TRADE-OFFS BETWEEN RISK DIFFERENTIATION AND CAPITAL OPTIMIZATION

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Abstract

I show that the best of two models predicting default, as determined by the Information Value (IV) metric, does not always result in the lowest capital requirements. This paper offers a framework to explore such inconsistencies. Under simple portfolio segmentations, inefficiencies may reach up to 4%.

1. Introduction

Capital requirements are the first pillar of the Basel accords, which mandate that financial institutions keep a minimum amount of capital in their balance sheets in relation to the credit risk in their portfolios. Those that follow the Internal Ratings Based (IRB) approach employ their internal models to estimate the risk parameters needed to calculate regulatory capital requirements. Based on their internal models, financial institutions assign a Probability of Default (PD) to each of their exposures and, under certain circumstances, a Loss Given Default (LGD) too. Using the estimation of these two parameters, institutions calculate their capital requirements using the risk weight formula found in CRE31. This equation is separable in such a way that LGD acts as a scaling factor of a function of PD. It can be expressed as shown in equation 1 and represented visually as in Figure 1.[1]

$$K = LGD \cdot f(PD) \tag{1}$$

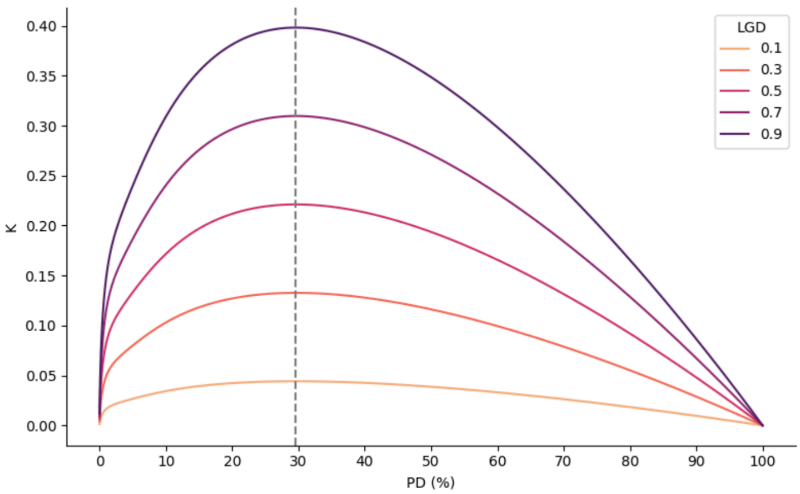

Figure 1. Plot of K as a function of PD. The vertical line indicates the maximum at $PD = 29.6\,\%$.

[1] This paper will use the formula for corporate, sovereign and bank exposures, and the values of EAD and M will be normalized to 1.

Understanding the properties of the risk weight function is crucial to seeing how model improvements lead to lower capital requirements. The risk weight function is concave down on PD, as seen in Figure 1. The function's concavity, by Jensen's inequality, implies that dividing a group of exposures into two sub-groups with different PDs will always reduce capital requirements. In other words, the total K of two PDs is always less than the K of the weighted average of the two. This is formally expressed in Equation 2, where α represents the share of exposure in the first grade.

$$\alpha\, K(PD_1) + (1-\alpha)\, K(PD_2) \leq K(\alpha\, PD_1 + (1-\alpha)\, PD_2),\ \text{where}\ \alpha \in (0,1) \tag{2}$$

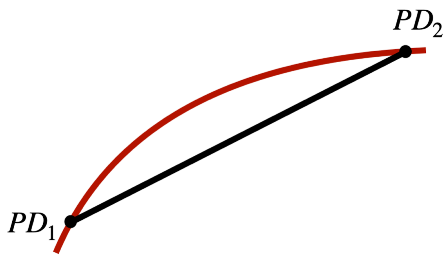

Figure 2. Concavity of the K function.

Everything follows from this principle. In order to minimize capital requirements, financial institutions develop models that differentiate between exposures, assigning different PDs to each grade. The complex statistical methods that are used for optimising capital can all be interpreted as ways of dividing up exposures into grades, making use use of the concavity of the risk weight function in order to reduce capital requirements.

For all practical purposes, credit risk modeling deals with PDs lower than 29.6%. In these circumstances, a segmentation of exposures will reduce capital requirements because the decrease caused by the lower PD will be of a larger magnitude than the increase caused by the higher PD. Thus, in general, more predictive models reduce capital requirements.

2. The trade-off

Even though dividing a grade into two sub-grades consistently leads to lower capital requirements, it's incorrect to assume that models with higher predictive power, as measured by metrics such as Area Under the Curve (AUC) or Information Value (IV), will always optimize capital. It is common in risk modeling to assume that model improvements imply capital optimization, but it not always the case. In fact, more accurate models can sometimes result in higher capital requirements than less accurate ones.

To illustrate this concept in a simple setting, consider a portfolio that is split into two equally-sized grades: a "good" grade with a low PD and a "bad" one with a high PD. Both grades can be split in half to create two sub-grades, each with a different PD. The better the information available, the more distant the PDs will be from one another. The quality of information will be modeled with the parameter Δ.

The set of two-dimensional vectors denoting the PDs of the sub-grades is formally expressed in Equation 3. The limits of Δ are set to guarantee that a PD does not reach zero and does not surpass 29.6%.

$$\mathscr{PD}_{grade} = \{(PD_{grade} - \Delta,\ PD_{grade} + \Delta)\} \text{ for grade} \in \{\text{good, bad}\} \tag{3}$$
$$\text{such that } 0 < \Delta \leq \min(PD_{grade},\ 29.6\% - PD_{grade})$$

Example. Let's consider the example of PD_{good}= 5% and PD_{bad}= 10%, as represented in Figure 3.

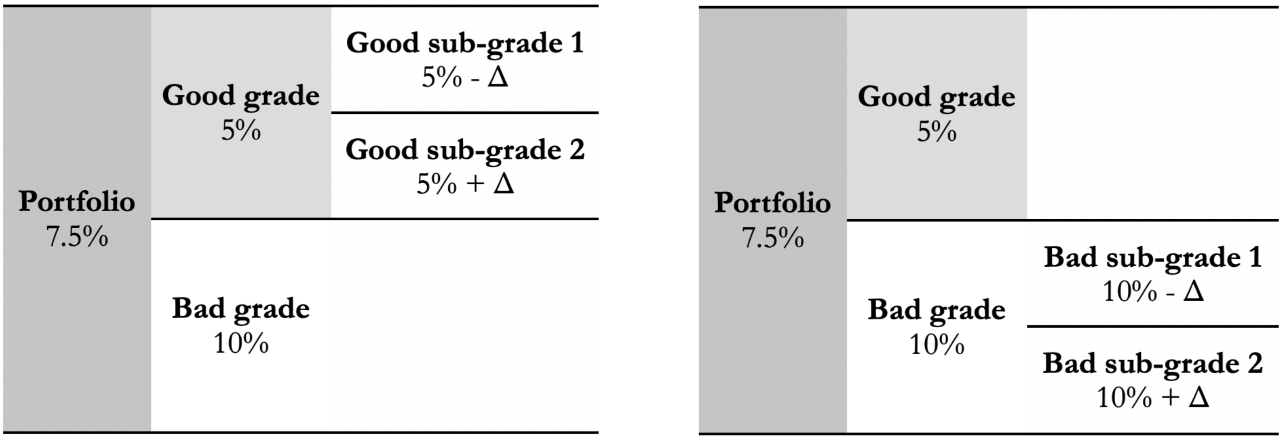

Figure 3. Representation of the portfolio segmentation example.

There are two segmentation strategies: dividing the good grade and dividing the bad one. When it comes to dividing a grade in half, a larger value of Δ is always preferred. A large Δ represents access to high quality information that allows the model to discriminate exposures with a high PD from those with a low PD. A large value of Δ means better risk differentiation and lower capital requirements due to the risk weight formula's concavity. In the face of a choice, the higher its value, the better.

It is, however, not straightforward to compare two segmentations across grades. Which grade is better to split for a given value of Δ? Let us explore this question with the help of Figure 4, where it is clear that the consequences of increasing Δ differ between each grade.

Regarding the K curve, it is clear that capital decreases as discrimination increases. For small values of Δ, the segmentations of the good and the bad grades follow a close trajectory, meaning that capital requirements are of a similar magnitude. However, as Δ increases, the reduction in capital becomes more pronounced in the segmentation of the good grade. Indeed, it takes a much larger value of Δ in the bad grade to reach the minimum K in the segmentation of the good grade.

Regarding the IV curve, it is worth noting that it does not track the capital curve with precision. The dotted green curve departs from the solid green one at the start, and they meet at the end. The dotted orange curve, by contrast, follows the solid orange line more closely.

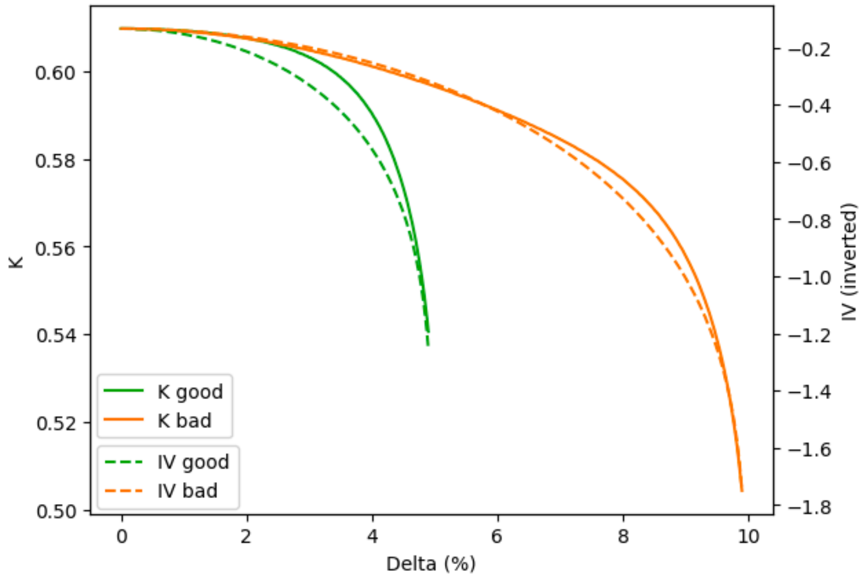

Figure 4. Plot of K and IV as a function of Δ. The segmentations of each grade are depicted in a different color.

The discrepancy between the slopes of the K and IV curves has implications for capital optimization efforts. Consider two segmentations: one for the good grade and one for the bad, both yielding the same IV. Capital requirements will be systematically different. In the following section, the magnitude of the discrepancies in capital requirements between equally predictive models will be measured.

3. Mapping the trade-offs

Not every mistake matters. Choosing a suboptimal strategy may be inconsequential if the excess capital is relatively small. It is not very relevant that a suboptimal model is chosen if the penalty to do so is, for instance, 0.1% higher capital requirements than it could otherwise be. In order to get a sense of how large this penalty can be, how big the capital discrepancy between equally predictive models, let us quantify the maximum penalty of a mistake over the space of the pair of initial PDs.

More formally, consider a function that selects the maximum penalty for every combination of $PD_{good} < PD_{bad} < 29.6\,\%$, which will offer a guide to discern which scenarios are more likely to penalize financial institutions for using a metric such as IV. This Maximum Penalty function is described in Equation 4 and graphically depicted in Figure 5.

$$MaxPenalty = \max_{i,j}(K(PD_{good}, \Delta_i) - K(PD_{bad}, \Delta_j)) \quad \forall\, \Delta_i, \Delta_j \tag{4}$$
$$\text{such that } IV(PD_{good}, \Delta_i) = IV(PD_{bad}, \Delta_j)$$

The plot below illustrates the function described above, with the color intensity representing its magnitude.

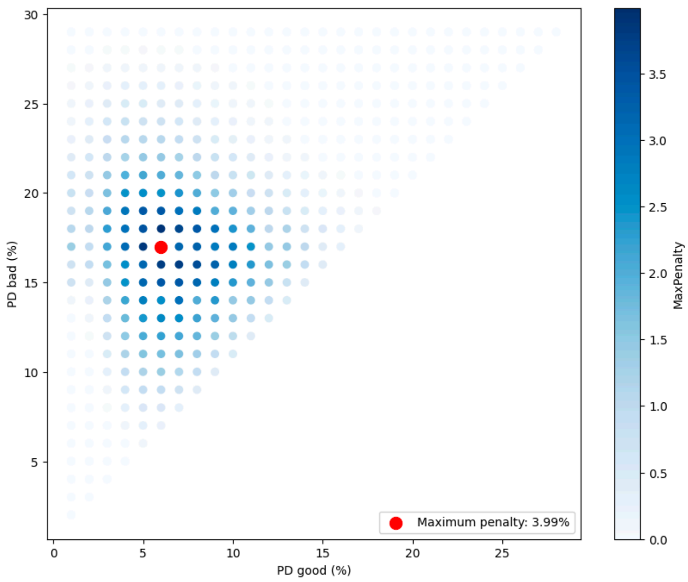

Figure 5. Plot of the Maximum Penalty function.

A maximum penalty is found at $PD_{good} = 6\,\%$, $PD_{bad} = 17\,\%$. An equal IV is achieved by dividing the good grade with $\Delta = 5\,\%$ and dividing the bad grade with $\Delta = 11.6\,\%$, as summarized in Table 1. Choosing the first one over the second one would result in a suboptimal level of capital that would be 3.99% higher than necessary (0.667 over 0.641).

Segmented Grade	PD	Δ	IV	K
Good	6%	5%	0.791	0.667
Bad	17%	11.6%	0.791	0.641

Table 1. The two segmentations that maximize capital inefficiencies.

4. Conclusions

This paper has shown that there can be a trade-off between risk differentiation and capital optimization. Under a simple setting, maximum capital inefficiencies have been found to be in the order of 4%. This figure could be even larger in more realistic settings, because actual practice diverges from the presented framework in various dimensions, namely the number of grades, the risk dispersion, and the distribution of exposure. Furthermore, this paper has used IV as a metric to evaluate the predictive power of a model, but qualitatively similar results are to be expected using AUC or entropy. Trade-offs are inevitable, as metrics do not follow the same functional form as the risk weight equation. It would be worthwhile to explore such additions in further research.

5. Acknowledgements

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