Advanced Macroeconomics 1, KU Leuven

Problem set 2 (Fall 2024)

Date posted: December 11, 2024
Deadline: January 6, 2025 (23:59)

General instructions

This is the second of the two assignments, which jointly determine 50% of your final grade. In this assignment, you will be asked to solve a NK model with sticky prices and sticky wages (presented in the Gali textbook in Chapter 6) and to perform numerical exercises on it.

Some general rules and procedures for the assignment:

- The classes on December 18 and 20 are canceled, this way you can devote the week to working on the assignment.
- There is an answer sheet (in LaTeX) attached on Toledo, and included below. You can either use the LaTeX template, or create a similar format in MS Word (I don't mind the exact formatting, but make it reasonable). The final answer should be sent as a pdf. This means that scanned handwriting is not allowed.
- Your solution should consist of a .pdf file that summarizes the figures, tables, and interpretations for all four questions, as well as a set of .mod or .mod and .m files for each of the questions (details specified in the questions).
- You are supposed to work in the same groups as for assignment 1. If there is some issue, please let me know.
- You do not need to read Chapter 6 of the book, but it might provide useful information and intuition.

Sticky wages in the model

In class, we discussed a model with sticky prices. Empirically, we observe that also wages change infrequently. This can be modeled by a similar wage stickiness á la Calvo as we do with the price stickiness. As we saw in class, a nominal rigidity requires also some market power or market imperfections, so we will extend the modeling of labor markets for that.

In particular, consider a model where each household provides a continuum of differentiated labor service types, indexed by $j \in [0,1]$. Firms i production function takes the familiar form

$$Y_t(i) = A_t N_t(i)^{1-\alpha}, \tag{1}$$

where the labor employed by the firm is a composite labor input index

$$N_t(i) \equiv \left(\int_0^1 N_t(i,j)^{1-\frac{1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}. \tag{2}$$

This gives rise to a type-specific labor demand function of firm i:

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} N_t(i), \tag{3}$$

where $W_t(j)$ is the wage paid for type j labor services. The equation resembles the demand function for the differentiated goods under monopolistic competition. Similarly, we can define the aggregate wage index in the economy, W_t as

$$W_t \equiv \left(\int_0^1 W_t(j)^{1-\epsilon_w} dj\right)^{\frac{1}{1-\epsilon_w}}. \tag{4}$$

The optimal wage setting problem becomes similar to the optimal price setting problem under Calvo pricing. For more details, please consult Chapter 6 of the textbook.

Equilibrium dynamics and calibration

In this assignment, we will focus on solving the model in Dynare. Hence, you will start with the log-linearized equilibrium equations of the model.[1] To facilitate this we will define some additional concepts:

- The natural real wage, w_t^n - the real wage that would prevail in the absence of nominal rigidities,
- The real wage gap, $\tilde{w}_t \equiv w_t - w_t^n$ - the difference between the actual real wage and the natural one.

The equilibrium of the model can be expressed as

$$\begin{aligned}
\pi_t^p &= \beta \mathbb{E}_t \left\{\pi_{t+1}^p\right\} + \varkappa_p \tilde{y}_t + \lambda_p \tilde{w}_t, && (5)\\
\pi_t^w &= \beta \mathbb{E}_t \left\{\pi_{t+1}^w\right\} + \varkappa_w \tilde{y}_t - \lambda_w \tilde{w}_t, && (6)\\
\tilde{w}_t &= \tilde{w}_{t-1} + \pi_t^w - \pi_t^p - w_t^n + w_{t-1}^n, && (7)\\
\tilde{y}_t &= -\frac{1}{\sigma}\left(i_t - \mathbb{E}_t\left\{\pi_{t+1}\right\} - r_t^n\right) + \mathbb{E}_t\left\{\tilde{y}_{t+1}\right\}, && (8)\\
i_t &= \phi_p \pi_t^p + \phi_w \pi_t^w + \nu_t, && (9)\\
w_t^n &= \psi_{wa} a_t, && (10)\\
r_t^n &= -\sigma(1-\rho_a)\psi_{ya} a_t + (1-\rho_z) z_t. && (11)
\end{aligned}$$

Equation (5) is the New-Keynesian Phillips curve, which we know from the standard NK model. Equation (6) is the wage Phillips curve, which determines the dynamics of wage inflation similar to what equation (5) does for price inflation. Equation (7) is an identity describing the dynamics of the wage gap. Equation (8) is the dynamic IS curve. Equations (10) and (11) define the natural real wage and the natural real interest rate. Monetary policy is determined by the Taylor rule in equation (9).

[1] You can follow the derivations of those equations in the book. They follow similar steps to our benchmark NK model.

The equations above use a number of complex parameters:

$$
\begin{aligned}
\psi_{wa} &\equiv \frac{1-\alpha\psi_{ya}}{1-\alpha}, \\
\psi_{ya} &\equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}, \\
\varkappa_p &\equiv \frac{\alpha\lambda_p}{1-\alpha}, \\
\lambda_p &\equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}, \\
\varkappa_w &\equiv \lambda_w\left(\sigma+\frac{\varphi}{1-\alpha}\right), \\
\lambda_w &\equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}.
\end{aligned}
$$

To solve a model numerically, you need to assume values for the parameters of the model. You can use the calibration described in Table 1.

Table 1: Model parameters

Parameter	**Value**	**Description**
α	0.25	Capital share
β	0.99	Quarterly discount factor
σ	2.0	CRRA utility coefficient
φ	2.0	Inverse of Frisch elasticity of labor supply
ϵ_p	9	Elasticity of substitution (goods)
ϵ_w	4.5	Elasticity of substitution (labor)
θ_p	0.75	Price rigidity
θ_w	0.75	Wage rigidity
ρ_ν	0.5	Persistence of the monetary policy shock
ρ_a	0.9	Persistence of the technology shock
ρ_z	0.5	Persistence of the preference shock
σ_ν	0.01	Standard deviation of ν
σ_a	0.01	Standard deviation of a
σ_z	0.02	Standard deviation of z
ϕ_p	1.5	Interest rate rule coefficient
ϕ_w	0.0	Interest rate rule coefficient

Solving a model with sticky prices and sticky wages in Dynare

1. (6 points) Solve the model in Dynare and plot IRFs to the three shocks. Interpret the results. [Provide a .mod file.]

2. (4 points) Simulate the model (using the parameters in Table 1) for $T = 10,000$ periods. Calculate the standard deviations of the output gap, price inflation, and wage inflation.

[Provide an .m file that uses the results from the .mod file in question 1.]

3. (4 points) Explore the sensitivity of the results with respect to the nominal rigidity parameters. To do so, please solve the model for three sets of parameters and plot the impulse response functions to a preference shock for the three different parametrizations on one graph (similar to Figure 6.2 in the textbook). The three sets of parameters you should consider are: $\{\theta_p, \theta_w\} = \{\{0.75, 0.75\}, \{0.25, 0.75\}, \{0.75, 0.25\}\}$. Please interpret the differences. [Provide one main .m file and one .mod file. The .m file should run the .mod file, which reads the parameters from a file you create in the .m file.]

4. (6 points) For the last question, you can choose the basic version A of the question for 4 points or the extended version B of the question for 6 points:[2]

   A. Solve the model for three different versions of the monetary policy rule:

   - Rule 1: $\phi_p = 1.5$ and $\phi_w = 0$,
   - Rule 2: $\phi_p = 0$ and $\phi_w = 1.5$,
   - Rule 3: $\phi_p = 0.5$ and $\phi_w = 1.0$.

   For the three rules: simulate the model for $T = 10,000$ periods (using the same set of shocks for the three different versions), calculate the volatility (standard deviations) of output gap, price inflation, and wage inflation. Using those moments calculate the the average period welfare loss, as given by equation (12). Present the results in a table. Which of the rules gives the best results? [Provide an .m and a .mod file.]

   B. Create a grid of monetary policy rules consisting of two sequences:

   - $\phi_p = 1.5$ and $\phi_w \in \{0, 0.1, 0.2, \ldots, 1.4\}$,
   - $\phi_p \in \{0, 0.1, 0.2, \ldots, 1.4\}$ and $\phi_w = 1.5$.

   Each of the two sequences encompasses 15 rules, so you have 30 rules in total. For each rule: simulate the model for $T = 10,000$ periods (using the same set of shocks for the thirty different versions), calculate the volatility (standard deviations) of output gap, price inflation, and wage inflation. Using those moments calculate the average period welfare loss, as given by equation (12). Present the results in a table. Which of the rules gives the best results? [Provide an .m and a .mod file.]

The average period welfare loss in this model can be calculated as:

$$\mathbb{L} = \frac{1}{2}\left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) var\left(\tilde{y}_t\right) + \frac{\epsilon_p}{\lambda_p} var\left(\pi_t^p\right) + \frac{\epsilon_w (1 - \alpha)}{\lambda_w} var\left(\pi_t^w\right)\right]. \tag{12}$$

[2] Please provide the solution to only one of the versions. If you solve both versions, only version B will be graded.

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Answer sheet for Assignment 2 (Fall 2024)
Students: Name (student number)

1 Basic model - IRFs

Plot and interpret the IRFs.

2 Simulated data

List the calculated moments in a table.

3 Role of nominal rigidities

Plot the IRFs (as described in the instructions) and interpret the differences.

4 In search of optimal rules

Provide a table with the results and answer the question.