} : t_s = -s \text{ for all } y \text{ with } y + s \leq z \text{ and some } s \geq 0\}.$$

For any two schedules $t_s, t_{s'} \in \mathscr{T}_{\succcurlyeq_z}$, a higher subsidy corresponds to a less impoverished distribution. Formally, $s \geq s'$ if and only if

$$(x_1^{u_1}(\omega_1, a_1, t_{s'}), \ldots, x_n^{u_n}(\omega_n, a_n, t_{s'})) \succcurlyeq_z (x_1^{u_1}(\omega_1, a_1, t_s), \ldots, x_n^{u_n}(\omega_n, a_n, t_s)) \tag{2}$$

for all endowment distributions $(\omega_1, \ldots, \omega_n) \in \mathbb{R}^n_+$, all wage distributions $(a_1, \ldots, a_n) \in \mathbb{R}^n_{++}$, all preference profiles $(u_1, \ldots, u_n) \in \mathscr{U}^n$, and all income solution functions $x_1^{u_1}, \ldots, x_n^{u_n}$.

This follows by examining the censored values $x_{s'}^z$ and x_s^z of each of i's coordinates for the vectors in equation (2). By Lemma 1, it suffices to show that these values satisfy

$$x_s^z \geq x_{s'}^z. \tag{3}$$

[4]In the absence of taxation, the solution functions are uniquely determined. However, when specific tax schedules are introduced, multiple equilibrium solutions can emerge.

[5]This is what happens, for example, in the context of inequality-reducing taxation (see, e.g., Carbonell-Nicolau, 2025a).

If $x_s^z = z$, inequality (3) holds trivially. Thus, consider the case where $x_s^z < z$. In this case, consumer i's gross income level y_s solves:

$$\max_{y \in [0, a_i]} u_i(y/a_i, y + s)$$

Since consumption is a normal good and $s \geq s'$, the solution $y_{s'}$ to the analogous problem

$$\max_{y \in [0, a_i]} u_i(y/a_i, y + s')$$

must satisfy

$$y_{s'} + s' \leq y_s + s = y_s - t_s(y_s) \tag{4}$$

Now observe that under a lump sum subsidy s', consumer i's feasible set of gross income-net income bundles strictly contains the set of bundles attainable under tax schedule $t_{s'}$, since $t_{s'}$ may impose non-zero marginal tax rates for gross incomes whose corresponding after-tax levels exceed the poverty line z. Therefore, since $y_{s'}$ remains feasible under $t_{s'}$, revealed preference implies that $y_{s'}$ also solves

$$\max_{y \in [0, a_i]} u_i(y/a_i, y - t_{s'}(y))$$

Combined with inequality (4), this yields

$$x_{s'}^z = y_{s'} - t_{s'}(y_{s'}) = y_{s'} + s' \leq y_s - t_s(y_s) = x_s^z,$$

which establishes (3).

Understanding the poverty measures covered by Theorem 1 is essential for delineating the scope of the result. To this end, we provide a detailed discussion below.

First, recall that for any two income distributions $\boldsymbol{x}$ and $\boldsymbol{y}$, the relation $\boldsymbol{x} \succcurlyeq_z \boldsymbol{y}$ holds if and only if $\boldsymbol{x}$ exhibits no less poverty than $\boldsymbol{y}$ under all poverty measures that satisfy the fundamental axioms of focus, symmetry, continuity, and monotonicity.

This well-established result implies that if a tax schedule t is $\succcurlyeq_z$-poverty-reducing, then it is also poverty-reducing according to *any* poverty measure that satisfies these four axioms.

Conversely, any tax schedule that reduces poverty under a measure satisfying the four basic axioms must also be $\succcurlyeq_z$-poverty-reducing.

These observations establish an equivalence of the notion of poverty reduction across all measures adhering to these axioms, providing a robust framework for evaluating tax-mediated poverty alleviation.

To understand why the converse assertion is true, observe that any poverty pre-order $\succcurlyeq$ satisfying the four axioms is an extension (or partial completion) of the core pre-order $\succcurlyeq_z$. That is, $\succcurlyeq \supseteq \succcurlyeq_z$ and $\succ \supseteq \succ_z$.

Now, suppose t is $\succcurlyeq$-poverty-reducing. Then, for any endowment distribution $(\omega_1, \ldots, \omega_n) \in \mathbb{R}_+^n$, wage distribution $(a_1, \ldots, a_n) \in \mathbb{R}_{++}^n$, preference profile $(u_1, \ldots, u_n) \in \mathscr{U}^n$, and income solution functions $x_1^{u_1}, \ldots, x_n^{u_n}$, we have

$$(x_1^{u_1}(\omega_1, a_1, 0), \ldots, x_n^{u_n}(\omega_n, a_n, 0)) \succcurlyeq (x_1^{u_1}(\omega_1, a_1, t), \ldots, x_n^{u_n}(\omega_n, a_n, t)).$$

Fix $(\omega_1, \ldots, \omega_n)$, $(a_1, \ldots, a_n)$, $(u_1, \ldots, u_n)$, and $x_1^{u_1}, \ldots, x_n^{u_n}$. Then, for each individual i,

$$(x_i^{u_i}(\omega_i, a_i, 0), \ldots, x_i^{u_i}(\omega_i, a_i, 0)) \succcurlyeq (x_i^{u_i}(\omega_i, a_i, t), \ldots, x_i^{u_i}(\omega_i, a_i, t)). \tag{5}$$

This implies

$$x_i^{u_i}(\omega_i, a_i, t) \geq x_i^{u_i}(\omega_i, a_i, 0). \tag{6}$$

If this were not the case, we would have

$$(x_i^{u_i}(\omega_i, a_i, t), \ldots, x_i^{u_i}(\omega_i, a_i, t)) \succ_z (x_i^{u_i}(\omega_i, a_i, 0), \ldots, x_i^{u_i}(\omega_i, a_i, 0)),$$

which would further imply

$$(x_i^{u_i}(\omega_i, a_i, t), \ldots, x_i^{u_i}(\omega_i, a_i, t)) \succ (x_i^{u_i}(\omega_i, a_i, 0), \ldots, x_i^{u_i}(\omega_i, a_i, 0))$$

(since $\succcurlyeq$ is an extension of $\succcurlyeq_z$), contradicting (5).

Since (6) holds, Lemma 1 implies

$$(x_1^{u_1}(\omega_1, a_1, 0), \ldots, x_n^{u_n}(\omega_n, a_n, 0)) \succcurlyeq_z (x_1^{u_1}(\omega_1, a_1, t), \ldots, x_n^{u_n}(\omega_n, a_n, t)).$$

Thus, t is also $\succcurlyeq_z$-poverty-reducing.

These insights are formalized in the following corollary to Theorem 1:

Corollary 1 (to Theorem 1)**.** *Suppose $\succcurlyeq$ is an extension of the core pre-order $\succcurlyeq_z$, i.e., $\succcurlyeq \supseteq \succcurlyeq_z$ and $\succ \supseteq \succ_z$. A tax schedule $t \in \mathcal{T}$ is $\succcurlyeq$-poverty-reducing if and only if $t(y) = -s$ for all y with $y + s \leq z$ and some $s \geq 0$.*

Finally, in Appendix B, we provide a complete characterization of poverty-reducing tax schedules in the absence of the monotonicity constraint, which precludes negative marginal tax rates. We find that while pure subsidies for individuals with post-tax incomes below the poverty line remain poverty-reducing, they are no longer the unique solution. Without the monotonicity requirement, the complete set of poverty-reducing tax schedules is characterized by subsidies with (weakly) negative marginal tax rates for the range of post-tax incomes below the poverty line.

Thus, even when we expand the feasible policy set to include non-monotonic tax schedules, our core findings remain qualitatively similar, with the key difference being the permissibility of negative marginal tax rates.

4.2. Informal proof of Theorem 1

The formal proof of Theorem 1 is presented in Appendix A. To provide intuition for the underlying logic, we offer here a visual heuristic argument.

To begin, we argue that if $t \in \mathcal{T}$ satisfies $t(y) = -s$ for all y with $y + s \leq z$ and some $s \geq 0$, then it is $\succcurlyeq_z$-poverty-reducing, meaning that for all endowment distributions $(\omega_1, \ldots, \omega_n)$, all wage distributions $(a_1, \ldots, a_n)$, all preference profiles $(u_1, \ldots, u_n) \in \mathcal{U}^n$, and all income solution functions $x_1^{u_1}, \ldots, x_n^{u_n}$, we have

$$(x_1^{u_1}(\omega_1, a_1, 0), \ldots, x_n^{u_n}(\omega_n, a_n, 0)) \succcurlyeq_z (x_1^{u_1}(\omega_1, a_1, t), \ldots, x_n^{u_n}(\omega_n, a_n, t)). \tag{7}$$

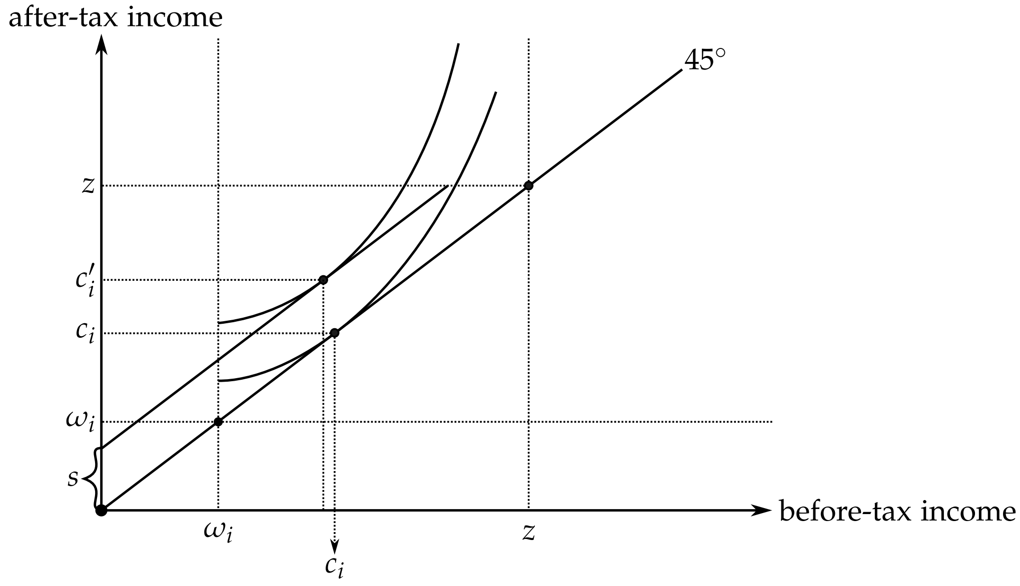

Figure 1: Individual i's problem.

For fixed parameters $(\omega_1, \ldots, \omega_n)$, $(a_1, \ldots, a_n)$, $(u_1, \ldots, u_n)$, and solution functions $x_1^{u_1}, \ldots, x_n^{u_n}$, let

$$\boldsymbol{c} = (c_1, \ldots, c_n) \quad \text{and} \quad \boldsymbol{c}' = (c_1', \ldots, c_n')$$

denote, respectively, the distributions

$$(x_1^{u_1}(\omega_1, a_1, 0), \ldots, x_n^{u_n}(\omega_n, a_n, 0)) \quad \text{and} \quad (x_1^{u_1}(\omega_1, a_1, t), \ldots, x_n^{u_n}(\omega_n, a_n, t))$$

censored at the poverty line z.

To see that (7) holds, it suffices to show that $\boldsymbol{c}' \succcurlyeq_D \boldsymbol{c}$, which is implied by the inequalities $c_i' \geq c_i$ for each $i \in \{1, \ldots, n\}$ (Lemma 1). We will illustrate this component-wise inequality through graphical analysis.

Fix an individual i. In the absence of income taxation, i's utility maximization problem is given by

$$\max_{l_i \in [0,1]} u_i(l_i, \omega_i + a_i l_i).$$

This problem can be reformulated in terms of i's gross (before-tax) income, y_i, and net (after-tax) income, x_i. Gross income, y_i, is related to labor choice through the following identity:

$$y_i = \omega_i + a_i l_i,$$

In the absence of taxation, we have $x_i = y_i$.

Using these relationships, we can rewrite i's maximization problem as follows:

$$\max_{y_i \in [\omega_i, \omega_i + a_i]} u_i((y_i - \omega_i)/a_i, y_i). \tag{8}$$

We present the solution to (8) graphically in Figure 1. In the figure, the 45° line truncated at the poverty line z represents individual i's budget line, in the absence of taxation, for incomes in the range $[0, z]$, and c_i denotes individual i's optimal income level. Since t acts as a pure subsidy for incomes within the range $[0, z]$ (i.e., $t(y) = -s$ for all $y \in [0, z]$ and some $s \geq 0$), the implementation of the tax shifts the budget line upward by the amount of the subsidy, s. Consequently, individual i experiences a positive income effect, leading to an increase in net consumption from c_i to c_i', as illustrated in Figure 1. Given the assumption that consumption is a normal good, it follows that $c_i' \geq c_i$, as we wanted to show.

For tax-free, non-poor incomes above the poverty line, the question remains whether after-tax incomes can fall below the poverty line z, potentially violating the inequality $c_i' \geq c_i$ for some individual i. Such a scenario would require individual i's budget line to contain a decreasing segment beyond z. However, this is impossible under our definition of a tax schedule, which must preserve income ranks. A decreasing portion of the budget line would imply that higher pre-tax incomes lead to lower post-tax incomes, violating rank preservation.

Next, we heuristically argue the converse assertion: a $\succcurlyeq_z$-poverty-reducing tax schedule t satisfy $t(y) = -s$ for all y with $y + s \leq z$ and some $s \geq 0$.

Suppose that t is $\succcurlyeq_z$-poverty-reducing. We first argue that t must (weakly) subsidize the poor, i.e., that $t(y) \leq 0$ for all $y \in [0, z]$. Proceeding by contradiction, suppose that $t(x^*) > 0$ for some $x^* \in [0, z]$. This scenario is illustrated in Figure 2, where the net income corresponding to x^*, $x^* - t(x^*)$, lies below the 45° line.

Set initial endowments to zero, $(\omega_1, \ldots, \omega_n) = \mathbf{0}$, and select a homogeneous wage distribution $(a, \ldots, a)$ with identical preferences $(u, \ldots, u)$ such that individual 1's optimal income in the absence of taxation is $x^u(0, a, 0) = x^*$. This optimal choice appears in Figure 2 at the tangency point between individual 1's budget line (the 45° line) and 1's indifference curve through (x^*, x^*).

Let $\boldsymbol{c} = (c_1, \ldots, c_n)$ and $\boldsymbol{c}' = (c_1', \ldots, c_n')$ denote the distributions

$$(x^u(0, a, 0), \ldots, x^u(0, a, 0)) \quad \text{and} \quad (x^u(0, a, t), \ldots, x^u(0, a, t))$$

censored at the poverty line z, respectively.

Since t is $\succcurlyeq_z$-poverty-reducing, it follows that $\boldsymbol{c}' \succcurlyeq_D \boldsymbol{c}$, which implies $c_1' \geq c_1$. However, as we shall demonstrate using Figure 2, this inequality does not hold.

This contradiction stems from our initial assumption that t does not (weakly) subsidize the poor. Therefore, this assumption must be invalid. Consequently, we can conclude that $t(y) \leq 0$ for all $y \in [0, z]$.

To argue that $c_1' < c_1$, we must first consider individual 1's budget line under the tax t. This budget line necessarily intersects the point $(x^*, x^* - t(x^*))$ as shown in Figure 2. Given this constraint, the remainder of the budget line must be confined to the gray shaded area in the figure. This is a direct result of the tax schedule's properties: t is non-decreasing and preserves income ranks.

Now, let's examine two possible scenarios. If individual 1's gross income under the tax, $y^u(0, a, t)$, were to fall below x^*, then the corresponding net income, $c_1' = x^u(0, a, t)$, would necessarily be less than c_1, since the gray area to the left of x^* lies entirely below c_1.

Conversely, if c_1' exceeds x^*, we can still conclude that $c_1' < c_1$ must hold. Proceeding by contradiction, let's assume $c_1' \geq c_1$ in this case. Figure 2 illustrates this scenario, depicting a portion of individual 1's budget constraint under the tax as a line passing through

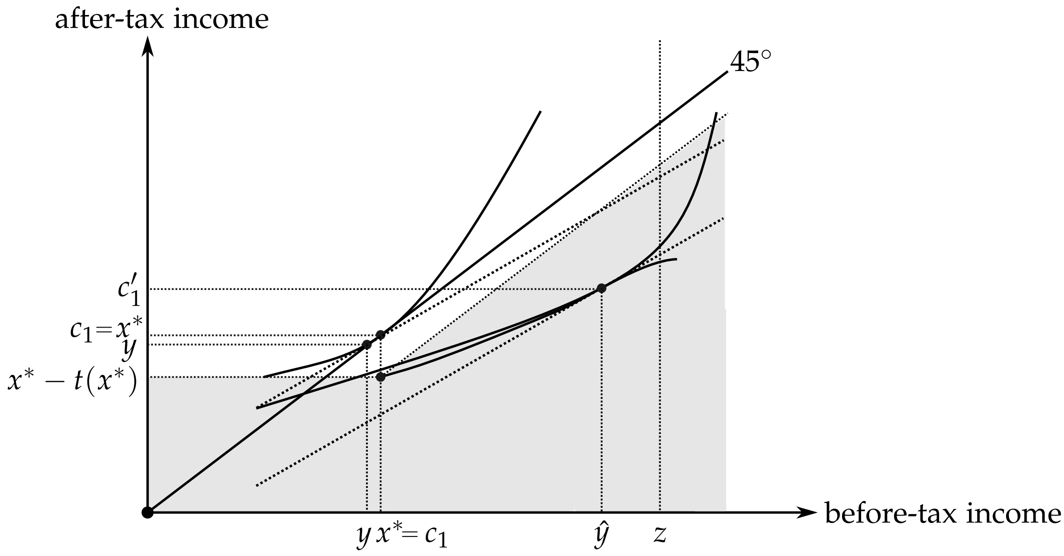

Figure 2: Individual 1's problem.

$(x^*, x^* - t(x^*))$ and $(\hat{y}, c_1')$. The optimal point for individual 1 is represented by the tangency between 1's indifference curve through $(\hat{y}, c_1')$ and the budget constraint. As shown, $c_1' > c_1$.

However, if this inequality were true, it would violate the assumption that consumption is a normal good. To understand why, consider the following: Starting from the point (y, y), which individual 1 would choose under the dotted budget line tangent to 1's indifference curve through (y, y), a reduction in income would shift the budget line inward to the tangent at $(\hat{y}, c_1')$. This would result in consumption increasing from y to c_1', contradicting the normality of consumption.

Therefore, we can conclude that $c_1' < c_1$ must hold true.

We have seen that if t is $\succcurlyeq_z$-poverty-reducing, then it must weakly subsidize the poor. We are now ready to argue that t must provide a fixed subsidy $s \geq 0$ to all individuals whose post-subsidy income remains at or below the poverty line z, i.e., $t(y) = -s$ for all y where $y + s \leq z$.

Proceeding by contradiction, let us consider a scenario where t does not have this property. Given that t weakly subsidizes the poor, a portion of the budget constraint faced by consumers under t must lie above the 45° line and have a slope less than unity, as illustrated in Figure 3 by the segment passing through the point (y, c_1').

In this case, we can construct two homogeneous distributions

$$(x^u(0, a, 0), \ldots, x^u(0, a, 0)) \quad \text{and} \quad (x^u(0, a, t), \ldots, x^u(0, a, t)),$$

with zero non-labor endowments and identical wage rates for all individuals, with the property that

$$z \geq x^u(0, a, 0) > x^u(0, a, t),$$

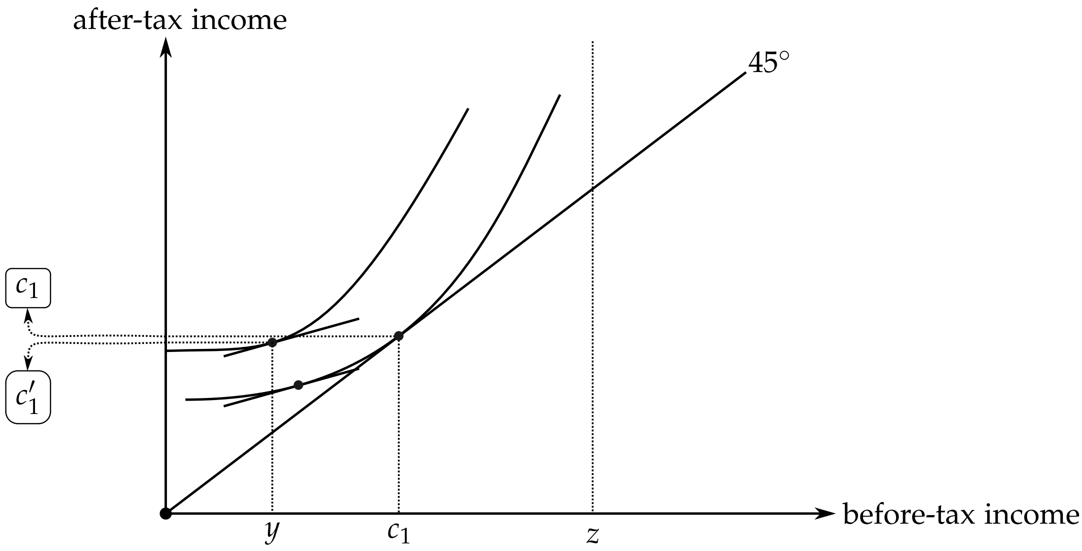

Figure 3: Individual 1's problem.

which implies

$$(x^u(0,a,0),\ldots,x^u(0,a,0)) \not\succcurlyeq_z (x^u(0,a,t),\ldots,x^u(0,a,t)),$$

contradicting that t is $\succcurlyeq_z$-poverty-reducing.

To see this, consider the preferences illustrated in Figure 3, which we can specify to satisfy consumption normality. The figure shows that under no taxation, the optimal income level $x^u(0,a,0)$ equals c_1, while under tax schedule t, the utility-maximizing after-tax income $x^u(0,a,t)$ equals c_1'. Since

$$z \geq c_1 > c_1',$$

these preferences demonstrate that the tax schedule reduces income while keeping it below the poverty line z. This contradicts our assumption, showing that t cannot be $\succcurlyeq_z$-poverty-reducing.

In essence, our universal poverty reduction mandate requires that any proposed tax schedule incorporate a fixed subsidy for all individuals whose net income falls below the poverty threshold. Without this constraint, it becomes possible to construct scenarios where, even with normal consumption preferences, the tax schedule reduces individuals' post-tax income relative to their tax-free earnings. Such an outcome exacerbates poverty instead of alleviating it.

5. Concluding remarks

This paper treats poverty reduction as an explicit non-utilitarian policy objective, a criterion that aligns with what Kanbur et al. (2006) identify as "behavioral public economics," wherein the government optimizes based on objectives distinct from individuals' utility functions.

By shifting toward a concrete and widely recognized policy goal, this approach avoids the fundamental challenge of interpersonal utility comparisons in classical welfare economics. However, since poverty measures are typically income-based, they do not capture the individual disutility of work effort.

From a positive perspective, this omission may be justified, to some extent, by the perceptions of taxpayers who finance poverty alleviation programs, as they may place greater emphasis on the income of the poor rather than their leisure choices (Besley and Coate, 1995).

This paper completely characterizes poverty-reducing nonlinear tax schedules within an endogenous income framework that is sufficiently flexible to accommodate heterogeneous wage rate distributions, individual-specific non-labor endowments, and idiosyncratic preferences.

In our study, the poverty reduction criterion stipulates that all poverty measures adhering to the essential principles of Focus, Symmetry, Continuity, and Monotonicity consistently align when comparing any two income distributions. Moreover, the effectiveness of a tax policy in alleviating poverty should be universal, meaning it must remain invariant to changes in the economic environment, which includes variations in wage rate distributions, non-labor endowments, and individual preference profiles.

When negative marginal tax rates are deemed unfeasible—perhaps due to concerns about vertical equity—poverty-reducing tax schedules take on a straightforward form: a fixed lump-sum subsidy for after-tax incomes that fall below the poverty line. These tax schemes can be systematically ranked by any poverty measure, with larger subsidies corresponding to greater poverty reduction.

In scenarios where negative marginal tax rates on income are permissible, tax schemes are poverty-reducing if and only if they subsidize the range of poor after-tax incomes at non-positive marginal tax rates. This finding introduces the possibility of using negative marginal tax rates as effective tools for poverty alleviation in this broader context. However, positive marginal tax rates remain ineffective at reducing poverty.

Looking beyond our current findings, we note that conventional poverty measures based on first-order stochastic dominance lack sensitivity to how income is distributed among the poor. This limitation stems from their reliance on the Monotonicity axiom, which views any income increase for poor individuals as beneficial, regardless of its distributional consequences. A more sophisticated approach would consider whether income growth exacerbates inequality among the poor. Social preferences that align with this view, such as those developed by Shorrocks (1983) in the context of income inequality, can be adapted to the context of poverty by applying them to censored income distributions (see, e.g., Carbonell-Nicolau, 2025b). A natural extension of this work would be to analyze poverty-reducing income taxation using these distribution-sensitive measures within the framework of the present paper.

A. Proof of Theorem 1

We begin with the following preliminary result.

Lemma 2. *Suppose that $t \in \mathscr{T}$ is $\succcurlyeq_z$-poverty-reducing. Then $t(y) \leq 0$ for all $y \in [0, z]$.*

Proof. Suppose that $t(x^*) > 0$ for some $x^* \in [0, z]$. We will show that t is not $\succcurlyeq_z$-poverty-reducing.

Fix $a > x^*$ and define $\alpha = x^*/a$. Set the non-labor endowments equal to zero, $(\omega_1, \ldots, \omega_n) = \mathbf{0}$, and pick the homogeneous wage rate distribution $(a, \ldots, a)$ and the vector of identical preferences $(u, \ldots, u)$, where

$$u(l, x) = x^\alpha (1 - l)^{1-\alpha}.$$

It is easy to see that the solution to the problem

$$\max_{y \in [0,a]} u(y/a, y),$$

which determines the individuals' gross income in the absence of taxation, is $x^u(0, a, 0) = x^*$.

When confronted with the tax schedule t, individuals choose their gross incomes to solve the problem

$$\max_{y \in [0,a]} u(y/a, y - t(y)). \tag{9}$$

If y^* denotes a solution to this problem and $y^* \leq x^*$, then the corresponding net income,

$$x^u(0, a, t) = y^* - t(y^*), \tag{10}$$

must satisfy $x^u(0, a, t) < x^*$. Indeed, this is clearly the case if $y^* = x^*$, since $t(x^*) > 0$ by assumption. If $y^* < x^*$, then

$$x^u(0, a, t) = y^* - t(y^*) \leq x^* - t(x^*) < x^*,$$

where the weak inequality holds because t preserves income ranks.

If $y^* > x^*$, the inequality $x^u(0, a, t) < x^*$ still holds. To see this, note first that, because y^* solves (9), there is a line $x = \beta y + \gamma$ with $\beta \leq 1$ that is tangent to the indifference curve

$$u(y/a, x) = u(y^*/a, y^* - t(y^*))$$

at $(y^*, y^* - t(y^*))$.

To see that the slope β is less than or equal to 1, consider first the case when y^* lies in the interior of a tax bracket. Then $x = \beta y + \gamma$ is the linear extension of the map from gross income to net income $y \mapsto y - t(y)$ restricted to a sufficiently small neighborhood of y^*. This restriction is linear with slope $1 - t'(y) \in [0, 1]$.

Alternatively, y^* may represent a kink point for the tax schedule t, where the marginal tax rate transitions from τ_0 to τ_1. For this to be the case, we must have $\tau_0 < \tau_1$; otherwise, the solution y^* would not occur at the kink point. Moreover, the marginal rate of substitution at $(y^*, y^* - t(y^*))$,

$$MRS^u(y^*, y^* - t(y^*)) = -(1/a) \cdot \frac{\partial u(y^*/a, y^* - t(y^*))}{\partial l} \Bigg/ \frac{\partial u(y^*/a, y^* - t(y^*))}{\partial x},$$

belongs to the interval $[1 - \tau_1, 1 - \tau_0]$. Since $\tau_0, \tau_1 \in [0, 1]$, the slope of the line $x = \beta y + \gamma$, β, which coincides with $MRS^u(y^*, y^* - t(y^*))$, is less than or equal to 1.

Now, since the line $x = \beta y + \gamma$ is tangent to the indifference curve

$$u(x, y/a) = u(y^* - t(y^*), y^*/a)$$

at $(y^*, y^* - t(y^*))$, y^* is also a solution to the problem

$$\max_{y \in [0,a]} u(y/a, \beta y + \gamma). \tag{11}$$

This implies that an individual with utility function u and wage rate a would choose the gross income-net income pair $(y^*, y^* - t(y^*))$ when faced with feasible consumption bundles of the form $(y, \gamma + \beta y)$.

Moreover, this chosen pair is less preferred than the pair (x^*, x^*). This preference ordering can be established as follows:

- Due to the monotonicity of t, we have $0 < t(x^*) \leq t(y^*)$. Consequently, $(y^*, y^* - t(y^*))$ is less preferred than $(y\prime y^*)$.

- In turn, (y^*, y^*) is less preferred than (x^*, x^*), since (x^*, x^*) is the optimal choice in the absence of taxation, while (y^*, y^*) remains available in that scenario.

This sequence of preferences demonstrates that the pair $(y^*, y^* - t(y^*))$ is indeed less preferred than (x^*, x^*).

Consider now a scenario where the net income in problem (11) is augmented by a lump sum subsidy, e. This subsidy is precisely calibrated to enable the consumer to attain a bundle that yields the same utility as (x^*, x^*). This lump sum addition represents a pure income effect. Let $(\hat{y}, \hat{x})$ denote an optimal gross income-net income bundle corresponding to the modified problem

$$\max_{y \in [0,a]} u(y/a, \beta y + \gamma + e),$$

where the feasible bundles now take the form $(y, \beta y + \gamma + e)$. Given our assumption that consumption is a normal good, we can assert that

$$\hat{x} > y^* - t(y^*), \tag{12}$$

Furthermore, we can establish that

$$\hat{x} \leq x^*. \tag{13}$$

This inequality can be demonstrated as follows:

- By construction, both $(\hat{y}, \hat{x})$ and (x^*, x^*) lie on the same indifference curve, indicating equal utility.

- The marginal rate of substitution at $(\hat{y}, \hat{x})$, $MRS^u(\hat{y}, \hat{x})$, is equal to β, which we previously established is less than or equal to 1.

- In contrast, $MRS^u(x^*, x^*) = 1$.

- Given that the marginal rate of substitution is increasing along an indifference curve, we can conclude that $\hat{y} \leq x^*$.

- Consequently, since both points lie on the same indifference curve and $\hat{y} \leq x^*$, it follows that $\hat{x} \leq x^*$.

Equations (10), (12), and (13) together yield

$$x^u(0, a, t) = y^* - t(y^*) < x^* = x^u(0, a, 0) \leq z,$$

implying that

$$(x^u(0, a, 0), \dots, x^u(0, a, 0)) \not\succcurlyeq_z (x^u(0, a, t), \dots, x^u(0, a, t)).$$

Hence, t is not $\succcurlyeq_z$-poverty-reducing. ■

Theorem 1. *A tax schedule $t \in \mathscr{T}$ is $\succcurlyeq_z$-poverty-reducing if and only if it $t(y) = -s$ for all y with $y + s \leq z$ and some $s \geq 0$.*

Proof. [$\Leftarrow$] Suppose that $t(y) = -s$ for all y with $y + s \leq z$ and some $s \geq 0$. We must show that t is $\succcurlyeq_z$-poverty-reducing, meaning that for all endowment distributions $(\omega_1, \dots, \omega_n)$, all wage distributions $(a_1, \dots, a_n)$, all preference profiles $(u_1, \dots, u_n) \in \mathscr{U}^n$, and all solution functions $x_1^{u_1}, \dots, x_n^{u_n}$, we have

$$(x_1^{u_1}(\omega_1, a_1, 0), \dots, x_n^{u_n}(\omega_n, a_n, 0)) \succcurlyeq_z (x_1^{u_1}(\omega_1, a_1, t), \dots, x_n^{u_n}(\omega_n, a_n, t)). \tag{14}$$

For fixed parameters $(\omega_1, \dots, \omega_n)$, $(a_1, \dots, a_n)$, $(u_1, \dots, u_n)$, and solution functions $x_1^{u_1}, \dots, x_n^{u_n}$, let

$$\boldsymbol{c} = (c_1, \dots, c_n) \quad \text{and} \quad \boldsymbol{c}' = (c_1', \dots, c_n')$$

denote, respectively, the distributions

$$(x_1^{u_1}(\omega_1, a_1, 0), \dots, x_n^{u_n}(\omega_n, a_n, 0)) \quad \text{and} \quad (x_1^{u_1}(\omega_1, a_1, t), \dots, x_n^{u_n}(\omega_n, a_n, t))$$

censored at the poverty line z.

To see that (14) holds, it suffices to show that $\boldsymbol{c}' \succcurlyeq_D \boldsymbol{c}$, which is implied by the inequalities

$$c_i' \geq c_i, \quad i \in \{1, \dots, n\} \tag{15}$$

(Lemma 1).

To see that (15) holds, choose $i \in \{1, \dots, n\}$ and note that either $c_i' < z$ or $c_i' = z$. The second case is easy to handle, since

$$c_i' = z \geq c_i.$$

Suppose that $c_i' < z$. Then $c_i' = y_i' + s$, where y_i' is the solution to the problem

$$\max_{y_i \in [\omega_i, \omega_i + a_i]} u_i((y_i - \omega_i)/a_i, y_i + s). \tag{16}$$

In the absence of taxation, individual i's income is the unique solution y_i^* to

$$\max_{y_i \in [\omega_i, \omega_i + a_i]} u_i((y_i - \omega_i)/a_i, y_i). \tag{17}$$

Letting

$$\widetilde{y}_i = y_i - \omega_i,$$

(16) and (17) can be reformulated as

$$\max_{\widetilde{y}_i \in [0,a_i]} u_i(\widetilde{y}_i/a_i, \widetilde{y}_i + \omega_i + s) \quad \text{and} \quad \max_{\widetilde{y}_i \in [0,a_i]} u_i(\widetilde{y}_i/a_i, \widetilde{y}_i + \omega_i).$$

Note that the only difference between these two problems is the fixed subsidy s, which represents a pure income effect. Consequently, since consumption is a normal good by assumption, it follows that

$$z > c_i' \geq y_i^* = c_i,$$

as we sought.

[$\Rightarrow$] Suppose that t is $\succcurlyeq_z$-poverty-reducing. We must show that $t(y) = -s$ for all y with $y + s \leq z$ and some $s \geq 0$.

Suppose, by contradiction, that t does not have this property. Then there exists a tax bracket overlapping with the set

$$\{y \in [0, z) : y - t(y) < z\}$$

where the marginal tax rate is strictly positive. If this were not true, the right derivative of t would be zero over the entire set $\{y \in [0, z) : y - t(y) < z\}$. This would imply that t is constant over this set and, by the continuity of t, constant over its closure $[0, \min\{y \in [0, z] : y - t(y) \leq z\}]$.

We analyze two exhaustive cases. In case (a), we consider tax brackets that overlap with the set of pre-tax incomes $\{y \in [0, z) : y - t(y) < z\}$, where at least one such bracket I has a marginal tax rate τ strictly between 0 and 1. In case (b), we also consider positive-rate brackets overlapping with this set, but we assume that all such brackets (of which there is at least one) have a marginal tax rate equal to 1.

First, we consider case (a). Select y' in the interior of I such that $y' < z$ and $y' - t(y') < z$. Furthermore, we can ensure that y' is chosen in a way that

$$y' - t(y') > y'. \tag{18}$$

This last condition is feasible because $t(y) \leq 0$ holds for all y in the interval $[0, z]$ (Lemma 2).

Consider the utility function

$$u(l, x) = \theta \left(\frac{x^{1-\alpha}}{1-\alpha} + \beta(1-l) \right) + (1-\theta) \left(\frac{(1-l)^{1-\alpha}}{1-\alpha} \right), \tag{19}$$

where $\alpha > 1$, $\beta > 0$, and $\theta \in (0, 1)$. This function belongs to the family of preferences $\mathscr{U}$.[6]

[6]The utility function u is continuous over its domain $\mathbb{R}_+ \times [0, 1]$. Consider the marginal rate of substitution of consumption for leisure in the interior of this domain:

$$MRS^u(l, x) = -\frac{\partial u(l, x)}{\partial l} \bigg/ \frac{\partial u(l, x)}{\partial x} = x^\alpha \left(\beta + \frac{1-\theta}{\theta}(1-l)^{-\alpha} \right)$$

This MRS is strictly increasing along any indifference curve, which establishes that u is strictly quasiconcave in the interior of $\mathbb{R}_+ \times [0, 1]$. Furthermore, we can verify that consumption is a normal good by examining

Since t is linear on the tax bracket I, and since $\tau \in (0,1)$ is the marginal tax rate corresponding to I, we can write

$$t(y) = -b + \tau y, \quad \text{for all } y \in I,$$

for some $b \geq 0$.

An individual with a wage rate of $a > 0$ chooses, under the linear tax schedule $-b + \tau y$, an optimal gross income level that solves the following optimization problem:

$$\max_{y \in [0,a]} u(y/a, b + (1-\tau)y).$$

Because the objective function is strictly concave, an interior solution to this problem exists if and only if

$$\theta(1-\tau)(b + (1-\tau)y)^{-\alpha} = \frac{\theta\beta}{a} + \frac{1-\theta}{a}(1 - y/a)^{-\alpha}.$$

Fixing $a > y' > 0$, plugging $y = y' \in (0,a)$ into this equation, and solving for β yields

$$\beta = a(1-\tau)(b + (1-\tau)y')^{-\alpha} - \frac{1-\theta}{\theta}(1 - y'/a)^{-\alpha}. \tag{20}$$

Note that for any $\alpha > 1$, and for any θ sufficiently close to 1, the corresponding β determined by (20) satisfies $\beta > 0$, ensuring it lies within the feasible parameter range.

Consequently, for any $\alpha > 1$, there exists $\theta_\alpha \in (0,1)$ such that for every $\theta \in (\theta_\alpha, 1)$ we have $\beta(\alpha,\theta) > 0$, where

$$\beta(\alpha,\theta) = a(1-\tau)(b + (1-\tau)y')^{-\alpha} - \frac{1-\theta}{\theta}(1 - y'/a)^{-\alpha},$$

and the optimization problem

$$\max_{y \in [0,a]} \theta\left(\frac{(b + (1-\tau)y)^{1-\alpha}}{1-\alpha} + \beta(\alpha,\theta)(1 - y/a)\right) + (1-\theta)\left(\frac{(1-y/a)^{1-\alpha}}{1-\alpha}\right) \tag{21}$$

admits y' as its unique solution.

Next, consider the behavior of the indifference curve passing through the gross income-net income bundle $(y', b + (1-\tau)y')$. The utility equality along this curve is given by

$$\begin{aligned} &\theta\left(\frac{x^{1-\alpha}}{1-\alpha} + \beta(\alpha,\theta)\left(1 - \frac{y}{a}\right)\right) + (1-\theta)\left(\frac{(1-y/a)^{1-\alpha}}{1-\alpha}\right) \\ &\qquad = \theta\left(\frac{(b + (1-\tau)y')^{1-\alpha}}{1-\alpha} + \beta(\alpha,\theta)\left(1 - \frac{y'}{a}\right)\right) + (1-\theta)\left(\frac{(1-y'/a)^{1-\alpha}}{1-\alpha}\right). \end{aligned}$$

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the partial derivatives of the MRS:

$$\frac{\partial MRS^u(l,x)}{\partial l} = x^\alpha \cdot \frac{\alpha(1-\theta)}{\theta}(1-l)^{-\alpha-1} > 0 \quad \text{and} \quad \frac{\partial MRS^u(l,x)}{\partial x} = \alpha x^{\alpha-1}\left[\beta + \frac{1-\theta}{\theta}(1-l)^{-\alpha}\right] > 0.$$

Both partial derivatives are strictly positive under our parameter assumptions, confirming that consumption is indeed a normal good.

The marginal rate of substitution at any point (y, x) on this indifference curve is:

$$\left(-\frac{\partial u(y/a,x)}{\partial y}\bigg/\frac{\partial u(y/a,x)}{\partial x}\right)\bigg|_{\beta=\beta(\alpha,\theta)} = (1-\tau)\left(\frac{x}{b+(1-\tau)y'}\right)^{\alpha} + \frac{x^{\alpha}}{a}\frac{1-\theta}{\theta}\left[(1-y/a)^{-\alpha}-(1-y'/a)^{-\alpha}\right],$$

where this expression is positive when $\beta(\alpha, \theta) > 0$.

For sufficiently large α and any correspondingly large $\theta \in (\theta_{\alpha}, 1)$, the marginal rate of substitution at any bundle (y, x) on the indifference curve can be made arbitrarily close to zero for $x < b + (1-\tau)y'$ and arbitrarily large for $x > b + (1-\tau)y'$.

This property ensures that for sufficiently large α and $\theta \in (\theta_{\alpha}, 1)$, the gross income-net income bundle

$$(y', y' - t(y') = b + (1-\tau)y')$$

is also optimal under the tax schedule t. The indifference curve through this bundle becomes arbitrarily steep for gross income levels to the right of y' and arbitrarily flat for levels to the left of y', making all other feasible bundles under tax schedule t less preferred.

Therefore, the solution to (21) coincides with the solution to the same problem where the linear tax schedule $b + \tau y$ is replaced by t:

$$\max_{y\in[0,a]} \theta\left(\frac{(y-t(y))^{1-\alpha}}{1-\alpha} + \beta(\alpha,\theta)\left(1-\frac{y}{a}\right)\right) + (1-\theta)\left(\frac{(1-y/a)^{1-\alpha}}{1-\alpha}\right).$$

Next, observe that because

$$\theta(1-\tau)(y'-t(y'))^{-\alpha} = \frac{\theta\beta(\alpha,\theta)}{a} + \frac{1-\theta}{a}(1-y'/a)^{-\alpha},$$

and since $1 - \tau < 1$, for sufficiently large $\theta \in (0, 1)$ we have

$$\theta(y'-t(y'))^{-\alpha} > \frac{\theta\beta(\alpha,\theta)}{a} + \frac{1-\theta}{a}\left(1-\frac{y'-t(y')}{a}\right)^{-\alpha}.$$

The expression $\theta y^{-\alpha}$ is decreasing in y while

$$\frac{\theta\beta(\alpha,\theta)}{a} + \frac{1-\theta}{a}\left(1-\frac{y}{a}\right)^{-\alpha}$$

is increasing in y. Moreover, for y sufficiently close to a, we have

$$\theta y^{-\alpha} < \frac{\theta\beta(\alpha,\theta)}{a} + \frac{1-\theta}{a}\left(1-\frac{y}{a}\right)^{-\alpha}.$$

By the Intermediate Value Theorem, there exists $y^* \in (y' - t(y'), a)$ satisfying

$$\theta y^{*-\alpha} = \frac{\theta\beta(\alpha,\theta)}{a} + \frac{1-\theta}{a}\left(1-\frac{y^*}{a}\right)^{-\alpha}.$$

This y^* is precisely the unique solution to the first-order condition of the tax-free optimization problem:

$$\max_{y\in[0,a]} \theta\left(\frac{y^{1-\alpha}}{1-\alpha}+\beta(\alpha,\theta)(1-y/a)\right)+(1-\theta)\left(\frac{(1-y/a)^{1-\alpha}}{1-\alpha}\right).$$

Therefore, for sufficiently large α and θ,

$$x^u(0,a,0)=y^*>y'-t(y')>y'=x^u(0,a,t), \tag{22}$$

where the second inequality follows from (18).

The crucial step in reaching our contradiction is as follows: we have constructed a utility function $u\in\mathscr{U}$ for which (22) holds. This implies that an agent with utility function u and wage rate a achieves lower after-tax income under tax regime t compared to a tax-free environment. Moreover, this lower after-tax income, y', satisfies

$$y'<y'-t(y')<z.$$

Consequently,

$$(x^u(0,a,0),\dots,x^u(0,a,0))\not\succcurlyeq_z(x^u(0,a,t),\dots,x^u(0,a,t)).$$

This directly contradicts our assumption that t is $\succcurlyeq_z$-poverty-reducing, completing the proof.

It remains to consider case (b). Recall that in this case t has at least one tax bracket overlapping with the set $\{y\in[0,z):y-t(y)<z\}$ where the marginal tax rate is positive. Moreover, all such overlapping brackets have a marginal tax rate of 1.

To begin, we show that the intercept $t(0)$ must be negative. If instead $t(0)\geq 0$, then since $t(y)\leq 0$ for all $y\in[0,z]$ (Lemma 2), the monotonicity of t would imply $t(y)=0$ for all $y\in[0,z]$. This would mean t has the property that $t(y)=-s$ for some $s\geq 0$ whenever $y+s\leq z$, contradicting our assumption.

Given $t(0)<0$, there are two possible subcases: (i) the first tax bracket has a marginal tax rate of 1; and (ii) the first tax bracket has a zero marginal tax rate, and at some point $\hat{y}<z$ where $\hat{y}-t(\hat{y})<z$, a second tax bracket begins with a marginal tax rate of 1.

The proof technique from case (a) can be adapted to handle both subcases, with appropriate modifications. Since the arguments are similar, we present only subcase (i) in detail.

Let $-\gamma=t(0)$ to simplify notation. Note that $\gamma<z$, otherwise, by rank preservation of t, there would be no overlap of at least one tax bracket with the set $\{y\in[0,z):y-t(y)<z\}$.

We return to the utility function specified in (19) and analyze the optimization problem faced by an agent with wage rate a under a linear tax schedule $-\gamma+\lambda y$, where $\lambda\in(0,1)$:

$$\max_{y\in[0,a]} \theta\left(\frac{(\gamma+(1-\lambda)y)^{1-\alpha}}{1-\alpha}+\beta(1-y/a)\right)+(1-\theta)\left(\frac{(1-y/a)^{1-\alpha}}{1-\alpha}\right). \tag{23}$$

Observe that $y=0$ emerges as the unique solution to (23) whenever

$$\theta(1-\lambda)\gamma^{-\alpha}\leq\frac{\theta\beta}{a}+\frac{1-\theta}{a}.$$

Consequently, for fixed $a > 0$ and any $\alpha > 1$, there exists $\overline{\theta} \in (0,1)$ such that for every $\theta \in (\overline{\theta}, 1)$, we have $\beta(\alpha, \theta) > 0$, where

$$\beta(\alpha,\theta) = a(1-\lambda)\gamma^{-\alpha} - \frac{1-\theta}{\theta}, \tag{24}$$

and the optimization problem

$$\max_{y\in[0,a]} \theta\left(\frac{(\gamma+(1-\lambda)y)^{1-\alpha}}{1-\alpha} + \beta(\alpha,\theta)(1-y/a)\right) + (1-\theta)\left(\frac{(1-y/a)^{1-\alpha}}{1-\alpha}\right) \tag{25}$$

admits $y = 0$ as its unique solution.

Following the reasoning from case (a), we can establish via revealed preference that for sufficiently large α and θ, the solution to (25) coincides with the solution to the analogous problem where the linear tax schedule $-\gamma + \lambda y$ is replaced by t:

$$\max_{y\in[0,a]} \theta\left(\frac{(y-t(y))^{1-\alpha}}{1-\alpha} + \beta(\alpha,\theta)(1-y/a)\right) + (1-\theta)\left(\frac{(1-y/a)^{1-\alpha}}{1-\alpha}\right). \tag{26}$$

This equivalence follows from two observations. First, the constraint set of gross income-net income pairs for problem (26) is strictly contained within that of problem (25) in a neighborhood of 0. Second, for sufficiently large α, this neighborhood of 0 becomes decisive for the agent's optimization in both problems, as all feasible gross income levels outside this region become strictly inferior choices. Since the pair $(y = 0, x = \gamma)$ is attainable in both problems and (25) permits a wider range of choices near $y = 0$, we can conclude that $y = 0$ must be optimal for both problems.

Note that (24) implies that for sufficiently large α and θ,

$$\theta(1-\lambda)\gamma^{-\alpha} = \frac{\theta\beta(\alpha,\theta)}{a} + \frac{1-\theta}{a}.$$

Moreover, since $1 - \lambda < 1$, for sufficiently large $\theta \in (0, 1)$ we have

$$\theta\gamma^{-\alpha} > \frac{\theta\beta(\alpha,\theta)}{a} + \frac{1-\theta}{a}\left(1-\frac{\gamma}{a}\right)^{-\alpha}.$$

Applying the Intermediate Value Theorem as in the previous case yields the existence of $y^* \in (\gamma, a)$ satisfying

$$\theta y^{*-\alpha} = \frac{\theta\beta(\alpha,\theta)}{a} + \frac{1-\theta}{a}\left(1-\frac{y^*}{a}\right)^{-\alpha}.$$

This y^* uniquely solves the first-order condition of the tax-free optimization problem:

$$\max_{y\in[0,a]} \theta\left(\frac{y^{1-\alpha}}{1-\alpha} + \beta(\alpha,\theta)(1-y/a)\right) + (1-\theta)\left(\frac{(1-y/a)^{1-\alpha}}{1-\alpha}\right).$$

Thus, for sufficiently large α and θ,

$$x^u(0,a,0) = y^* > \gamma = x^u(0,a,t),$$

which leads to the same contradiction as in case (a), since $\gamma < z$. ∎

B. Relaxing monotonicity

This section provides a complete characterization of poverty-reducing tax schedules in the absence of the monotonicity constraint, extending the results of Theorem 1 to this more general setting.

For the remainder of this section, a *tax schedule* is defined as a continuous and rank-preserving function $t : \mathbb{R}_+ \to \mathbb{R}$, where $t(y)$ represents the tax liability corresponding to the pre-tax income level y.

Note that dropping the monotonicity assumption renders tax schedules flexible enough to accommodate negative marginal tax rates.

Let $\mathscr{T}^*$ denote the set of all tax schedules according to this weaker definition.

We first state and prove the analogue of Lemma 2.

Lemma 3. *Suppose that $t \in \mathscr{T}^*$ is $\succcurlyeq_z$-poverty-reducing. Then $t(y) \leq 0$ for all $y \in [0, z]$.*

Proof. Suppose there exists some $x^* \in [0, z]$ where $t(x^*) > 0$. We will prove that t cannot be $\succcurlyeq_z$-poverty-reducing.

To establish this, it suffices to demonstrate the existence of a utility function $u \in \mathscr{U}$ and a positive constant $a > 0$ such that

$$z > x^u(0, a, t) < x^u(0, a, 0). \tag{27}$$

Indeed, this inequality immediately implies

$$(x^u(0, a, 0), \ldots, x^u(0, a, 0)) \not\succcurlyeq_z (x^u(0, a, t), \ldots, x^u(0, a, t)),$$

thereby proving that t is not $\succcurlyeq_z$-poverty-reducing.

First, note that there is no loss of generality in assuming that x^* belongs to the interior of a tax bracket.

Fix $a > x^*$ and define the constant elasticity of substitution (CES) utility function

$$u(l, x) = \left((1 - l)^\rho + \beta^\rho x^\rho\right)^{\frac{1}{\rho}},$$

where $\rho < 0$ and $\beta > 0$.

If an individual with a wage rate of a were to choose among all the gross income-net income pairs of the form

$$(y, \eta + (1 + \gamma)y),$$

where $\eta = -t(x^*) - \gamma x^*$ and $\gamma > 0$, the optimal gross income level would solve the following optimization problem:

$$\max_{y \in [0,a]} u(1 - y/a, \eta + (1 + \gamma)y). \tag{28}$$

Here, the parameter η has been chosen so that the line $\eta + (1 + \gamma)y$ passes through the point $(x^*, x^* - t(x^*))$.

Note that γ can be chosen large enough to ensure that

$$z \geq y > x^* \Rightarrow \eta + (1 + \gamma)y > y - t(y). \tag{29}$$

Now consider the behavior of the utility function u as the parameter ρ converges to $-\infty$. The limit function is the Leontief-type utility function $\min\{1-l, \beta x\}$, whose L-shaped indifference curves have kink points lying on the ray

$$x = \frac{1}{\beta}(1 - y/a).$$

This ray passes through the point $(x^*, x^* - t(x^*))$ provided that

$$\beta = (1 - x^*/a)/(x^* - t(x^*)). \tag{30}$$

For this β, the ray intersects with the 45° line at

$$\left(\frac{a(x^* - t(x^*))}{a - t(x^*)}, \frac{a(x^* - t(x^*))}{a - t(x^*)}\right).$$

Under (30), the solution to the tax-free optimization problem

$$\max_{y \in [0,a]} u(1 - y/a, y)$$

converges to $y = \frac{a(x^* - t(x^*))}{a - t(x^*)}$ as $\rho \to -\infty$, whereas the solution to (28) converges to $y = x^*$ as $\rho \to -\infty$.

Moreover, by the condition (29), no solution to the problem

$$\max_{y \in [0,a]} u(1 - y/a, y - t(y))$$

can possibly exceed the solution to (28).

Consequently, given (29) and (30), and for any sufficiently negative ρ, there exists a small enough $\epsilon > 0$ such that

$$x^u(0, a, t) \leq x^* - t(x^*) + \epsilon < \min\left\{z, \frac{a(x^* - t(x^*))}{a - t(x^*)} \approx x^u(0, a, 0)\right\},$$

which yields the desired contradiction, (27). ∎

Theorem 2. *A tax schedule $t \in \mathcal{T}^*$ is $\succcurlyeq_z$-poverty-reducing if and only if it $t(y) \leq 0$ for all $y \in [0, z]$ and t is non-increasing over the set $[0, \min\{y \in [0, z] : y - t(y) \leq z\}]$.*

Remark 1. Note that if a tax schedule is non-increasing over an interval $[0, \overline{y}]$ for some $\overline{y} > 0$, then t is rank-preserving over $[0, \overline{y}]$.

Proof of Theorem 2. [$\Leftarrow$] Suppose that $t(y) \leq 0$ for all $y \in [0, z]$ and t is non-increasing over the set $[0, \min\{y \in [0, z] : y - t(y) \leq z\}]$. We must show that t is $\succcurlyeq_z$-poverty-reducing, meaning that for all endowment distributions $(\omega_1, \ldots, \omega_n)$, all wage distributions $(a_1, \ldots, a_n)$, all preference profiles $(u_1, \ldots, u_n) \in \mathcal{U}^n$, and all solution functions $x_1^{u_1}, \ldots, x_n^{u_n}$, we have

$$(x_1^{u_1}(\omega_1, a_1, 0), \ldots, x_n^{u_n}(\omega_n, a_n, 0)) \succcurlyeq_z (x_1^{u_1}(\omega_1, a_1, t), \ldots, x_n^{u_n}(\omega_n, a_n, t)). \tag{31}$$

For fixed parameters $(\omega_1, \ldots, \omega_n)$, $(a_1, \ldots, a_n)$, $(u_1, \ldots, u_n)$, and solution functions $x_1^{u_1}, \ldots, x_n^{u_n}$, let

$$\boldsymbol{c} = (c_1, \ldots, c_n) \quad \text{and} \quad \boldsymbol{c}' = (c_1', \ldots, c_n')$$

denote, respectively, the distributions

$$(x_1^{u_1}(\omega_1, a_1, 0), \ldots, x_n^{u_n}(\omega_n, a_n, 0)) \quad \text{and} \quad (x_1^{u_1}(\omega_1, a_1, t), \ldots, x_n^{u_n}(\omega_n, a_n, t))$$

censored at the poverty line z.

To see that (31) holds, it suffices to show that $\boldsymbol{c}' \succcurlyeq_D \boldsymbol{c}$, which is implied by the inequalities

$$c_i' \geq c_i, \quad i \in \{1, \ldots, n\} \tag{32}$$

(Lemma 1).

To see that (32) holds, choose $i \in \{1, \ldots, n\}$ and note that either $c_i' < z$ or $c_i' = z$.

The second case is easy to handle, since

$$c_i' = z \geq c_i.$$

Suppose that $c_i' < z$. Then $c_i' = y_i' - t(y_i')$, where y_i' solves

$$\max_{y_i \in [\omega_i, \omega_i + a_i]} u_i((y_i - \omega_i)/a_i, y_i - t(y_i)). \tag{33}$$

In the absence of taxation, individual i's income is the unique solution y_i^* to

$$\max_{y_i \in [\omega_i, \omega_i + a_i]} u_i((y_i - \omega_i)/a_i, y_i). \tag{34}$$

It suffices to show that

$$y_i' - t(y_i') \geq y_i^*. \tag{35}$$

First, observe that $y_i' \leq z$ must hold since $y_i' - t(y_i') < z$ and $t(y) \leq 0$ for each $y \in [0, z]$ (Lemma 3); otherwise, t would violate the rank-preservation property.

Given that $y_i' \leq z$ and $t(y) \leq 0$ for $y \in [0, z]$, individual i achieves a higher indifference curve when solving problem (33) with solution y_i' compared to the solution y_i^* of problem (34).

We can therefore introduce a lump sum subsidy $b > 0$ to problem (34) that is precisely large enough for individual i to reach the same indifference curve achieved at y_i'. This pure income effect transforms the optimization problem to:

$$\max_{y_i \in [\omega_i, \omega_i + a_i]} u_i((y_i - \omega_i)/a_i, y_i + b). \tag{36}$$

Let $(\hat{y}_i, \hat{y}_i + b)$ denote a solution to (36). Since consumption is a normal good, we have

$$y_i^* \leq \hat{y}_i + b. \tag{37}$$

Moreover, since t is non-increasing over $[0, \min\{y \in [0, z] : y - t(y) \leq z\}]$ and $y_i' - t(y_i') < z$, the budget line slope in problem (36) is less than or equal to that in problem (33). As both $(\hat{y}_i, \hat{y}_i + b)$ and $(y_i', y_i' - t(y_i'))$ lie on the same indifference curve, we can compare their marginal rates of substitution:

$$MRS^{u_i}(y_i', y_i' - t(y_i')) = -(1/a) \cdot \frac{\partial u((y_i' - \omega_i)/a, y_i' - t(y_i'))}{\partial l} \bigg/ \frac{\partial u((y_i' - \omega_i)/a, y_i' - t(y_i'))}{\partial x}$$
$$\geq -(1/a) \cdot \frac{\partial u((\hat{y}_i - \omega_i)/a, \hat{y}_i + b)}{\partial l} \bigg/ \frac{\partial u((\hat{y}_i - \omega_i)/a, \hat{y}_i + b)}{\partial x} = MRS^{u_i}(\hat{y}_i, \hat{y}_i + b).$$

This implies $y_i' - t(y_i') \geq \hat{y}_i + b$, which together with (37) establishes (35), completing our proof.

[⇒] Suppose that t is $\succcurlyeq_z$-poverty-reducing. By Lemma 3, $t(y) \leq 0$ for all $y \in [0, z]$, so we only need to show that t is non-increasing over the set $[0, \min\{y \in [0, z] : y - t(y) \leq z\}]$.

Suppose, by contradiction, that t does not have this property. Then there exists a tax bracket overlapping with the set

$$\{y \in [0, z) : y - t(y) < z\}$$

where the marginal tax rate is strictly positive. If this were not true, the right derivative of t would be zero over the entire set $\{y \in [0, z) : y - t(y) < z\}$. This would imply that t is constant over this set and, by the continuity of t, constant over its closure $[0, \min\{y \in [0, z] : y - t(y) \leq z\}]$.

As in the proof of Theorem 1, we consider two main cases. Case (a) examines tax brackets intersecting with the set $\{y \in [0, z) : y - t(y) < z\}$, where at least one bracket has a marginal tax rate strictly between 0 and 1. The argument deriving a contradiction for this case proceeds exactly as in the proof of Theorem 1.

Case (b) considers positive-rate brackets intersecting with the same set, but assumes all such brackets (of which at least one exists) have a marginal tax rate of 1. This case is further divided into two subcases: (i) the first tax bracket has a marginal tax rate of 1; and (ii) the first k tax brackets have a non-positive marginal tax rate, and at some point $\hat{y} < z$ where $\hat{y} - t(\hat{y}) < z$, the k-th tax bracket begins with a marginal tax rate of 1.

In subcase (i), because $t(y) \leq 0$ for all $y \in [0, z]$ (Lemma 3), we must have $t(0) < 0$, as in the proof of Theorem 1. This case can be resolved using the same argument as in that proof.

Subcase (ii) differs from Theorem 1, where case (b) necessarily implied $t(0) < 0$ due to the exclusion of negative marginal tax rates. The current framework, allowing such rates, cannot rule out the equality $t(0) = 0$. However, this subcase remains qualitatively similar to subcase (i) and can be handled with an analogous argument, which we omit for brevity. ■

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